

MORPHOLOGY OF LAVA DOMES INFERRED FROM NUMERICAL MODELING

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Mathematical description of lava dome evolution

$$\frac{\partial(\rho\mathbf{u})}{\partial t} + \langle \mathbf{u}, \nabla(\rho\mathbf{u}) \rangle = -\nabla p + \nabla \cdot (\eta(T, \mathbf{u}, \phi)(\nabla\mathbf{u} + \nabla\mathbf{u}^T)) + \rho\mathbf{g}$$

$$\mathbf{u}(0, \mathbf{x}) = \mathbf{u}_0(\mathbf{x})$$

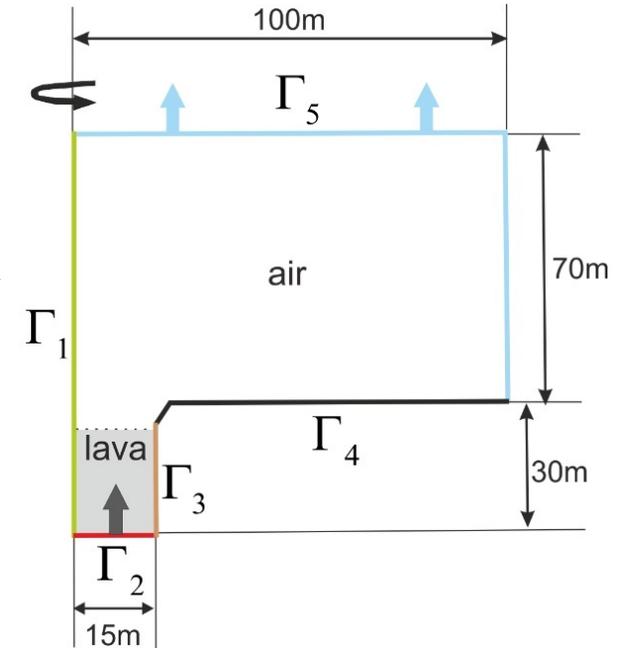
$$\frac{\partial\alpha}{\partial t} + \nabla \cdot (\alpha\mathbf{u}) = 0, \quad \alpha(0, \mathbf{x}) = \alpha_0(\mathbf{x})$$

$$\rho(t, \mathbf{x}) = \rho_A(1 - \alpha(t, \mathbf{x})) + \rho_L\alpha(t, \mathbf{x})$$

$$\eta(t, \mathbf{x}) = \eta_A(1 - \alpha(t, \mathbf{x})) + \eta_L\alpha(t, \mathbf{x})$$

$$\nabla \cdot \mathbf{u} = 0$$

Supplemented
by relevant
boundary
conditions



$$\frac{\partial(c\rho T)}{\partial t} + \nabla \cdot (c\rho T\mathbf{u}) = \nabla \cdot (k\nabla T), \quad T(0, \mathbf{x}) = T_0(\mathbf{x})$$

where \mathbf{x} is the Cartesian coordinates; t is time; $\rho = \rho(t, \mathbf{x})$ is the density; \mathbf{u} is the velocity; T is the temperature; $\eta(T, \mathbf{u}, \phi)$ is the viscosity depending on temperature, velocity and the concentration of crystals ϕ (described by a set of equations); p is the pressure; \mathbf{g} is the acceleration due to gravity; $\alpha(t, \mathbf{x}) \in [0, 1]$ is the volume of lava in the model; ρ_A, η_A are the air's characteristic density and viscosity; ρ_L, η_L are the lava's characteristic density and viscosity; $c(t, \mathbf{x})$ is the specific heat; and $k(t, \mathbf{x})$ is the coefficient of heat conduction.

Lava Rheology

Temperature-dependent viscosity η_1 (Dragoni, 1989)

$$\eta_1(T) = \exp\left(n(T_* - T)\right),$$

where T is the temperature in Kelvin; T_* is the typical lava melting temperature; and $n = 4 \times 10^{-2}$.

Temperature- and water-dependent viscosity η_2 (Giordano and Dingwell, 2003)

$$\log_{10} \frac{\eta_2(T, W)}{\eta_*} = -4.643 + \frac{5312.44 - 427.04 \times W}{T - 499.31 + 28.74 \times \ln W} - \log_{10} \eta_*,$$

where W is the water content in wt%, and η_* is the typical lava viscosity.

Lava Rheology

Lava viscosity depends on temperature and the volume fraction of crystals

(e.g., Griffiths, 2000; Costa et al., 2009)

$$\eta(\phi) = (1 + \phi^\delta) [1 - F(\phi, \xi, \gamma)]^{-B\phi_*}$$

where

$$F(\phi, \xi, \gamma) = (1 - \xi) \cdot \operatorname{erf} \left(\frac{\sqrt{\pi}}{2(1 - \xi)} \phi(1 + \phi^\gamma) \right)$$

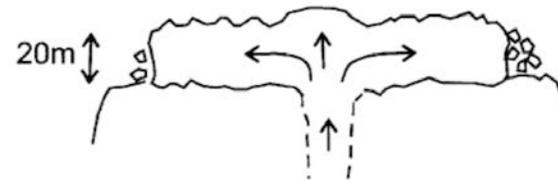
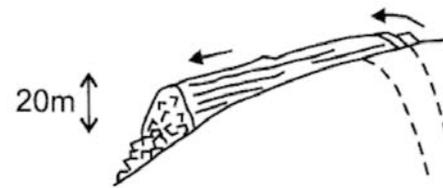
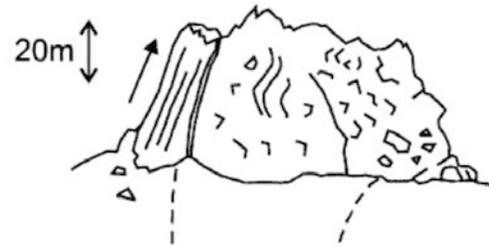
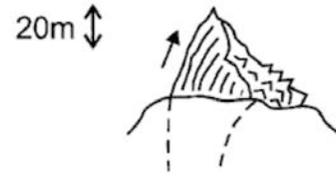
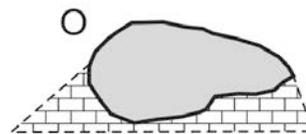
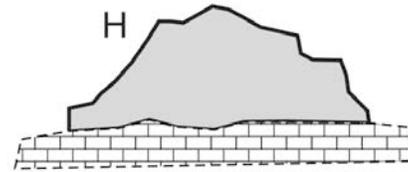
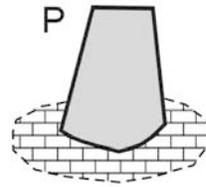
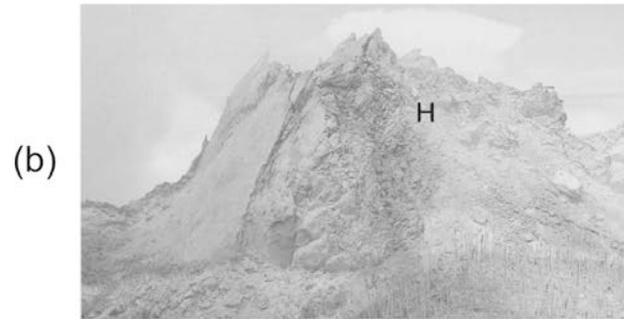
$$\phi = \phi / \phi_* = 0.5 \left[1 - \operatorname{erf} \left(b_1 (\theta - \theta_1) / b_2 \right) \right] / \phi_* \quad \text{(LR1)}$$

$$\frac{\partial \phi}{\partial t} + \nabla \cdot (\alpha \phi \mathbf{u}) = -\alpha \frac{\phi - \phi_{eq}}{\tau} \quad \text{(LR2)}$$

ϕ is the volume fraction of crystals; ϕ_* (= 0.384) is the specific volume fraction of crystals; $\theta = (T - T_s) / (T_* - T_s)$, T_s (= 1053 K) is the temperature of crystallization; B (=2.5) is the Einstein coefficient's theoretical value (1.5 to 5; Jeffrey and Acrivos, 1976);

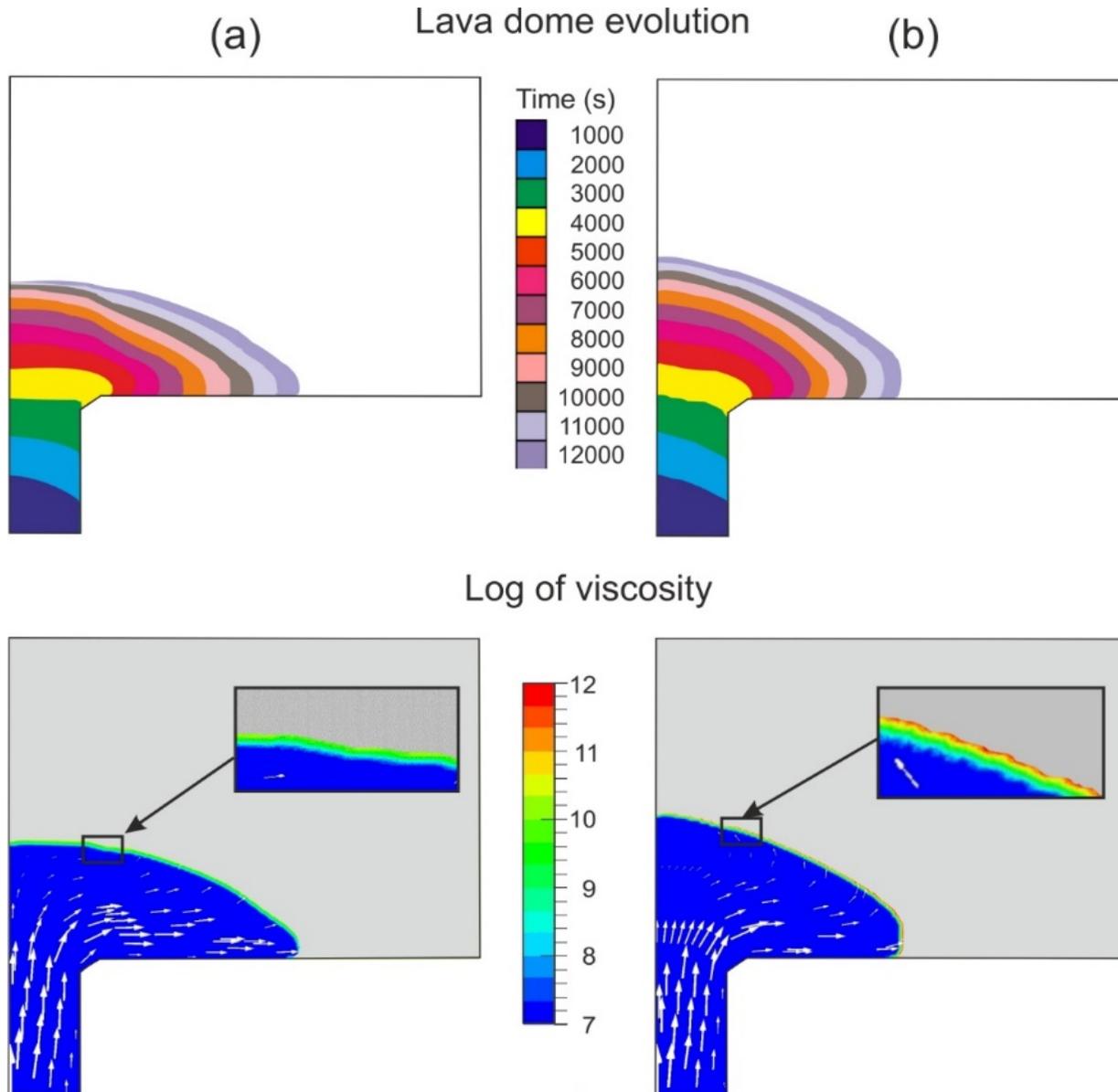
$\delta = 13 - \gamma$, $\gamma = 7.701$, $\theta_1 = 0.5$, $b_2 = 3/2$, $\xi = 2.0 \times 10^{-4}$, $b_1 = \sqrt{30}$
 (Wright and Okamura, 1977; Marsh, 1981; Lejeune and Richet, 1995; Costa et al., 2009);
 ϕ_{eq} is the equilibrium value of the volume fraction of crystals, and τ is the characteristic time of the crystal content growth (CCGT).

Morphological Structures of Lava Domes



Left panel: emplacement of the lava dome at Soufrière Hills Volcano, Montserrat (photos from Watts et al., 2002); **central panel:** the shapes of relevant structures; **right panel:** cartoons of the emplacement features by Watts et al. (2002). (a) Lava spines P (about 40 m high and 35 m broad) and Q; (b) megaspine H (about 40 m high and 100 m broad); (c) whaleback structure S; and (d) lobe O of a topped lava (about 20m thick) resting on rock debris.

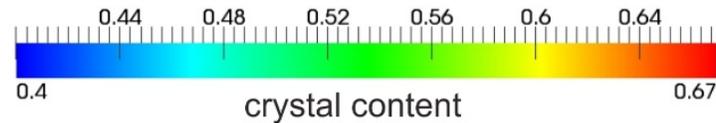
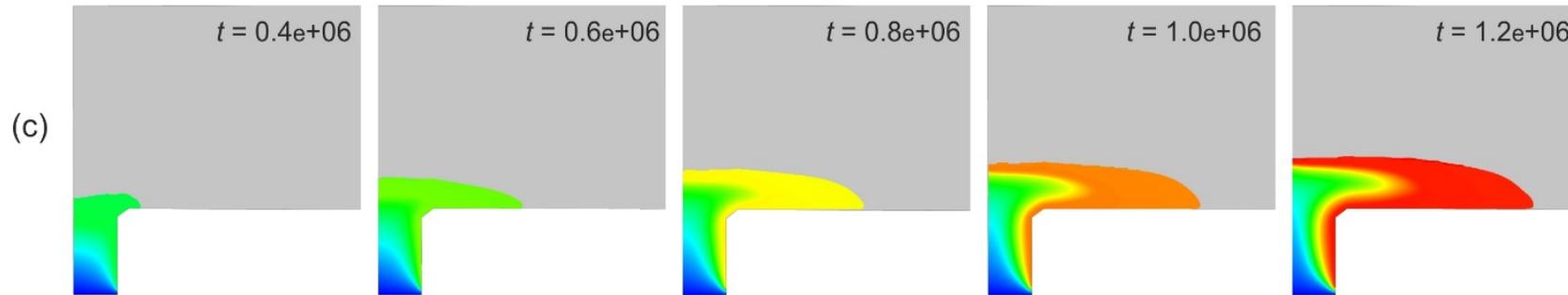
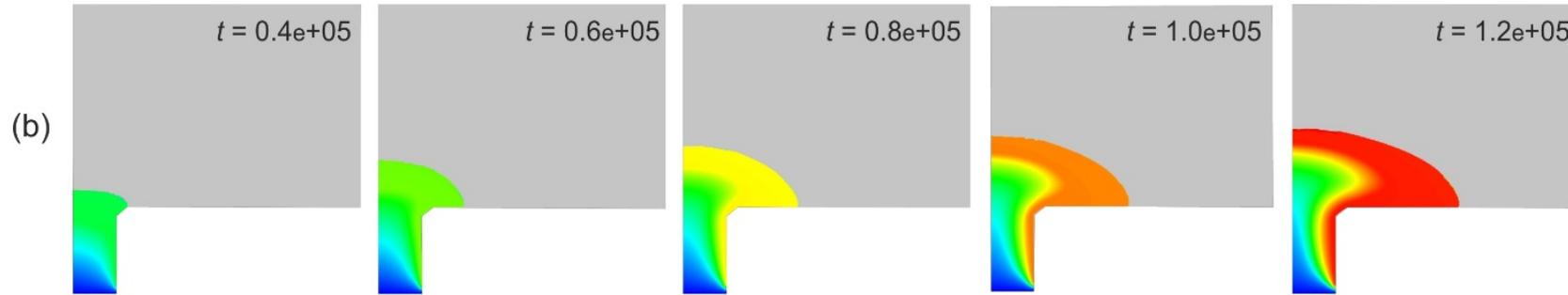
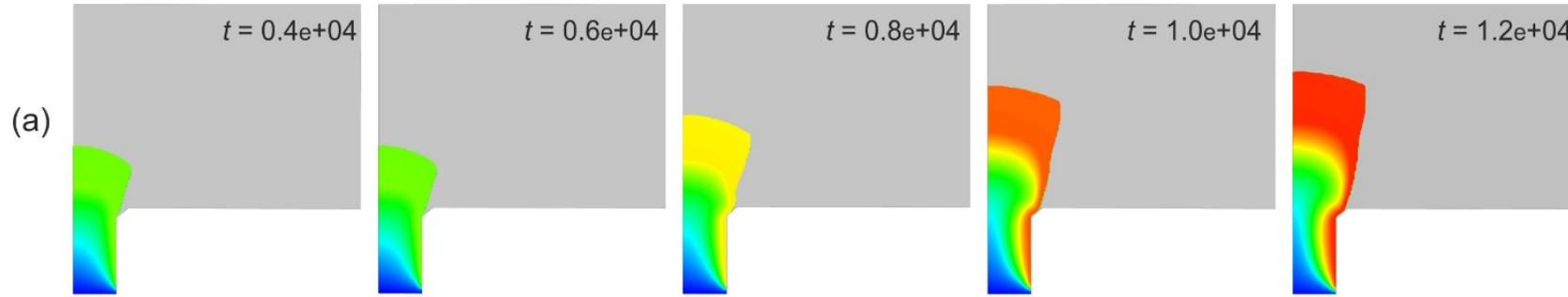
Lava Dome Growth: Thermal Influence (using LR1)



Lava dome growth (upper panels) and viscosity (lower panels) at time 1.2×10^4 s: Lava viscosity is less in the **left panels** by two orders of magnitude compared to the **right panels**.

A part of the thin carapace (high viscous layer) zoomed in the inserts.

Crystal content growth during lava dome evolution (using LR2)



extrusion rate (U) decreases
CCGT (τ) increases

The numerical experiments differ by the magnitude of extrusion rate and the crystal content growth time (CCGT).

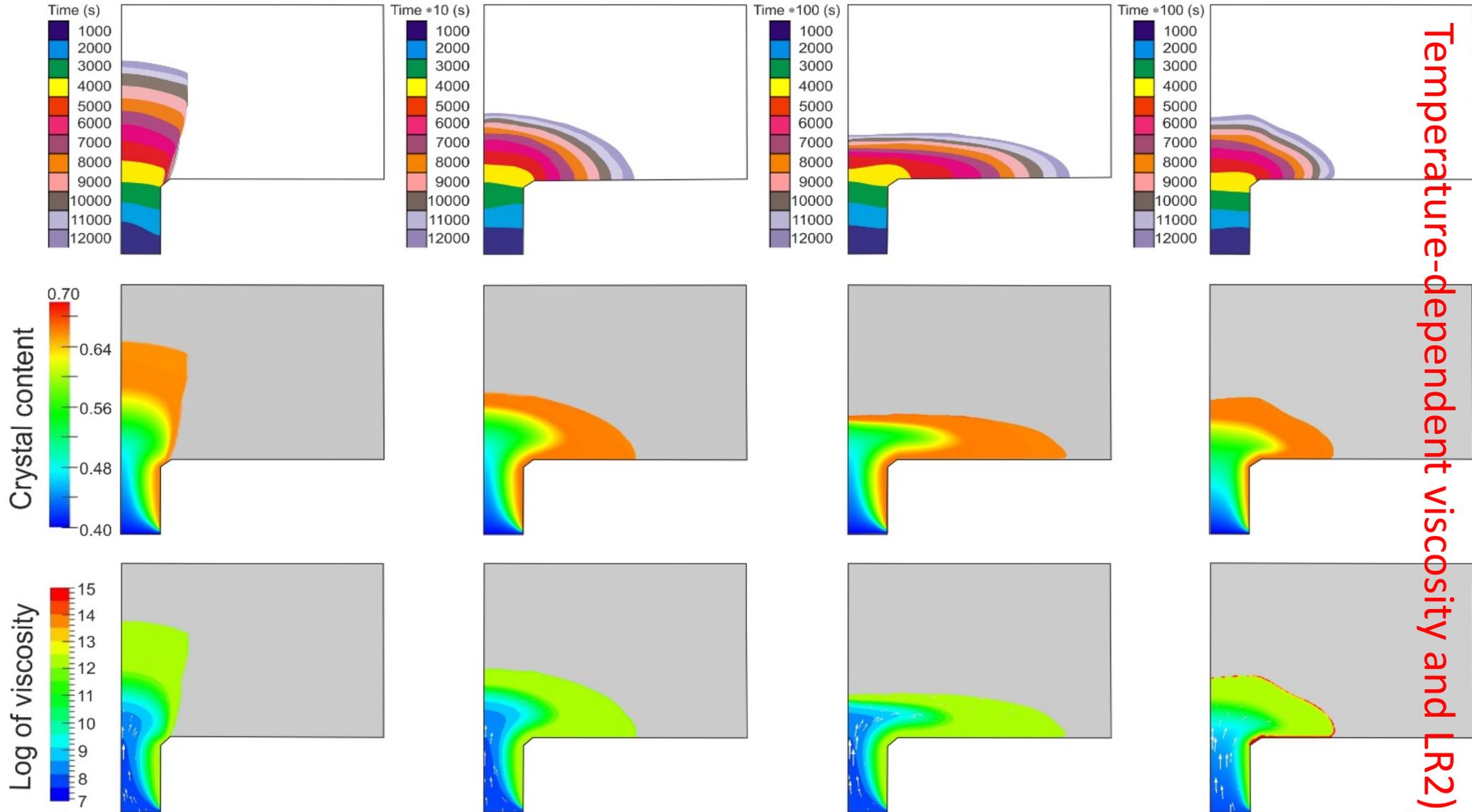
Morphology of evolving lava domes (using LR2)

(a) $U = 0.01$

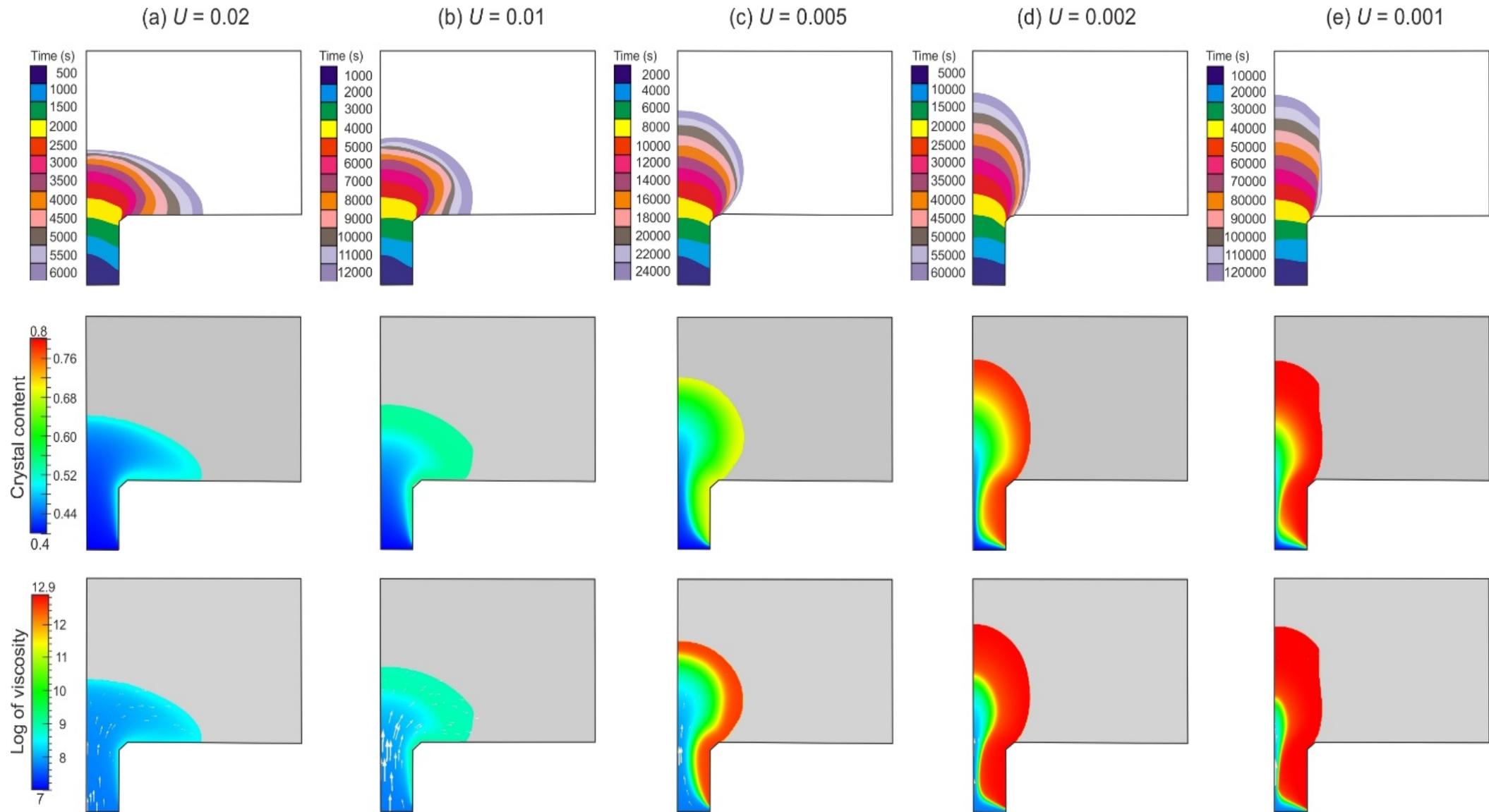
(b) $U = 0.001$

(c) $U = 0.0001$

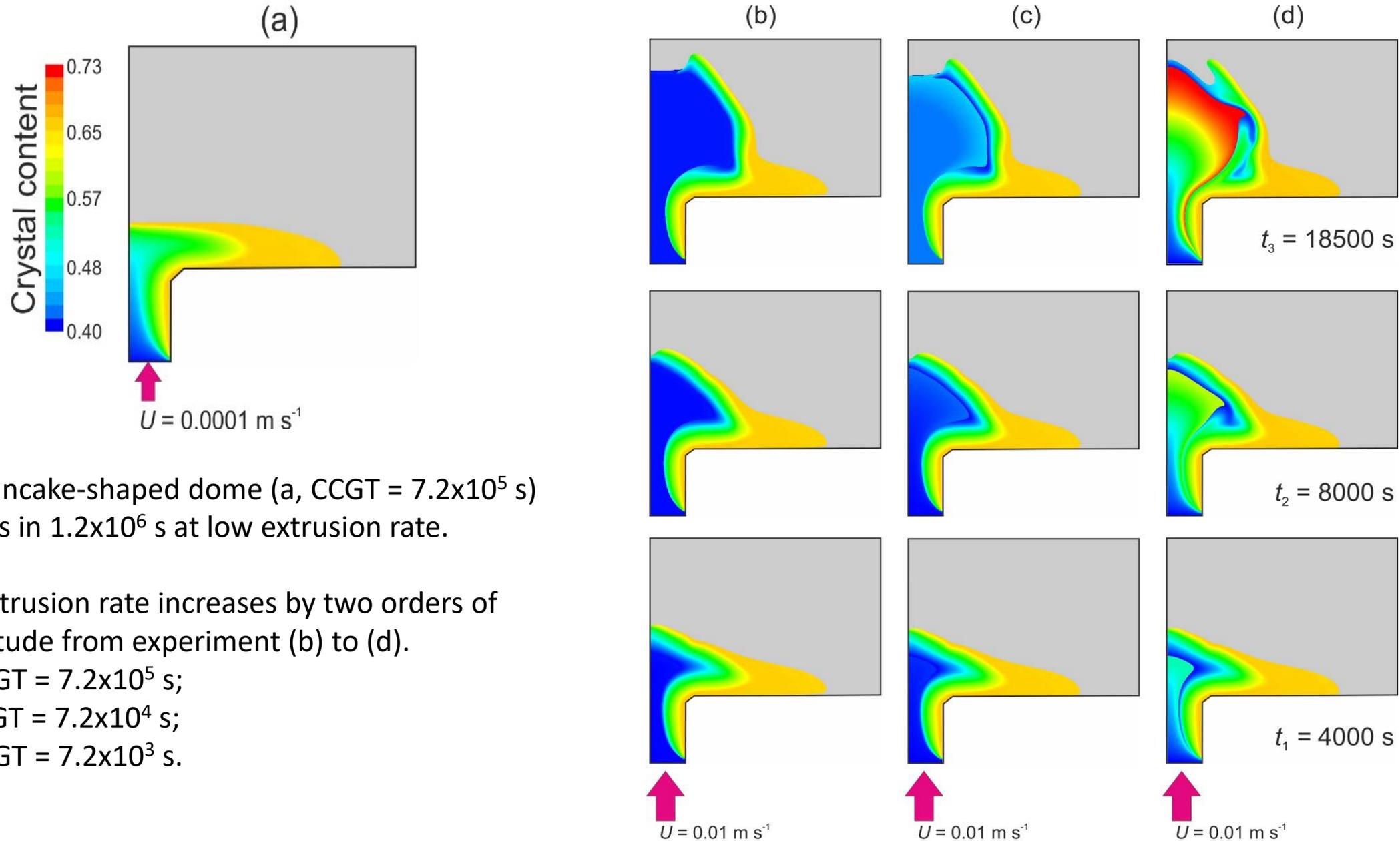
(d) $U = 0.0001$



Morphology of evolving lava domes (using LR2) $\tau = 1.8 \times 10^4$ s



Morphology of the evolving lava dome



The pancake-shaped dome (a, CCGT = $7.2 \times 10^5 \text{ s}$) evolves in $1.2 \times 10^6 \text{ s}$ at low extrusion rate.

The extrusion rate increases by two orders of magnitude from experiment (b) to (d).

(b) CCGT = $7.2 \times 10^5 \text{ s}$;

(c) CCGT = $7.2 \times 10^4 \text{ s}$;

(d) CCGT = $7.2 \times 10^3 \text{ s}$.

CONCLUSION

A rapid magma ascent rate reduces the time for magma crystallization, and hence the magma behaves as a less viscous fluid and extrudes on the surface in a more fluid-like manner. The results of our numerical modelling show that with the decrease of the extrusion rate, the crystalline magma switches from a fluid-type flow to a near solid-type advancement. The formation of obelisk-shaped structures takes place at low rates of extrusion, and the development of lobes and pancake-shaped structures occurs at the higher rates. Meanwhile, spike- and obelisk-shaped domes can be developed at the high extrusion rates as well, if the time of crystal content growth is small allowing the crystal content to grow rapidly, hence, increasing the lava viscosity, which does not promote lava flow but dome upbuilding. Also, we have shown that the volume-of-crystal-dependent rheology rather than the temperature-dependent rheology (only) allows for obelisk- and spine-shaped morphology of lava domes. The numerical models developed in this study can be applied to constrain the evolution of natural lava domes with varying rates of magma extrusion and to assess viscosity of lava base on observations of topography of lava domes and extrusion rates. **[The authors acknowledge a support from the Russian Science Foundation, grant RSF-19-17-00027]**