

ABSTRACT

Seismic waves lose energy during propagation in heterogeneous Earth media. Their decrease of amplitude, defined as seismic attenuation, is central in the description of seismic wave propagation. The attenuation of coherent waves can be described by the total quality factor, Q , and it is defined as the fractional energy lost per cycle, controlling the decay of the energy density spectrum with lapse time. The coda normalization (CN) method is a method to measure the attenuation of P- or S-waves by taking the ratio of the direct wave energy and late coda wave energy in order to remove the source and site effects from P- and S-wave spectra. One of the main assumptions of the CN method is that coda attenuation, i.e. the decay of coda energy with lapse time measured by the coda quality factor Q_c is constant. However, several studies showed that Q_c is not uniform in the crust for the lapse times considered in most attenuation studies. In this work, we propose a method to overcome this assumption, measuring coda attenuation for each source-station path and evaluating the effect of different scattering regimes on the corresponding imaging. The data consists of passive waveforms from the fault network in the Pollino Area (Southern Italy) and Mount St. Helens volcano (USA).

1. ATTENUATION IMAGING

- Seismic attenuation describes the energy loss of seismic waves as they propagate through the Earth. It is described by the total quality factor, Q , which controls the energy decay of seismic waves with lapse time.
- In the crust, Q is dependent on frequency as $Q^{-1} = Q_0^{-1} \cdot f^n$
- There are more than one methods to estimate the attenuation factor of seismic waves. The coda normalization method (CN), which was developed by Aki & Chouet (1975) is what we use in this study.
- The CN method takes the ratio of the direct wave energy (amplitude) and late coda-wave energy in order to remove the source and site effects from P- and S-wave spectra. Coda waves (CW): wave-trains following direct-wave phases.

$$\frac{1}{\pi f} \ln \left[\frac{A_S(f, t_c)}{A_C(f, t_c)} \right] = -Q_S^{-1} v_S^{-1} r - \gamma \left[\frac{\ln(r)}{\pi f} \right] + \frac{\ln P(f)}{\pi f} \quad [1]$$

where $P(f) = t^{-n} \exp(-2\pi f Q_C^{-1} t_c)$, A_S and A_C are the S-wave amplitude and coda wave amplitude respectively, f is the frequency, t_c is the coda wave lapse time, v_S is the S-wave speed, r is the source-receiver distance, γ is the geometrical spreading factor, n is the envelope spectral decay and Q_C is the coda quality factor.

- Assumptions of the coda normalization method:
 - The effect of the source radiation pattern is negligible if the lengths and azimuths of rays span an extensive range
 - Coda attenuation is constant
- While the first assumption is generally valid under certain conditions (De Siena et al., 2010), several studies have shown that Q_c is heterogeneous in the crust and thus the second assumption is not valid.
- In this study, we propose a method to overcome the second assumption by measuring coda attenuation for each source-receiver path (De Siena et al., 2016, Napolitano et al., 2019 among others).

2. METHODOLOGY

- Equation [1] can be used as a forward model in tomography to solve for the total quality factor, Q , variations, either for P- or S-waves.
- The standard technique (developed by Del Pezzo et al., 2006), hereafter called CN1, takes $K_C(f) = \ln(P)/\pi f$ as a constant. Equation [1] is solved by linear regression and values for an average quality factor $Q_{P,S}$, geometrical spreading γ and K_C are obtained.
- The area under study is divided into a grid of M blocks. The forward model is derived from Equation [1] using a data vector $d_{P,S}^k$ (which is obtained by equalizing it to Equation [1] and taking everything on the RHS) equal to

$$d_{P,S}^k = -\sum_{B=1}^M l_{P,S}^{kB} s_{P,S}^B [\delta(Q_{P,S}^B)^{-1}] = \sum_{B=1}^M G_{P,S}^{kB} [\delta(Q_{P,S}^B)^{-1}] \quad [2]$$

where k refers to the source-receiver path, B indicates the B^{th} of the M blocks that the k^{th} ray crosses, s_b is the slowness of segment of length l_{kB} crossing the B^{th} block, $G_{P,S}^{kB}$ is the inversion matrix and $\delta(Q_{P,S}^B)^{-1}$ are the variations from the average value of $Q_{P,S}^{-1}$ in each block. For the derivation of this formula please refer to De Siena et al. (2014a).

- Equation [2] is solved using a zero-order Tikhonov regularization. The total quality factor in each block is obtained as Q_B^{-1}
- $(Q_{P,S}^B)^{-1} = (Q_{P,S})^{-1} + \delta(Q_{P,S}^B)^{-1} \quad [3]$
- where $(Q_{P,S})^{-1}$ is the average inverse quality factor obtained from Eq. [1]
- The updated CN method we propose (CN2) has the same data vector $d_{P,S}^k$ but in this case K_C is not a constant. It is calculated before for each source-receiver pair.

3. DATASETS

- The data consists of waveforms from the Pollino area (Italy) and Mount St. Helens volcano (USA).
- The datasets are different in terms of ray coverage, quality of the waveforms and accuracy of their pre-processing (phase picking and event relocation).

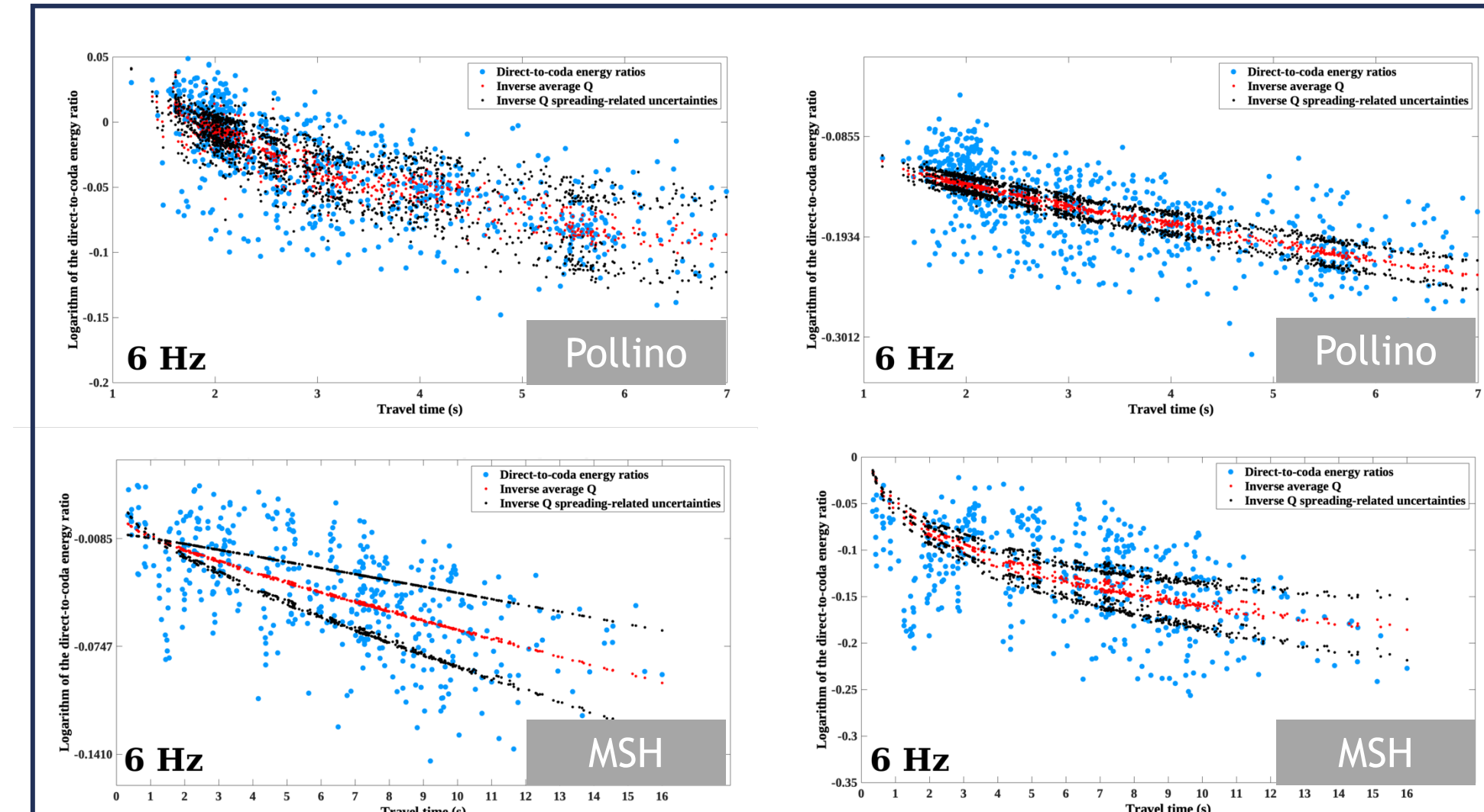


Fig. 1: Average inverse Q for Pollino (top panels) and MSH (lower panels) using the standard coda normalization method (left) and the updated coda normalization method (right). The cyan dots represent the LHS of Eq. [1], while the red dots represent the estimated model (RHS of Eq. [1])

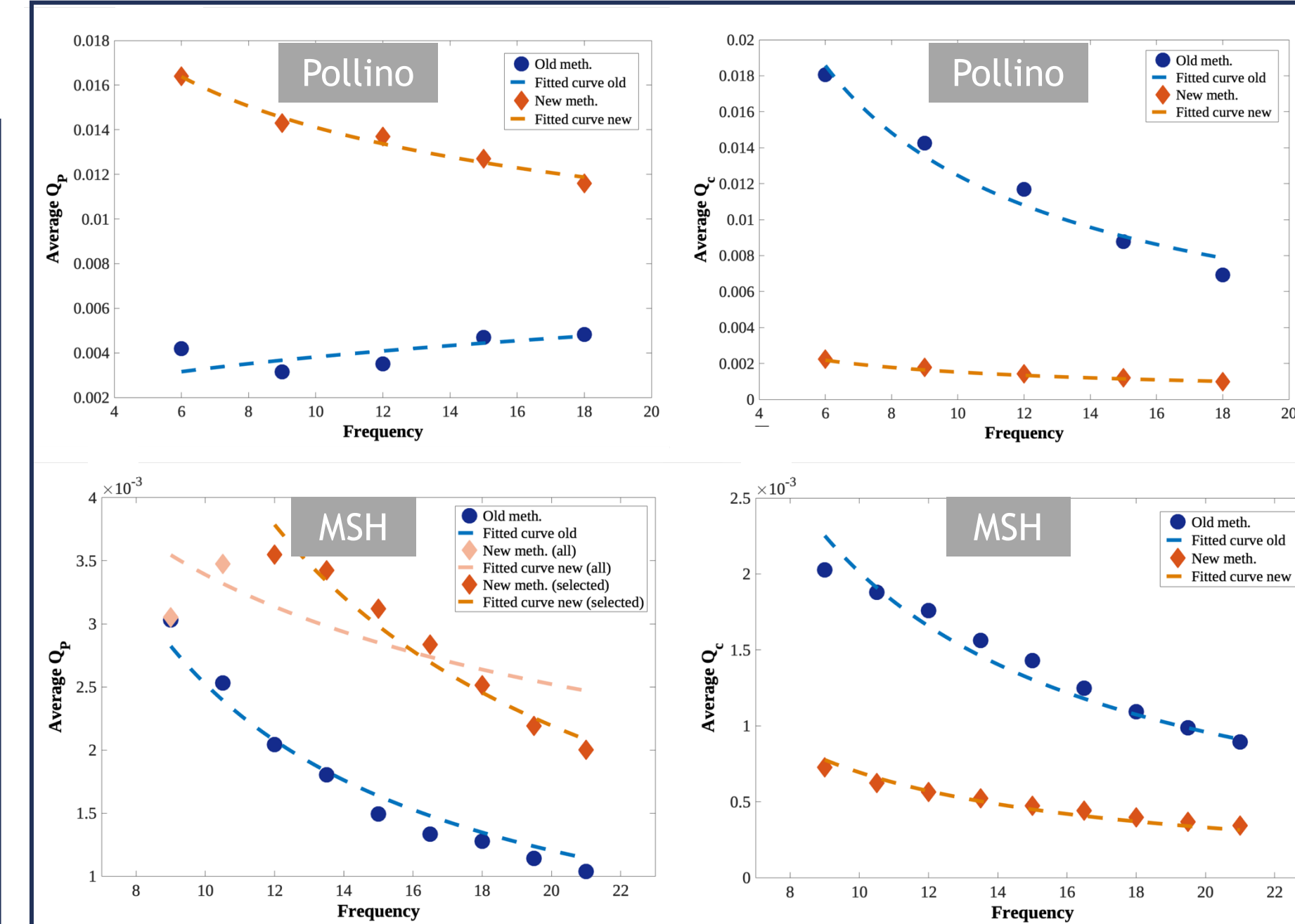


Fig. 2: Frequency dependence of Q^{-1} (left panels) and Q_c^{-1} (right panels) obtained for the two datasets, as indicated. Blue dots represent the results obtained using CN1 while orange diamonds the results obtained using CN2. For MSH, the transparent orange diamonds are the ones obtained using CN2 between 9 and 21 Hz, the darker orange diamonds are the results obtained using CN2 between 12 and 21 Hz. We fitted these two sets of points using the equation $Q^{-1} = a \cdot f^b$, from which we estimated a and b .

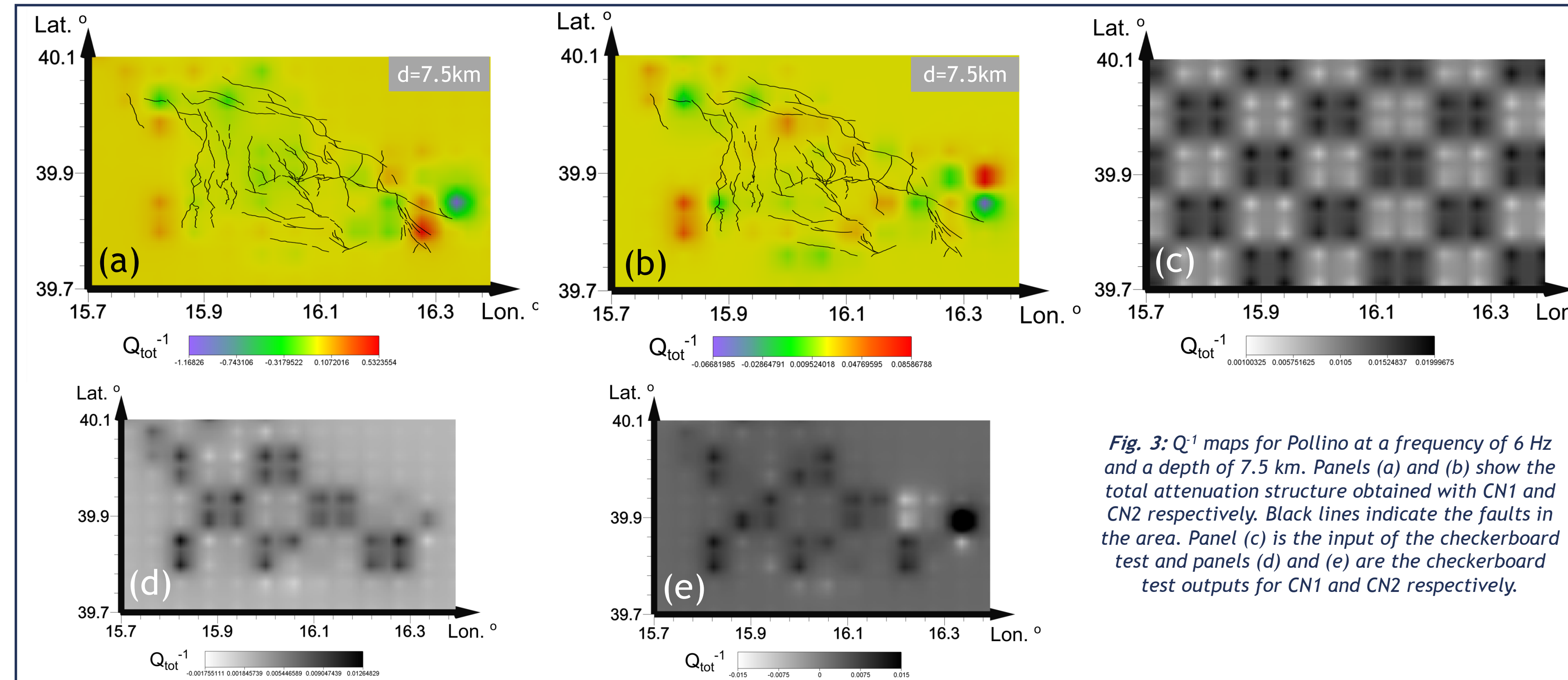


Fig. 3: Q^{-1} maps for Pollino at a frequency of 6 Hz and a depth of 7.5 km. Panels (a) and (b) show the total attenuation structure obtained with CN1 and CN2 respectively. Black lines indicate the faults in the area. Panel (c) is the input of the checkerboard test and panels (d) and (e) are the checkerboard test outputs for CN1 and CN2 respectively.

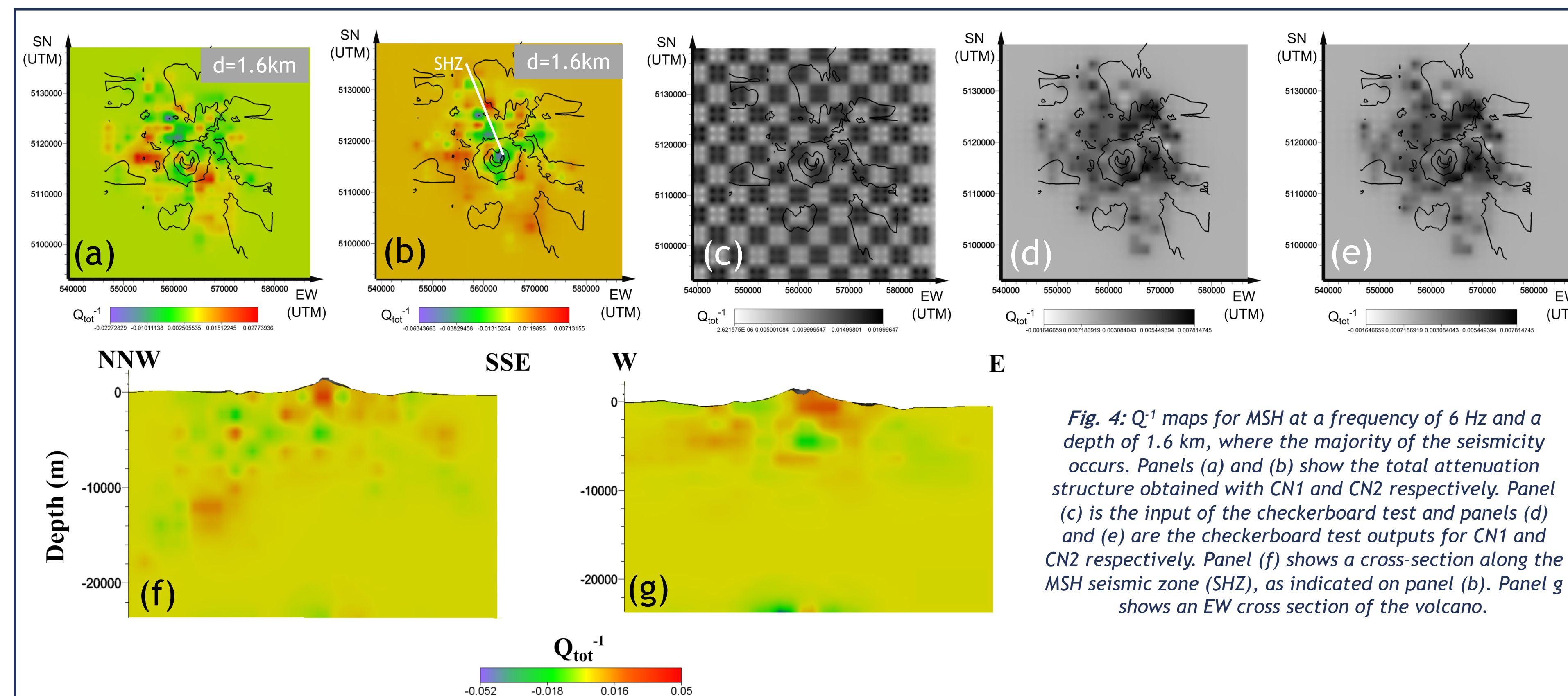


Fig. 4: Q^{-1} maps for MSH at a frequency of 6 Hz and a depth of 1.6 km, where the majority of the seismicity occurs. Panels (a) and (b) show the total attenuation structure obtained with CN1 and CN2 respectively. Panel (c) is the input of the checkerboard test and panels (d) and (e) are the checkerboard test outputs for CN1 and CN2 respectively. Panel (f) shows a cross-section along the MSH seismic zone (SHZ), as indicated on panel (b). Panel (g) shows an EW cross section of the volcano.

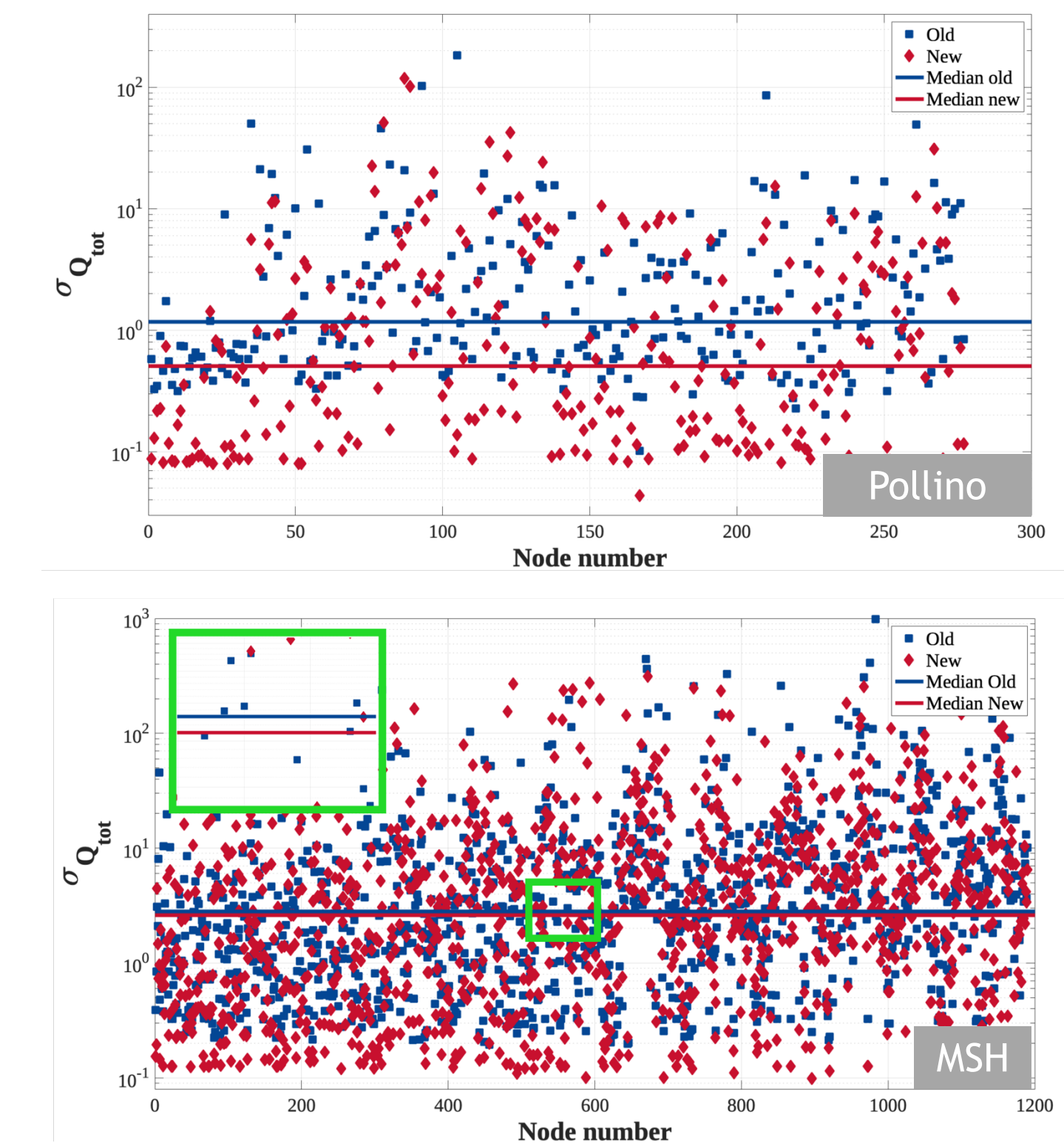


Fig. 5: Relative error $\sigma_{Q_{tot}}$ associated to Q^{-1} , the inverse total quality factor in each block B of the grid used for mapping. Upper panel shows the results for the Pollino, bottom panel for MSH. Red squared dots represent the relative error obtained using the CN1, the red diamonds the one obtained using the CN2. The number of points depends on the number of nodes of the grid. The straight lines represent the median of each set of values. The median of red points, associated to CN2, is always lower than the blue line. In the case of Pollino the separation between the two lines is wider than in the case of MSH, for which an enlargement is needed (green box) to appreciate the difference.

4. RESULTS

- The main advantage of the new coda normalization methodology (CN2) is the reduction of the estimated model parameters in Eq. [1] from three to two, since the coda quality factor is calculated individually for each source-receiver pair.
- A more reliable representation of Q^{-1} allows for a more realistic mapping of the variations of the inverse coda quality factor Q_c^{-1} .
- In Fig. 1 it is evident that using CN2, the model (red dots, RHS of Eq. [1]) fits better the data (cyan dots, LHS of Eq. [1]). The model data look less spread as a function of travel time for Pollino while for MSH the model fits better the trend of the data.
- The dependence of Q^{-1} on frequency is shown in Fig. 2. The resulting relationship for Pollino using CN1 is $Q^{-1} = (0.0222 \pm 0.0020) f^{(0.24 \pm 0.35)}$ and $Q^{-1} = (0.0277 \pm 0.0026) f^{(0.29 \pm 0.05)}$ using CN2. The resulting relationship for MSH using CN1 is $Q^{-1} = (0.052 \pm 0.001) f^{(1.30 \pm 0.08)}$ and $Q^{-1} = (0.022 \pm 0.0024) f^{(0.74 \pm 0.40)}$ using CN2 between the frequencies 12 and 21 Hz.
- CN2 gives an average Q^{-1} value which is higher than the one obtained using CN1 for both datasets. Comparing the average Q^{-1} values with their associated relative error obtained from CN1 and CN2, it is evident that the relative error of CN2 is lower. This is also clear in Fig. 5 where the relative error $\sigma_{Q_{tot}}$ associated to Q^{-1} , the inverse total quality factor in each block B of the grid used for mapping is plotted.
- The Q^{-1} structure of Pollino using CN1 and CN2 is shown in Fig. 3. The Q^{-1} anomalies differ between panels (a) and (b), where we notice the central part of the fault network changes from low attenuation to higher attenuation compared to the surrounding area. It is also evident that the total variation of the Q^{-1} values is lower for the CN2 case, which is more physically meaningful. The checkerboard test outputs indicate that the attenuation structure of almost the whole fault network area has been recovered reliably. (The maps are subject to interpretation.)
- The Q^{-1} structure of MSH using CN1 and CN2 is shown in Fig. 4. The Q^{-1} anomalies between panels (a) and (b) differ, however the cone of the volcano is mostly average attenuation with some high attenuation parts. The SHZ line is similar in the two images. The total variation of the Q^{-1} values is similar for the two methodologies. The checkerboard test outputs indicate that the attenuation structure is not very reliably recovered but this is likely a problem of the seismicity distribution since it is mostly located in the cone of the volcano and thus the ray coverage in the area is limited. (The maps are subject to interpretation.)

REFERENCES

- Aki, Keiiti, and Bernard Chouet. "Origin of coda waves: source, attenuation, and scattering effects." *Journal of geophysical research* 80.23 (1975): 3322-3342.
- De Siena, L., E. Del Pezzo, and F. Bianco. "Seismic attenuation imaging of Campi Flegrei: Evidence of gas reservoirs, hydrothermal basins, and feeding systems." *Journal of Geophysical Research: Solid Earth* 115.B9 (2010).
- De Siena, L., C. Thomas, and R. Aster. "Multi-scale reasonable attenuation tomography analysis (MuRAT): An imaging algorithm designed for volcanic regions." *Journal of volcanology and geothermal research* 277 (2014): 22-35.
- De Siena, Luca, et al. "Seismic scattering and absorption mapping of debris flows, feeding paths, and tectonic units at Mount St. Helens volcano." *Earth and Planetary Science Letters* 442 (2016): 21-31.
- Del Pezzo, Edoardo, et al. "Small scale shallow attenuation structure at Mt. Vesuvius, Italy." *Physics of the Earth and Planetary Interiors* 157.3-4 (2006): 257-268.
- Napolitano, Ferdinando, et al. "Scattering and absorption imaging of a highly fractured fluid-filled seismogenic volume in a region of slow deformation." *Geoscience Frontiers* (2019).

Acknowledgements:

I would like to thank the Natural Environment Research Council (NERC) of the UK as well as the Centre for Doctoral Training in Oil and Gas for funding the project part of which is this study.