Intra-sonic propagation of sliding zones in a fault

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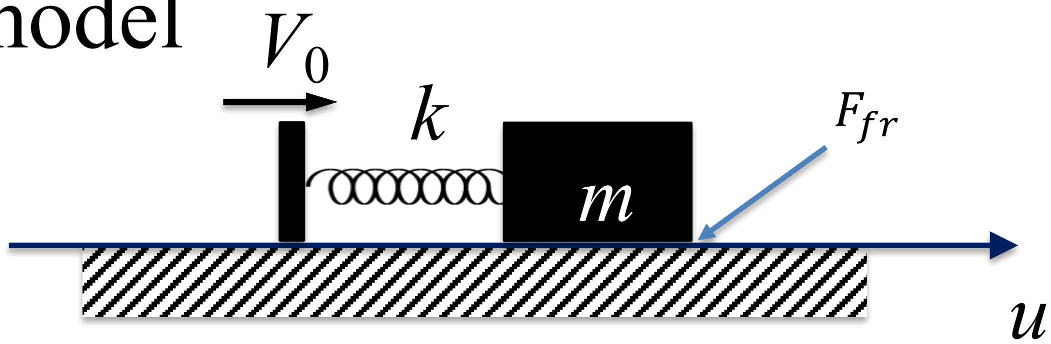
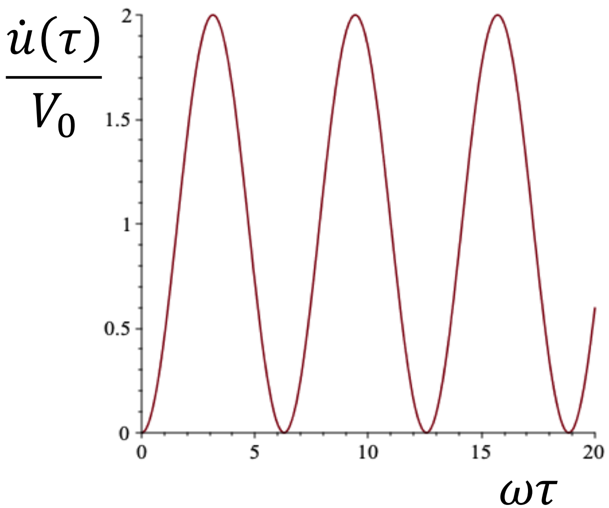
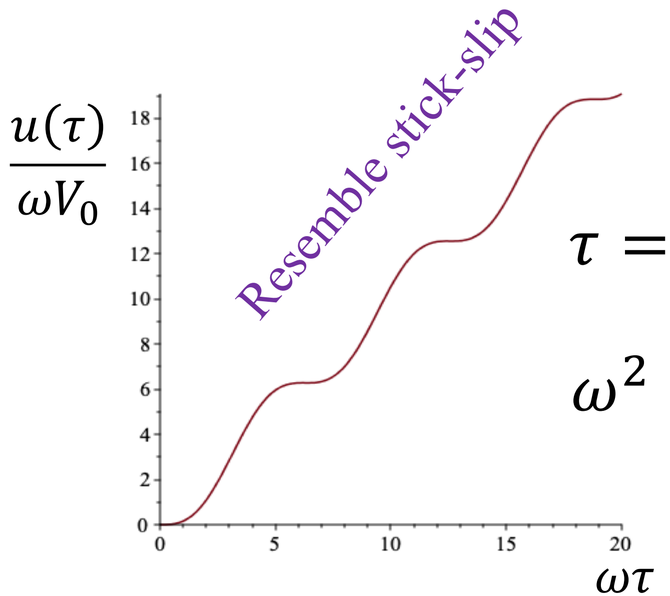
The effect of elasticity of the surrounding rocks. Self-oscillations

- Simple conceptual model



- Self-oscillations





Beginning of sliding

$$\tau = t - t_0,\ t_0 = \frac{F_{fr}}{kV_0}$$

$$\omega^2 = k/m$$

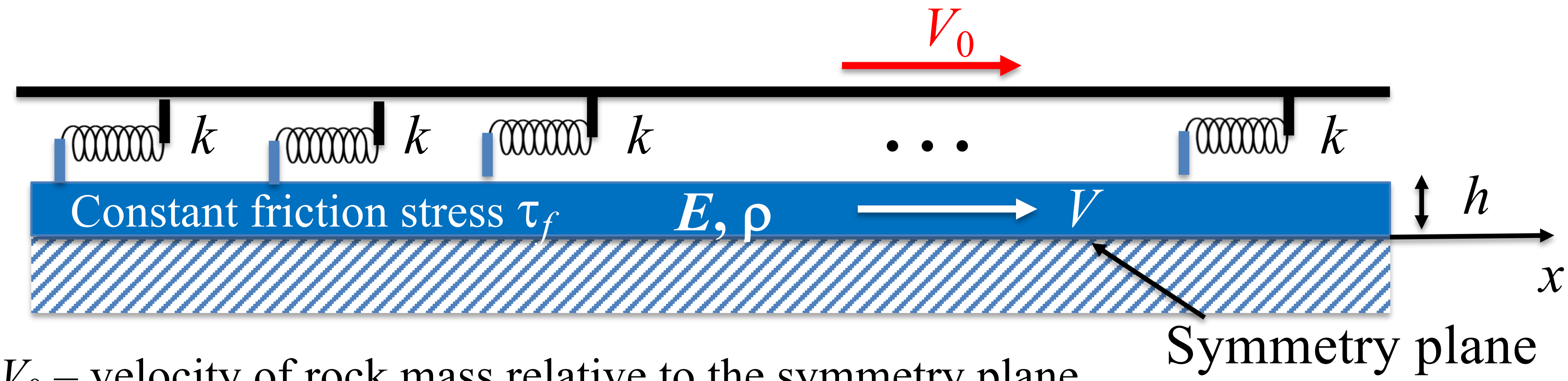
Stick-slip-type of sliding

- Friction is assumed to be rate-independent
- The force driving the block is passed through a system with finite stiffness
- It is the combination of the mass of the block and a spring stiffness that determine the frequency of the oscillations in sliding velocity
- The displacement curve resembles stick-slip

Intra-sonic propagation of sliding zone in a fault

- A simple 1D model
  - Friction is assumed to be constant
  - Symmetry plane coinciding with the middle plane of the fault
  - The rock near the fault is modelled as an elastic rod of thickness h
  - The shear provided by the rock mass moving with a constant velocity is modelled through a distribution of springs of equal stiffness k
- The model leads to telegraph equation with respect to the relative velocity of the sliding zone

Model of sliding zone propagation



V_0 – velocity of rock mass relative to the symmetry plane

k – the spring stiffness, Pa/m

E, ρ – Young's modulus and density of the rock at the fault

Telegraph equation

$$\boxed{\frac{\partial^2 \Delta V}{\partial t^2} = c_l^2 \frac{\partial^2 \Delta V}{\partial x^2} - \omega^2 \Delta V} \qquad \Delta V = V - V_0$$

p-wave velocity

$$c_l = \sqrt{Eh/\rho} \qquad \omega = \sqrt{k/(h\rho)}$$

Sliding zone moves with **p-wave** velocity, relative to the rock mass

Intra-sonic propagation of sliding zone in a fault

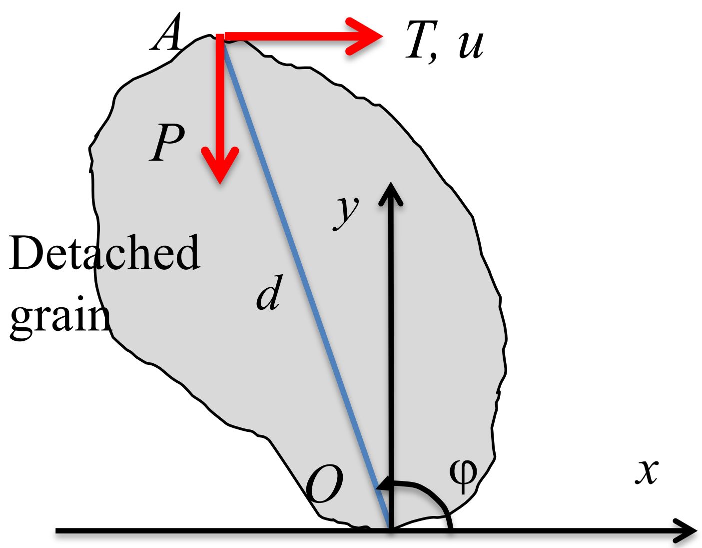
- When friction is independent of velocity (rate independent) the sliding zone propagates with the p-wave velocity

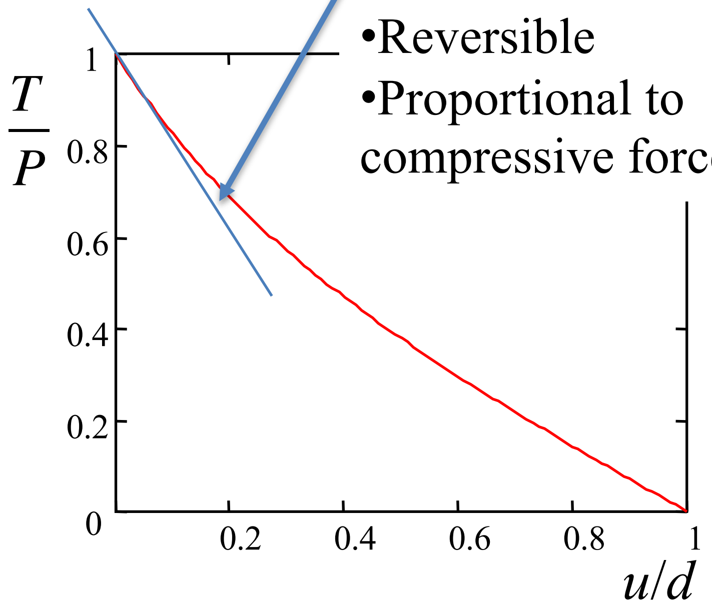
$$\Delta V = c_l$$

- When friction reduces with velocity the sliding zone propagation is intra-sonic

$$\Delta V < c_l$$

Rotation of non-spherical particle in compression. Negative stiffness

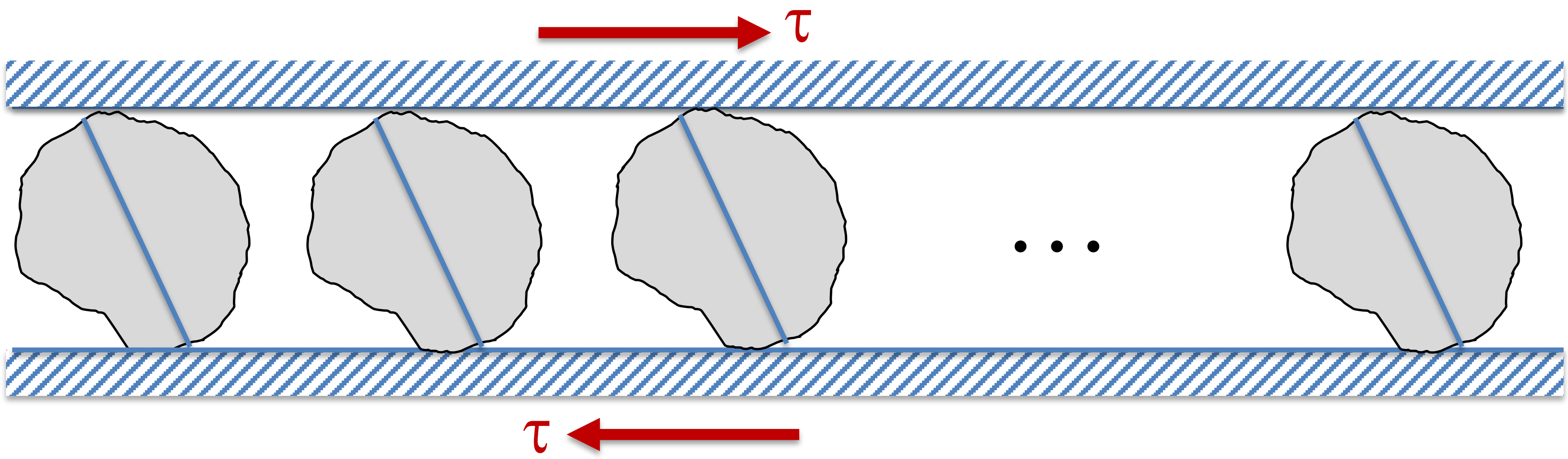




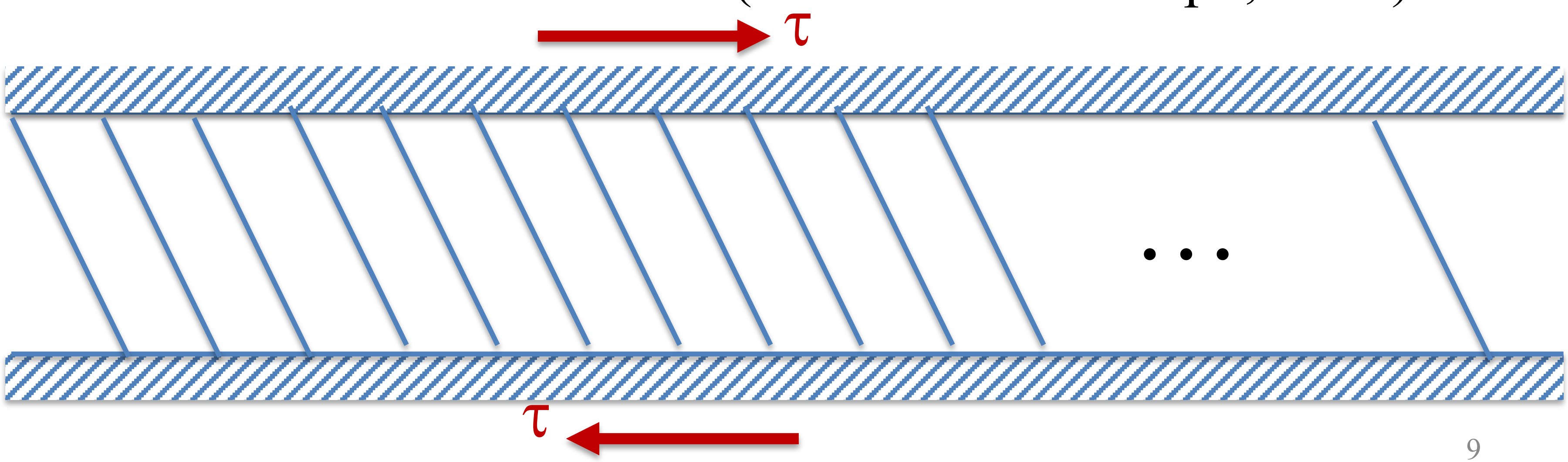
At onset of rotation:
- Apparent negative stiffness
- Reversible
- Proportional to compressive force

$$dT = kdx; \quad k = -\frac{P}{d\sin^3\varphi}$$ ← Negative stiffness

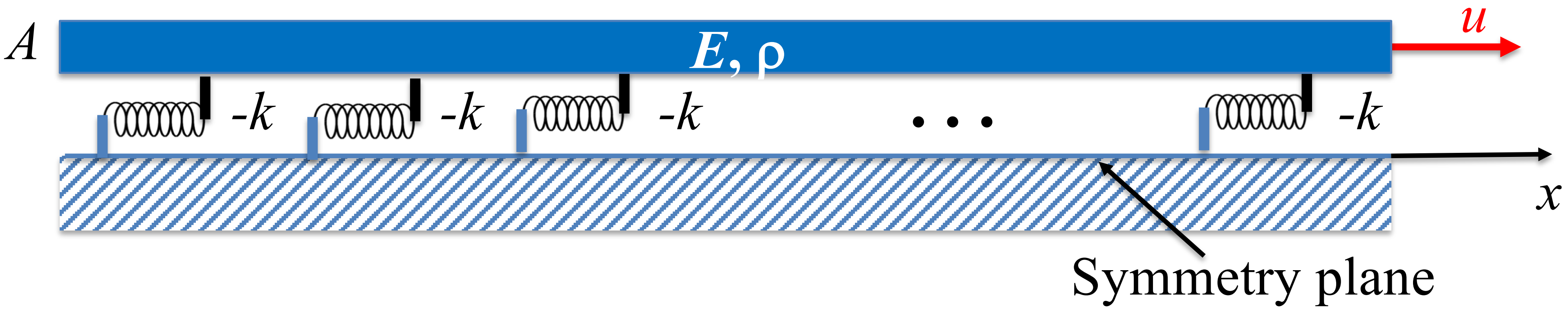
Rotation of non-spherical particles in gouge



Schematics: Tarasov's "fan" (Tarasov & Randolph, 2008)



Sliding over a frictionless fault with negative stiffness gouge



u – displacement of the rock at the fault relative to the symmetry plane
(-k) – the negative spring stiffness, Pa/m
E, ρ – Young's modulus and density of the rock at the fault
A – cross-section of the part of the rock at the fault

Telegraph equation

$$\boxed{\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + \omega^2 u} \quad c^2 = \frac{E}{\rho}, \omega^2 = \frac{k}{\rho A} > 0$$

Telegraph equation as a universal model of sliding zone propagation

- Friction: Variable – relative velocity, ΔV; sign $\boxed{-\omega^2}$

$$\frac{\partial^2 \Delta V}{\partial t^2} = c^2 \frac{\partial^2 \Delta V}{\partial x^2} - \omega^2 \Delta V$$

$$-\infty < x < \infty, \qquad \Delta V\Big|_{t=0} = f(x), \qquad \frac{\partial \Delta V}{\partial t}\Bigg|_{t=0} = F(x)$$

- Negative stiffness: relative displacement, u; sign $\boxed{+\omega^2}$

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + \omega^2 u$$

$$-\infty < x < \infty, \qquad u\Big|_{t=0} = f(x), \qquad \frac{\partial u}{\partial t}\Bigg|_{t=0} = F(x)$$

Solution of the telegraph equations

$$\frac{\partial^2 w_\mp}{\partial t^2} = c_l^2 \frac{\partial^2 w_\mp}{\partial x^2} \mp \omega^2 w_\mp$$

$$-\infty < x < \infty$$

$$w_{\mp\, t=0} = f(x), \qquad \left.\frac{\partial w_\mp}{\partial t}\right|_{t=0} = F(x)$$

$w_- = \Delta V$ (friction model), $w_+ = u$ (negative stiffness model)

$$w_\mp(\xi,\tau) = \frac{1}{2}\left[f(\xi-\tau) + f(\xi+\tau)\right] + \frac{\tau}{2}\int_{-1}^{1} F(\xi+\tau p) G_\mp\left(\tau\sqrt{1-p^2}\right) dp + \frac{\tau^2}{2}\int_{-1}^{1} f(\xi+\tau p) G_\mp^{(1)}\left(\tau\sqrt{1-p^2}\right) dp$$

$$G_-(z) = J_0(z),\ G_+(z) = J_0(iz),\ G_-^{(1)} = -\frac{1}{z}\frac{dG_-(z)}{dz},\ G_+^{(1)} = -\frac{1}{iz}\frac{dG_+(z)}{dz}$$

Bessel function of 1st kind

$$\xi = \frac{x\omega}{c_l}, \qquad \tau = t\omega,$$

Analysis

- Term $\frac{1}{2}[f(\xi-\tau)+f(\xi+\tau)]$ together with $\xi = x\omega/c_l$, $\tau = t\omega$ suggests sliding with velocity c_l.
- Kernels:

  $G_-(z) \to 0$ as $|z| \to \infty, G_+(z) \to \infty$ as $|z| \to \infty$
- This suggests that in the case of negative stiffness the amplitude of sliding zone increases, which corresponds to energy supply associated with negative stiffness.
- This energy comes from the work of the compression effecting negative stiffness in the case of rotating non-spherical particles

Conclusions

- Frictional sliding is produced by the force applied through the surrounding rock which is not absolutely stiff.
- Elasticity of the rock leads to self-oscillations in velocity. The way displacement increases resembles stick-slip even if the friction force is rate independent
- The force causing sliding over a fault is transmitted through longitudinal deformation of the rock near the fault. This makes the sliding zone propagate with the p-wave velocity.
- Rotations of (usually) non-spherical gauge particles in the presence of compression lead to the effect of negative stiffness.
- Sliding over a fault with negative stiffness material in the fault produces the sliding velocity also equal to p-wave velocity

Acknowledgements

AVD acknowledges the support from the School of Civil and Transportation, Faculty of Engineering, Beijing University of Civil Engineering and Architecture.

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