Hydraulic fracture oscillations in response to strong impulse

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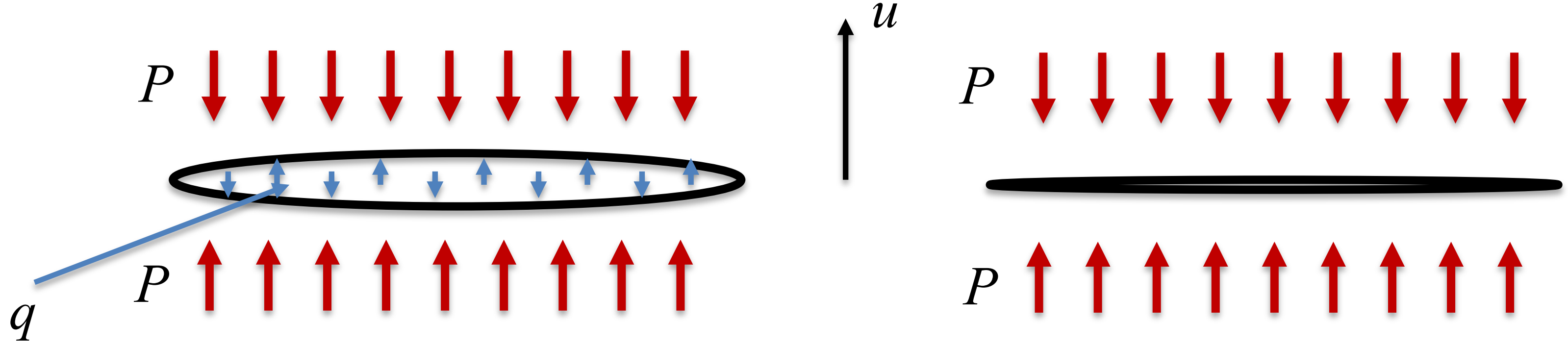
Large hydraulic fracture as a bilinear oscillator



$P<q$ Rock moduli: E_0, ν_0 $P>q$

Average displacement discontinuity, u, of a disc-like crack of radius R

$$\bar{u} = \frac{16}{3\pi}\frac{1-\nu_0^2}{E_0}R(q-P)$$

$$\bar{u} = -4\pi\frac{1-\nu_0^2}{k_c}R^2(q-P)$$

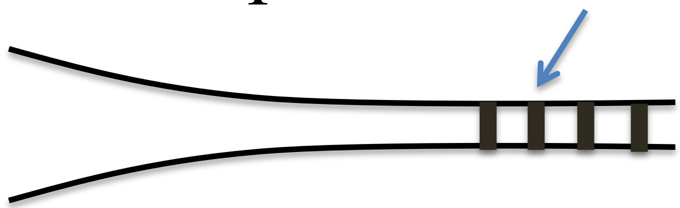
Stiffness of the contact (k_c)

Stiffness ratio (compression to tension)

$$K = \frac{k_-}{k_+} = \frac{3}{4}\pi^2\frac{E_0 R}{k_c} \to \infty \text{ as } R \to \infty$$ - Strong bilinearity

Fractures with constraint opening

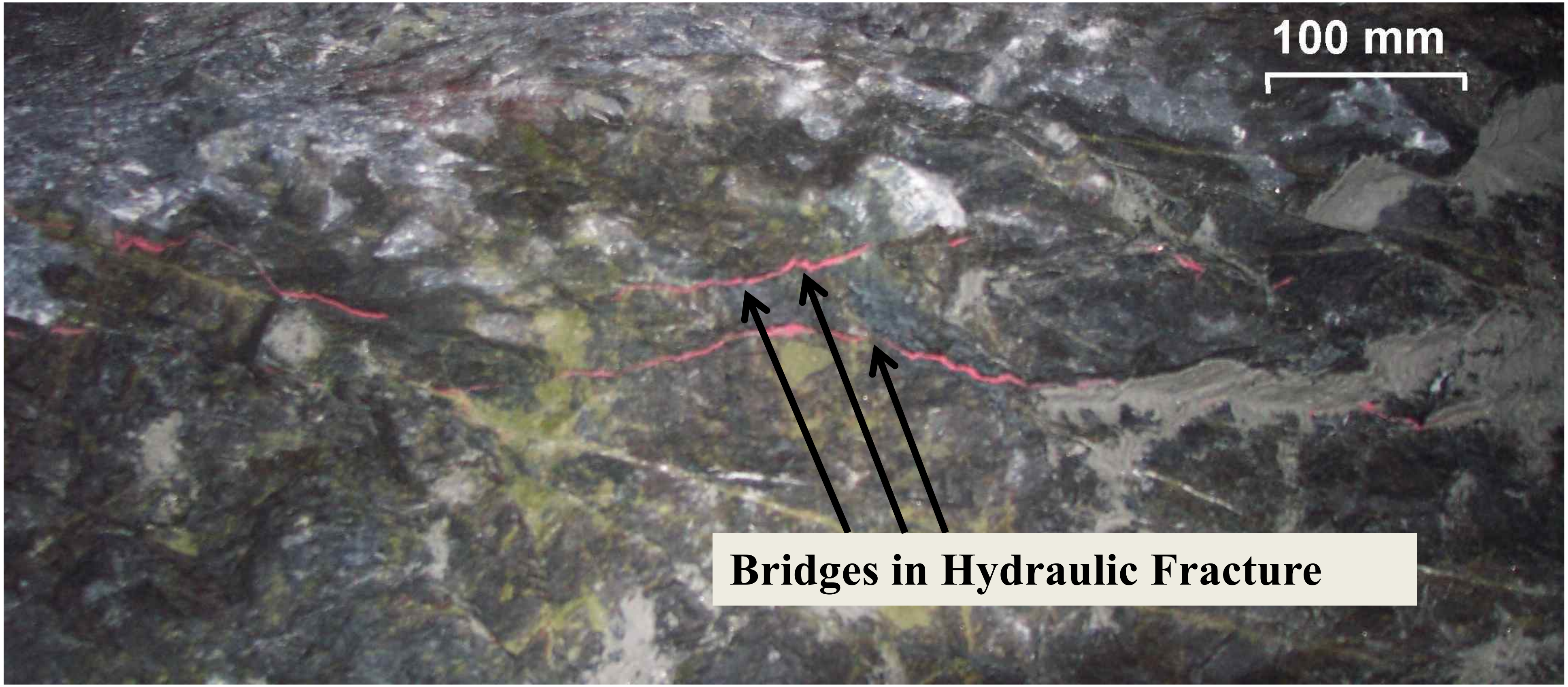
- Parts of unbroken rock connecting the opposite faces of the fracture
- Traditionally considered as obstacles for fluid flow
- Traditionally considered as a part of the process zone



- What was overlooked before:

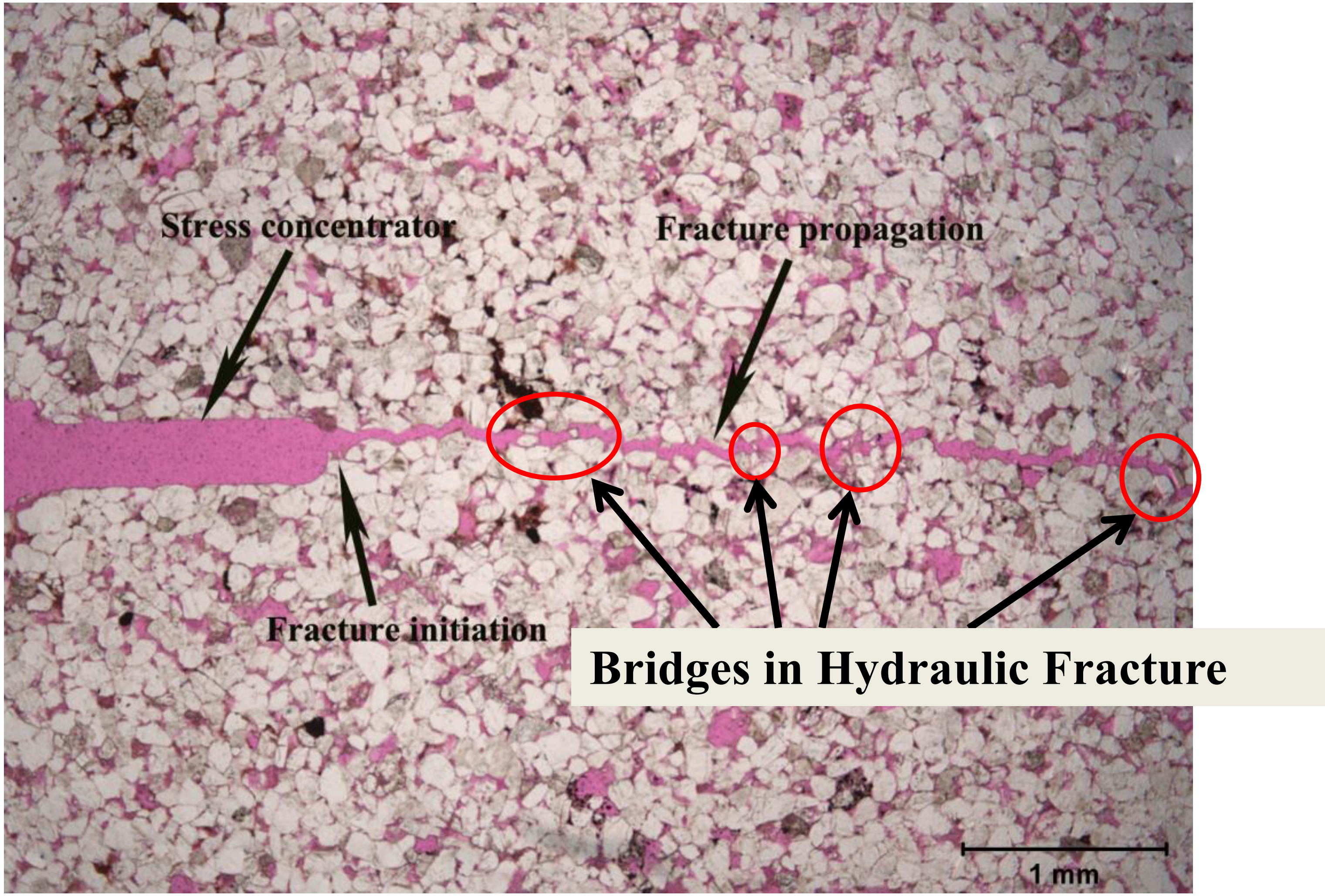
Bridges are all over the crack and can constrict the opening

Hydraulic fracture in mining



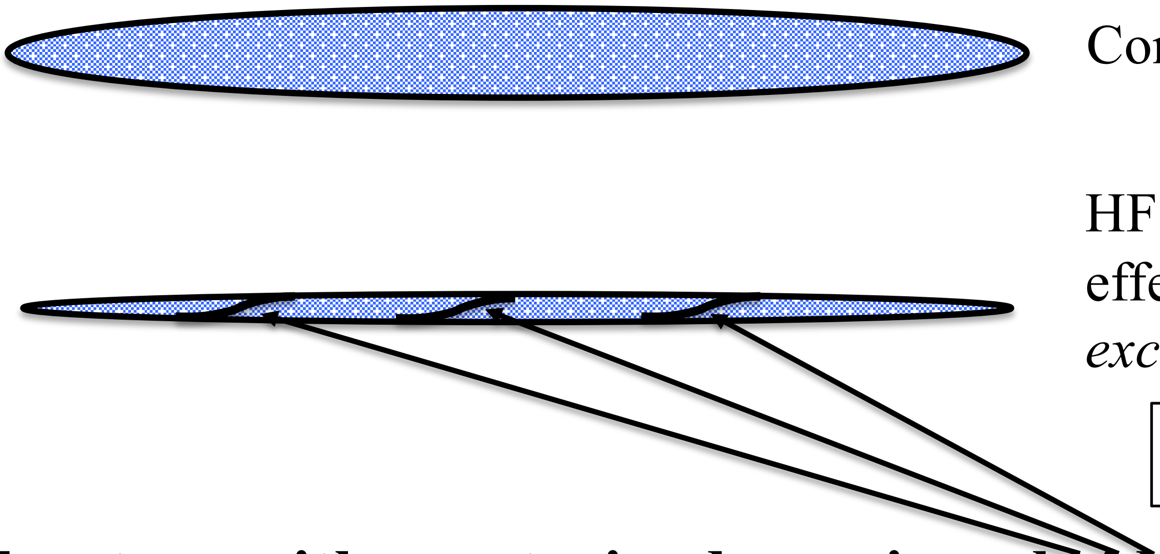
Northparkes mine, courtesy of Rob Jeffrey

Bridge Cracks



(Surdi 2010)

Large hydraulic fracture with constrained opening (HFCO) as a bilinear oscillator



Conventional fracture

HF with constrained opening, effective when the crack radius, *R*, *exceeds* a constriction length, λ:

$$R >> \lambda = E_0/k_b$$

Fracture with constrained opening - bridges distributed all over the fracture. Effective stiffness of bridges is low (He et al., 2020), hence $k_b << k_c$.

$$K = k_c/k_b \gg 1$$

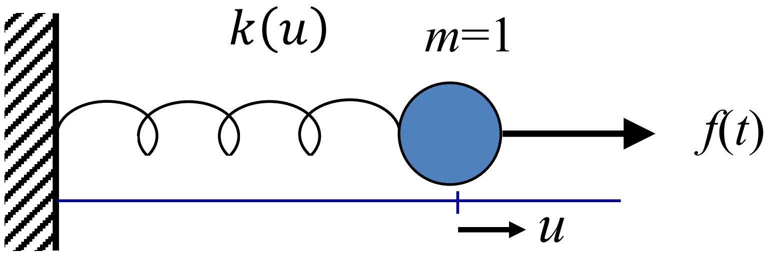
- Strong bilinearity

He, J., E. Pasternak and A.V. Dyskin, 2020. Bridges outside fracture process zone: Their existence and effect. *Engineering Fracture Mechanics,* 225, 106453

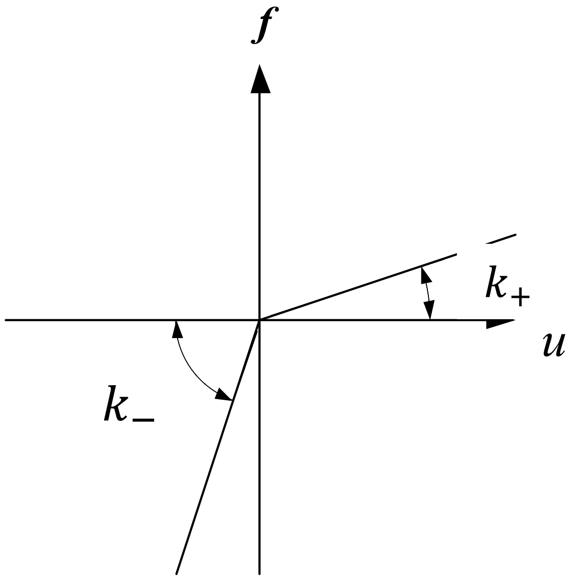
Bilinear oscillator and multiple resonances

- Bilinear oscillator as a simplification of strong non-linearity
- Resonance frequency
- Multiple resonances
- Half resonance

Bilinear oscillators



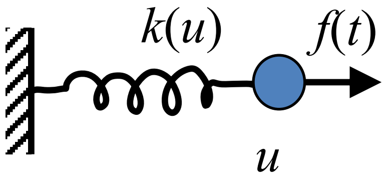
$$\ddot{u} + k(u)u = f(t)$$



$$k(u) = \begin{cases} \omega_+^2, & u > 0 \\ \omega_-^2, & u < 0 \end{cases}$$

Dyskin, A.V., E. Pasternak and E. Pelinovsky, 2012. Periodic motions and resonances of impact oscillators. *Journal of Sound and Vibration* 331(12) 2856-2873

Bilinear oscillator

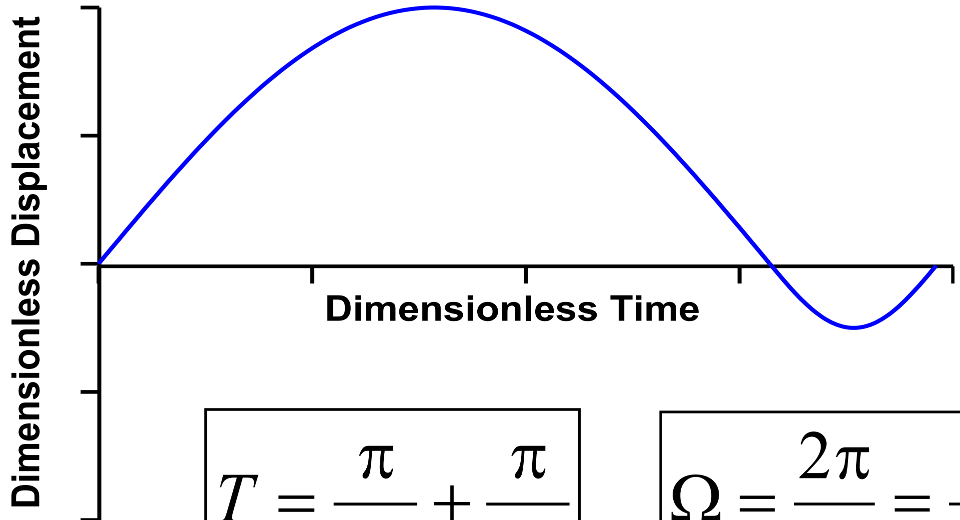


$$\ddot{u} + k(u)u = f(t)$$

$$k(u) = \begin{cases} \omega_+^2, & u > 0 \\ \omega_-^2, & u < 0 \end{cases}$$

Solution, one cycle

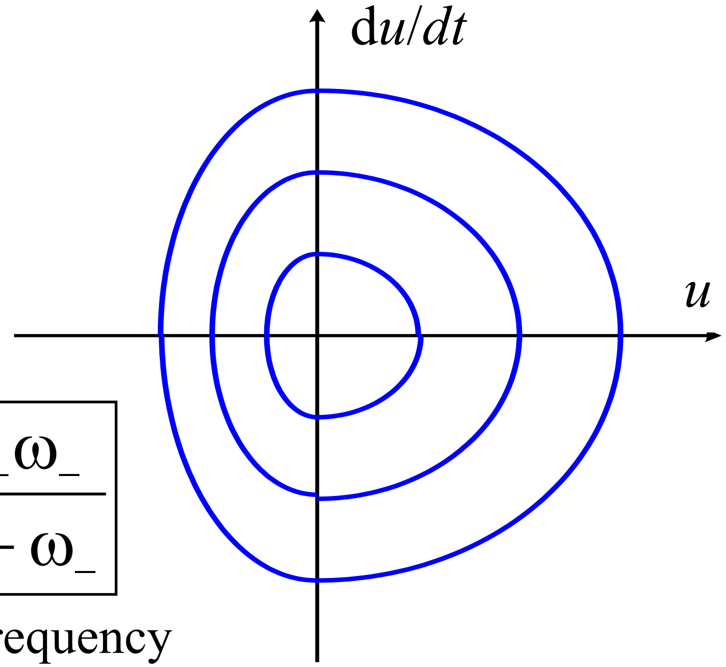
$$u(t) = A\begin{cases} \sin(\omega_+ t), & 0 < \omega_+ t < \pi, \\ \dfrac{\omega_+}{\omega_-}\sin[\omega_- t + \varphi], & \pi < \omega_- t + \varphi < 2\pi, \end{cases}$$



$$T = \frac{\pi}{\omega_+} + \frac{\pi}{\omega_-}$$

$$\Omega = \frac{2\pi}{T} = \frac{2\omega_+\omega_-}{\omega_+ + \omega_-}$$

Resonance frequency



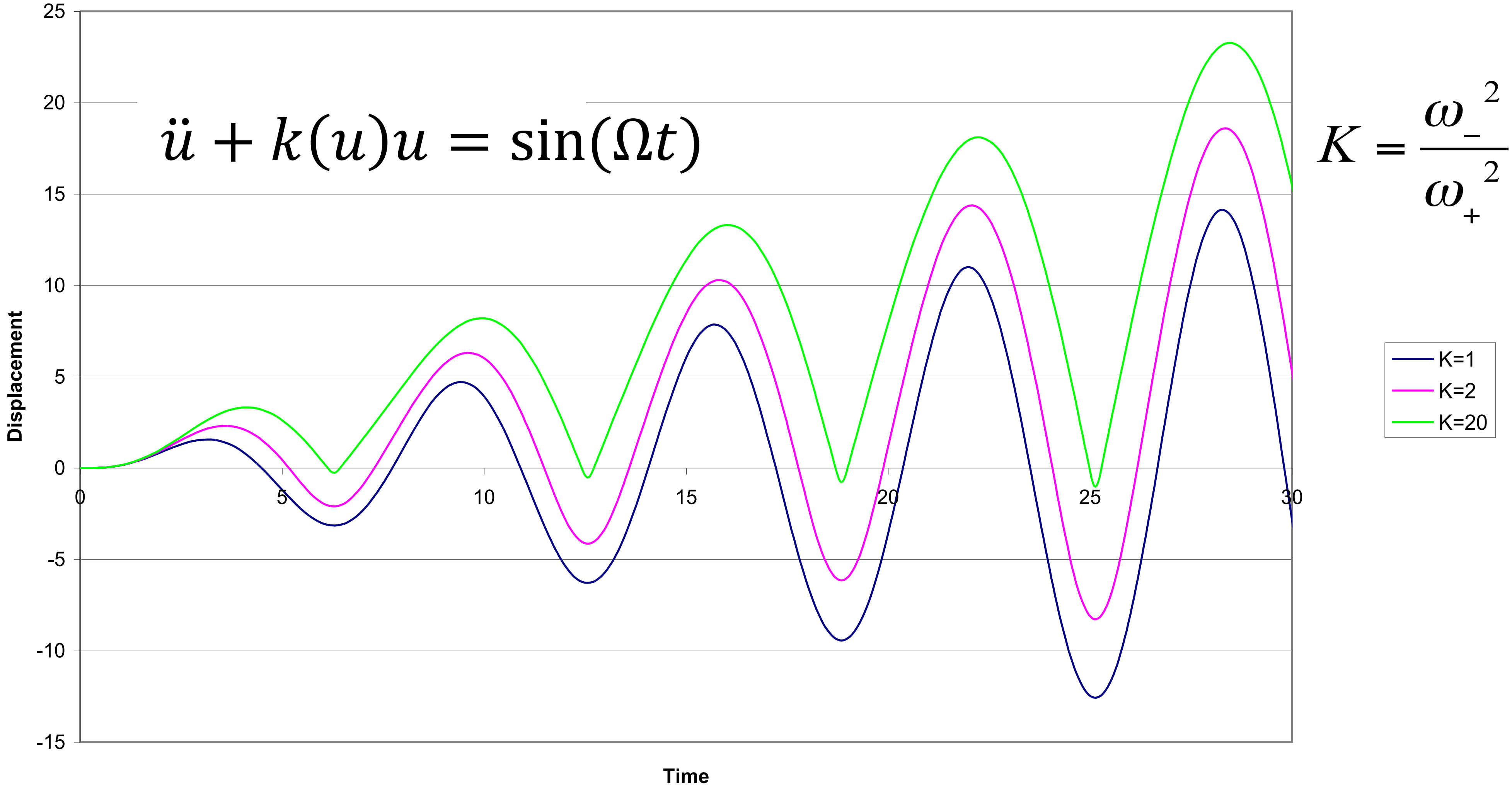
Thomson, J.M.T. (1983)

Dyskin, A.V., E. Pasternak and E. Pelinovsky, 2012. Periodic motions and resonances of impact oscillators. *Journal of Sound and Vibration* 331(12) 2856-2873

Basic resonance

$$\ddot{u} + k(u)u = \sin(\Omega t)$$

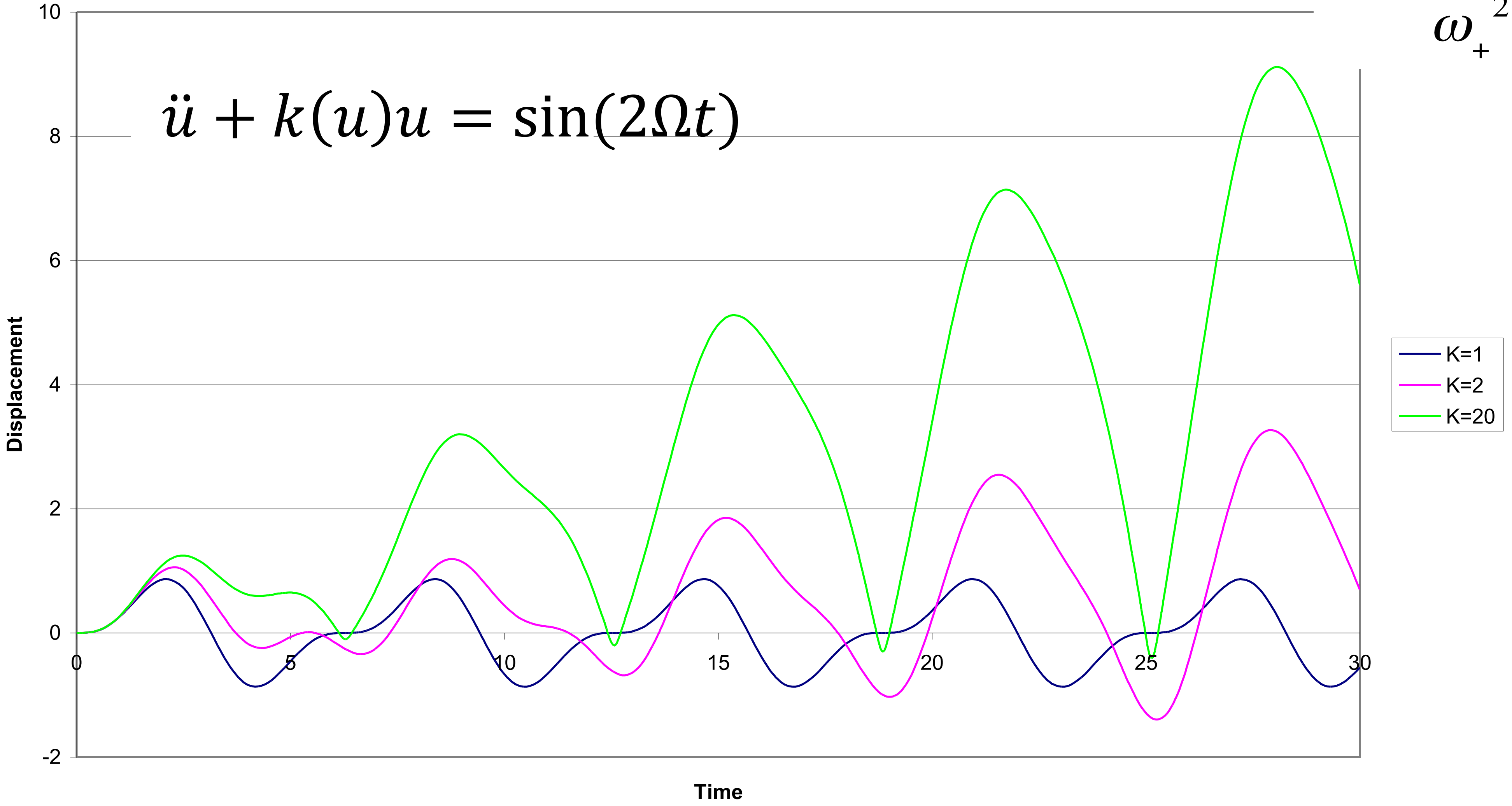
$$K = \frac{\omega_-^2}{\omega_+^2}$$



Second resonance (excitation at **double** resonance frequency)

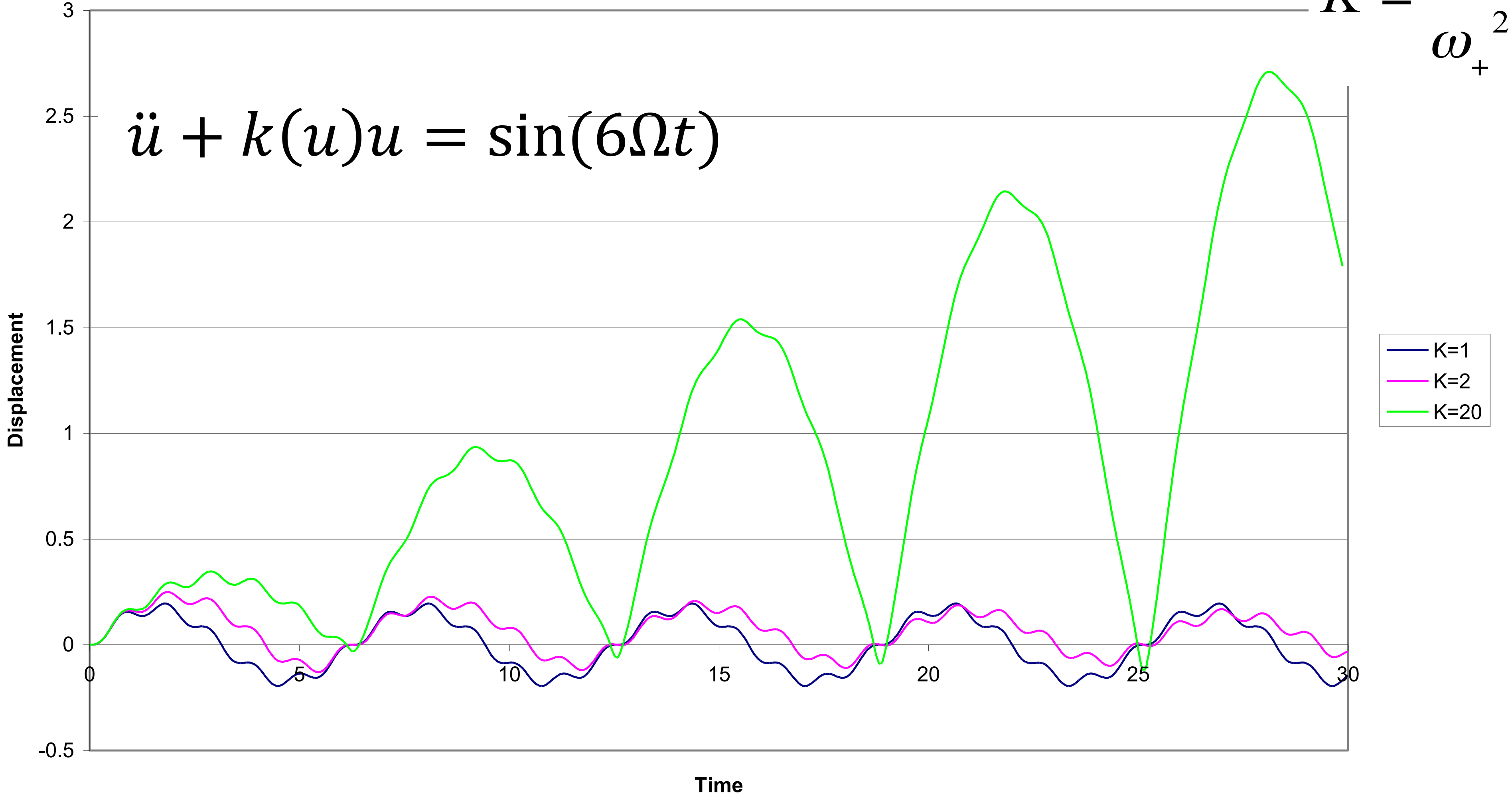
$$K = \frac{\omega_-^2}{\omega_+^2}$$

$$\ddot{u} + k(u)u = \sin(2\Omega t)$$



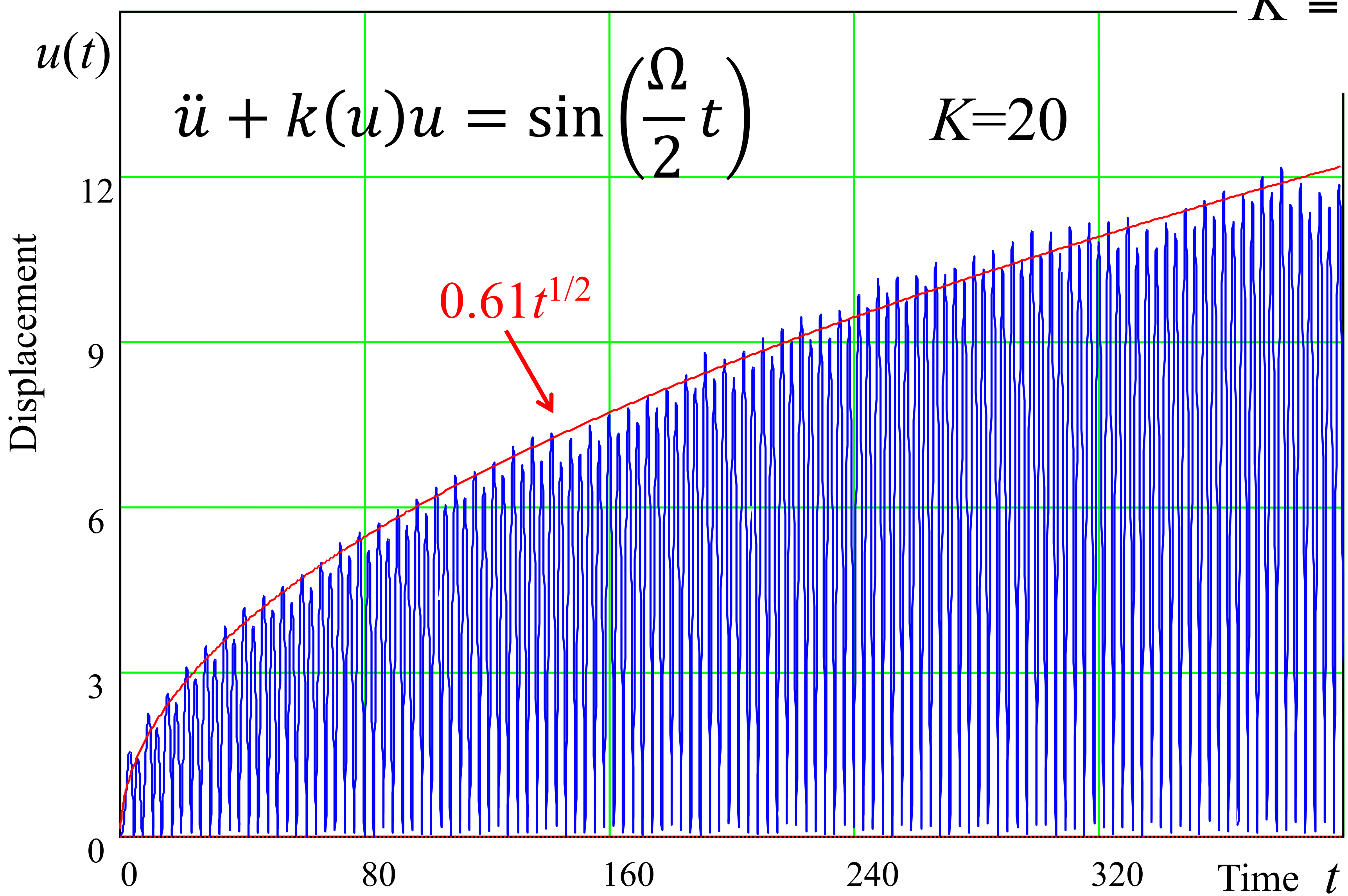
Sixth resonance (excitation at **6 x** resonance frequency)

$$K = \frac{\omega_-^{\ 2}}{\omega_+^{\ 2}}$$



Half resonance (excitation at **half** resonance frequency)

$$K = \frac{\omega_-^2}{\omega_+^2}$$



Spectrum of bilinear oscillator

- Period $T = \frac{\pi}{\omega_+} + \frac{\pi}{\omega_-}$

- Fourier series for displacement discontinuity

$$u(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left[a_k \cos\left(\frac{2\pi k t}{T}\right) + b_k \sin\left(\frac{2\pi k t}{T}\right) \right]$$

- Spectral amplitude

$$c_k = \sqrt{a_k^2 + b_k^2}$$

First two harmonics

- Introduce $\tau = \dfrac{\pi}{T\omega_+}$

- Ratio of the first two harmonics

$$\frac{c_1}{c_2} = \frac{\cos\left(\frac{\pi\tau}{2}\right)}{\cos(\pi\tau)} \frac{|4\tau(2-\tau)-3||4\tau(2-\tau)+5|}{|\tau(2-\tau)-1||\tau(2-\tau)+3|}, \qquad \frac{1}{2} < \tau \leq 1$$

Recovery of the stiffness ratio

- Find period T
- Find ratio of two first harmonics $\frac{c_1}{c_2}$
- Find τ
- Find stiffness ratio $K = \frac{k_-}{k_+} = \frac{\omega_-^2}{\omega_+^2} = \frac{\tau^2}{(1-\tau)^2}$

Interpretation of the stiffness ratio

- Conventional HF

$$K = \frac{3}{4}\pi^2\frac{E_0 R}{k_c}$$

  - If rock modulus E_0 and contact stiffness k_c are known, the fracture radius R is determinable

- HF with constrained opening (large crack radius), $R >> \lambda = \frac{E_0}{k_b}$

$$K = \frac{k_c}{k_b}$$

  - Effective stiffness k_b of bridges is determinable

Conclusions

- Large hydraulic fractures can behave as bilinear oscillators with stiffness in compression being considerably higher than stiffness in tension
- Bilinear oscillators exhibit multiple resonances
- Spectrum of bilinear oscillator is controlled by the period and stiffness ratio $K = \frac{k_-}{k_+} = \frac{\omega_-^2}{\omega_+^2}$
- Finding spectrum of oscillations in response to external excitation allows the estimation of hydraulic fracture dimensions

Acknowledgements

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