# USING A FIRST-ORDER AUTONOMOUS DYNAMICAL SYSTEM TO EVALUATE RESIDENCE TIME OF THE GREENLAND FRESHWATER ANOMALY IN THE SUBPOLAR GYRE

Dmitry Dukhovskoy





COAPS Florida State University, Tallahassee, USA, ddukhovskoy@fsu.edu



Subpolar Gyre (green shading). The grey contours are isobath drawn every 1000 m. The blue arrows schematically indicate fluxes with mean transports (km<sup>3</sup> yr<sup>-1</sup>) of the Greenland Freshwater Anomaly (GFWA). For Greenland, the mean annual freshwater flux anomalies are shown for 4 regions. The blue shaded area over the southwestern shelf designates approximate release location of Lagrangian particles.

Estimates of the Greenland freshwater fluxes accumulation of the GFWA in the subpolar seas is derived from the numerical experiment of Dukhovskoy et al., 2019.

## JGR Oceans

**RESEARCH ARTICLE** 10.1029/2018JC014686

orum for Arctic Modeling

and Observational Synthes (FAMOS) 2: Beaufort Gyre

Special Section:

#### Role of Greenland Freshwater Anomaly in the Recent Freshening of the Subpolar North Atlantic

#### D. S. Dukhovskoy<sup>1</sup> [0], I. Yashayaev<sup>2</sup> [0], A. Proshutinsky<sup>3</sup> [0], J. L. Bamber<sup>4</sup> [0], I. L. Bashmachnikov<sup>5,6</sup> [0], E. P. Chassignet<sup>1</sup> [0], C. M. Lee<sup>7</sup>, and A. J. Tedstone<sup>4</sup> [0]

<sup>1</sup>Center for Ocean-Atmospheric Prediction Studies, Florida State University, Tallahassee, FL, USA, <sup>2</sup>Bedford Institute of Oceanography, Fisheries and Oceans, Dartmouth, Nova Scotia, Canada, <sup>3</sup>Woods Hole Oceanographic Institution, Woods



### 2 Definitions and methodology

2.1 Greenland Freshwater Anomaly

Annual Greenland freshwater discharge rate can be expressed as a sum of the mean discharge  $(\bar{F}_G)$  and its anomaly  $(F'_G)$  (Figures 1b and 1c)

$$F_G(t) = \bar{F}_G + F'_G \tag{1}$$

The mean Greenland freshwater discharge rate over 1958–1993 is 818.3 km<sup>3</sup> yr<sup>-1</sup>. Average increase of the Greenland freshwater discharge rate during 1993–2016 is  $209 \pm 30$  km<sup>3</sup> yr<sup>-1</sup>. The discharge anomaly is not constant but is slowly accelerating during this time period (Figure 1c) that can be approximated by a linear trend

$$F'_{G}(t) \approx \hat{F}'_{G}(t) = F_{0} + p \cdot t \tag{2}$$

where  $F_0 = 21.8 \text{ km}^3 \cdot \text{yr}^{-1}$  and  $p = 15.9 \text{ km}^3 \cdot \text{yr}^{-2}$ .

The GFWA is defined as time-integrated Greenland freshwater discharge anomaly from time  $t_0$  to t (Figure 1d)

$$V_{\rm GFWA}(t) = \int_{t_0}^{t} F'_{G}(\tilde{t}) d\tilde{t}.$$
(3)

In this study, the GFWA combines all components of the Greenland freshwater flux (Figure 1d). However, the increase in glacier meltwater discharge has dominated the contributions from the solid and tundra runoff discharges since 1994 (~65% since 2000, Figure 1d; 84% since 2009 in Enderlin et al., 2014). Integrated over the time period 1993–2016, the GFWA is  $5007 \pm 390$  km<sup>3</sup>.

(b) Annual total Greenland freshwater discharge (km<sup>3</sup> yr<sup>-1</sup>) derived from the monthly gridded product by Bamber et. (2018). The horizontal solid line is the mean flux over 1958–1993 ( $\overline{F}_G$  = 818.3 km<sup>3</sup> yr<sup>-1</sup>) used as a reference for calculating the GFWA. (c) Annual Greenland freshwater flux anomaly ( $F'_G$ ). The dashed line is a linear regression fitted to the time series. The grey solid curve is the fraction of the surplus Greenland freshwater flux into the SPG (section 2.3). (d) Greenland freshwater anomaly (equation 3) by components. The numbers indicate the fraction of meltwater in the total GFWA. (e) Volume of the GFWA accumulated in the SPG estimated from the tracer numerical experiments (black – HYCOM, grey – ICMMG). (f) Monthly climatology of the GFWA net transport across the Denmark and Davis straits estimated from the HYCOM tracer experiments.



## 2.4 Dynamical System

We are interested in analytical description of the GFWA accumulation in the SPG forced by the increased Greenland discharge in order to estimate a time scale required for the analyzed system to adjust to the perturbed freshwater flux. The following first-order autonomous dynamical system can be used to describe time-evolving changes in the system caused by a change in external conditions (e.g., Skogestad, 2009; Teschl, 2012)

$$\frac{dV(t)}{dt} + kV(t) - F(t) = 0,$$
(4)  
 $V(t_0) = V_0,$ 
(5)

where F(t) is a forcing function describing external conditions (input), V(t) is dependent variable (output), and  $k = \tau^{-1}$  where  $\tau$  is the system time constant (time scale) that we want to estimate for the SPG. In general,  $\tau$  determines how fast the system adjusts to the change in the forcing function. For the steady forcing conditions,  $\tau$  provides a time scale required for the system state to change from  $V_0(t_0)$  to  $V(\infty)$  when the system approaches a new steady state (more discussion is in section 4).

The model (4) with initial condition (5) describes a dynamical system that is at rest at time  $t_0$  and is disturbed by a change in the forcing function F(t). The model is employed to describe the change in the freshwater content of the SPG caused by accumulated GFWA. We consider an idealized case with no other salt fluxes but GFWA, i.e. V(t) is defined as

$$V(t) = \oiint \frac{S_0(\mathbf{x},t) - S(\mathbf{x},t)}{S_0(\mathbf{x},t)} d\Omega,$$
(6)

where  $S_0(\mathbf{x}, t)$  is the initial salinity field not impacted by the GFWA. Thus, V(t) is the volume of the GFWA accumulated in the domain  $\Omega$  (Figure 1e).

Changes in V(t) are driven by F(t) that is the influx of the GFWA into the system driven by the Greenland freshwater discharge anomaly. According to section 2.3,  $F(t) = \alpha F'_G(t)$  and  $\alpha = 0.8$ . The second term (kV) describes export of the GFWA out of the study region. The general solution of (4) with the initial condition (5) is determined by the forcing function F(t). Two cases are considered here. First, the Greenland freshwater discharge anomaly is a constant function  $\Phi = \alpha F'_{G}$  imposed as a step function

$$F(t) = \begin{cases} \Phi, & t \ge t_0 \\ 0, & t < t_0 \end{cases}$$
(7)

Second, F(t) is a linearly increasing function given by equation (2).

#### 3.1 Solution for a constant discharge rate

For a constant forcing function F(t) imposed as a step function at time  $t_0$  (equation 7) the solution that satisfies initial condition (5) is

$$V(t) = V_0 e^{-kt} + \frac{\phi}{k} (1 - e^{-kt})$$
(8)

where  $\Phi = 167 \text{ km}^3 \cdot \text{yr}^{-1}$  (section 2.2). Under a constant F(t) the volume of the GFWA within the study region grows initially and then asymptotes a steady-state (Figure 2a). Parameter kt in the solution (9) is a dimensionless quantity and  $(1 - e^{-kt})$  describes how fast the system approaches the new steady state (V( $\infty$ )). Coefficient k also determines the value of the new steady state, i.e. it defines how much GFWA is accumulated in the study domain. In agreement with expectations, a longer time scale results in a higher volume of the GFWA accumulated in the domain at a given time.

The choice of *k*, which is unknown in our case, determines a solution for the given problem. To determine *k*, the solutions *V*(t) are compared to the estimated  $V_{SPG}(t)$  derived from the tracer model experiments (D2019). Based on the tracer budget, the estimated volume of the GFWA accumulated in the SPG (V<sub>SPG</sub>) by 2016 is 2240 km<sup>3</sup> (Figure 1e) that is roughly 45% of the  $V_{GFWA}(t = 2016)$ . The analytical solution matches the tracer-based estimate for  $k \approx 1/17$  (Figures 2a and 2c) suggesting that the time scale  $\tau$  is about 17 years.

#### 3.2 Solution for a linearly increasing discharge rate

The surplus Greenland freshwater discharge is not constant and has been accelerating at a nearly constant rate during 1993–2016 (Bamber et al., 2012; 2018). Therefore, a more realistic solution is obtained by using a linearly increasing discharge rate. The general solution of (4) with  $F(t) = \hat{F'}_{G}(t)$  given by (equation 2) and with the initial condition (equation 5) is

$$V(t) = \frac{F_0}{k} + \frac{p}{k^2}(kt - 1) + \left(V_0 + \frac{p}{k^2} - \frac{F_0}{k}\right)e^{-kt}.$$
(9)

The solution (equation 9) has two parts, the exponential part (third term) and a linear trend with the offset  $F_0/k$ . The exponential term quickly decays and approaches zero for time exceeding the time scale ( $t > \tau$ ) making the linear trend term being the dominant part (Figure 2c). Hence, the GFWA volume accumulated in the SPG does not reach a steady state but continue to grow driven by the linearly increasing Greenland freshwater discharge. Comparison of the analytical solutions (Figures 2c and 2e) with the estimated volume of the GFWA accumulated in the SPG from the tracer model experiment suggests  $k \approx 1/14$  and the time scale  $\tau$  is about 14 years.



Analysis of the residence time of Lagrangian particles in the SPG. Distribution of particles after 1 year (a) and 24 years (b). The colors designate particles released at 3 depths. The red box denotes the SPG. (c) Mean age distribution for overall time the particles spent in the SPG. (d) Transit time distribution. The inset diagrams in (c) and (d) show the median, the 10<sup>th</sup> and 90th percentiles of the residence time of the particles advected at 3 depths.



#### **5** Discussion

Two qualitatively different responses of the SPG to the changing Greenland freshwater flux are suggested by analytical solutions of the first-order autonomous system representing accumulation of the GFWA in the SPG for constant and linearly increasing discharge rates. For the constant Greenland freshwater discharge anomaly, volume of the accumulated GFWA reaches a steady state ( $V(\infty)$ ) after *t* exceeds time scale  $\tau$ . For the linearly increasing discharge rate, the system never reaches a steady state and GFWA accumulation is unbounded. Considering the linear increasing scenario as a good approximation of the GFWA flux during 1993-2016, we conclude that the GFWA content has been slowly increasing in the SPG during this time period. Recent estimates of the freshwater flux from the Greenland ice sheet and Arctic glaciers by Bamber et al. (2018) indicate a slowdown of the freshwater discharge during 2012-2016. If the hiatus continues long enough (compared to the time scale  $\tau$ ), the system will reach a new steady state and GFWA content in the SPG will stop increasing. After substituting  $V_0=V_{SPG}=2240 \text{ km}3$ ,  $k=1/17 \text{ yr}^{-1}$ ,  $F_0=300 \text{ km}^3 \text{ yr}^{-1}$  (surplus discharge rate in 2012–2016, Figure 1c) into (equation 8), a new steady state value of the GFWA content in the SPG is ~5000 km^3.

Estimated time scales of the SPG response to the increased Greenland freshwater flux are 17 and 14 years. These estimates agree well with decadal time scales of the negative salinity anomalies propagating in the SPG reported in the previous studies (Dickson et al., 1988; Dickson et al., 2002; Belkin, 2004; Yashayaev et al., 2015; Yashayaev & Loder, 2016). In addition, the time scales have reasonable agreement with the mean age and mean transit times estimated from

the Lagrangian particles, except for the particles advected in the upper 50 m. These particles have short residence time in the SPG (< 5 years). The result suggests that accumulation of the GFWA in the SPG occurs in the subsurface layers. Indirectly, this result is supported by observations of the GSA in the SPG that was well-defined only in the subsurface layers (100–400 m) (Malmberg and Kristmannsson, 1993; Dickson et al., 1988; Ellet and Bllindheim, 1992; Ellet, 1994; 1995). The subsurface accumulation of freshwater anomaly could be related to the differences in the large-scale ocean surface vs subsurface circulation in the region. Surface circulation in the SPG is cyclonic and thus strongly divergent precluding accumulation of the surface water masses in the interior regions. The large-scale circulation in the deeper layers is less divergent and even could be convergent compensating the mass loss in the surface layers. Thus, in the long term, GFWA would be carried out of the gyre in the upper 50 m and to a lesser degree in the deeper layers.