Analogies in stochastic behaviour from the microscale of turbulence to the large-scale hydrometeorological processes under the Hurst-Kolmogorov dynamics

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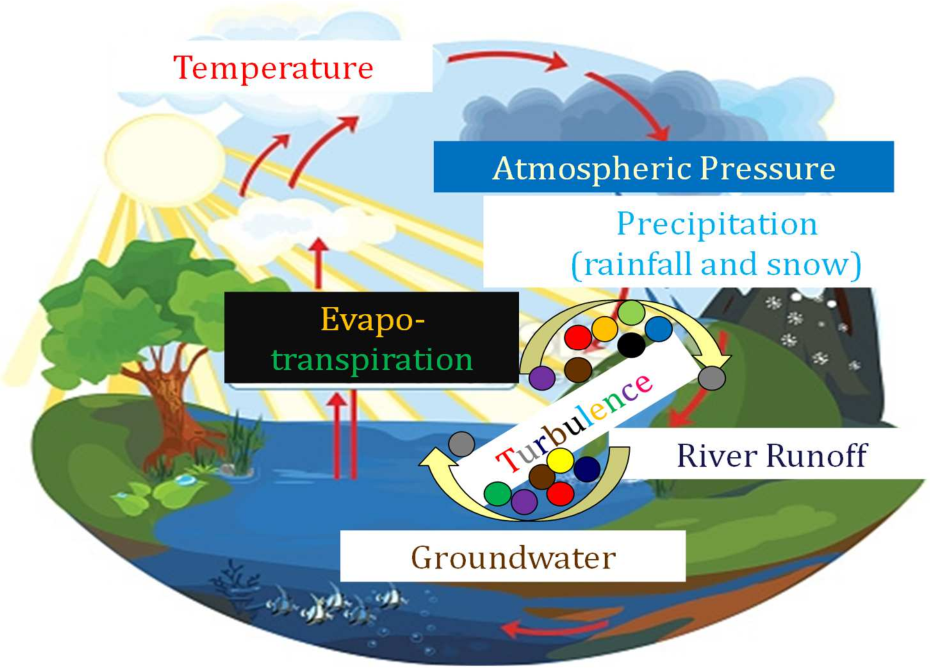
Aim

- Hydro-meteorology: combination of meteorology and hydrology that studies the water cycle from the groundwater to the lower atmosphere (typically up to 10,000 m), with the most important processes to be Temperature, Dew point - Humidity, Precipitation, Atmospheric Pressure and Wind.
- Hurst-Kolmogorov dynamics: long-term changes of Nature result in the clustering of events (e.g., successive dry years followed by successive wet ones not in a white-noise or Markovian way).
- Turbulence (stochastic): the variance of the scaled flow velocity exhibits HK behaviour (similarly, the laminar flow exhibits Markov behaviour).

Are there any stochastic similarities?



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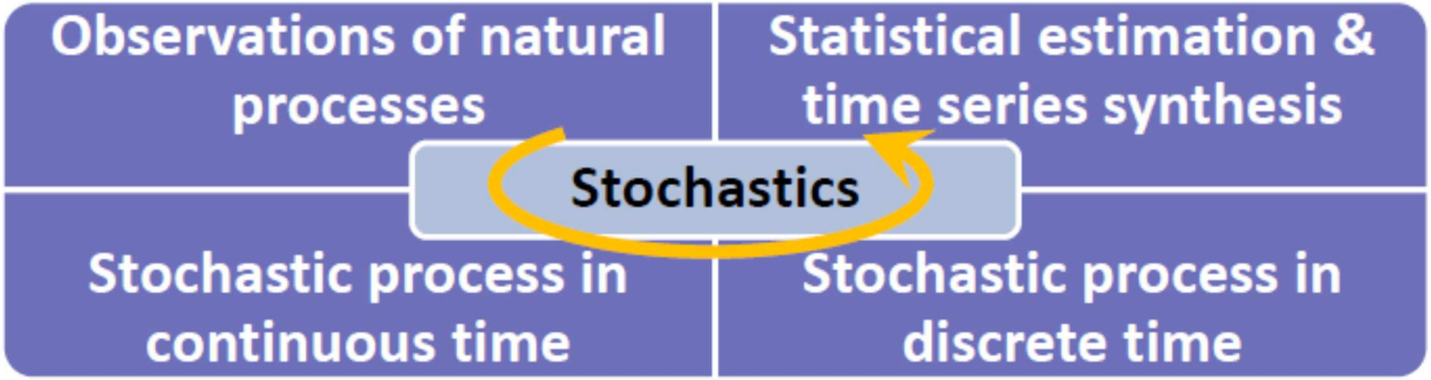
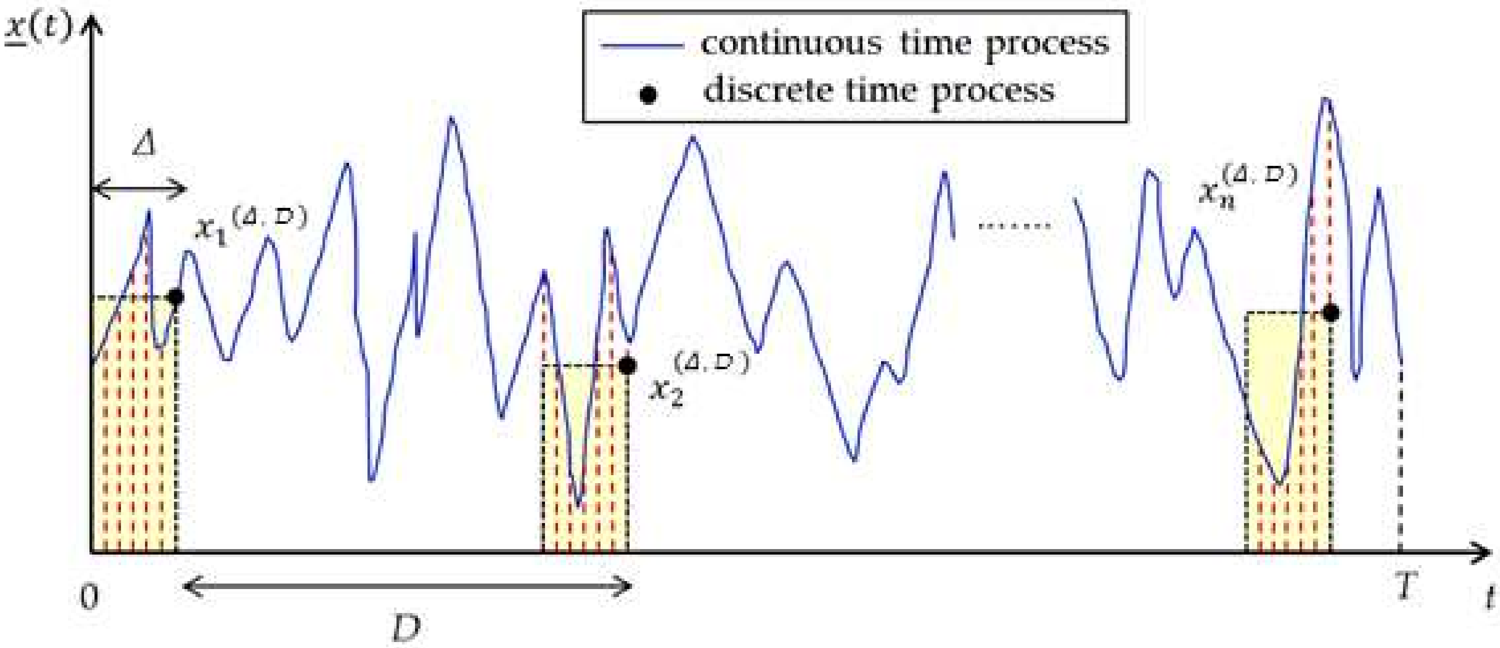
Deterministic vs. Stochastic approach

Can we predict the outcome of something as simple as dice? No, but that's ok! We can quantify the embedded uncertainty. (Source: Dimitriadis et al., 2016)

Stochastic framework





Preservation of the 2nd order dependence structure

- The 2nd order dependence structure of a model can be equivalently expressed by any stochastic metric (e.g. autocovariance $c(h)$, power spectrum $s(w)$ etc.) since each one is a function of the other; only the point of view changes from *lag* (h) to *frequency* (w) and vice versa.
- We choose an alternative estimator for the 2nd order dependence structure, that of the variance of the averaged process vs. *scale* (or else simply called the climacogram; Koutsoyiannis, 2010).
- The climacogram has additional advantages in stochastic analysis such as:
  - identical expression for the continuous-discrete time-scale
  - simpler bias expression
  - smaller statistical uncertainty

Therefore, all calculations are based on the climacogram.

The climacogram as a robust stochastic metric

The climacogram is defined as (Dimitriadis and Koutsoyiannis, 2015):

$$\gamma(k) := \mathrm{Var}\left[\int_0^k \underline{x}(y)\mathrm{d}y\right]/k^2$$

where k is the continuous-scale

and with the estimator

$$\hat{\gamma}(\kappa) = \frac{1}{\lfloor n/\kappa \rfloor - 1} \sum_{i=1}^{\lfloor n/\kappa \rfloor} (\overline{x_i} - \overline{x})^2$$

where $\lfloor n/\kappa \rfloor$ is the integer part of n/κ

$\underline{\bar{x}}_\kappa = \left(\sum_{l=\kappa(i-1)+1}^{\kappa i} \underline{x}_l\right)/\kappa$ is the sample average

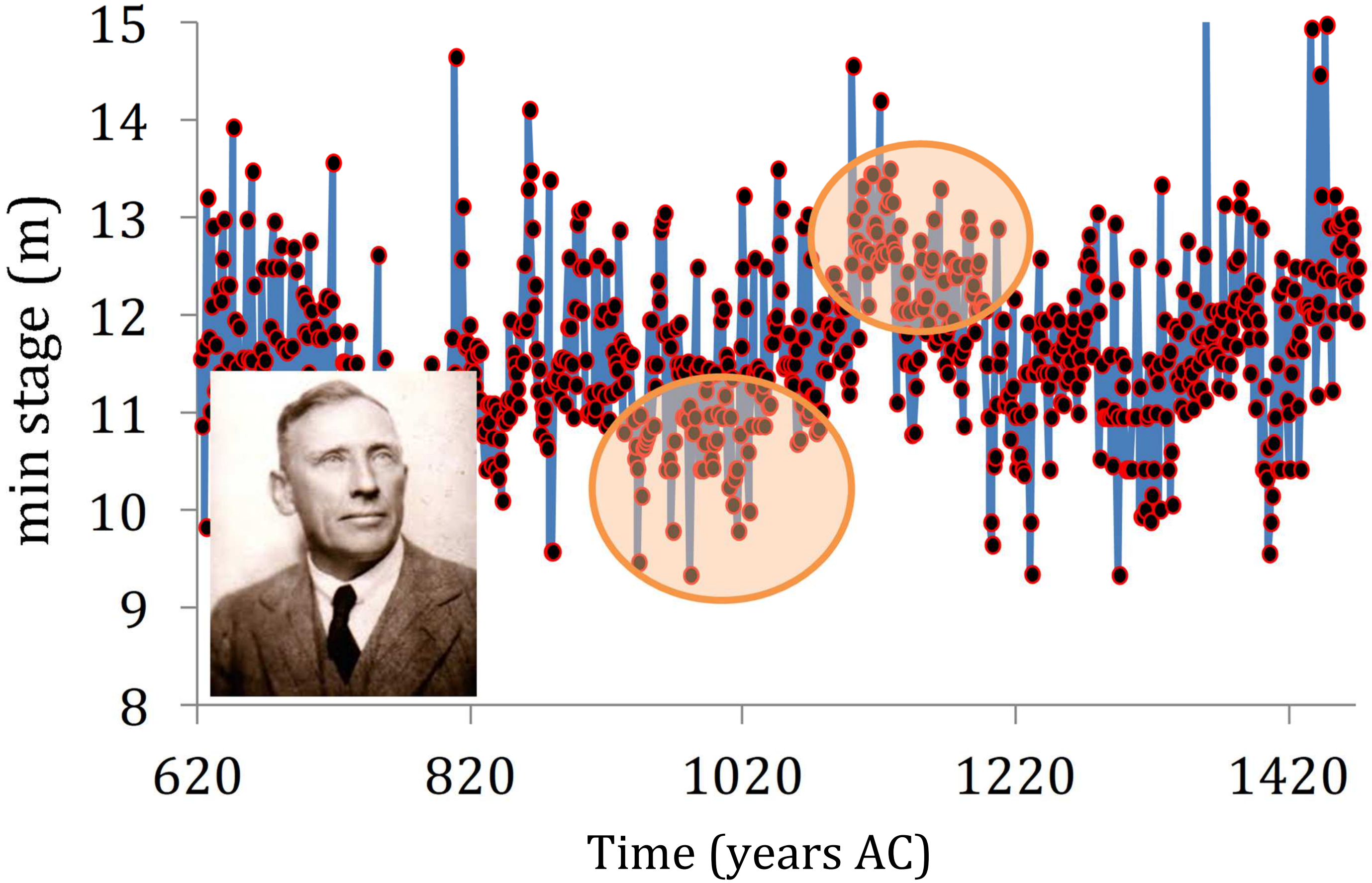
of the time-averaged process $\underline{x}_\kappa$ at scale $\kappa = k$ and

$\underline{\overline{x}} = \sum_{l=1}^{n} \underline{x}_l / n$ is the sample average at scale $\kappa = 1$.

- $c(h) := \mathrm{cov}\left[\underline{x}(t), \underline{x}(t+h)\right] := \dfrac{\partial^2\left(h^2\gamma(h)\right)}{2\partial h^2}$
- $s(w) := 4\int_0^\infty c(h)\cos(2\pi w h)\,\mathrm{d}h$

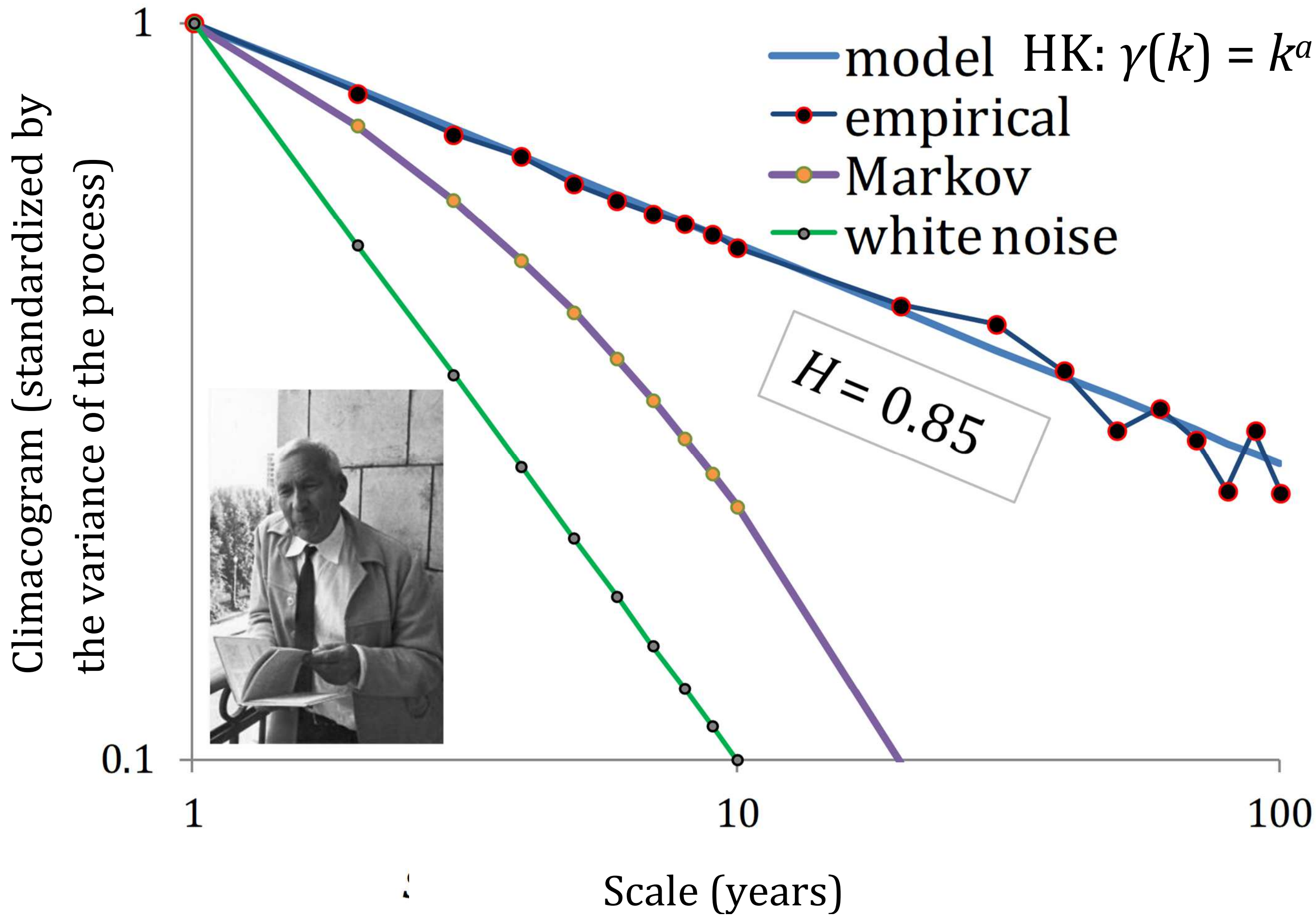
Hurst-Kolmogorov behaviour (I)

Hurst-Kolmogorov (HK) behaviour: long-term change resulting in the clustering of events (e.g., successive dry years followed by successive wet ones like in the example below of the minimum stage of the river Nile):



Hurst-Kolmogorov behaviour (II)

The long-term persistent behaviour can be quantified through the logarithmic slope a = 2H-2. For the Nile stage we estimate H >> 0.5 (H = 0.5 corresponds to a white-noise process).



Simulation of long-term persistence

- For HK process we use the symmetric-moving-average (SMA) scheme (Koutsoyiannis, 2019).
- Sum of products of coefficients (not parameters) a_j
- White noise terms $\underline{v}$i, where for simplicity and without loss of generality we assume that $\mathrm{E}[\underline{x}] = \mathrm{E}[\underline{v}] = 0$ and $\mathrm{E}[\underline{v}^2] = \mathrm{Var}[\underline{v}] = 1$

$$\underline{x}_i = \sum_{j=-l}^{l} a_{|j|} \underline{v}_{i+j}$$

where l theoretically equals infinity but a finite number can be used for preserving the dependence structure up to lag l.

For the HK process:

$$a_j = C\left(\frac{|j+1|^{H+\frac{1}{2}} + |j-1|^{H+\frac{1}{2}}}{2} - |j|^{H+\frac{1}{2}}\right) \text{ and } C = \sqrt{\frac{2\Gamma(2H+1)\sin(\pi H)\gamma(D)}{\Gamma^2(H+1/2)\big(1+\sin(\pi H)\big)}}$$

where $\Gamma(x)$ is the gamma function.

Simulation of intermittency and extremes through joint-statistics

- The effects of HK behaviour to intermittency and extremes can be simulated for non-Gaussian processes by an explicit algorithm (see several applications in Dimitriadis and Koutsoyiannis, 2018, and in extreme precipitation in Iliopoulou and Koutsoyiannis, 2019).
- The coefficient of skewness and kurtosis for the white noise process is:

$$C_{\mathrm{s},v} = \frac{\left(\sum_{j=-l}^{l} a_{|j|}^{2}\right)^{3/2}}{\sum_{j=-l}^{l} a_{|j|}^{3}} C_{\mathrm{s},x} \text{ and } C_{\mathrm{k},v} = \frac{\left(\sum_{j=-l}^{l} a_{|j|}^{2}\right)^{2}}{\sum_{j=-l}^{l} a_{|j|}^{4}} C_{\mathrm{k},x} - 6\frac{\sum_{j=-l}^{l-1}\sum_{k=j+1}^{l} a_{|j|}^{2} a_{|k|}^{2}}{\sum_{j=-l}^{l} a_{|j|}^{4}}$$

where $C_{\mathrm{s},x}$ and $C_{\mathrm{k},\underline{x}}$ are the coefficient of skewness and kurtosis of $\underline{x}_i$.

- For higher-order moments:

$$\mathrm{E}\left[\underline{x}_i{}^{p}\right] = \mathrm{E}\left[\left(\sum_{j=-l}^{l} a_{|j|}\underline{v}_{i+j}\right)^{p}\right] = \sum_{k_{-l}+k_{1-l}+\cdots+k_{l}=p} \binom{p}{k_{-l}, k_{1-l}, \dots, k_{l}} \mathrm{E}\left[\prod_{-l\le j\le l} \left(a_{|j|}\underline{v}_{i+j}\right)^{k_j}\right]$$

where $\binom{p}{k_{-l}, k_{1-l}, \dots, k_{l}} = \frac{p!}{k_{-l}!k_{1-l}!\dots k_{l}!}$, is a multinomial coefficient, e.g.

$$\mathrm{E}\left[\underline{x}^{5}\right] = \mathrm{E}\left[\underline{v}^{5}\right]\sum_{j=-l}^{l} a_{|j|}{}^{5} + 10\sum_{j=-l}^{l}\sum_{k=-l;k\neq j}^{l} a_{|j|}^{2}\, a_{|k|}^{3}$$

Seeking theoretical consistency in analysis of geophysical data

Test: Stochastic similarities among hydrometeorological timeseries extracted from a global database of surface stations (overall, several billions of records).

Project

Seeking Theoretical Consistency in Analysis of Geophysical Data (Using Stochastics)

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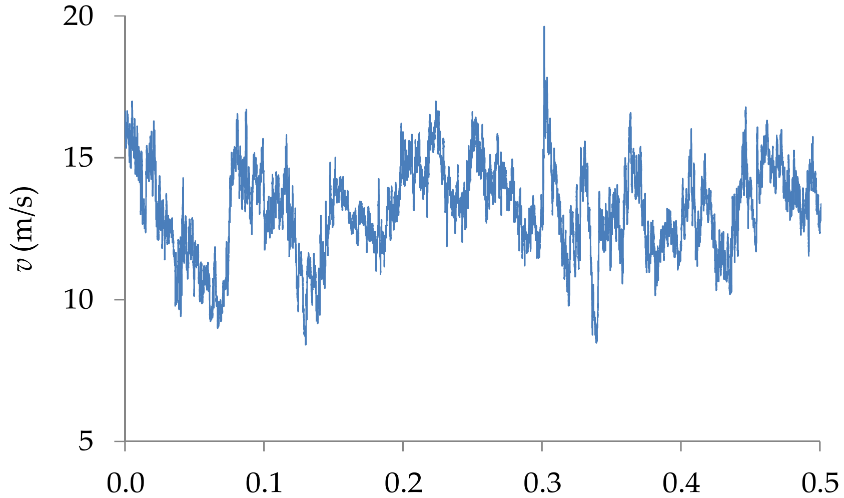
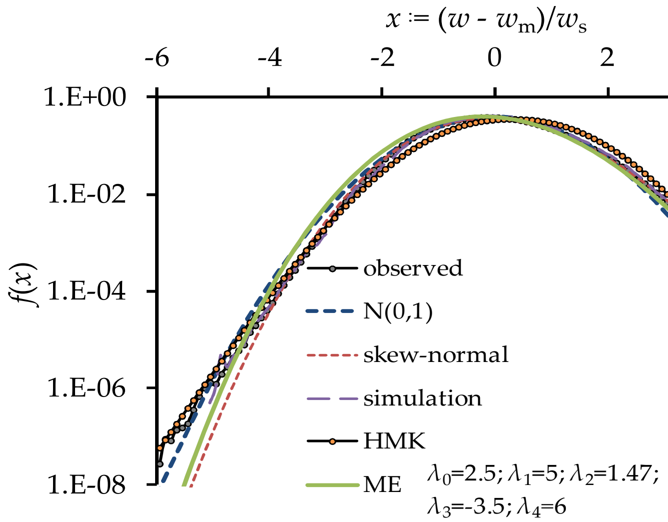
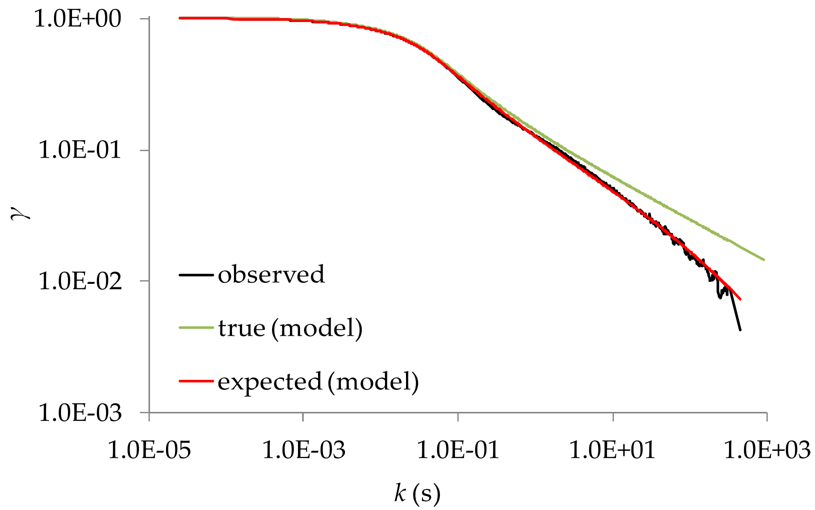
Goal: Analysis of geophysical data is (explicitly or implicitly) based on stochastics, i.e. the mathematics of random variables and stochastic processes. These are abstract mathematical objects, whose properties distinguish them from typical variables that take on numerical values. It is important to understand these properties before making calculations with data, otherwise the results may be meaningless (not even wrong).

Application to grid-turbulence

- Consists of nearly isotropic (isotropy ratio 1.5) and homogeneous turbulent wind stream-wise velocity data.
- Measured by X-wire probes at several locations downstream of an active grid (normalization required).
- 40 timeseries of 36×10^6 data each.

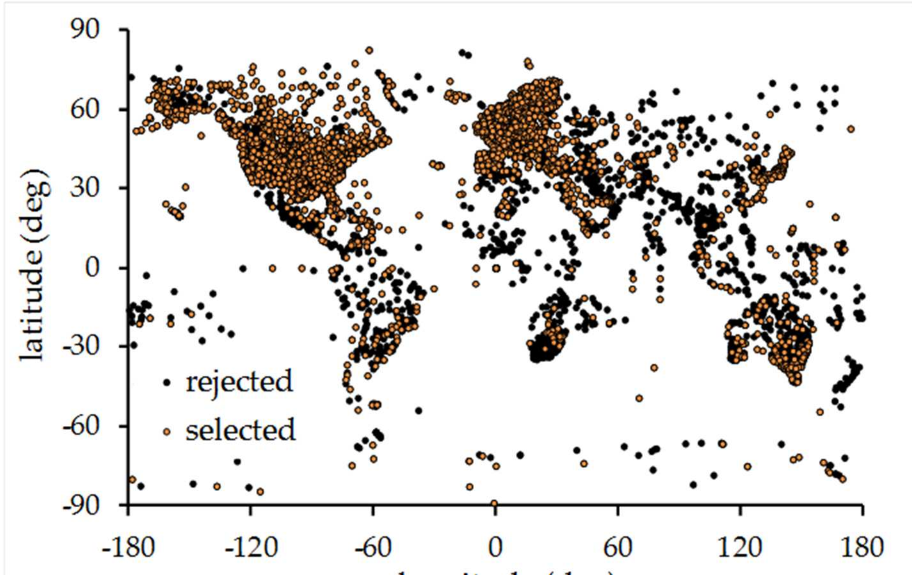


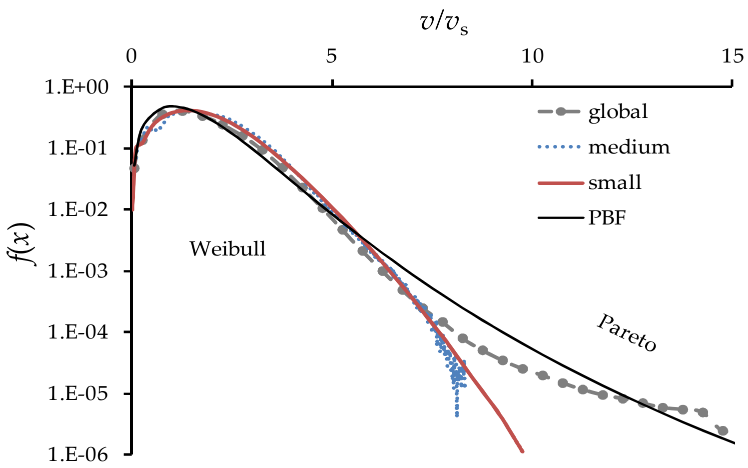


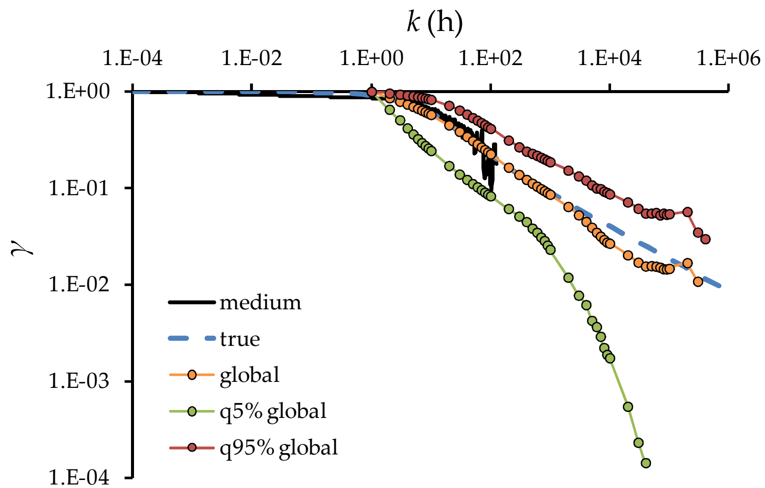


Application to atmospheric wind speed

- Microscale structure: make use of the grid-turbulence data.
- Medium scale structure: sonic anemometer (10 Hz resolution) timeseries recorded for two months at Beaumont USA.
- Macroscale structure: global database (GHCN) of hourly surface wind (over 4000 stations from different sites and climatic regimes).

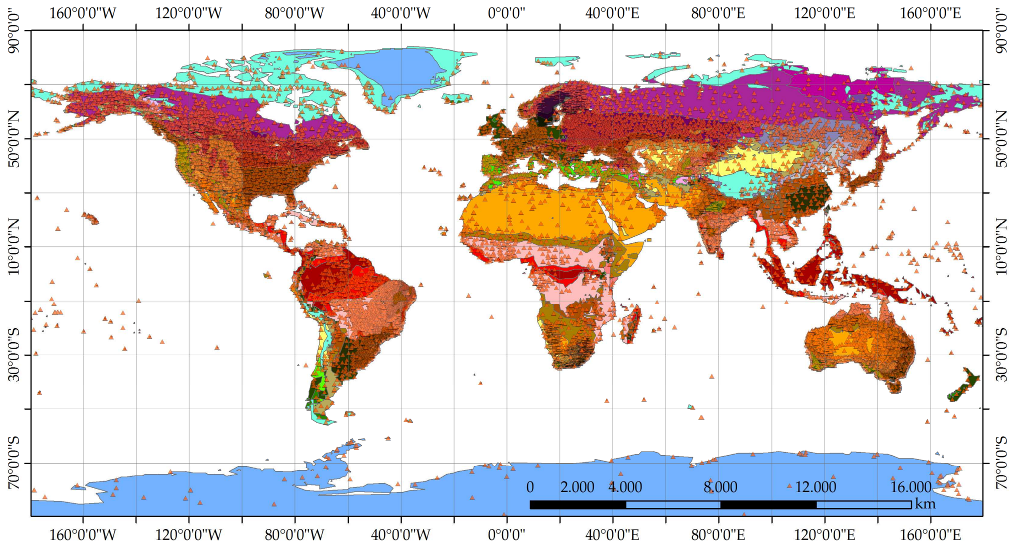


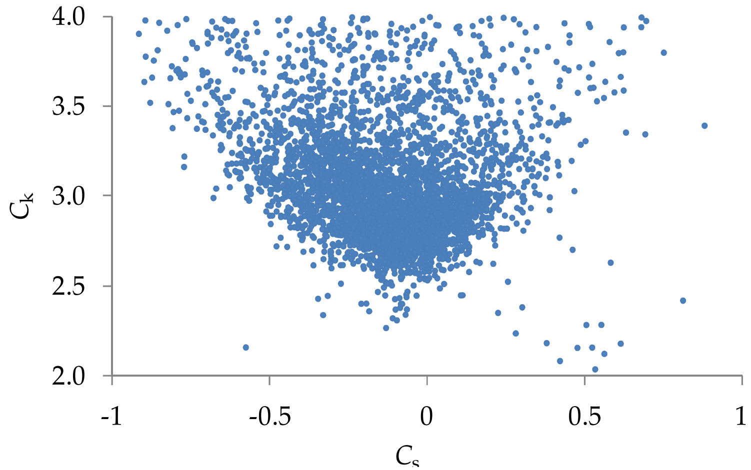


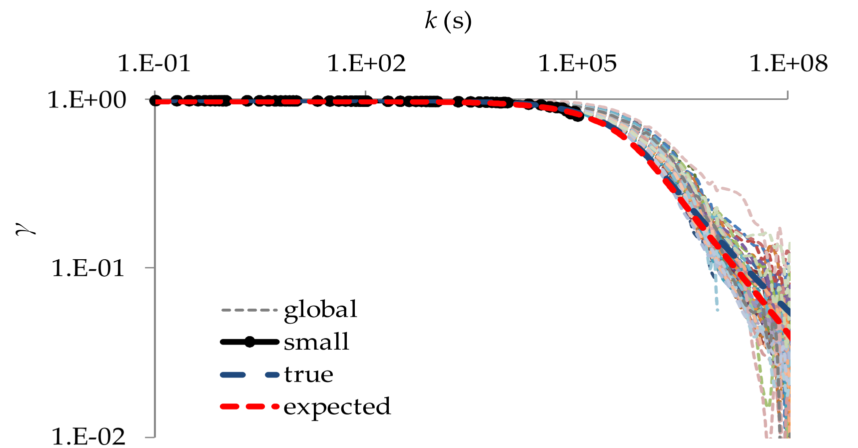


Application to surface temperature

- Microscale structure: same stations as in the medium scale of wind.
- Macroscale structure: global database of hourly air temperature (over 5000 stations from different sites and climatic regimes).
- Assuming temperature follows a Gaussian distribution, we estimate the correlation structure (after homogenization).

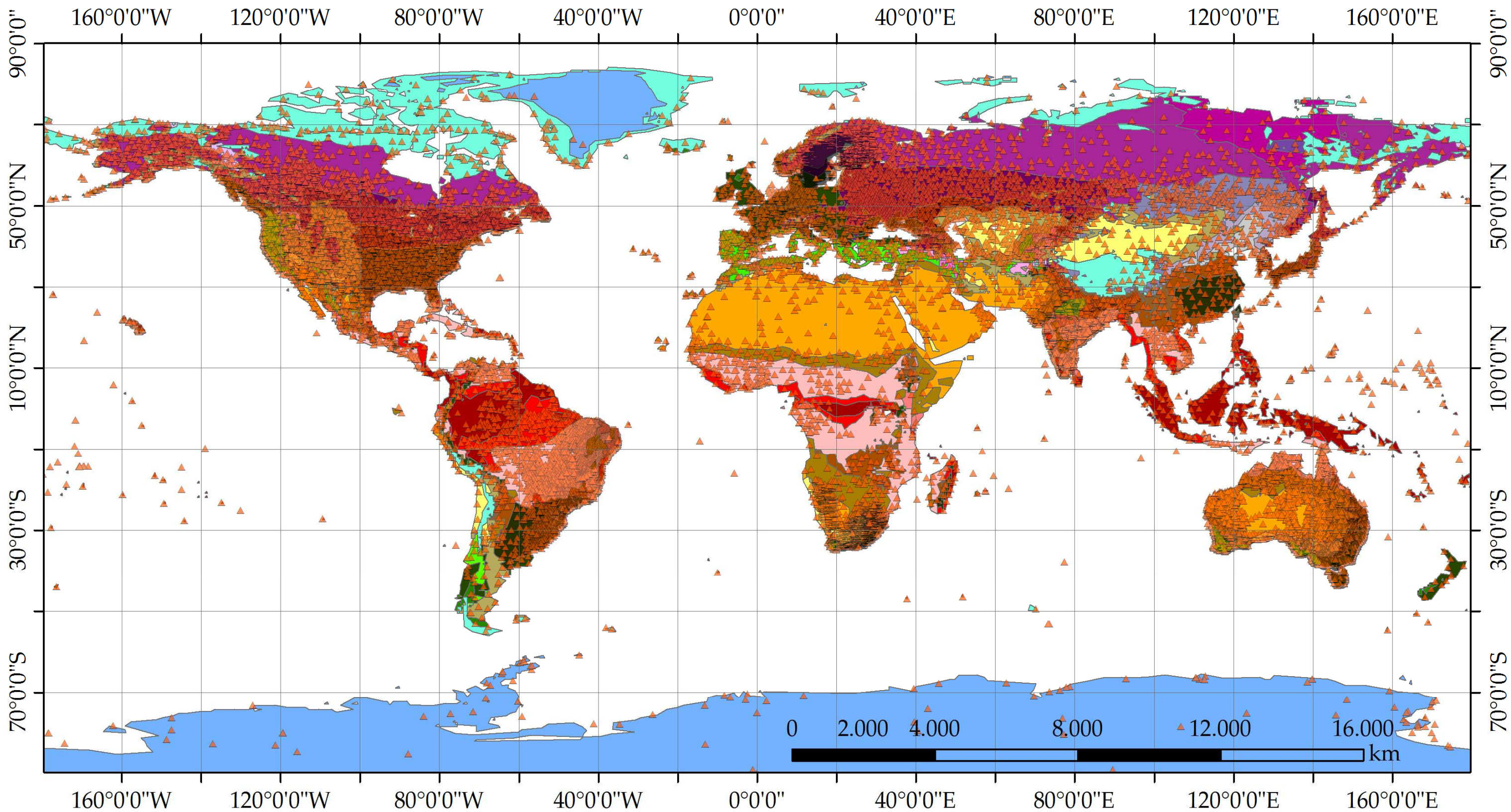






Application to large-scale hydrometeorological processes (I)

Macroscale structure: global database GHCN of NOAA of land-based stations of temperature, wind speed, dew-point, precipitation and atmospheric pressure (after quality controls the analysis included over 5000 stations from different sites and climatic regimes; see analysis in Koutsoyiannis et al., 2018).



Application to large-scale hydrometeorological processes (II)

- Make full use of the GHCN database.
- Estimate the Hurst parameter (H) by Koppen-Geiger classification (simplified climatic regime).

Hurst parameter (H) / Koppen-Geiger classification	temperature	dew-point	surface wind speed	precipitation	atmospheric pressure
A	0.79	0.78	0.84	0.62	0.71
B	0.73	0.77	0.82	0.59	0.72
C	0.70	0.71	0.87	0.65	0.73
D	0.72	0.68	0.85	0.66	0.65
E	0.68	0.65	0.70	0.83	0.71

Conclusions

- A stochastic framework is often necessary in analysing non-linear processes.
- An explicit scheme should be preferred for the direct simulation of both marginal and dependence structures, in order to adequately preserve the intermittent and the HK behaviours
- HK behaviour (H >> 0.5) is apparent in the isotropic turbulence and in key hydrometeorological processes in both small and large scales.
- Further observations and processes are required to test whether the stochastic similarities identified in several geophysical processes can be unified into a single framework.

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