Analytical Model for Tsunami Propagation including Source Kinematics

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   - The underwater disturbance
   - Analytical solution
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Summary

There are only a few analytical 2+1 D models for tsunami propagation, in which most of them treat tsunami generation from static deformation field isolated from the kinematics of the rupture. This work examines the behavior of the tsunami propagation in a simple set-up including a source time function which accounts for a time description of the rupture process on the tsunami source. An analytical solution is derived in the wavenumber domain, which is quickly inverted to space with the Fast Fourier Transform. The solution is obtained in closed form in the 1+1D case. The inclusion of temporal parameters of the source such as rise time and rupture velocity reveals a specific domain of very slow earthquakes that enhance tsunami amplitudes and produce non-negligible shifts on the arrival times. The obtained results confirm that amplification occurs when the rupture velocity matches the long-wave tsunami speed and the static approximation corresponds to a limit case for (relatively) fast ruptures.

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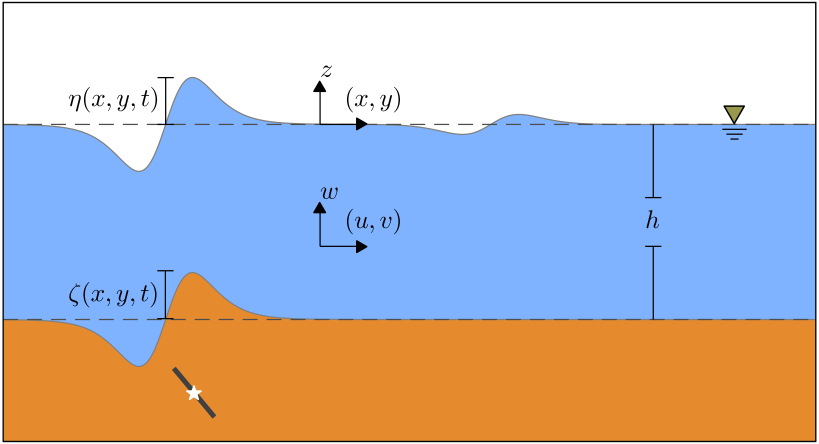
Problem set-up



Governing Equations

In this study, Tsunamis are governed by the Linear potential theory.
In a constant depth ocean, the system of equations takes the form:

$$
\begin{aligned}
\Delta\varphi &= 0 \\
\varphi_z &= \zeta_t, \quad && \text{at } z = -h \\
\varphi_z &= \eta_t, \quad && \text{at } z = 0 \\
\varphi_t &= -g\eta, \quad && \text{at } z = 0.
\end{aligned}
$$

where g is the gravity acceleration and $(u, v, w) = \nabla\varphi(x, y, z, t)$

Governing Equations

In terms of the Laplace-Fourier Transform, the well-known solution is:

Governing Equations

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$$\overline{\widehat{\eta}}(k_x, k_y, s) = \frac{s}{s^2 + \omega^2} \left\{ \frac{s\overline{\widehat{\zeta}}(k_x, k_y, s) - \widehat{\zeta}(k_x, k_y, 0^-)}{\cosh(kh)} + \widehat{\eta}_0(k_x, k_y) \right\}$$

where $\omega^2 = (ck)^2 \frac{\tanh(kh)}{kh}$, $k^2 = k_x^2 + k_y^2$, $c = \sqrt{gh}$, and $\eta_0(x, y)$ is the initial condition. In this work, $\eta_0(x, y)$ is set to zero.

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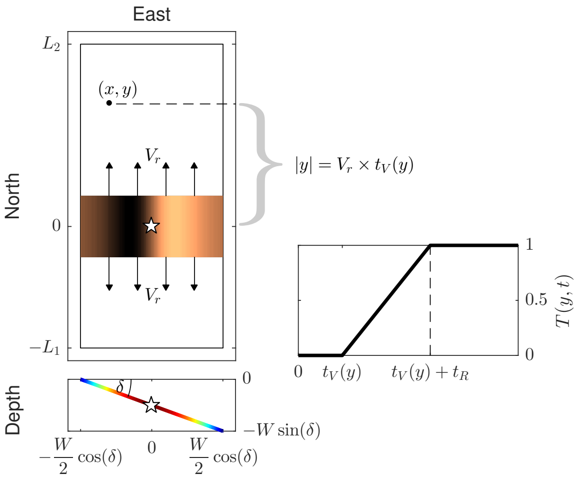
Definition of the Forcing term

The underwater disturbance evolution defines the forcing term that makes the tsunami evolve from its source.
If this source term is defined by an earthquake, the kinematics of the rupture is included:

$$\zeta(x, y, t) = \zeta_0(x, y)T(y, t)$$

The final shape of the bottom disturbance is $\zeta_0(x, y)$ and starts to evolve when the rupture front reaches the point (x, y). Thus, it varies linearly from zero to its final value in a fixed rise-time, as shown in the next slide.

Definition of the Forcing term



$$T(y,t) = S\left(\frac{t - t_V(y)}{t_R}\right)$$

$$S(x) = x\mathcal{H}(x) - (x-1)\mathcal{H}(x-1)$$

$\mathcal{H}(x)$ is the Heaviside step function.

The spatial source is modeled by uncoupling variables

$$\zeta_0(x,y) = \zeta_0^x(x)\zeta_0^y(y)$$

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To derive an analytical solution in the wavenumber domain, the source is constant along y: $\zeta_0^y(y) = \mathcal{H}(L_2 - y)\mathcal{H}(L_1 + y)$
The next auxiliary functions are defined:

$$q(a,b,t) =: \frac{\mathcal{H}(t)}{a^2 - b^2}\left(\frac{\sin(bt)}{b} - \frac{\sin(at)}{a}\right), \text{ with } a = \omega \text{ and } b = V_r k_y.$$

$$p(a,b,t,t_0) =: \mathcal{L}^{-1}\{se^{-st_0}\overline{q}(a,b,s)\}(t) = \partial_t q(a,b,t-t_0).$$

Note there are removable singularities in functions p and q when $\omega = V_r k_y$.

By using the properties of the Laplace transform:

$$\widehat{\eta}(k_x, k_y, t) = \frac{\widehat{\zeta_0^x}(k_x)}{\cosh(kh)} \frac{V_r^2}{t_R} \left[\phi(t) - \phi(t - t_R)\right]$$

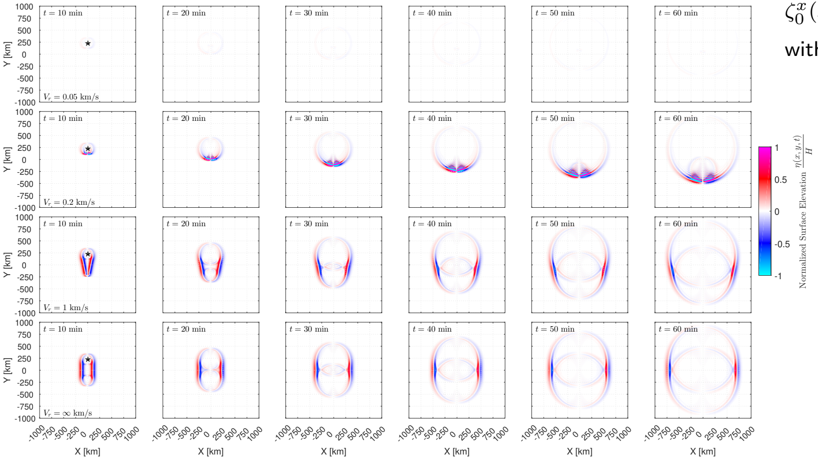
where

$$\begin{aligned}\phi(t) = \frac{2}{V_r} p(t,0) - \frac{1}{V_r} \cos(L_2 k_y) p(t, t_2) + k_y \sin(L_2 k_y) q(t - t_2) - \frac{1}{V_r} \cos(L_1 k_y) p(t, t_1) \\ + k_y \sin(L_1 k_y) q(t - t_1) + i \left\{ -\frac{1}{V_r} \sin(L_2 k_y) p(t, t_2) - k_y \cos(L_2 k_y) q(t - t_2) \right. \\ \left. + \frac{1}{V_r} \sin(L_1 k_y) p(t, t_1) + k_y \cos(L_1 k_y) q(t - t_1) \right\}\end{aligned}$$

and $t_i = \frac{L_i}{V_r}$, $i \in \{1, 2\}$.

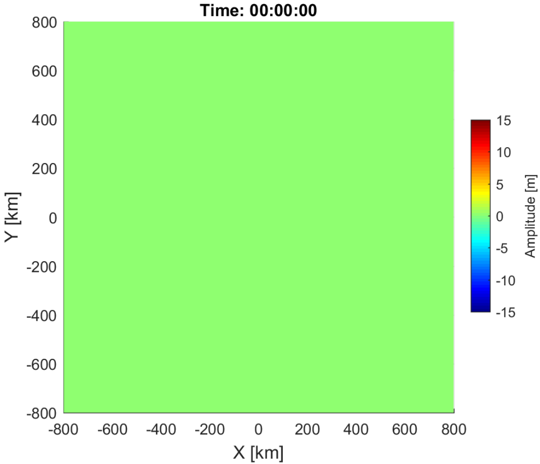
Finally, $\eta(x, y, t)$ is numerically retrieved with the FFT.

Example: Unilateral rupture



$\zeta_0^x(x)$ is modeled with a N-wave.

Propagation when $\nu = 1$



1+1 D Non-dispersive case

- $\zeta_0^x(x) = H$ • $t_R = 0$ • $\omega \approx ck$

In this specific case, a closed form is obtained:

$$\eta(y,t) = \frac{\nu^2 H}{\pi(1-\nu^2)} \Bigg[\psi\left(y - V_r t, 0\right) + \psi(ct - y, 0) + \left(\frac{1}{\nu} - 1\right) \psi(ct, y) + \mathcal{H}(t') \left\{ \psi\left(L + V_r t' - y, 0\right) - \frac{1}{2}\left(\frac{1}{\nu} - 1\right) \psi\left(y - L + ct', 0\right) + \frac{1}{2}\left(\frac{1}{\nu} + 1\right) \psi\left(y - L - ct', 0\right) \right\} \Bigg]$$

where ν is the ratio between velocities, $\frac{V_r}{c}$, $t' = t - \frac{L}{V_r}$, and,

$$\psi(x,y) = \arctan\left(\frac{\sinh\left(\frac{\pi x}{2h}\right)}{\cosh\left(\frac{\pi y}{2h}\right)} \right).$$

Radial case

- $\zeta_0(x,y) = \zeta_0(r)$

$$\eta(r,t) = \frac{\mathcal{M}(r,t) - \mathcal{M}(r,t-t_R)}{t_R}$$

with

$$\mathcal{M}(r,t) = \int_0^\infty \frac{J_0(kr)k}{\omega(k)\cosh(kh)} \int_0^{\max(V_r t,0)} J_0(k\xi)\xi\zeta_0(\xi)\sin[\omega(k)(t-\xi/V_r)]d\xi dk$$

$\mathcal{M}$ can be efficiently computed with numerical algorithms.

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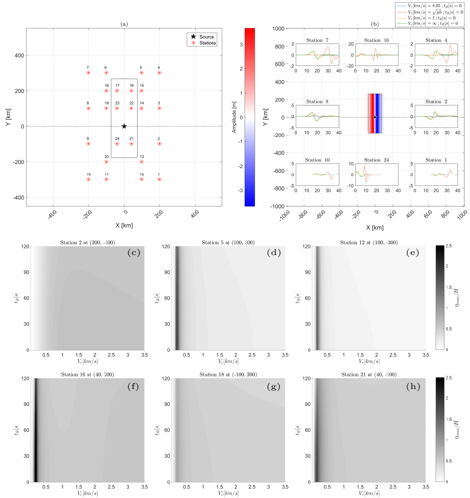
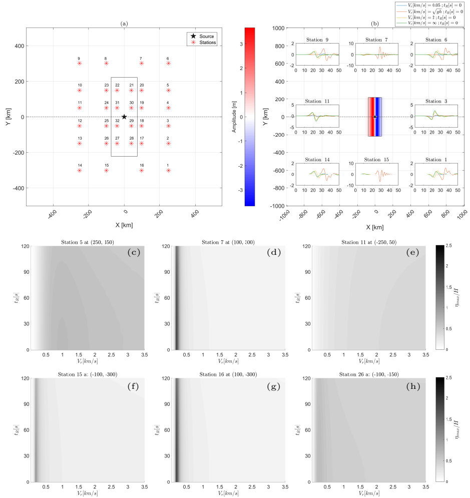
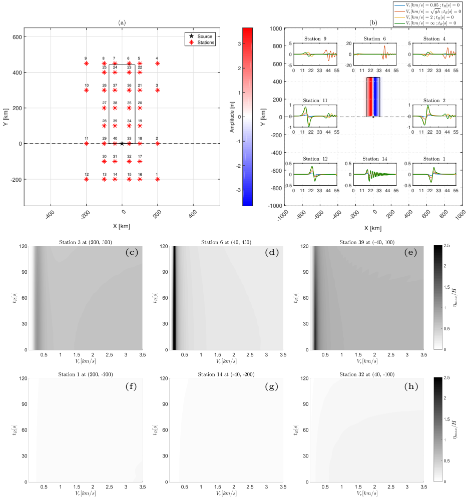
Asymmetric rupture propagation



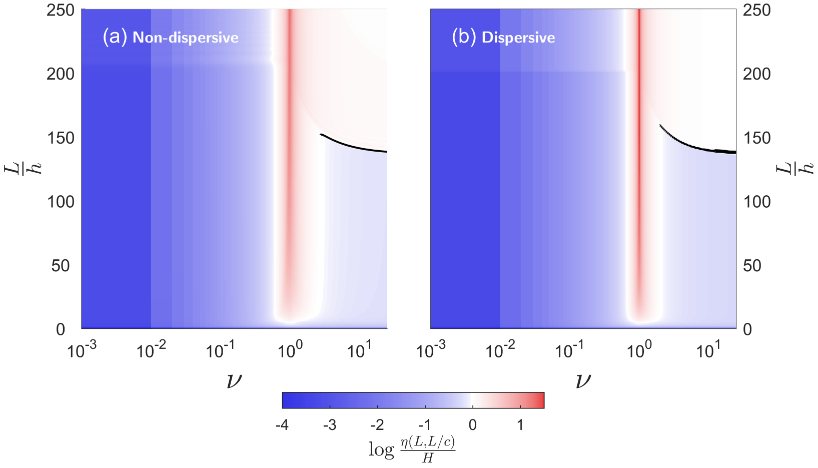
Symmetric rupture propagation



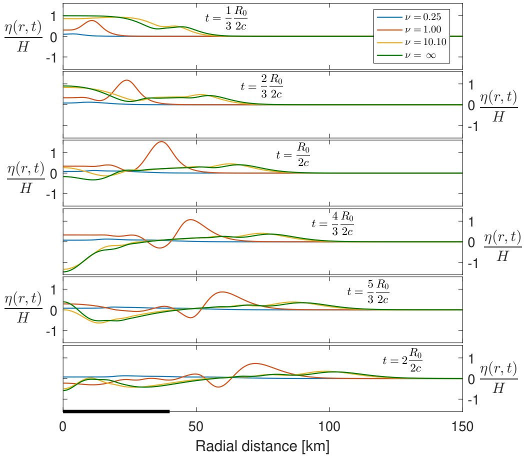
Unilateral rupture propagation



1+1 D Maximum amplification



Radial source



A simple permanent deformation is taken:

$$\zeta_0(r) = H\mathcal{H}(R_0 - r)$$

$$R_0 = \frac{W}{2} = 40\,km$$

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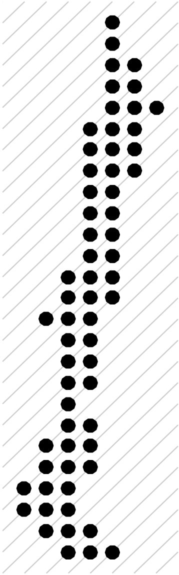
Conclusions

- Tsunami amplification is stronger when ν approaches to 1.
- The kinematics on the tsunamigenic source process can strongly influence the waveforms, especially in the near field, close to the source area.
- Wave amplitude is optimal for unilateral ruptures since there is a longer distance where the constructive interference process is taking place and grows proportionally to the fault length.
- Amplification becomes important for lower values of ν: 2.0 and 2.7 for the dispersive and non-dispersive cases, respectively. For $\nu < 0.1$, there is no amplification because in this region tsunami waves are not excited.



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Thanks for reading!

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