Design storm estimation in Tuscany (Italy) through regional frequency analysis and generalized additive modelling

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Dataset description

The complete dataset includes annual maxima of daily rainfall recorded:

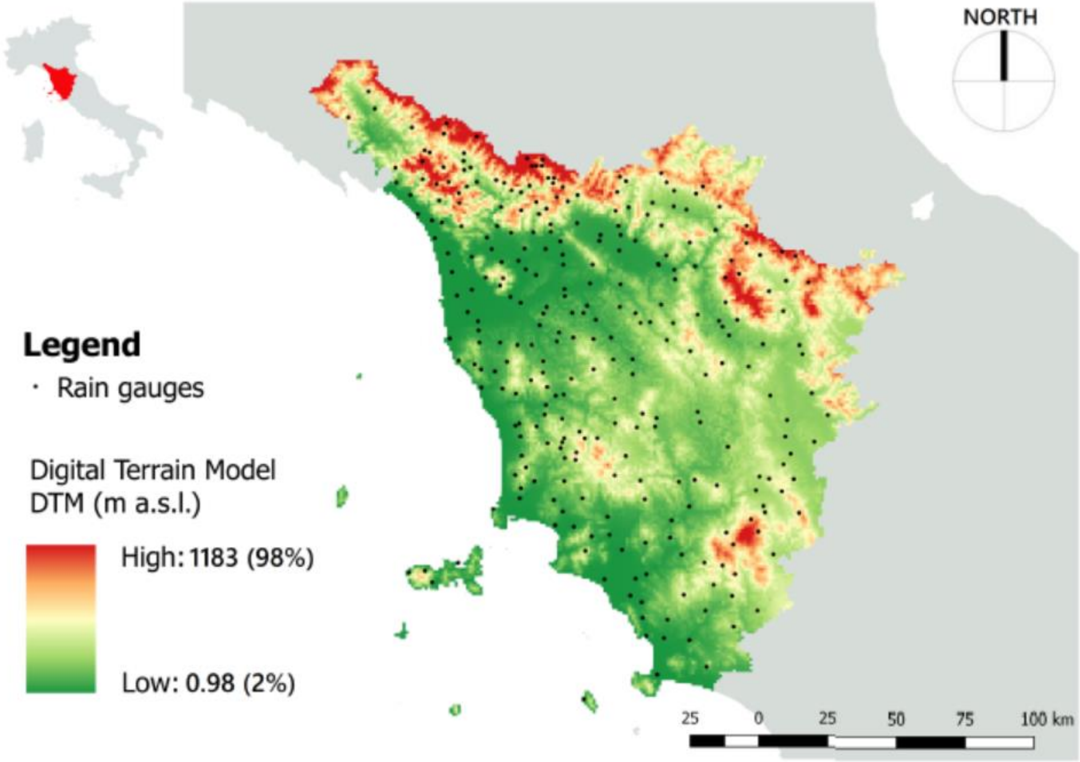
- in 876 rainfall gauges in Tuscany;
- over the period 1916-2017.

They have been considered:

- only stations with 30 or more years of data;
- only the post Second World War period.

The final dataset includes annual maxima of daily rainfall recorded:

- in 289 rainfall gauges in Tuscany;
- over the period 1951-2017;
- 14455 observations.



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Regional frequency analysis

METHODOLOGY

1. Regions Subdivision ⇒

Preliminary hypothesis of subdivision

Spatial distribution of sample coefficient of skewness G (Lsk) [I level] and of variation Cv (Lcv) [II level]

2. Growth Factor ⇒

TCEV probability distribution function of the standardized variable $X' = X/\mu$

$$F_{X'}(x') = exp\left[-\Lambda_1\, exp(-\eta x') - \Lambda^* \Lambda_1^{\frac{1}{\theta^*}} exp\left(-\frac{\eta x'}{\theta^*}\right)\right]$$

Three levels hierarchical approach of parameters estimation

3. Index Raifall ⇒

For every homogenous region and every rainfall duration multivariate model *(Caporali et al., Environmentrics 2008)*

$$\mu_d = a_0 + a_1 \cdot \ln(M.A.P.) + a_2 \cdot z + a_3 \cdot \left[\sin\left(\frac{Asp}{2} - \frac{\pi}{2}\right) + \pi\right] \cdot |Asp| + a_4 \cdot h_{m,d}$$

as function of climatic and geomorphological characteristics

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Regional frequency analysis

The **index variable method** *(Dalrymple, Geol. Survey 1543-A, 1960)*

The studied territory is divided into **homogeneous regions** where the **pdf** of the random variable observed at the measurement sites **is the same for all the stations unless of** a characteristic **scale factor** of each site called **INDEX VALUE**

$$x_i(F) = \mu_i \cdot x'(F)$$

“local” Index Rainfall “regional” Growth Factor

The **TCEV - Two Component Extreme Value probabilistic model** *(Rossi et al., WRR, 1984)*, with its hierarchical regionalization procedure on three levels *(Fiorentino et al., Reidel Pub., 1987)*.

$$F_{X'}(x') = exp\left[-\Lambda_1\, exp(-\eta x') - \Lambda^* \Lambda_1^{\frac{1}{\theta^*}} ex\,p\left(-\frac{\eta x'}{\theta^*}\right)\right]$$

where L_1 and L_2, with $L_1 > L_2 \geq 0$, represent the **annual number of events**, on average, due to the basic and the outliers component, while the parameters q_1 and q_2, with $q_2 > q_1 > 0$, represent the **annual mean value of the events**

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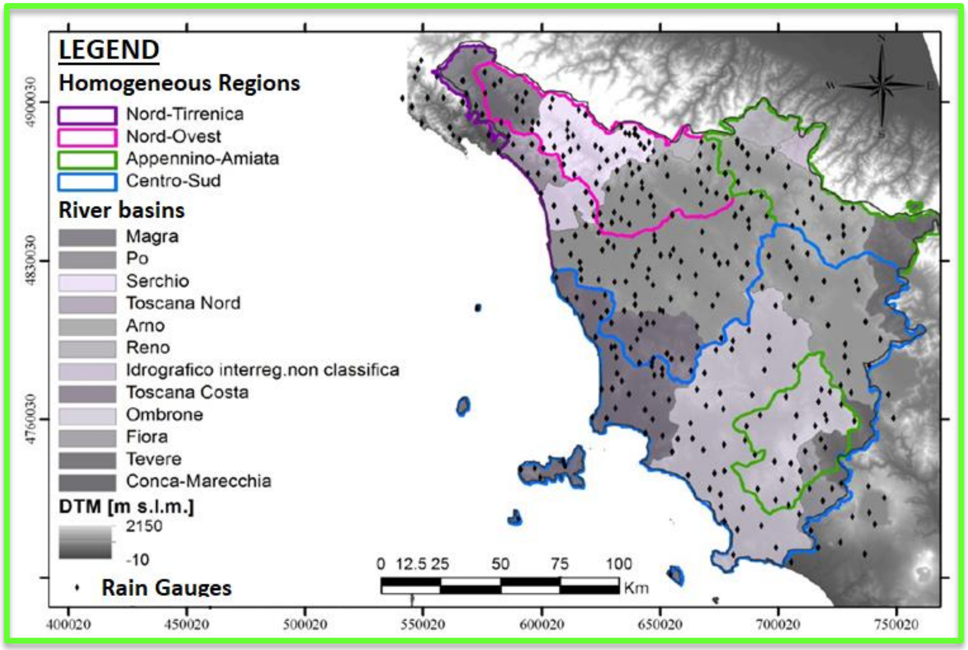
Regional frequency analysis: identification of Homogeneous Regions

Subdivision in homogeneous regions and subregions	
Hypothesis 1	UNIQUE homogeneous region and subregion
Hypothesis 2	UNIQUE homogeneous region and 3 subregion NORTH-APPENNINE, NORTH-TYRRHENIAN, CENTER-SOUTH
Hypothesis 3	3 regions NORTH-APPENNINE, NORTH-TYRRHENIAN, CENTER-SOUTH coincident with the 3 sub-regions
Hypothesis 4	4 regions NORTH-TYRRHENIAN, NORTH-WEST, APPENNINE-AMIATA, CENTER-SOUTH coincident with the 4 sub-regions



Comparison between the observed cdf of coefficients G (Lsk) e Cv (Lcv) and the theoretical ones obtained with a Monte Carlo techniques as:

- Difference between ***m(x)obs – m(x)th*** *and* ***σ(x)obs - σ(x)th.***
- ***t Student*** test for the mean, ***Wilcoxon*** test for the mean, the ***χ2*** test.
- ***Discordancy D*** and ***Heterogeneity H*** test (Hosking & Wallis, Cambridge Un. Press, 1997).
- ***Gumbel probability plot test*** of the observed and the theoretical of the TCEV model growth curves.

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Generalized additive model for Location, Scale and Shape (GAMLSS)

(Rigby and Stasinopoulos, Journal of the Royal Statistical Society, 2005)

- a model for each GEV parameter *(Chavez Demoulin and Davison, Journal of the Royal Statistical Society, 2005)*
  - a univariate smoothing component on the elevation x
  - a bivariate smoothing component on the geographical coordinates s

$$\begin{cases} y_i \sim \mathrm{GEV}\left(\mu(x_i, \mathbf{s}_i), \sigma(x_i, \mathbf{s}_i), \xi(x_i, \mathbf{s}_i)\right) \\ \mu(x_i, \mathbf{s}_i) = g_1(x_i) + f_1(\mathbf{s}_i) \\ \log(\sigma(x_i, \mathbf{s}_i)) = g_2(x_i) + f_2(\mathbf{s}_i) \\ \xi(x_i, \mathbf{s}_i) = g_3(x_i) + f_3(\mathbf{s}_i) \end{cases}$$

- Estimation method: Penalized ML via P-IRLS.
- **Number of knots**: We estimate various models increasing progressively the number K and H of knots until the degrees of freedom of each smoothing component are stable.
- **Type of basis**: Penalized thin plate, with smoothing parameter selected via REML estimation.

Final model ⇨

$$\begin{cases} y_i \sim \mathrm{GEV}\left(\mu(x_i, \mathbf{s}_i), \sigma(x_i, \mathbf{s}_i), \xi(x_i, \mathbf{s}_i)\right), \\ \mu(x_i, \mathbf{s}_i) = g(x_i, K = 20) + f(\mathbf{s}_i, H = 60) \\ \log(\sigma(x_i, \mathbf{s}_i)) = g(x_i, K = 20) + f(\mathbf{s}_i, H = 40) \\ \xi(x_i, \mathbf{s}_i) = g(x_i, K = 20) + f(\mathbf{s}_i, H = 30) \end{cases}$$

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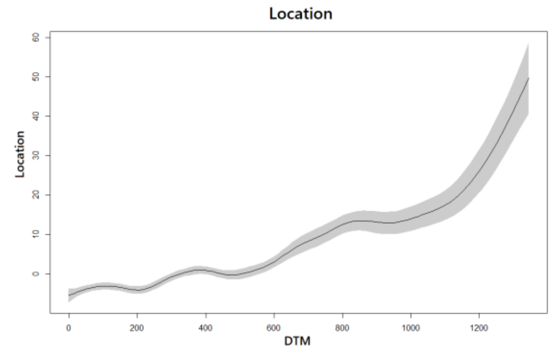
Generalized additive model for Location, Scale and Shape (GAMLSS)

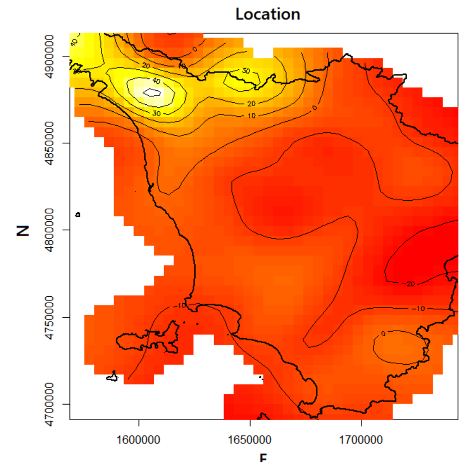
Final model coefficients and terms

Parametric coefficients			
Parameter	Estimate	Standard error	P-value
intercept μ	60.22	0.19	$< 2e-16$ ***
intercept σ	18.17	0.01	$< 2e-16$ ***
intercept ξ	0.13	0.01	$< 2e-16$ ***

Smooth terms		
Term	Degrees of freedom	P-value
Location: DTM	11.28	$< 2e-16$ ***
Location: Coordinates	56.9	$< 2e-16$ ***
Scale: DTM	5.72	2.94e-16 ***
Scale: Coordinates	31.39	$< 2e-16$ ***
Shape: DTM	2.42	0.023 *
Shape: Coordinates	15.27	8.19e-08 ***

Smoothing components in the model for the location parameter





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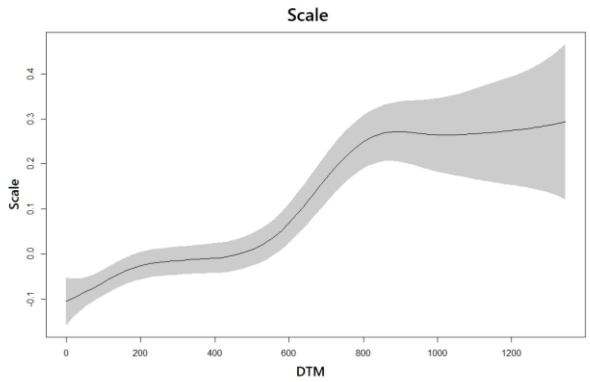
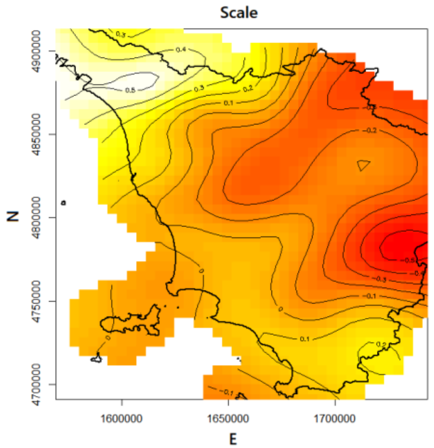
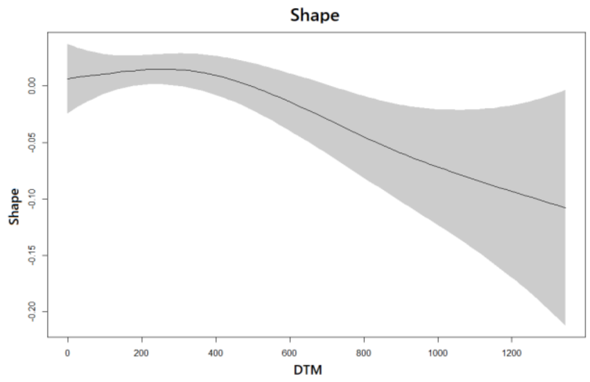
Generalized additive model for Location, Scale and Shape (GAMLSS)

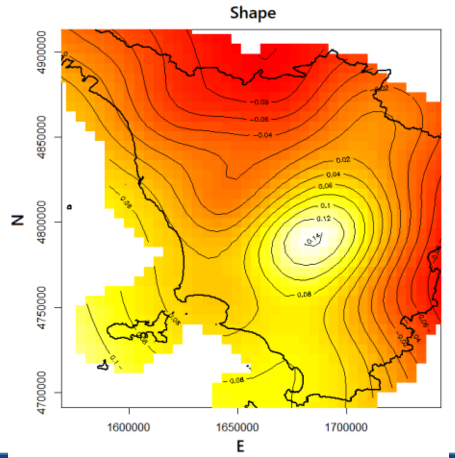
Smoothing components in the model for the scale parameter





Smoothing components in the model for the shape parameter





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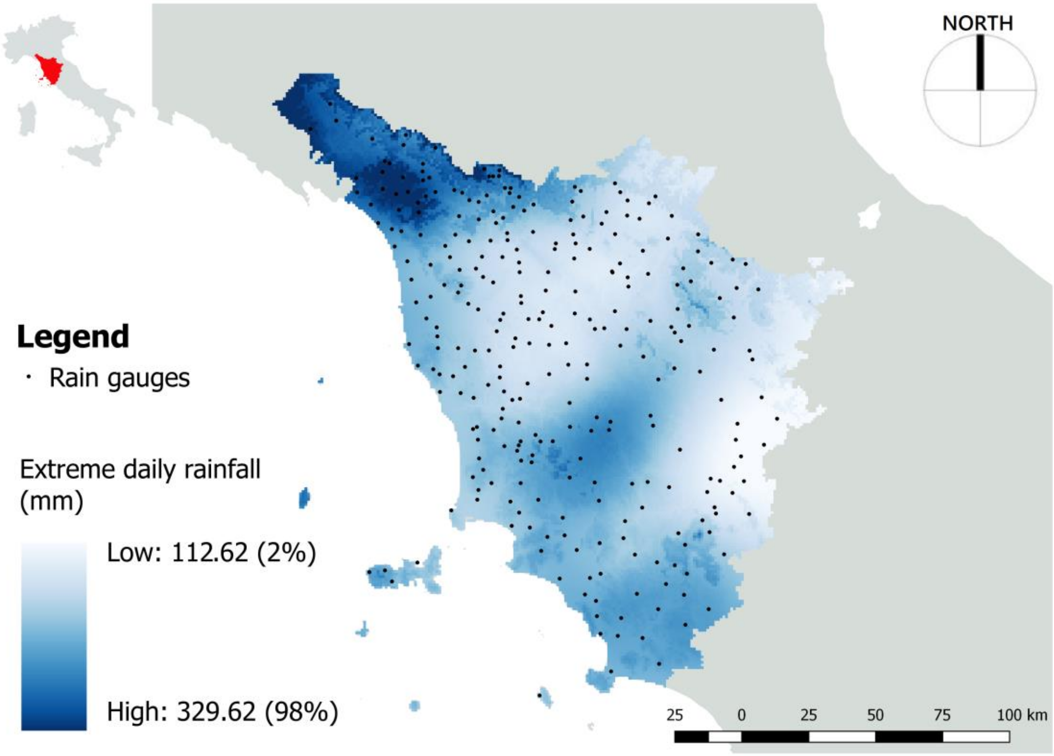
Generalized additive model for Location, Scale and Shape (GAMLSS)

Design rainfall with daily storm duration and 200 year return period



Min	100.6
10%	143.59
1st Quart.	160.90
Median	194.30
Mean	199.60
3rd Quart.	228.70
90%	265.45
Max	444.00

In addition, the magnitudes of design extreme rainfall that are expected to be exceeded in Tuscany once every 2, 20, 50, 100 years, has been predicted

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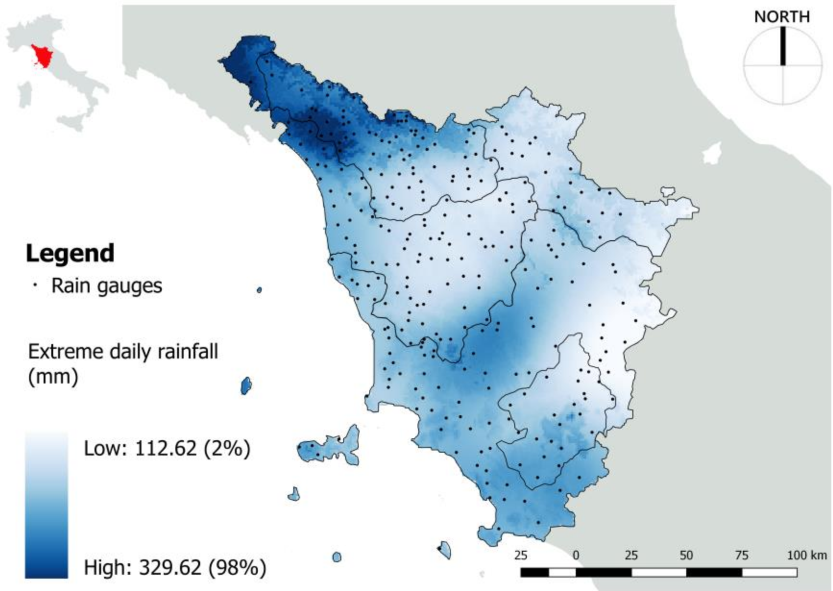
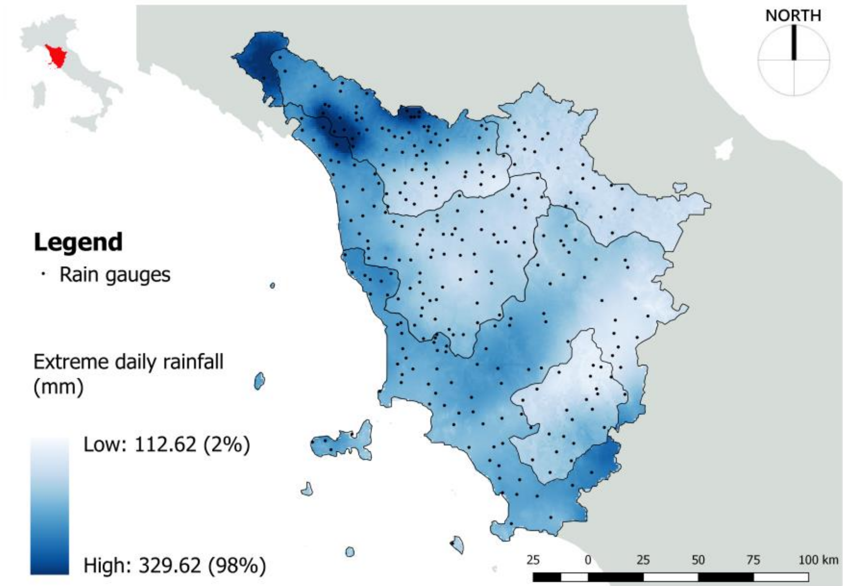
Generalized additive model compared to Regional Frequency Analysis

Design rainfall with daily storm duration and 200 year return period





GAMLSS

return period	correlation
2	0.88
20	0.91
50	0.89
100	0.88
200	0.87

RFA, using the index method

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Conclusions

- This analysis referred to the annual maxima of daily rainfall recorded in Tuscany, over the period 1951-2017.
  Aim of the study: to model the distribution of rainfall maxima to predict
  - extreme rainfalls
  - how often they are expected to occur in each location of the region
- A Generalized Additive Model for spatial extreme values has been used
  - Considering the class of Generalized Additive Models for Location, Scale and Shape (GAMLSS).
- **Objective of main interest in the field of hydrology**: prediction of extreme rainfalls associated 2-year, 20-year, 50-year, 100-year, 200-year return periods.
- **Highest values**: Appennines, Alpi Apuane in the North; Colline Metallifere and Maremma in the South
- **Lowest values**: central flat zone including Empoli, Pistoia, Prato and Firenze and Val di Chiana in the East
- **Comparison with the Index method**: the correlation coefficient is nearly 0.9 for all the return periods, detecting a consistence between the two methods.
- GAMLSS method allows for homogeneous subregions to be identified a posteriori, subsequently to the estimation process, which is free from any a priori assumption on the phenomenon spatial behaviour.
- **A future development**: given the additive formulation of our regression models, we could easily consider to model extreme value series with spatio-temporal non-stationarity.