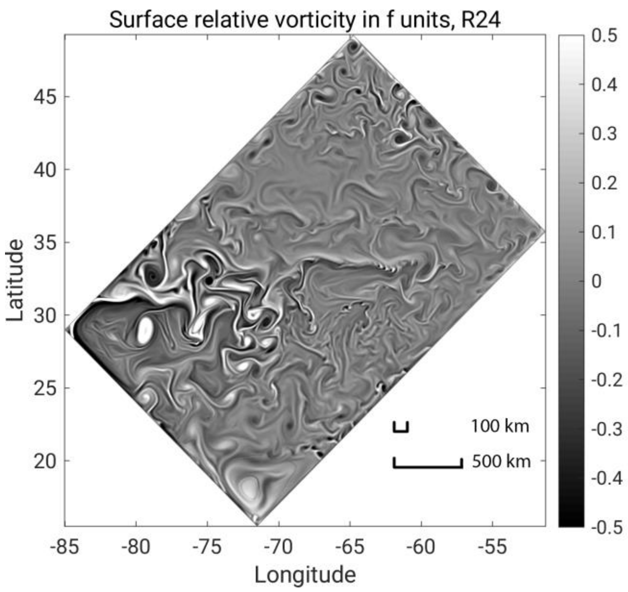

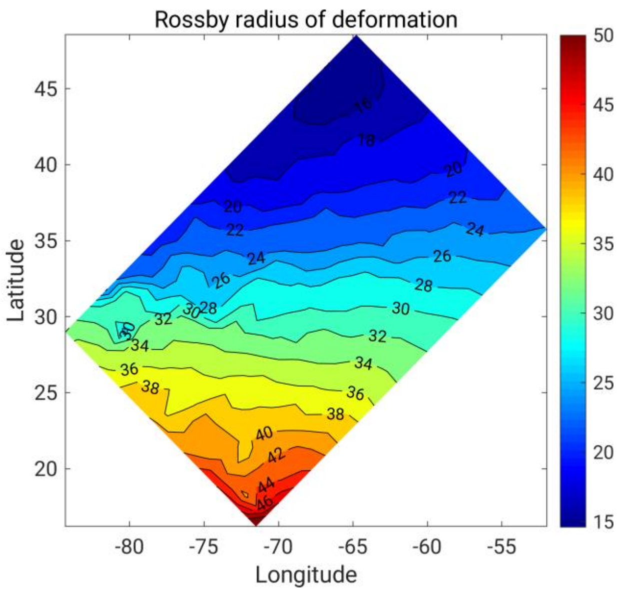

A new data-driven subgrid 2d turbulence parameterization and comparison with conventional kinetic energy backscatter (KEB) parameterizations in NEMO ocean model

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Problem statement

- Ocean models at "eddy-permitting" resolution begin to resolve mesoscale eddies partly
- They can be viewed as Large eddy simulations (LES) of quasi-2d turbulence
- Modern Kinetic energy backscatter (KEB) parameterizations rely on phenomenological property of 2d turbulence: they prohibit downscale energy cascade
- Nevertheless, more accurate subgrid scale (SGS) models can be constructed via regression methods with the use of artificial neural networks (ANN)
- However, the key challenge for ANN SGS model is to keep numerical stability of the simulation, which is usually supported by projecting SGS model onto dissipative direction
- But for mesoscale eddies, we know that energy exchange with subgrid scales is two-directional and roughly equal
- The **presentation essence** is to extract subgrid forces responsible for backscatter only. It allows to formulate a regression problem for KEB parameterization, while usual eddy viscosity is predefined and maintains numerical stability
- To extract KEB SGS tendency, we apply nudging of coarse resolution model to reference one. Nudging will restore energy balance in the system for **any** eddy viscosity, while in conventional approach eddy viscosity should be first estimated from subgrid forces

NEMO Double Gyre

$$\frac{dT}{dt} = F_T, \frac{dS}{dt} = F_S$$

$$\rho = \rho_0(1 - a(T - T_0) + b(S - S_0))$$

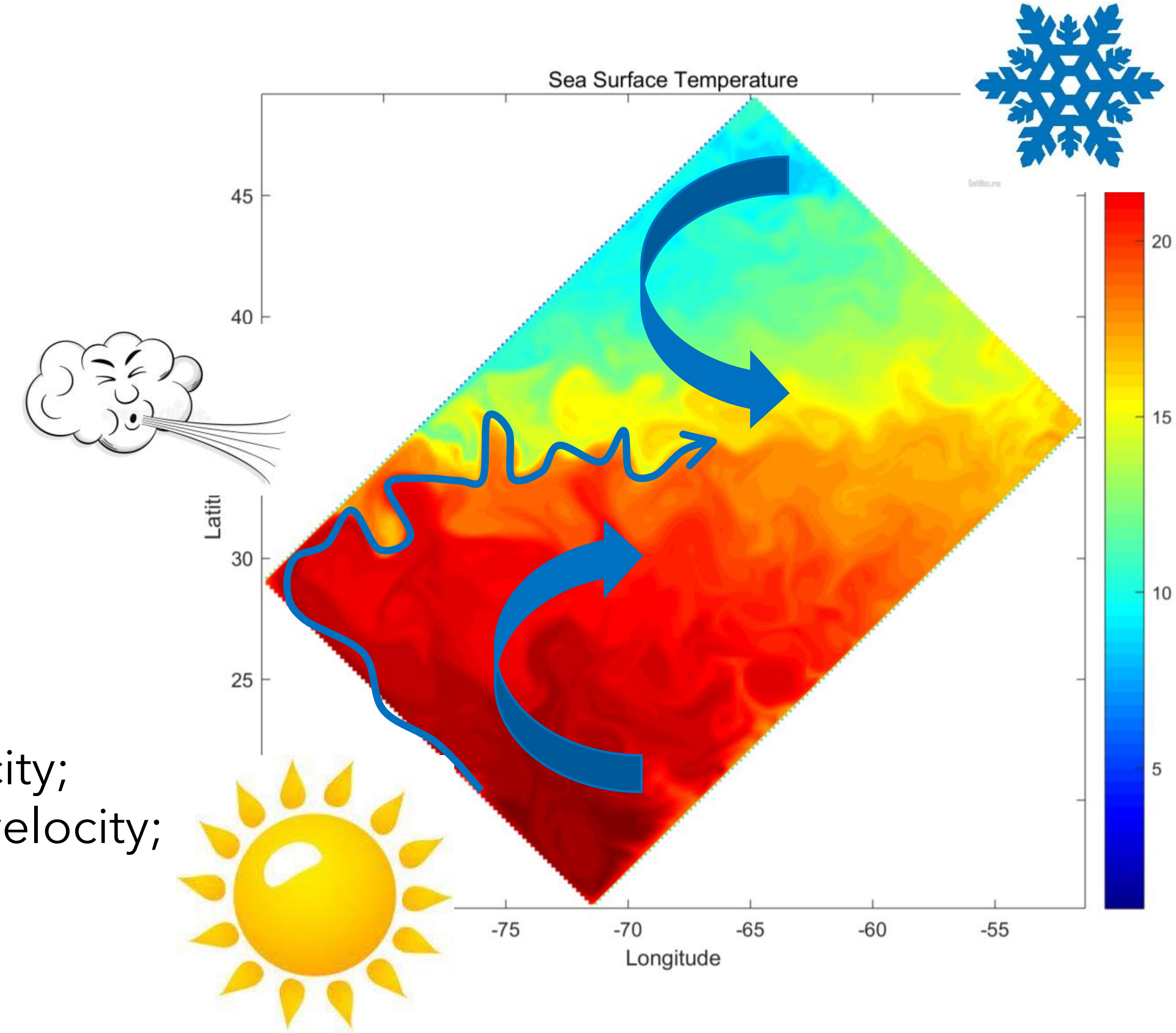
$$\frac{\partial p}{\partial z} = -\rho g$$

$$\frac{\partial \boldsymbol{U}_h}{\partial t} + \mathbf{adv}_h + \mathbf{cor}_h = -\frac{1}{\rho_0}\nabla_h p + F_U$$

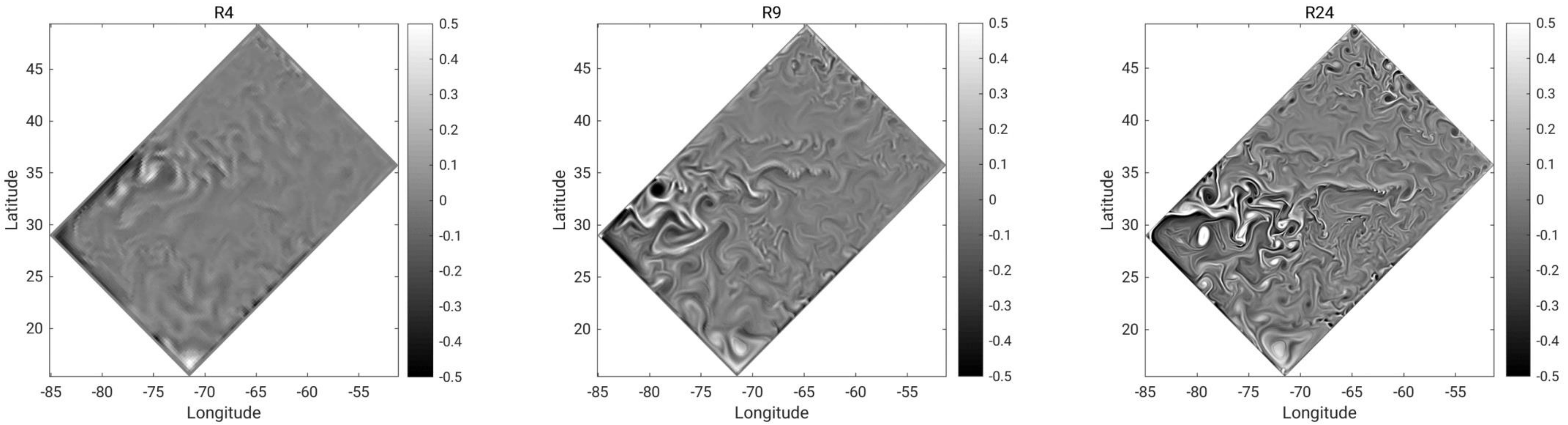
$$\nabla \cdot \boldsymbol{U} = 0$$

$$\frac{\partial \eta}{\partial t} = -H\nabla_h \cdot (\overline{\boldsymbol{U}_h})$$

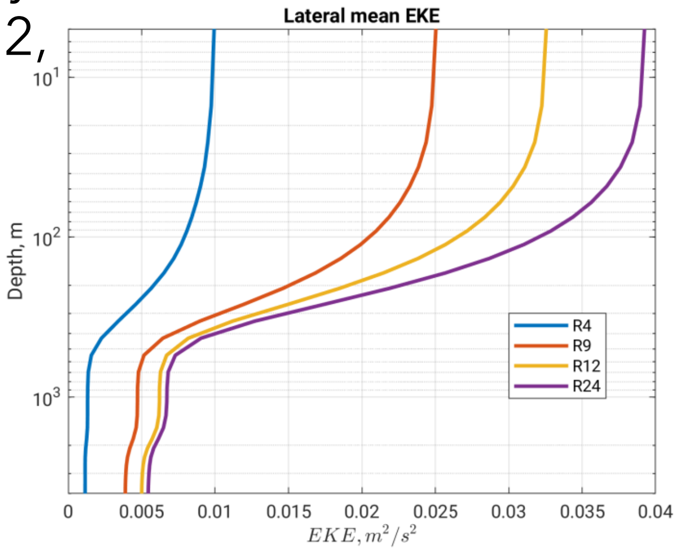
$T, S, \boldsymbol{U}, \boldsymbol{U}_h, \overline{\boldsymbol{U}_h}, p, \eta, \rho, H$ – temperature; salinity; velocity; horizontal velocity; vertically-averaged horizontal velocity; pressure; surface elevation; density; depth.

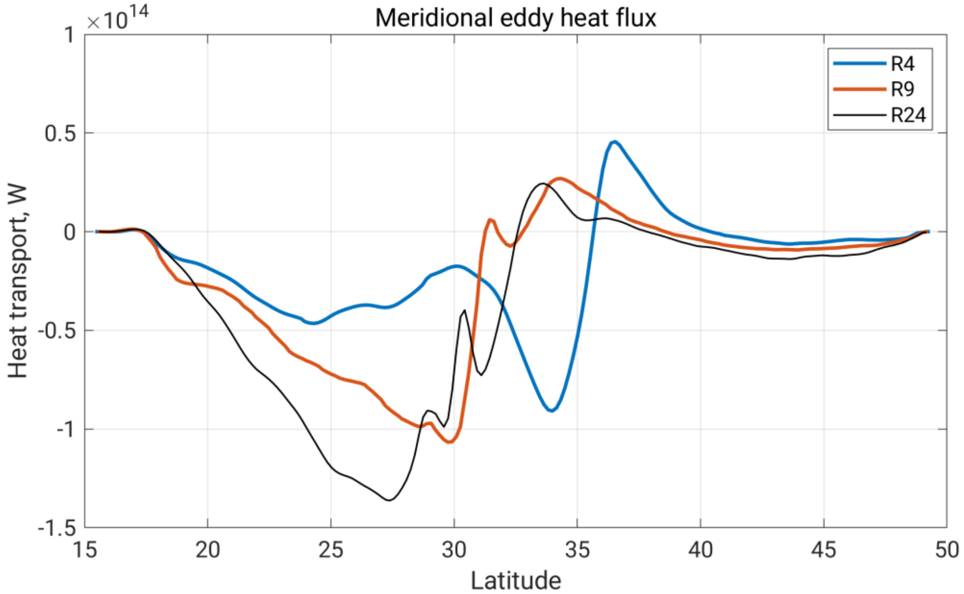


Convergence with resolution



Turbulent eddy field (vorticity) and meridional eddy heat flux strongly depend on resolution (R4, R9, R12, R24 – $1/4^o, 1/9^o, 1/12^o, 1/24^o$)





Conventional KEB parameterizations[1,2]

- Dissipation by biharmonic damping is compensated by KEB parameterization.
- Negative viscosity KEB:

$\frac{\partial \boldsymbol{U}_h}{\partial t} = \cdots + \nu_4 \Delta_h^2 \boldsymbol{U}_h + \nabla_h(\nu_2 \nabla_h \boldsymbol{U}_h)$ $\nu_2 = -\Delta x \cdot c_{back}\sqrt{\max(e,0)}$ $\frac{de}{dt} = \dot{E}_{diss} + \dot{E}_{back} + \nu_e \Delta e$	$\nu_4 = const < 0, \nu_2 \leq 0$ $\dot{E}_{diss} = \nu_4 \nabla_h \boldsymbol{U}_h \cdot \nabla_h(\Delta_h \boldsymbol{U}_h)$ $\dot{E}_{back} = \nu_2 \nabla_h \boldsymbol{U}_h \cdot \nabla_h \boldsymbol{U}_h$

(1) https://www.degruyter.com/view/journals/rnam/35/2/article-p69.xml
(2) https://iopscience.iop.org/article/10.1088/1755-1315/386/1/012025/meta

Conventional KEB parameterizations[1,2]

- Dissipation by biharmonic damping is compensated by KEB parameterization.
- Stochastic KEB:

- $\psi(x,y,z) = \phi(x,y) \cdot \sqrt{\max(\dot{E}_{diss}, 0)}$
- $\phi(x,y)$ – white noise in space and time, $N(0,1)$

$$\frac{\partial U_h}{\partial t} = \cdots + \nu_4 \Delta_h^2 \boldsymbol{U}_h \ \boxed{+\alpha \nabla^{\perp} \hat{\psi}}$$

Where $\widehat{(\cdot)}$ - 6 applications of spatial filter nullifying chess-noise

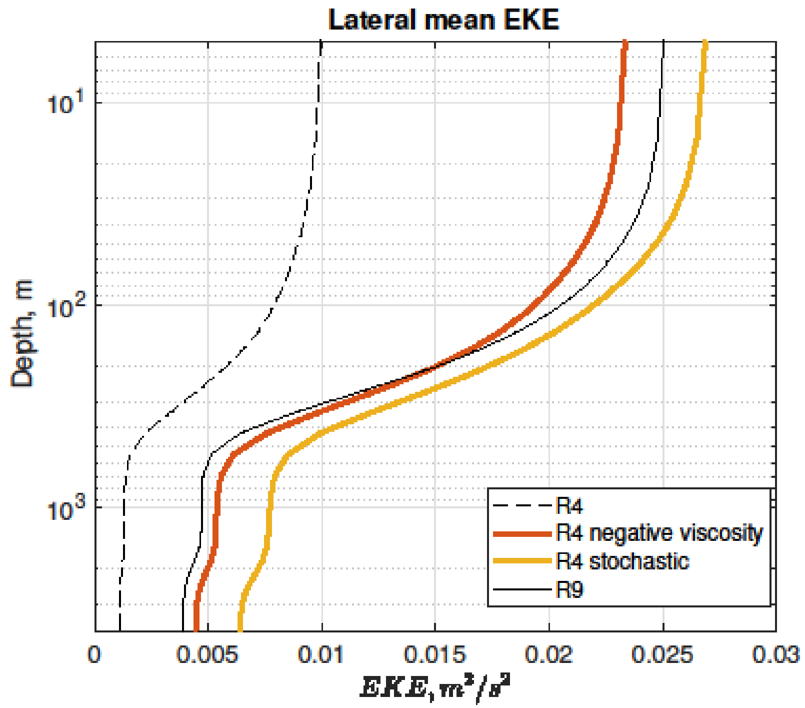
- Condition on amplitude α representing global energy balance:

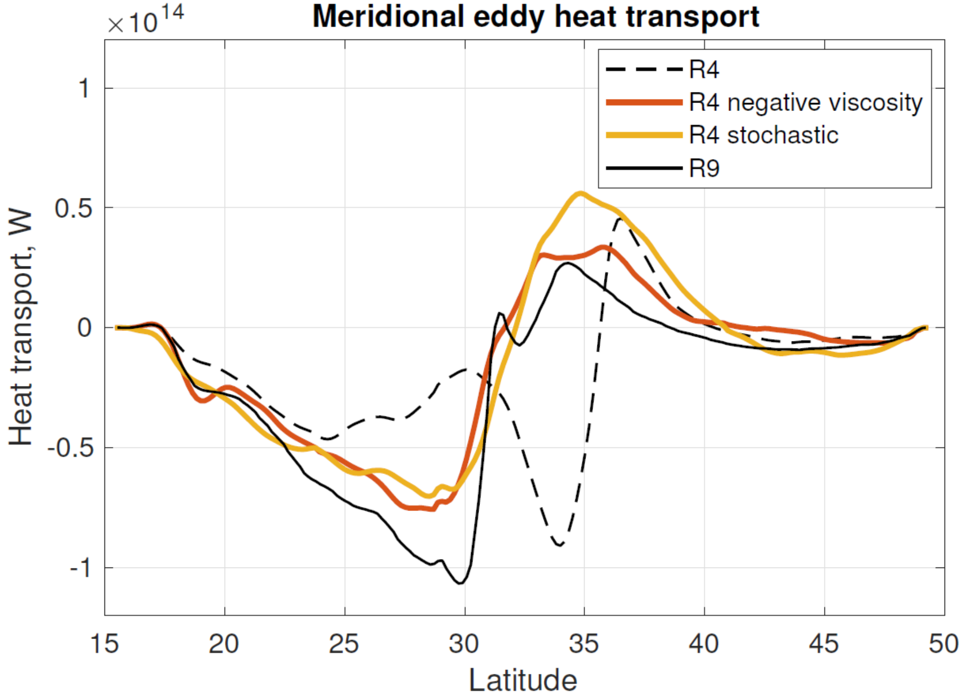
$$\frac{\alpha^2 \Delta t}{2} \int \left| \nabla^{\perp} \hat{\psi} \right|^2 dV = \int \dot{E}_{diss} \, dV$$

(1) https://www.degruyter.com/view/journals/rnam/35/2/article-p69.xml
(2) https://iopscience.iop.org/article/10.1088/1755-1315/386/1/012025/meta

Conventional KEB parameterizations

- Usual KEBs allow to increase eddy kinetic energy.
- It affects resolved eddy heat flux. Latitude of maximum southward flux is shifted southward in accordance with reference simulation.

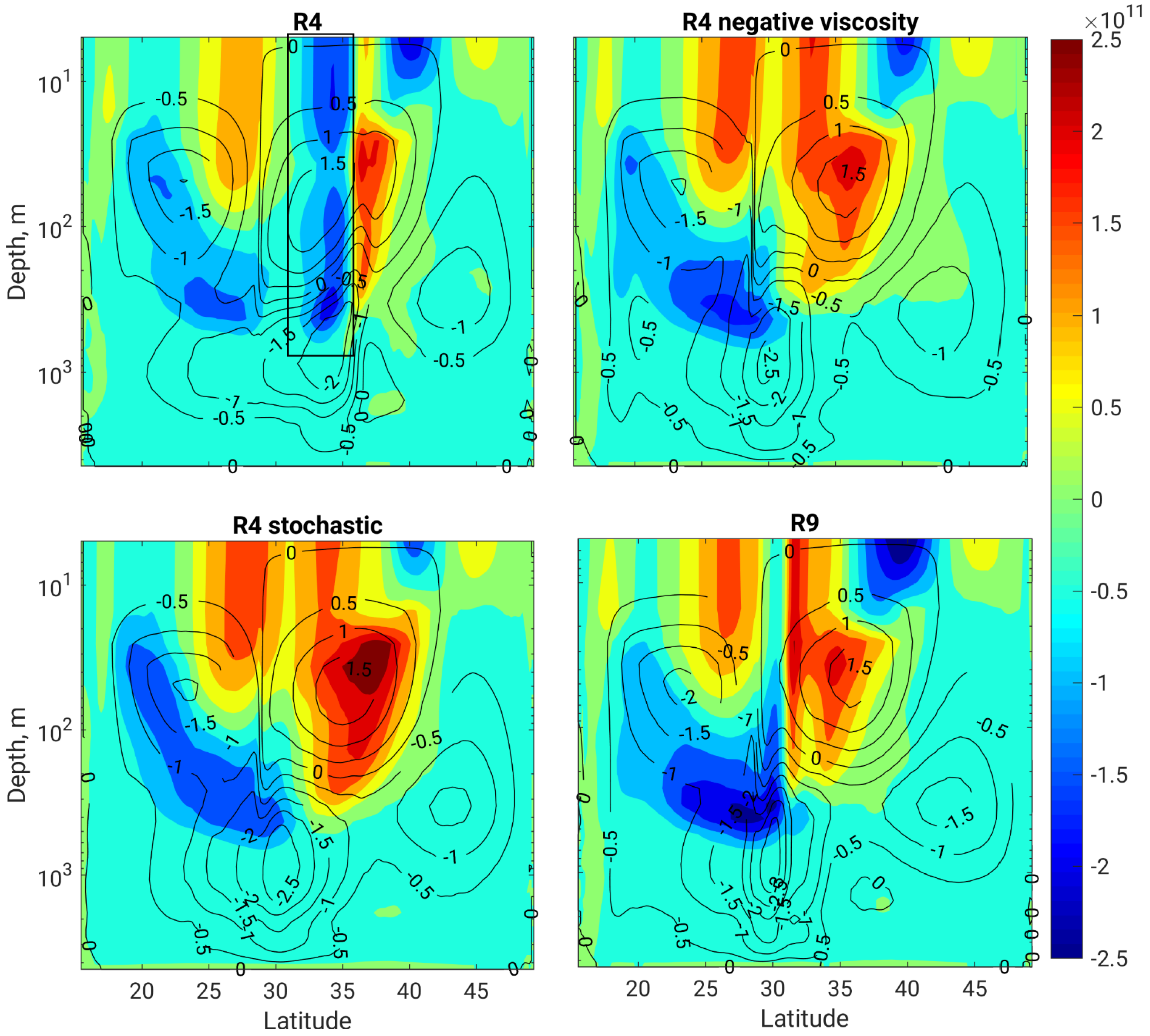




Conventional KEB parameterizations

- In color: distribution of meridional eddy heat flux in depth
- In contours: MOC streamfunction

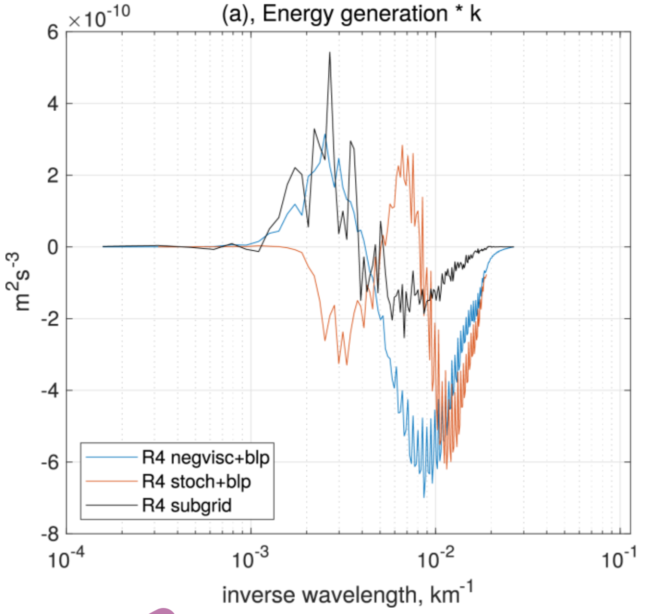
- Applying KEBs, wrong southward flux marked with square disappears and latitude of bottom MOC cell is shifted southward

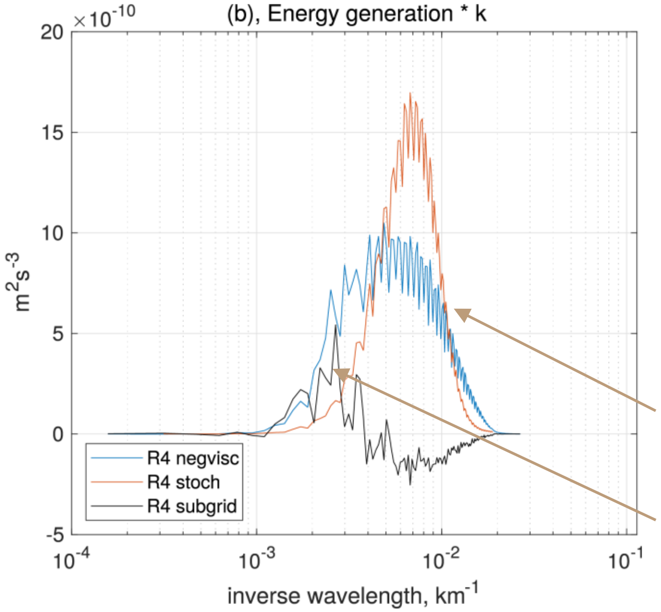


Spectral analysis of <u>conventional</u> subgrid forces and KEB parameterizations

- Let P be a projection operator from R9 grid to R4 grid. Then unresolved advection (subgrid forces):

$$S = -P\big((\boldsymbol{U} \cdot \nabla)\boldsymbol{U}_h\big) + (P(\boldsymbol{U}) \cdot \nabla)P(\boldsymbol{U}_h)$$





The key idea is to **split** subgrid generation spectrum into forward and backward energy scatter using **nudging**

KEB in parameterizations

KEB in subgrid forces

(a), Energy generation spectra by full subgrid model (KEB + biharmonic dissipation) vs subgrid forces
(b), Energy generation spectra by KEB parameterizations vs subgrid forces

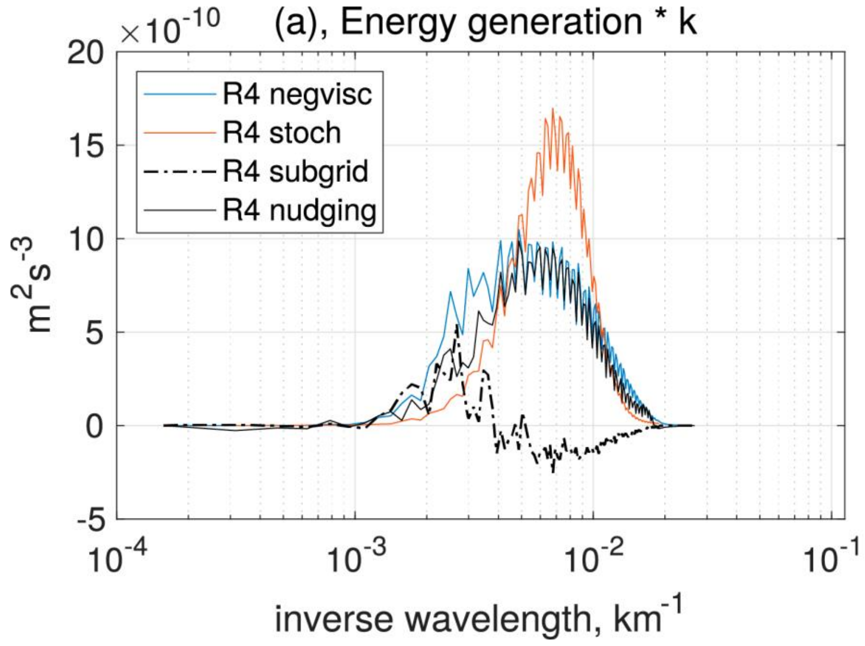
Alternative subgrid forces (nudging)

- $(T^c, S^c, \boldsymbol{U}_h^c)$ – fields of coarse R4 model, $(T, S, \boldsymbol{U}_h)$ – fields of reference R9 model
- $\tau = 1 day$, enough for mesoscale eddies
- Nudging experiment was performed over a year, all fields was saved each 6 hours
- Coarse model has only biharmonic viscosity as SGS model on momentum
- We consider momentum nudging tendency as a subgrid force restoring energy balance and representing KEB
- Nudging as a technique revealing unrepresented physics is proposed here for the first time. Recently, it was used as a parameter estimation tool[1]
- Nudging tendency gives energy source spectrum similar to KEBs presented above

$$\frac{\partial T^c}{\partial t} = \dots + \frac{1}{\tau}(P(T) - T^c)$$

$$\frac{\partial S^c}{\partial t} = \dots + \frac{1}{\tau}(P(S) - S^c)$$

$$\frac{\partial \boldsymbol{U}_h^c}{\partial t} = \dots + \frac{1}{\tau}(P(\boldsymbol{U}_h) - \boldsymbol{U}_h^c)$$



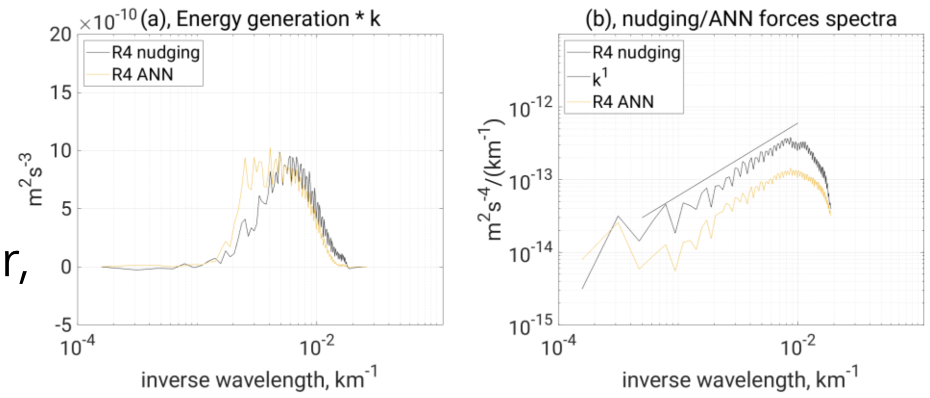
(1), https://journals.aps.org/prfluids/abstract/10.1103/PhysRevFluids.3.104604

Regression of nudging tendency with ANN

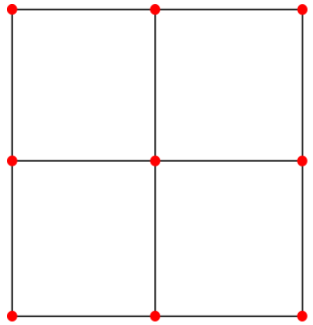
- Regression problem:

$$F(\nabla(\boldsymbol{U}_h^c)) = \frac{1}{\tau}(P(\boldsymbol{U}_h) - \boldsymbol{U}_h^c) + \boldsymbol{\varepsilon}$$

- $\nabla(\boldsymbol{U}_h^c)$ – resolved velocity gradient tensor, $\boldsymbol{\varepsilon}$ – regression error, $F(\cdot)$ – general functional dependency represented by ANN



$$\begin{bmatrix} U_x & U_y \\ V_x & V_y \end{bmatrix}$$



$$\begin{bmatrix} U_x^{i-1,j-1} \\ U_x^{i,j-1} \\ \vdots \\ U_y^{i-1,j-1} \\ U_y^{i,j-1} \\ \vdots \\ V_x^{i-1,j-1} \\ V_x^{i,j-1} \\ \vdots \\ V_y^{i-1,j-1} \\ V_y^{i,j-1} \\ \vdots \end{bmatrix} = x \in R^{36}$$

$$F(x) = A_2\sigma(b_1 + A_1 x) + b_2$$

$$\sigma(y) = \frac{2}{1+\exp(-2y)} - 1$$

- ANN corresponds to 36 input neurons, 20 hidden neurons and 2 output neurons with the following coefficients to be optimized numerically:

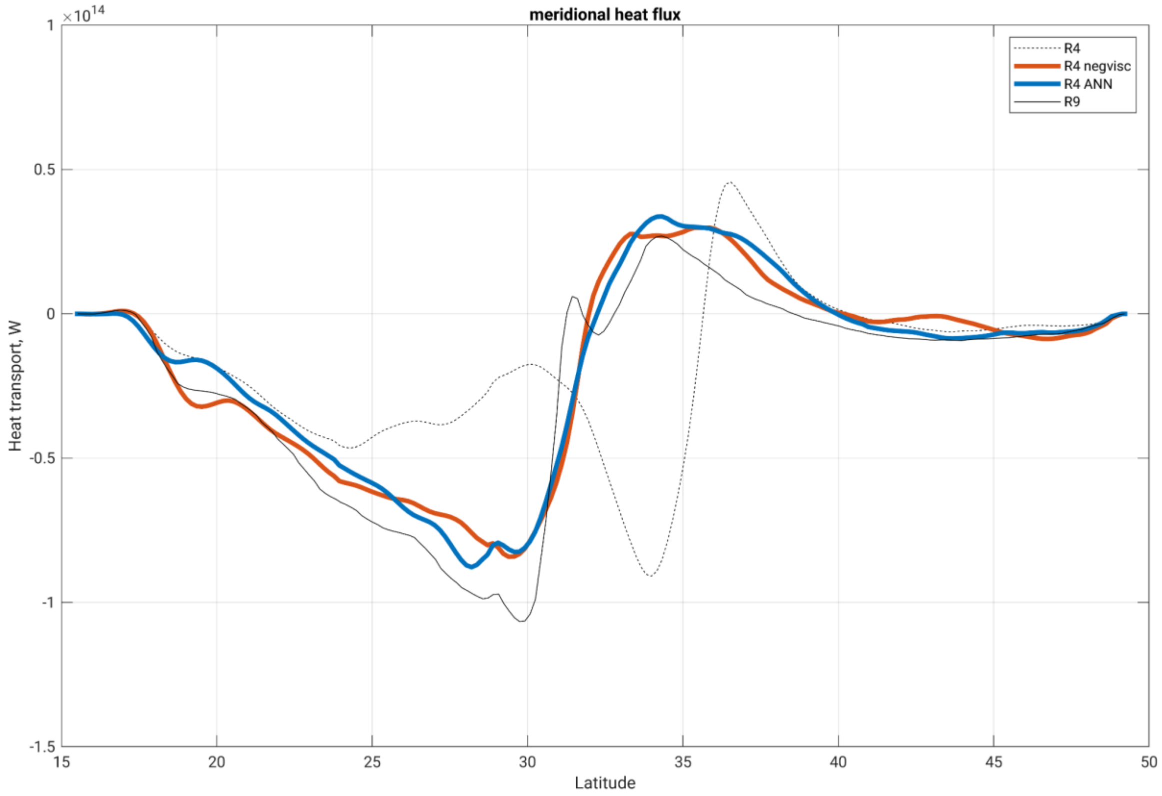
$$b_2 \in R^2, b_1 \in R^{20}$$
$$A_2 \in R^{2\times 20}, A_1 \in R^{20\times 36}$$

- Training set is composed of 1000 randomly selected points in computational domain 120x80x30 at each of 1440 time moments. In sum, 1440000 pairs of feature–target. MSE = 50%

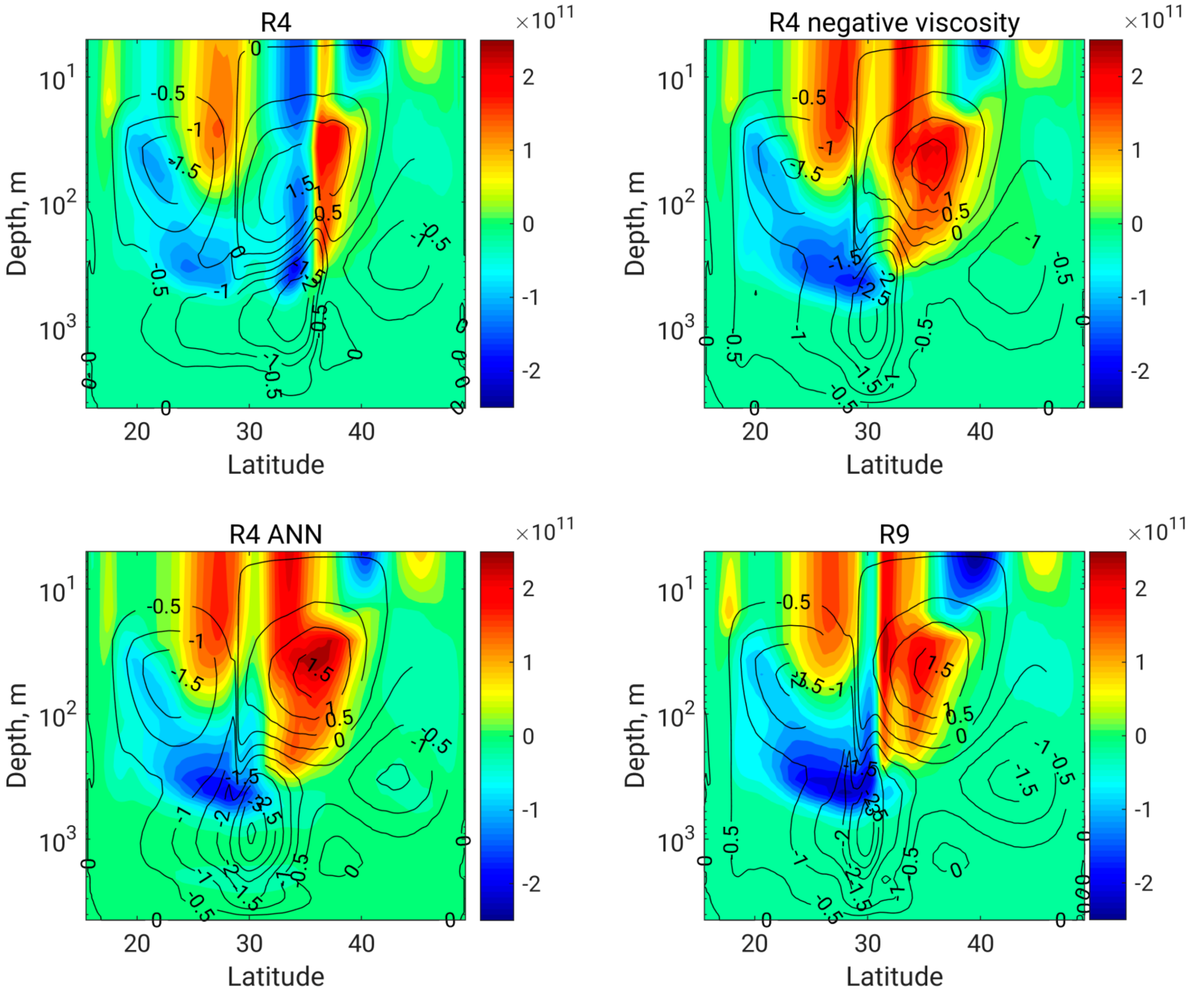
Experiments with ANN KEB

- As neg.visc and stochastic KEBs, ANN KEB is multiplied by a constant to produce energy source equals 80% of biharmonic dissipation (as in *a posteriori* experiments ANN slightly overestimates energy source (+50%))
- Two points adjacent to boundary are excluded (ANN cannot produce adequate tendency near boundary and accelerates boundary currents)

ANN KEB model reproduces meridional eddy heat flux as well as negative viscosity model



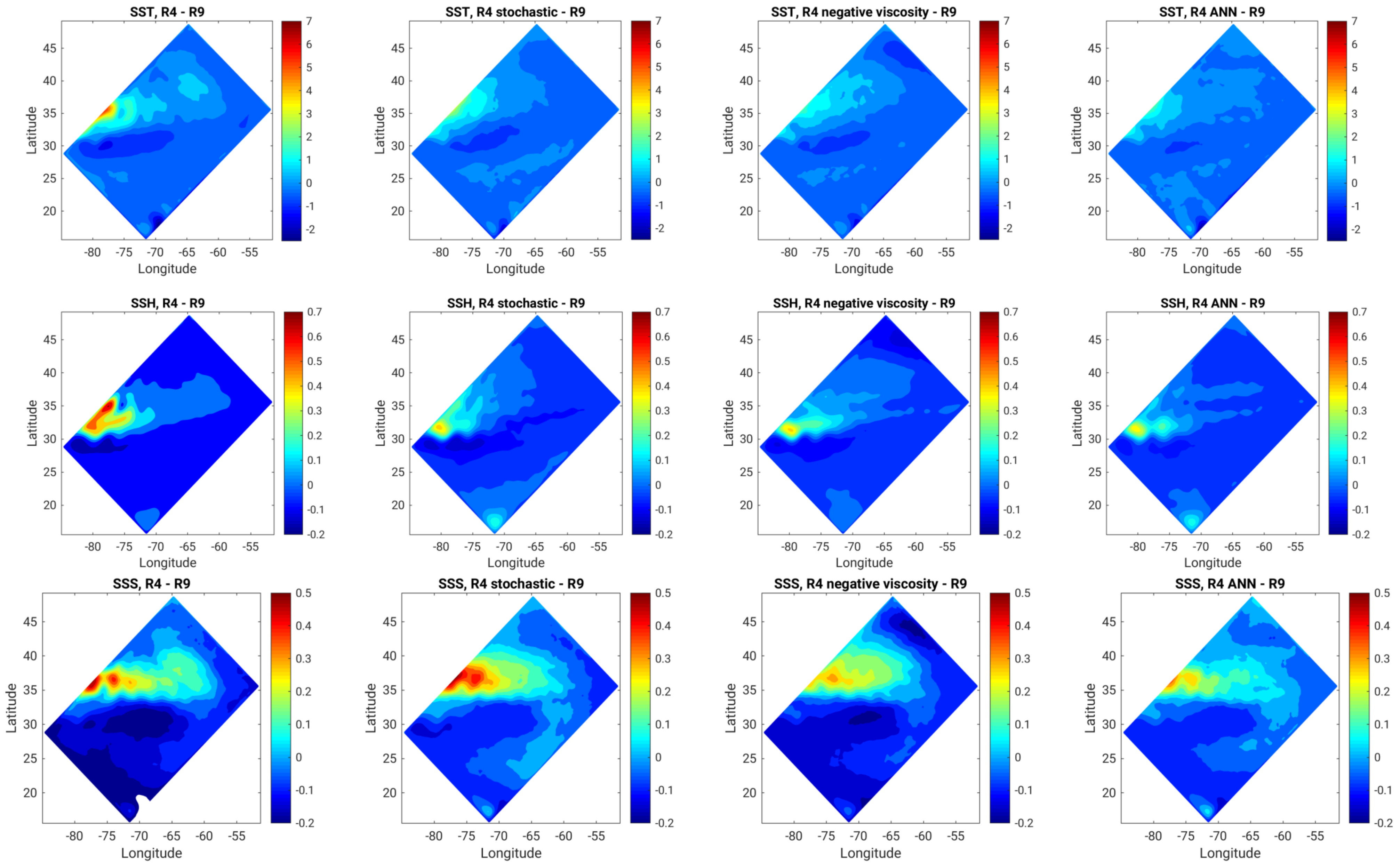
Experiments with ANN KEB (MOC)



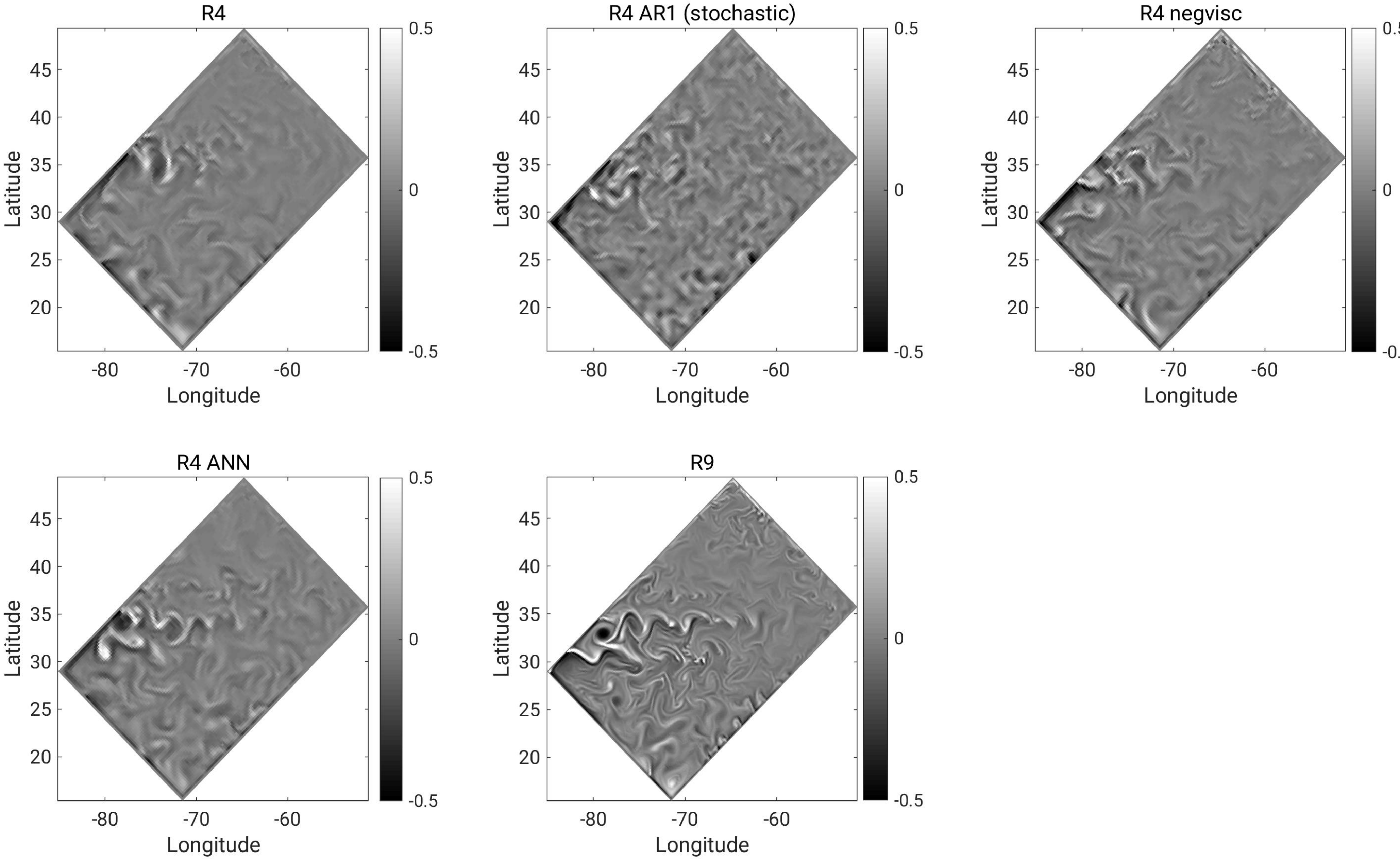
ANN KEB model performs slightly better than conventional counterparts:

- More intense bottom MOC cell
- More intense northern MOC cell

Experiments with ANN KEB (mean fields error)



Experiments with ANN KEB (vorticity snapshot)



Conclusions

- Kinetic energy backscatter (KEB) process is formulated as a regression problem using nudging
- Nudging produces correct energy source spectra and represents purely KEB process (without dissipation)
- Nudging forces are predicted by the values of velocity gradient tensor on 3x3 grid pattern using ANN
- ANN is applied to the whole domain. No space/depth variation of parameters is necessary
- ANN KEB performs as well as conventional KEBs
- Drawbacks: wrong prediction near boundary, non-flux form, computational cost (about 1000 operations per grid point)