Investigations on the power-law burst lifetime distribution characteristics of the magnetospheric system

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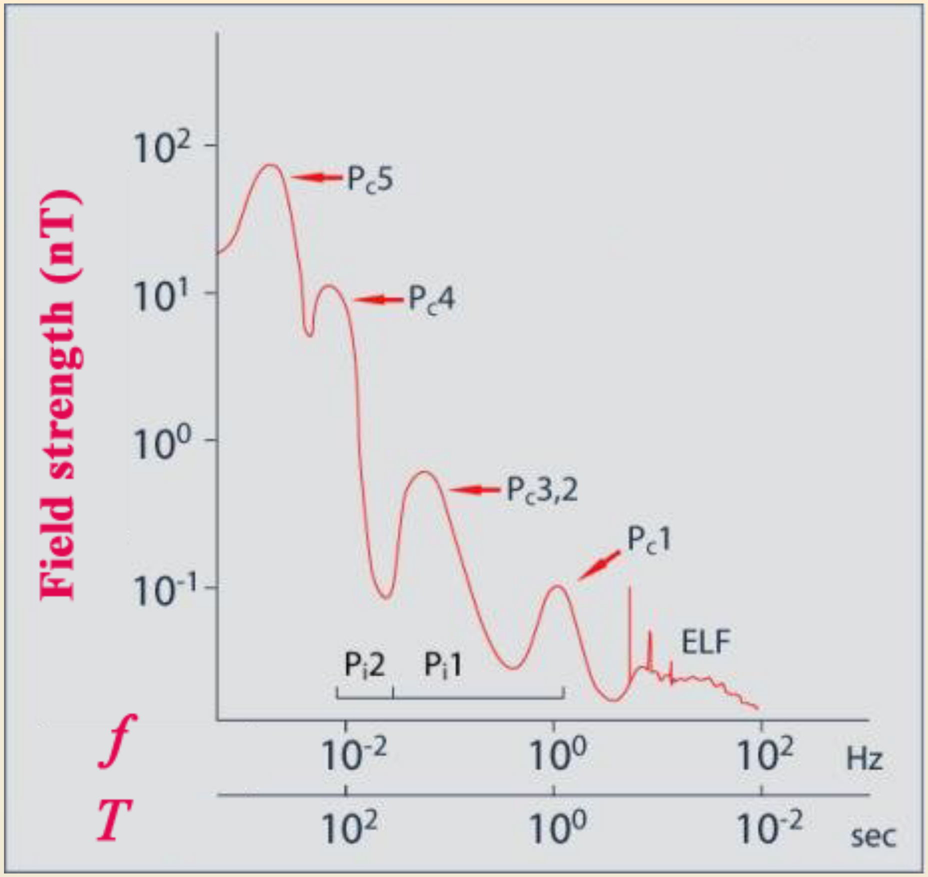


Geomagnetic pulsations are the signatures of magnetic fields associated with ultra low frequency (ULF) waves in the Earth's magnetosphere. These oscillations have short periods (usually of the order of seconds to minutes) and small amplitudes. They serve as extremely useful diagnostics of the Earth's magnetosphere. They can also play a significant role in space weather forecasting.

Figure 1: Graph of the amplitude spectrum of geomagnetic pulsations as a function of frequency. Amplitudes depicted are typical values observed at mid latitudes, during periods of moderate geomagnetic activity.

(Courtesy : http://roma2.rm.ingv.it/en/themes/22/magnetic_pulsations)

Pulsations	T, s	f	Amplitudes, nT
Pc 1	0.2–5	0.2–5 Hz	0.01–0.1
Pc 2	5–10	0.1–0.2 Hz	0.1–1
Pc 3	10–45	22–100 mHz	1–10
Pc 4	45–150	7–22 mHz	5–50
Pc 5	150–600	2–7 mHz	50–500
Pi 1	1–40	0.025–1 Hz	0.2–1
Pi 2	40–150	2–25 mHz	10–100

(Kozlovskaya and Kozlovsky, 2010)

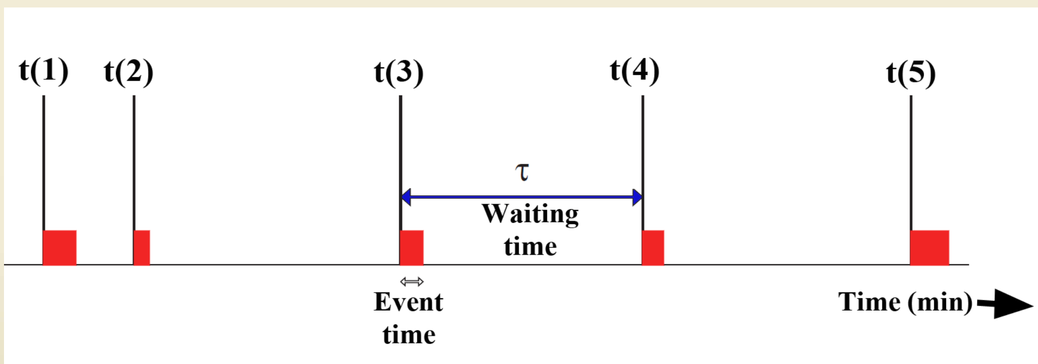
Wp index can serve as a magnetospheric proxy whose waiting time statistics helps us to study the Pi2 power related magnetospheric waiting time and associated dynamics.

The study helps to classify the properties of the Wp burst lifetime distributions which are probed by the combined effort of those stimulations that have both solar wind as well as magnetospheric origin, which in turn are responsible for the intermittence in the Wp fluctuations [Freeman et al., 2000; Wanliss and Weygand, 2007].

The burstiness and associated waiting time distributions whose statistical relationships can be considered as a general strategy for analyzing space weather and inner magnetospheric complexity to a large extent.

It measures the distribution of delay times between subsequent hopping events in such processes.

In a physical system the time duration between two events is called a waiting-time, like the time between avalanches.



If there is a lack of a characteristic time scale, the probability densities vary with power law relations

$$P(\tau) \sim \tau^{-\gamma}$$

where Υ is the scaling constant, and τ is the time interval during which fluctuations follow one of the above definitions

The 1-min digital data of the Wp index for the solar minima years (2007 and 2008) and solar maxima years (2013 and 2014) have been taken from Substorm Swift archive.

The AE index is taken from geomagnetic data archive at World Data Center, Kyoto.

The burst lifetimes with constant thresholds as defined by Freeman et al. [2000] is used for the present analysis. The probability distributions P(τ) have been computed by considering bin sizes which are alike in logarithm space.

The distribution function is fitted with a model consisting of the product of an inverse power law with an exponential cutoff [Freeman et al., 2000; Wanliss and Weygand, 2007]

$$P(\tau) = (A/\tau^{\gamma})\exp(-\tau/T_C)$$

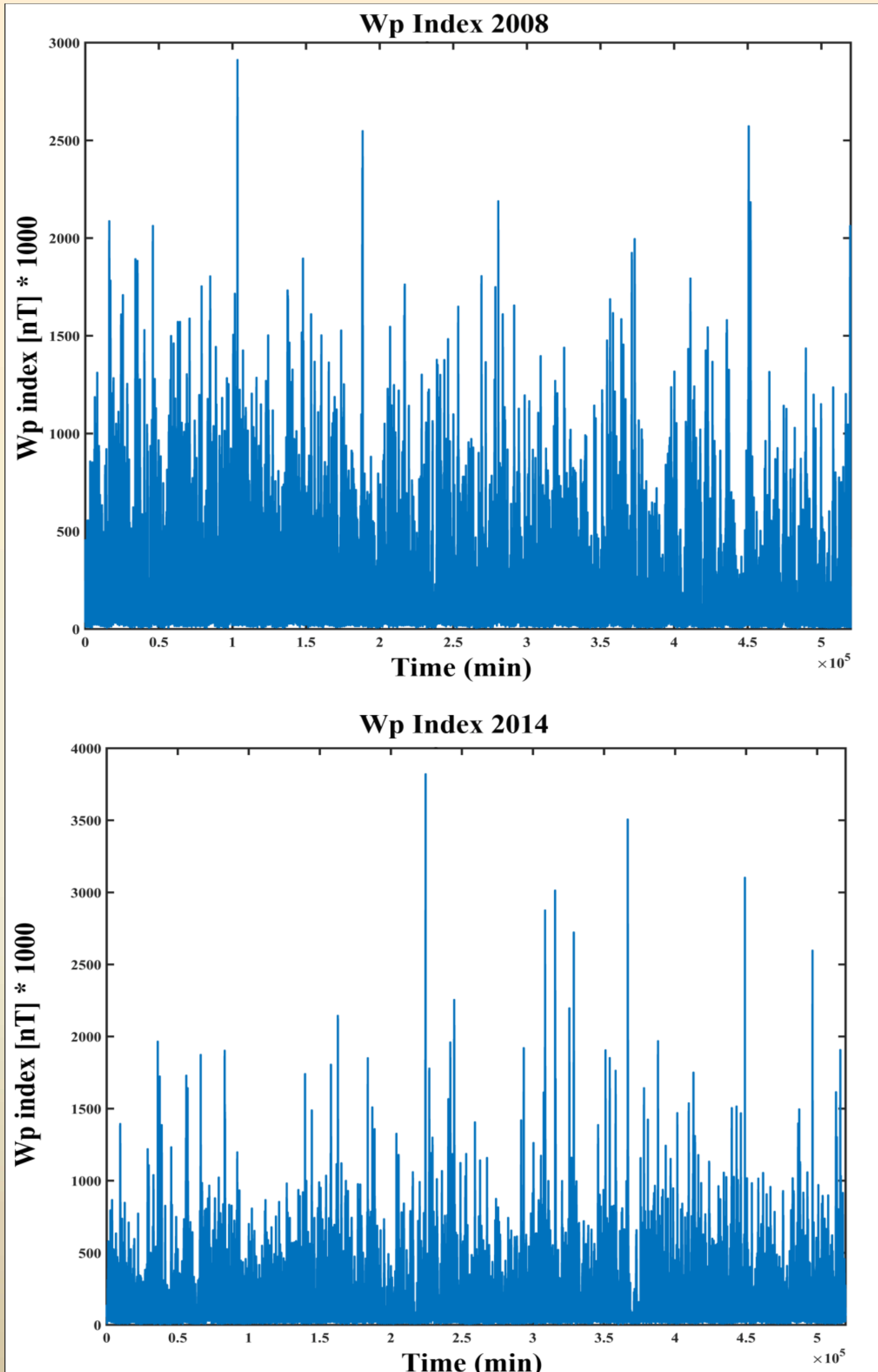


Figure 2. **(left)** The variations in the Wp index for the solar minimum year 2008 and for the solar maximum year 2014.

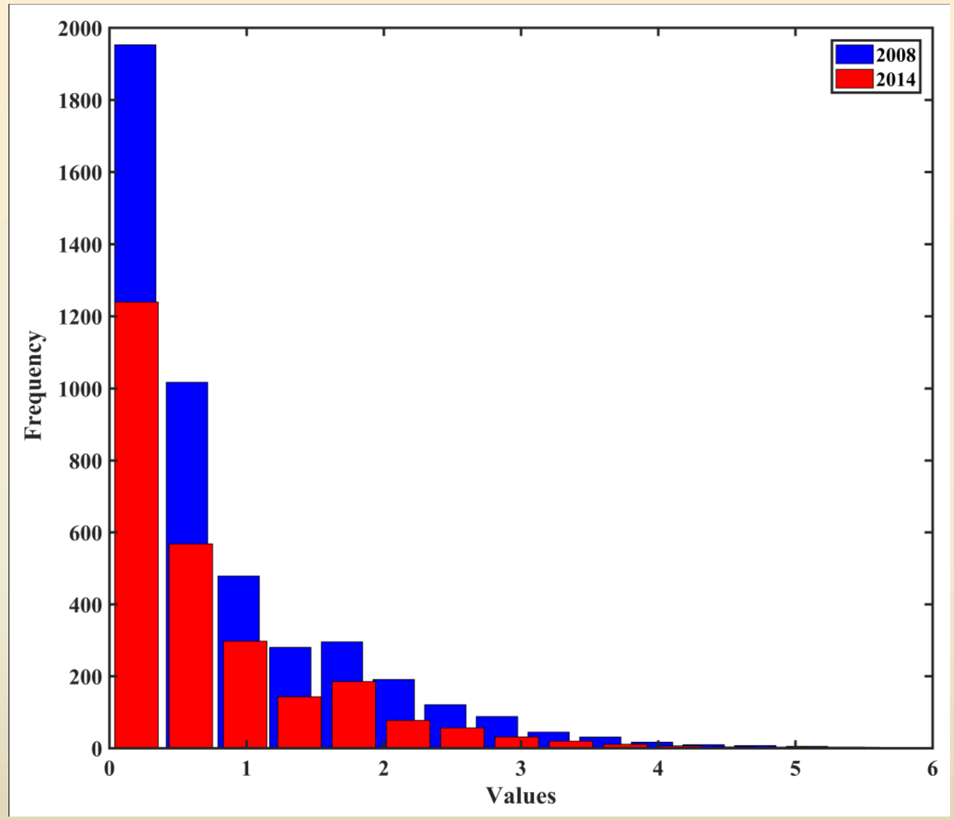
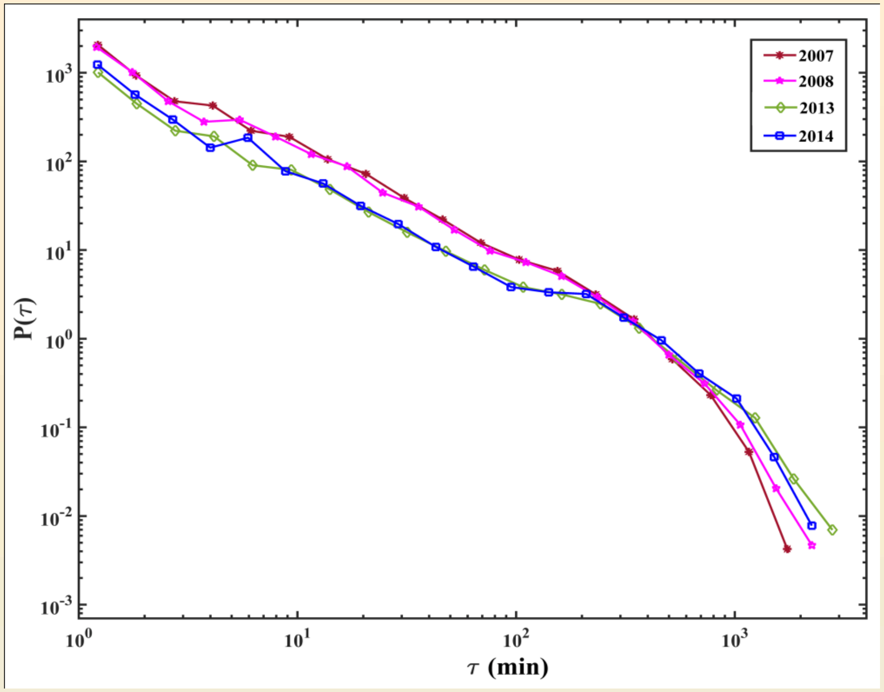


Figure 3. (top) Logarithmic binning of waiting times for the **solar minimum (2008)** and **maximum (2014)** years.



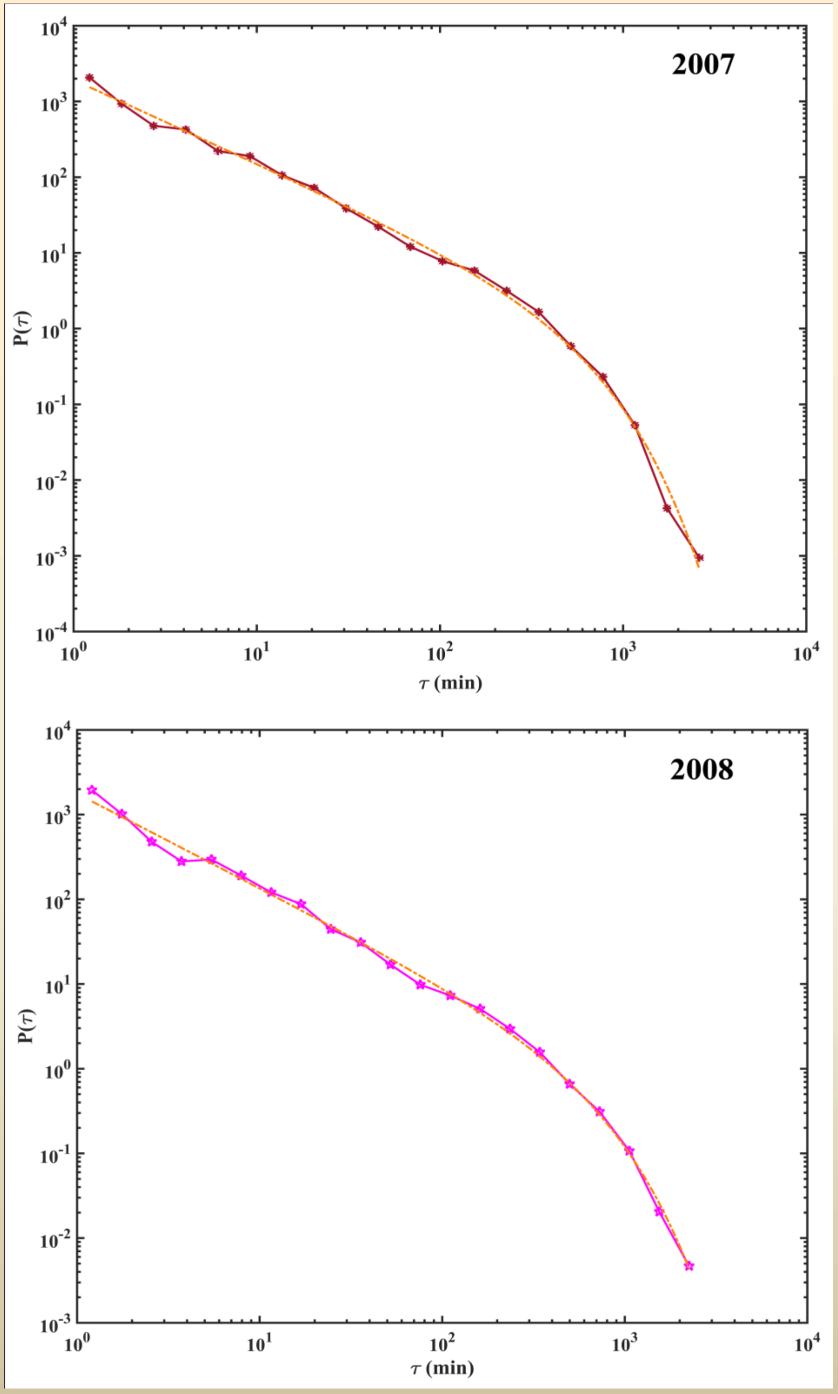


Figure 4. (above) The probability distribution P(τ) versus waiting time τ for the solar minimum (2007 & 2008) and solar maximum (2013 & 2014) for a threshold of 250 nT.

Figure 5. (right) The model fit on the probability distribution curves for the years 2007 and 2008 (for a threshold of 250 nT).

The **Auroral** Electrojet **Index**, AE, is designed to provide a global, quantitative measure of **auroral** zone magnetic activity produced by enhanced ionospheric currents flowing below and within the **auroral** oval.

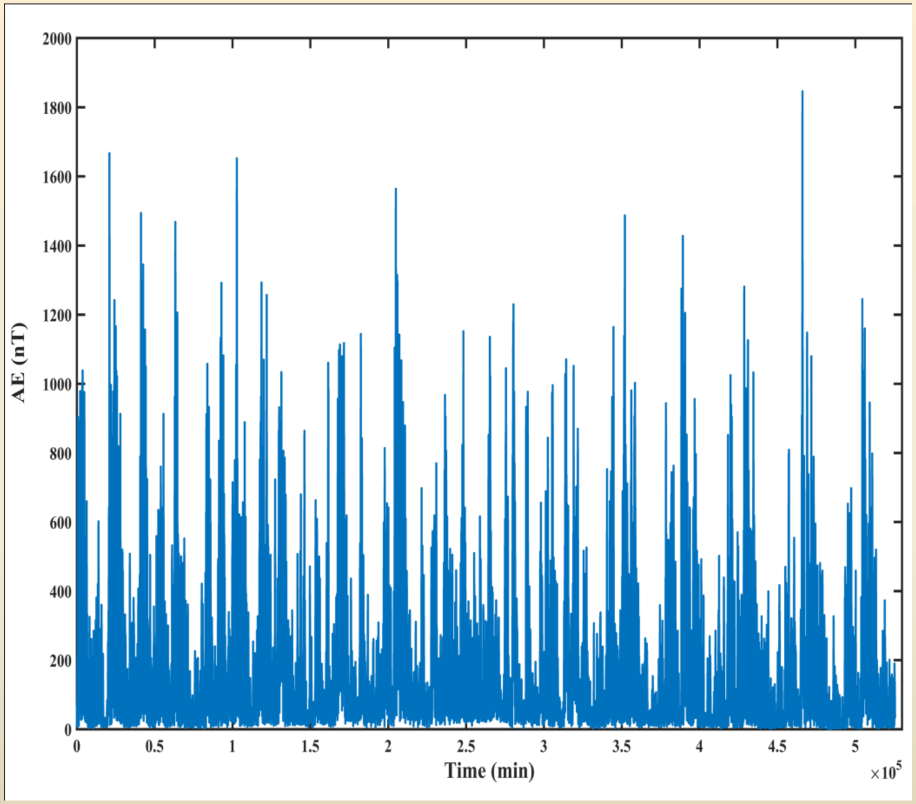


Figure 6. Auroral electrojet index (AE index) data for the year 2007.

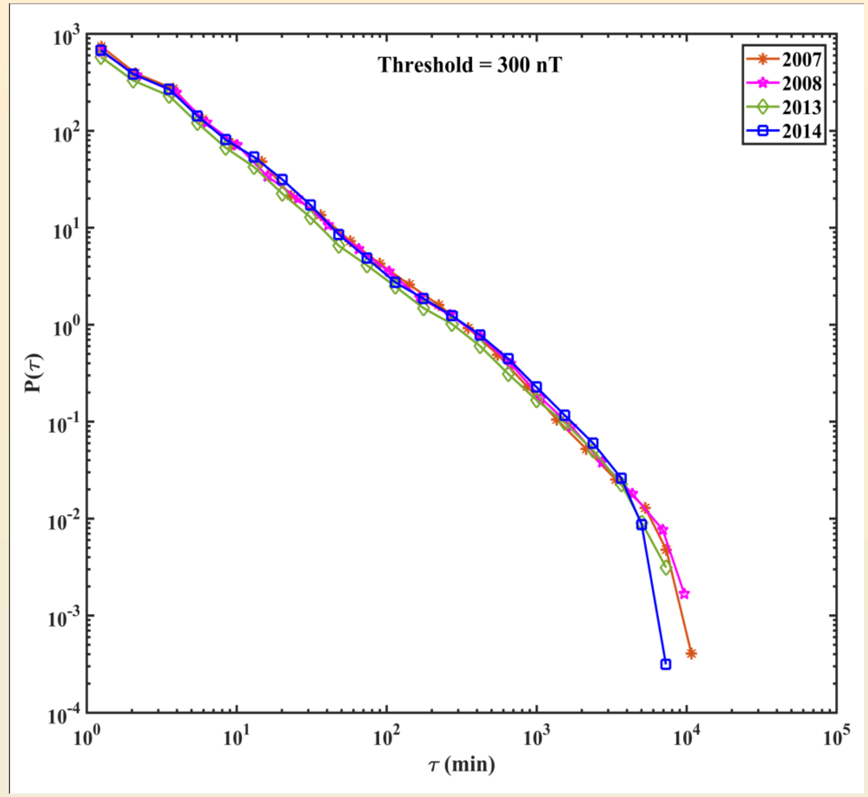


Figure 9. The probability distribution P(τ) versus waiting time τ of AE indices for the solar minimum (2007 & 2008) and solar maximum (2013 & 2014) for a threshold of 300 nT.

Summary

- The average power-law scaling exponent for **Wp index** of scaling region for
(a) **solar maximum periods ϒ = 1.0633.**
(b) **solar minimum periods ϒ = 1.10815.**

- The average power-law scaling exponent for **AE index** of scaling region for
(a) **solar maximum periods ϒ = 1.14355.**
(b) **solar minimum periods ϒ = 1.19055.**

- The influence of solar activity on Wp index variations is quite different from that of AE index. Quite different from that of Wp index, the PDF curves that belong to low activity periods almost overlap with the ones that belong to high activity periods, especially in the power-law region. Thus, **unlike Wp index, it is not possible to recognize any variation that might have contributed by solar activity forcings**.

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The waiting time distributions and associated statistical relationships can be considered as a general strategy for analyzing space weather and inner magnetospheric processes to a large extent. It measures the distribution of delay times between subsequent hopping events in such processes. In a physical system the time duration between two events is called a waiting-time, like the time between avalanches. The burst lifetime can be considered as the time duration when magnitude of fluctuations are above a given threshold intensity. If a characteristic time scale is absent then the probability densities vary with power-law relations having a scaling exponent. The burst lifetime distribution of the substorm index called as the Wp index (Wave and planetary), which reflects Pi2 wave power at low-latitude is considered for the present analysis. Our analysis shows that the lifetime probability distributions of Wp index yield power-law exponents. Even though power-law exponents are observed in magnetospheric proxies for different solar activity periods, not many studies were made to analyze whether these features will repeat or differ depending on sunspot cycle. We compare the variations of power-law exponents of Wp index and other magnetospheric proxies, such as AE index, during solar maxima and solar minima. Thus the study classifies the activity bursts in Wp and other magnetospheric proxies that may have different dynamical critical scaling features. We also expect that the study sheds light into certain stochastic aspects of scaling properties of the magnetosphere which are not developed as global phenomena, but in turn generated due to inherent localized properties of the magnetosphere.

1. INTRODUCTION

The waiting time distributions and associated statistics are common methods for analyzing stochastic solar-terrestrial processes which measure the distribution of delay periods between subsequent hopping events in such processes. Since power laws were consistently observed associated with several features of global as well as local magnetosphere system for different solar activity periods, it is possible to say that these will repeat, even though scaling properties can differ depending on sunspot cycle. Wp index can serve as a magnetospheric proxy whose waiting time statistics helps us to study the $Pi2$ power related magnetospheric waiting time and associated dynamics. The study sheds light to certain aspects of scaling properties of the magnetosphere which are characterized by the solar activity dependence. It also helps to classify the properties of the Wp burst lifetime distributions which are probed by the combined effort of those stimulations that have both solar wind as well as magnetospheric origin, which in turn are responsible for the intermittence in the Wp fluctuations [Freeman et al., 2000; Wanliss and Weygand, 2007].

In a process or associated time series, the time interval between two ‘events’ is called a waiting time, for instance, the duration between avalanches. It can also be stated as the time period between event triggering or the time interval between maximum intensity or the time period from the end of a burst and the start of the next one or the time interval when intensity fluctuations are above a threshold intensity [Freeman et al., 2000]. It is possible to monitor these respectively as the waiting times, the inter-peak, quiet, and burst lifetimes. If there is a lack of a characteristic time scale, the probability densities vary with power law relations

$$P(\tau) \sim \tau^{-\Upsilon} \qquad (1)$$

where Υ is the scaling constant, and τ is the time interval during which fluctuations follow one of the above definitions.

The burst lifetimes with constant thresholds is considered for the present work as defined by Freeman et al. [2000]. $Pi2$ pulsations are typical geomagnetic fluctuations with a period ranging from 40 to 150 seconds. $Pi2$ pulsations usually register maximum amplitude near midnight and to increase the availability, ground stations have to be distributed accordingly around the Earth for their detection. Fortunately, now it is possible to arrange a number of geomagnetic stations likewise to record geomagnetic field changes with 1-sec resolution. From the collected data, an index can be derived that measures $Pi2$ power, called as the Wp index (the wave and planetary index). It is proposed as superior in identifying the occurrence of substorms when comparing with the AE (auroral electrojet) and ASY indices [Nosé et al., 2009; 2012].

2. DATA AND METHODS

The 1-min digital data of the Wp index for the solar minima years (2007 and 2008) and solar maxima years (2013 and 2014) have been taken from Substorm Swift archive. The AE index is taken from geomagnetic data archive at World Data Center, Kyoto. The burst lifetimes with constant thresholds as defined by Freeman et al. [2000] is used for the present analysis. A constant threshold is more suitable since if the magnetosphere is in a SOC state, the gradient of the power-law section of the burst lifetime distribution of associated proxy will be independent of the threshold level [Takalo et al., 1994; Consolini, 1997].

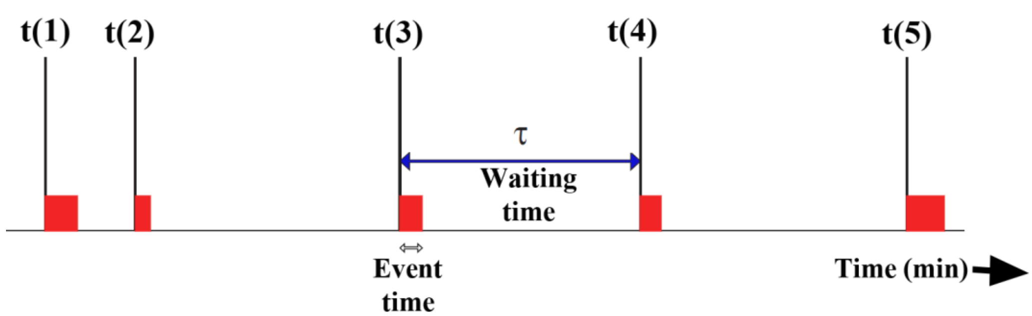

Figure 1. The event time and waiting time (τ) in a Wp index time series.

Generally, the probability distribution of waiting times was calculated by sorting the waiting times into approximately equally-spaced bins and calculating the probability density for each bin, say i, by dividing the number of waiting times in each bin, n_i, by $N\Delta_i$, where N is the total number of waiting times and Δ_i is the bin width. But, it is necessary to identify the appropriate size Δ_i of the i^{th} bin. As the waiting time distribution will be an inverse power law, the bins that correspond to large times will have only less data fallen on them, if we choose equal bin sizes in linear space. This in turn results in an inaccurate calculation of the power law exponent. Hence, for the present work, the probability distributions $P(\tau)$ have been computed by considering bin sizes which are alike in logarithm space. According to this, $log(\tau_i) - log(\tau_{i-1})$ is invariable, where τ_i and τ_{i-1} are the mid-points of consecutive bins. The size of the i^{th} bin, $\Delta_i = \tau_i - \tau_{i-1}$, will outweigh the reduction in the density of the data. The probability density for each bin is given as

$$P(\tau_i) = n_i/\mathrm{N}\Delta_i \tag{2}$$

where n_i is the number of data points fall in i^{th} bin, and N is the total number of waiting times computed from the actual Wp series.

The distribution function is fitted with a model consisting of the product of an inverse power law with an exponential cutoff [Freeman et al., 2000; Wanliss and Weygand, 2007],

$$P(\tau) = (A/\tau^{\Upsilon})\, exp\,(-\tau/T_c) \tag{3}$$

3. RESULTS AND DISCUSSION

The figure 1 shows a general picture of the event time and waiting time (τ) in a Wp index time series. The variations in the Wp index for the solar minimum year 2008 and for the solar maximum year 2014 is shown in figure 2. For convenience in choosing the threshold values, the magnitude is scaled by a factor of 1000 and the threshold value is fixed as 250 nT. The magnitude of actual Wp index is having units in nanotesla whilst time is taken as points (where each point is a minute). Unlike the waiting time of ε, VB_s or SYM-H, the Wp index statistics can be different in the sense that event duration is very small compared with the waiting time interval. The sample histogram plots of Wp index for the year 2008 and 2014 are shown in figure 3. The histogram binning is adopted in the logarithm space to maintain the density of the data in higher bins. The waiting times in the lower bins of 2008 are very high compared to the waiting times in the lower bins of 2014.

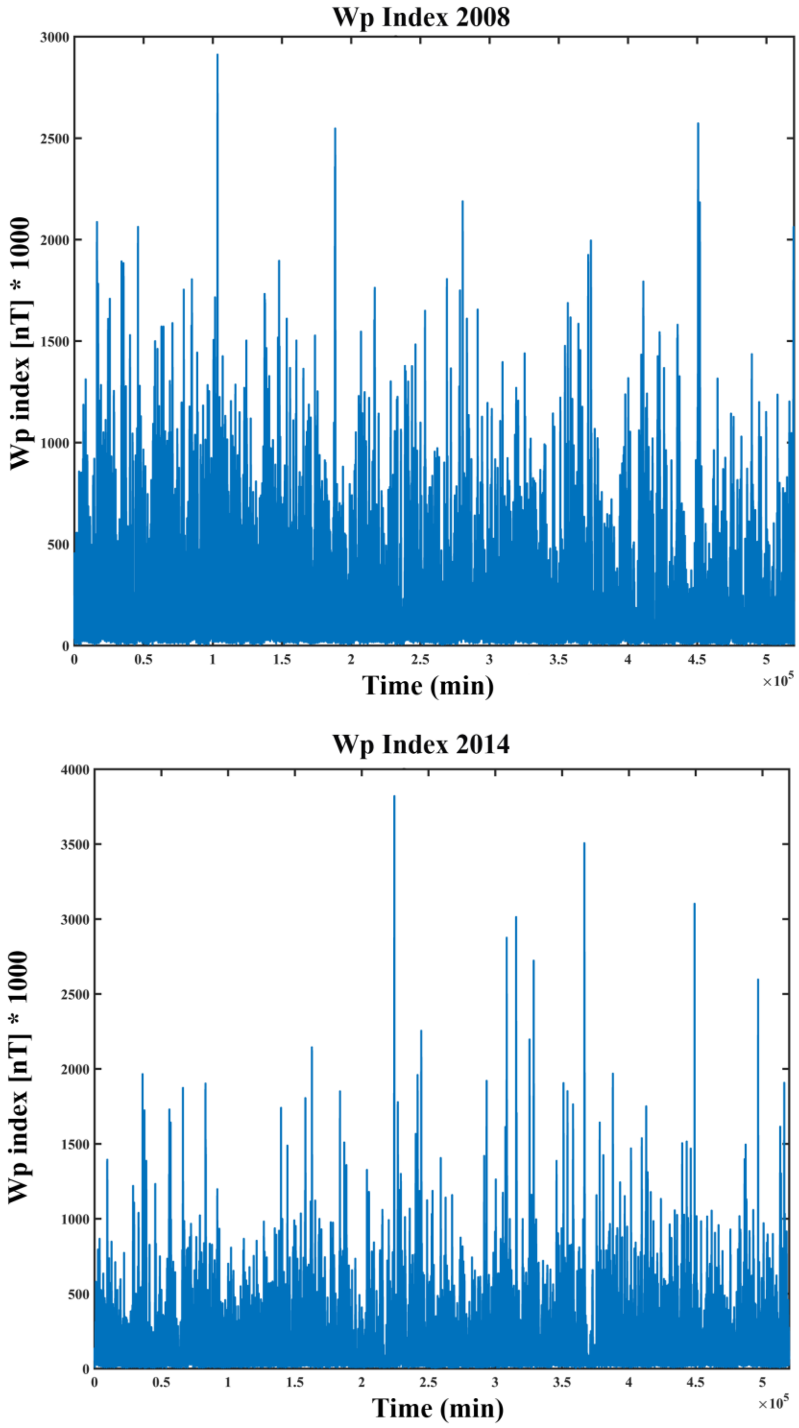

Figure 2. The variations in the Wp index for the solar minimum year 2008 and for the solar maximum year 2014.

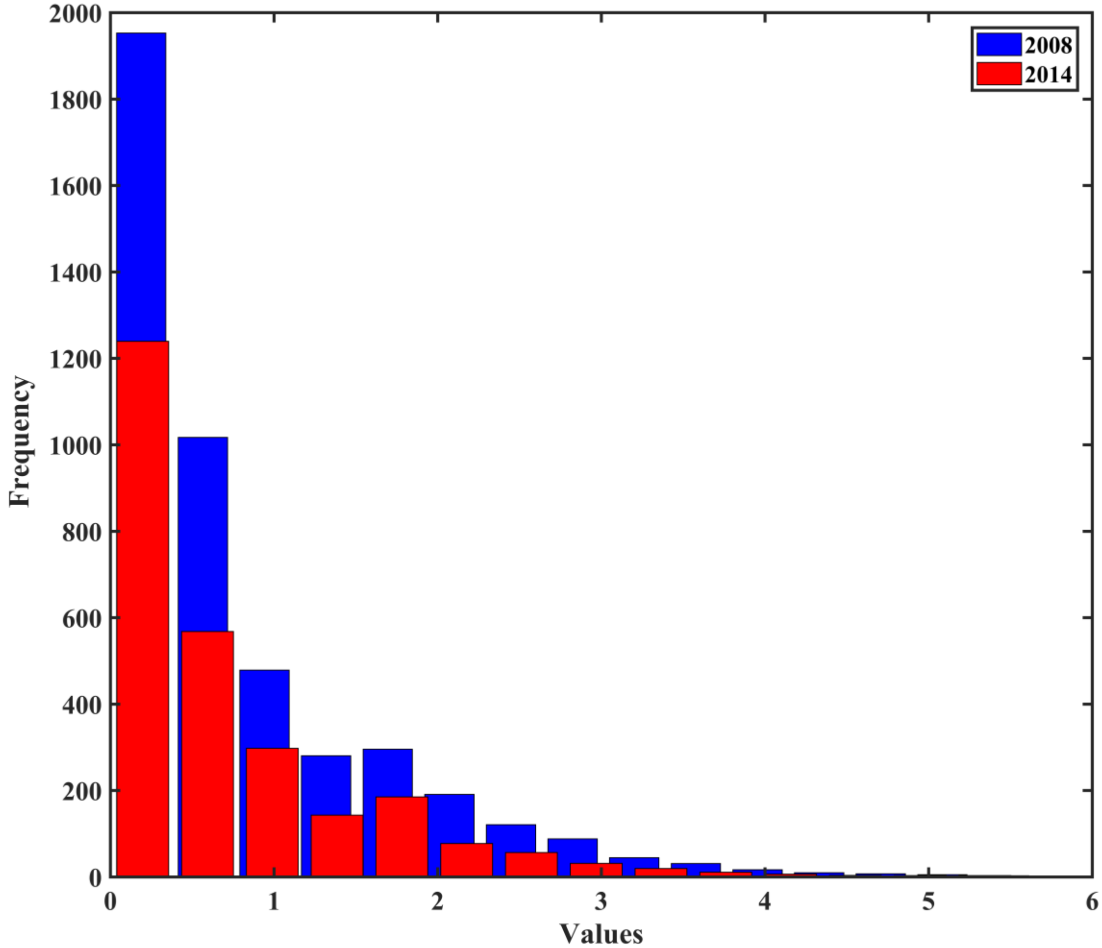

Figure 3. Logarithmic binning of waiting times for the solar minimum (2008) and maximum (2014) years.

From the logarithmically binned data it is straightforward to calculate the probability distribution function versus the waiting time statistics. The $P(\tau)$ versus τ for the solar minimum years 2007 and 2008 as well as solar maximum years 2013 and 2014 have been shown in figure 4. From the figure it is evident that the curves show power-law decay of probability distributions for all years. From figure 4 it is evident that the Wp waiting times have a power law region that stretches to approximately three orders of magnitude, from the least time periods to ~240 minutes for all years. At large waiting times, the power-law begins to break which suggests an exponential roll off.

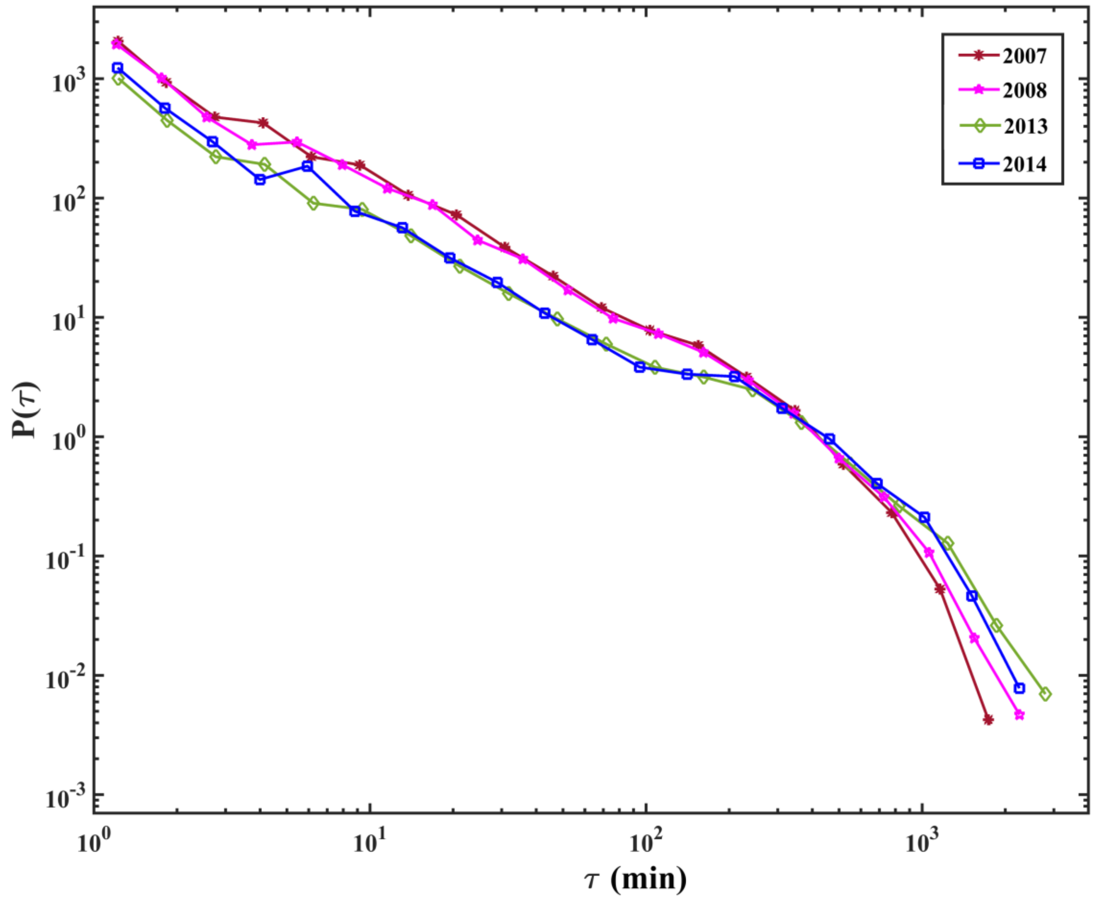

Figure 4. The probability distribution $P(\tau)$ versus waiting time τ for the solar minimum (2007 & 2008) and solar maximum (2013 & 2014) for a threshold of 250 nT.

The time scale of 240-480 minutes can be compared to the characteristic scale of substorm periods. The roll off is comparatively earlier for solar minimum periods (2007 & 2008) which usually marks low solar activity when compared with solar maximum periods (2013 & 2014) with profound solar activity. The PDF curves that belong to low activity periods are distinct from that belong to high activity periods especially in the power-law region.

The extent of lifetimes of Wp index is similar to the one reported by Wanliss and Weygand (2007) for SYM-H and not like the ones for solar wind parameters. Figure 5 shows the probability distribution curves for threshold levels 500 nT and 750 nT. Thus for all threshold levels viz. 250 nT, 500 nT and 750 nT, the power law region typically spans nearly over three decades, and is distinctly stable irrespective of the chosen threshold values.

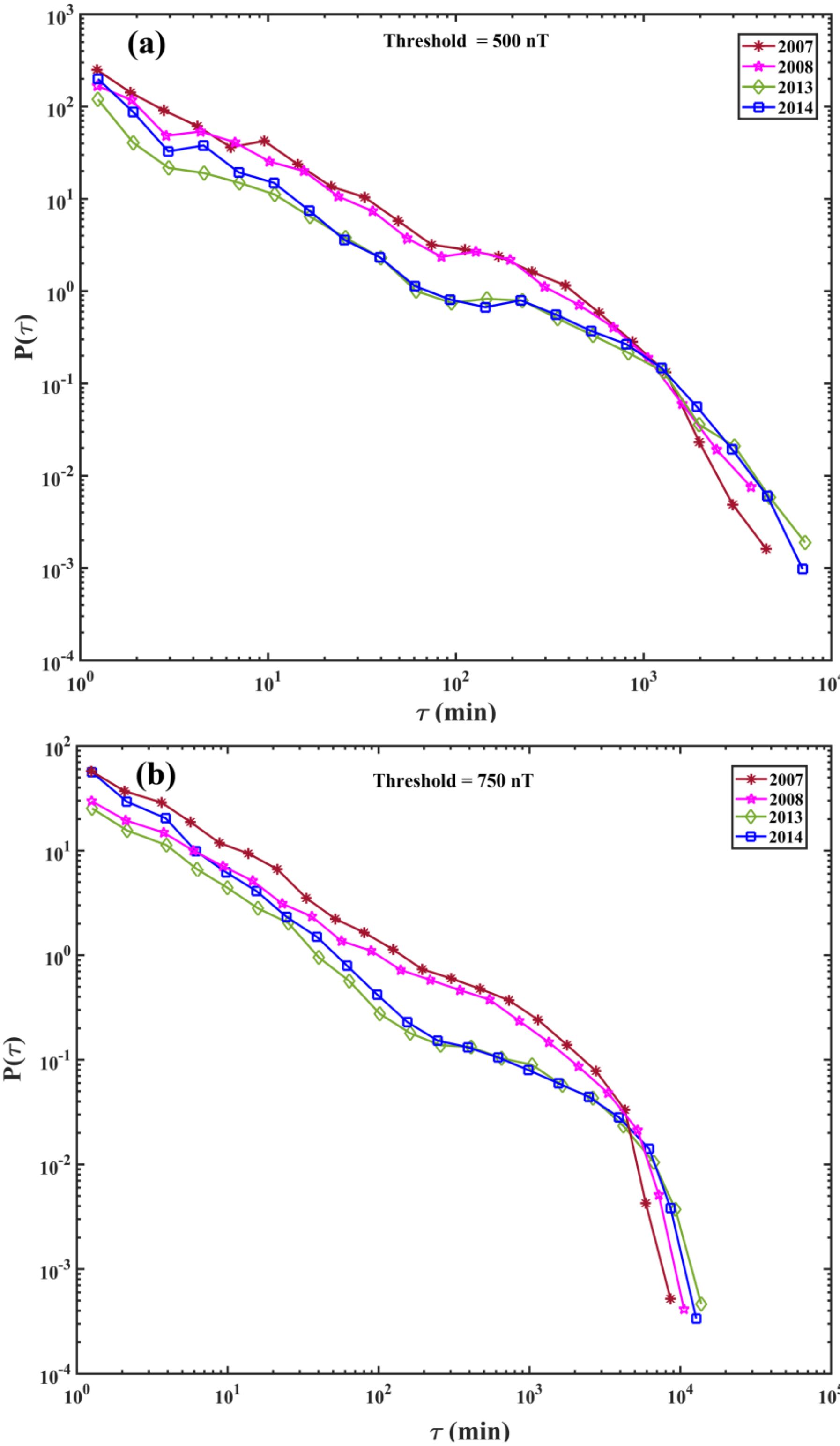

Figure 5. The probability distribution $P(\tau)$ versus waiting time τ for the solar minimum and maximum years for thresholds (a) 500 nT and (b) 750 nT.

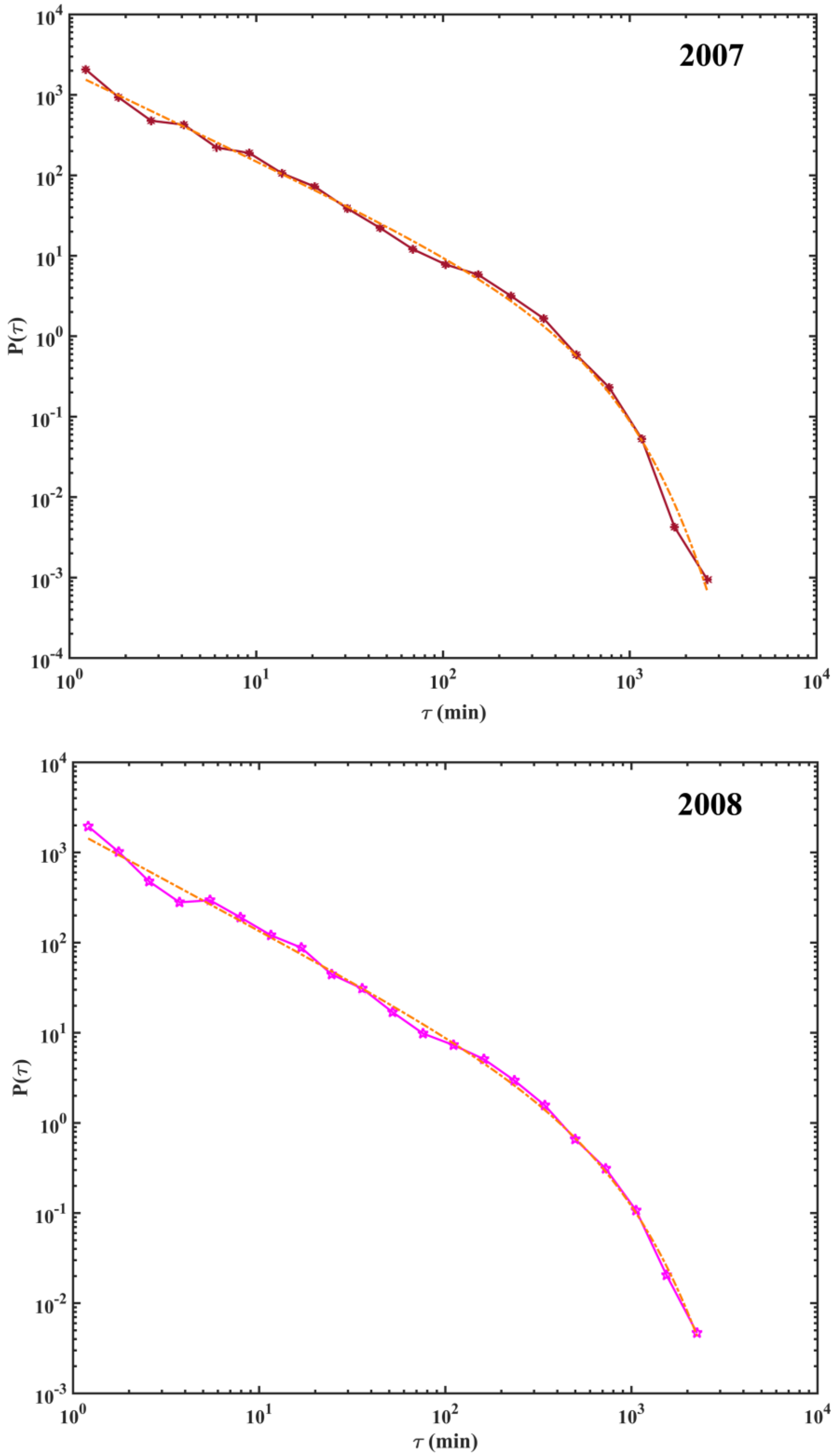

Figure 6. The model fit on the probability distribution curves for the years 2007 and 2008 (for a threshold of 250 nT).

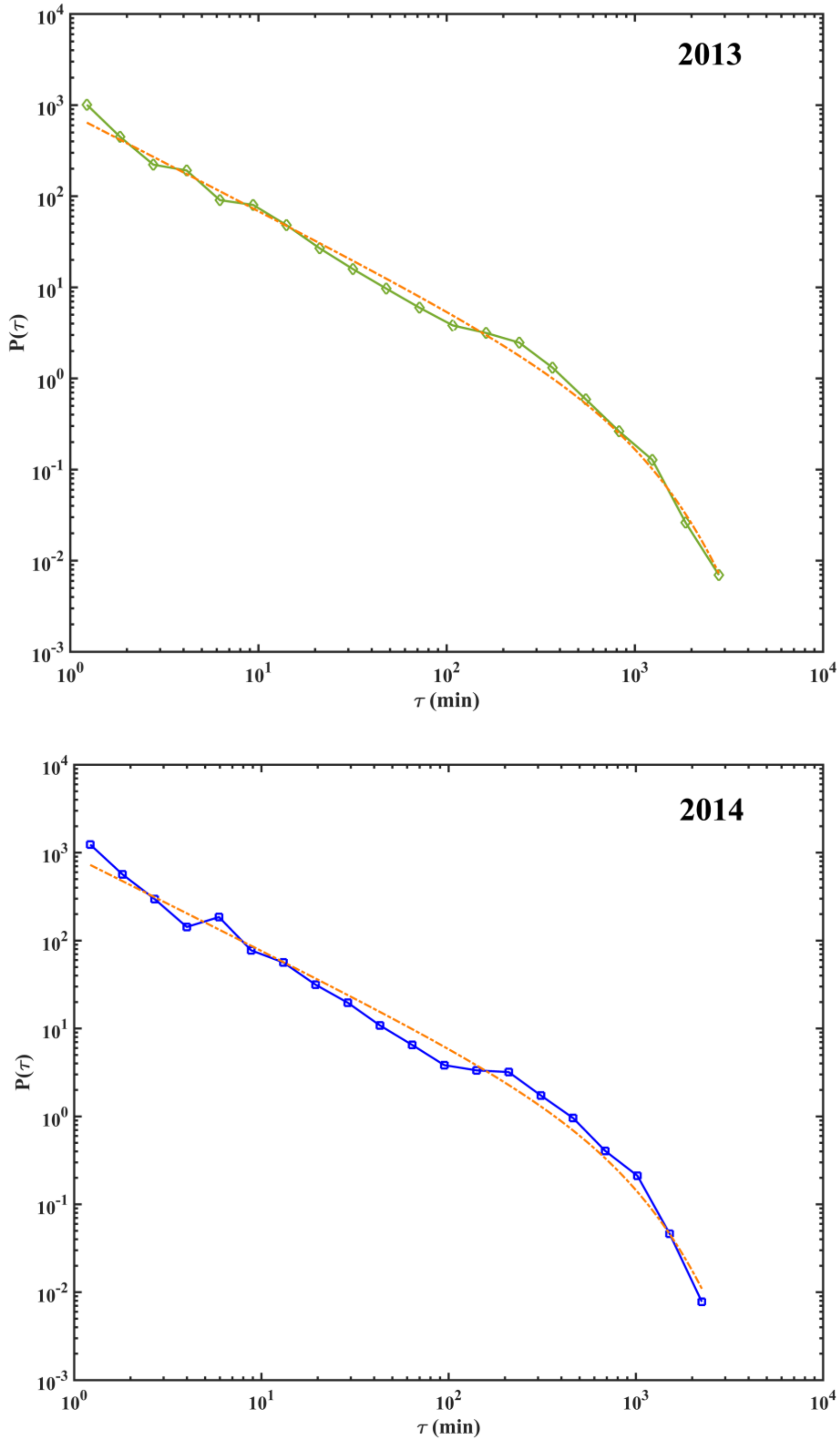

Figure 7. The model fit on the probability distribution curves for the years 2013 and 2014 (for a threshold of 250 nT).

Hence, our a priori assumption regarding the choice of a constant threshold is validated by the steadiness in power-law regime, which in turn draws the conclusion that Wp statistical features could be the result of a SOC system [Wanliss and Weygand, 2007]. But the effect of lognormal component as reported earlier by Freeman et al. (2000) become more pronounced in higher thresholds of 500 nT and 750 nT. In spite of that a good fit with the sum of an inverse power law with an exponential cutoff with an additional lognormal component could not be obtained.

Table 1. The values of fitted parameters for solar maximum and minimum periods

Year	A	ϒ	T_c	R (log value)
2007	1939.9	1.1051	421.61	0.99681
2008	1770.4	1.1112	519.12	0.99779
2013	800.3	1.0622	880.21	0.99560
2014	898.4	1.0644	727.65	0.99058

The probability distribution functions for all the years are fitted with inverse power law with exponential cutoff model described earlier. The figures 6 and 7 shows fitting of the probability distribution curves to the mode function. The values of scaling exponent is given in table 1 where T_c is expressed in minutes. From the values of T_c it is clear that for solar minimum periods, the cutoff occurred earlier at ~ 7 hours in 2007 and at ~ 8.7 hours in 2008. But for solar maximum periods the cutoff was occurred much later ~ 15 hours in 2013 and ~ 13 hours in 2014.

If we consider the waiting time statistics of another prominent geomagnetic index, auroral electrojet index (sample AE index variations for the year 2007 is shown in figure 8), we get probability density functions corresponding to the solar minimum and maximum years as shown in figure 9. From the graphs, it is evident that the AE waiting times have a power law region that stretches to approximately four orders of magnitude, from the least time periods to ~720 minutes for all years. At large waiting times, the power-law begins to break which suggests an exponential roll off.

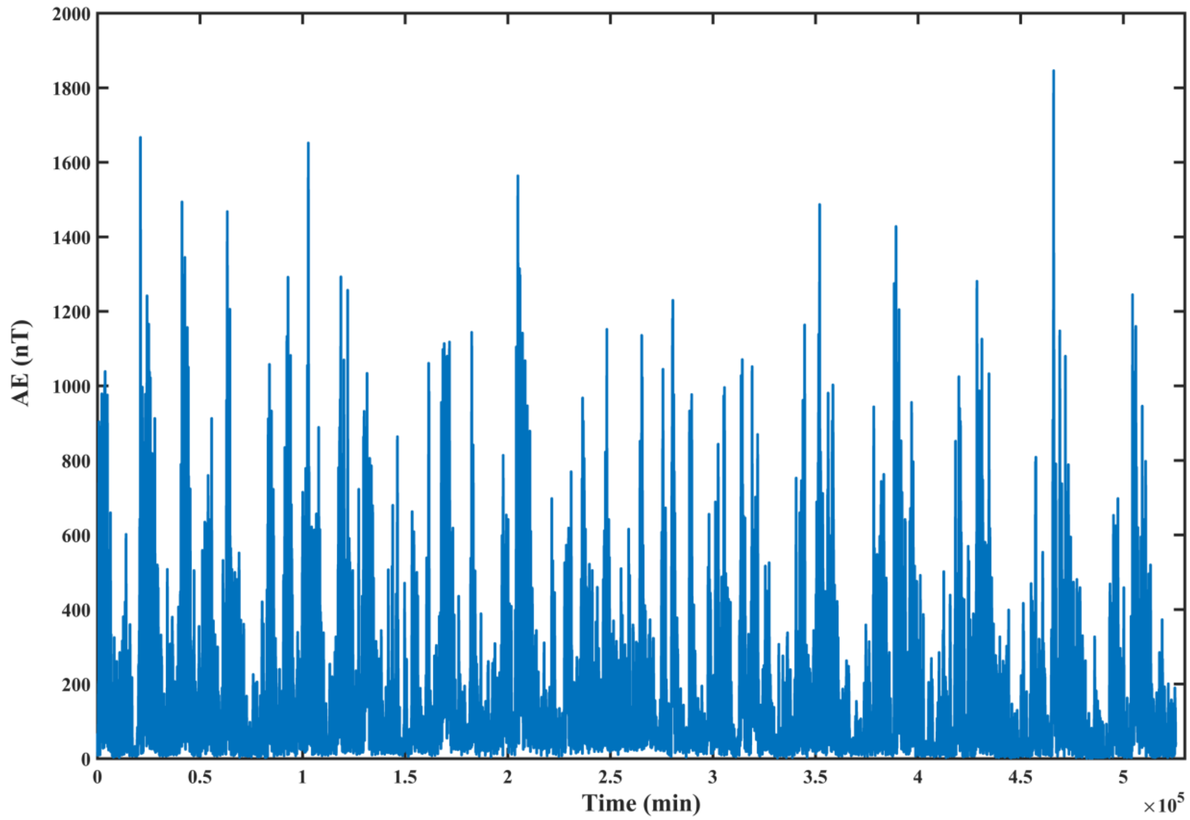

Figure 8. Auroral electrojet index (AE index) data for the year 2007.

But, quite different from that of Wp index, the PDF curves that belong to low activity periods almost overlap on the ones that belong to high activity periods, especially in the power-law region. Thus, unlike Wp index, it is not possible either to recognize any variation that might have contributed by solar activity forcings. The probability distribution functions for all the years are fitted with inverse power law with exponential cutoff model described earlier. The values of scaling exponent are given in table 2. The power law index (Υ) values are consistent except for the year 2014. Even though, the average scaling exponent during solar maximum is less that of minimum, we cannot safely conclude that there is a remarkable variation in power law scaling between both activity times. The values of T_c are also not showing any pattern regarding solar activity periods. Hence, it is certain that the influence of solar activity on Wp index variations is quite different from that of AE index. This goes well with earlier findings on the independence of AE index fluctuations on solar activity as reported earlier by Gopinath (2016), Gopinath and Prince (2017) and Gopinath and Prince (2018).

Table 2. The values of fitted parameters for solar maximum and minimum periods (of AE index data)

Year	A	ϒ	T_c	R (log value)
2007	1017.5	1.1857	3346.3	0.99709
2008	998.21	1.1954	4560.2	0.99869
2013	815.2	1.1893	4104.8	0.99893
2014	805.76	1.0978	1796.7	0.99100

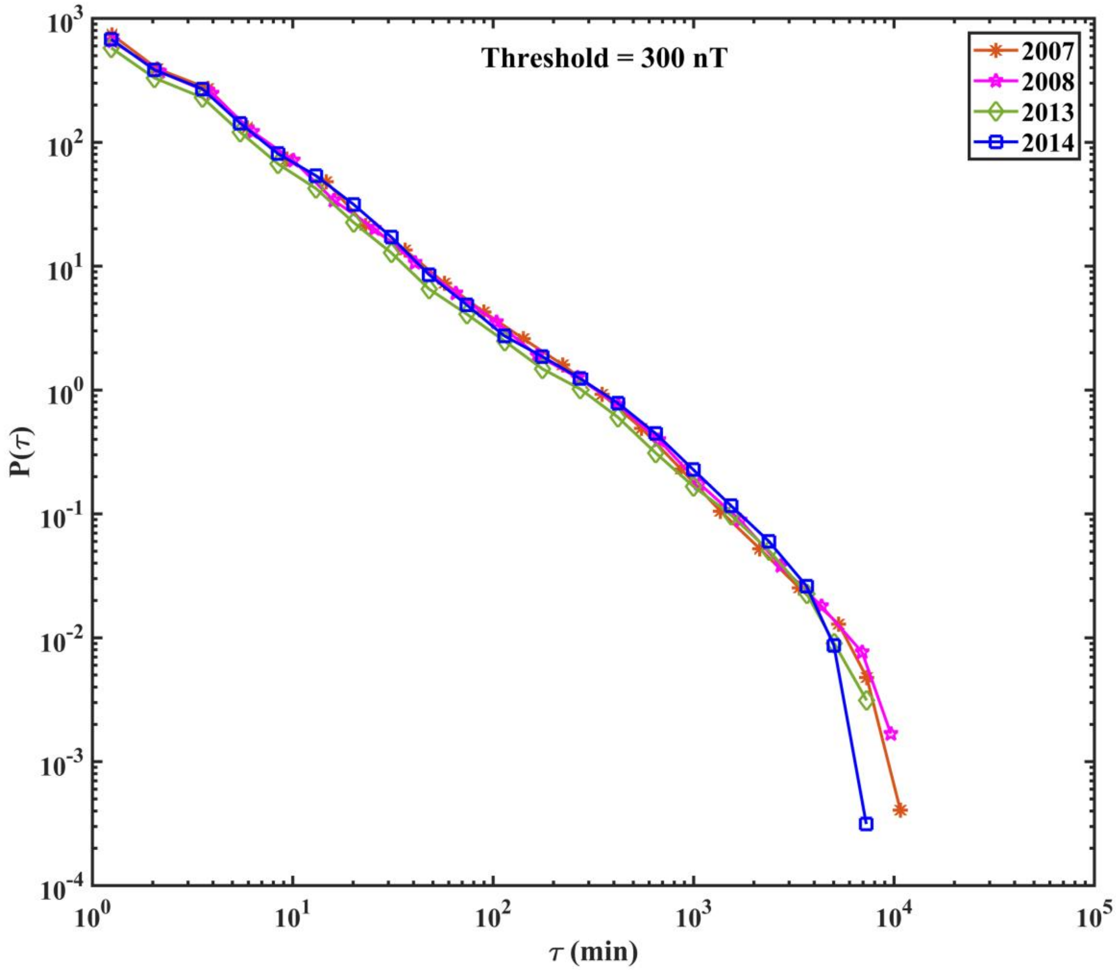

Figure 9. The probability distribution $P(\tau)$ versus waiting time τ of AE indices for the solar minimum (2007 & 2008) and solar maximum (2013 & 2014) for a threshold of 300 nT.

4. CONCLUSION

The waiting time distributions of magnetospheric Wp index has been compared during solar maximum and minimum periods. Thus the variations in power-law scaling of waiting time probability distributions during solar maxima and solar minima are being analyzed.

- The average power-law scaling exponent for Wp index of scaling region for solar maximum periods Υ = **1.0633**.
- The average power-law scaling exponent for Wp index of scaling region for solar minimum periods Υ = **1.10815.**
- It is found that Wp waiting time distributions show power-law scaling with dissimilar average scaling exponents during solar minima and maxima.
- The average power-law scaling exponent for AE index of scaling region for solar maximum periods Υ = **1.14355**.
- The average power-law scaling exponent for AE index of scaling region for solar minimum periods Υ = **1.19055.**
- Even though it is found that AE waiting time distributions show power-law scaling with dissimilar average scaling exponents during solar minima and maxima but from the closeness of probability density functions as well as from the T_c values, we cannot safely conclude that solar activity forcings modulate AE index waiting time properties.

Even though, the average results show dissimilar exponents in solar maxima and minima, it may not completely reflect whether the magnetospheric activity is solar-driven. It could be that solar wind never acts as a direct driver but indirectly influences long-term magnetospheric activities. Since, Wp index is related with Pi2 power, may be solar wind has less control over the triggering of such geomagnetic pulsations. Similar statements can be drawn for the case of AE index too. Hence, more studies of several solar cycles should be conducted to get a clear picture whether Wp index related magnetospheric activities are having any relationship with solar activity driving. But, a major drawback is that the Wp index is available only from 2005, hence previous solar cycle variations cannot be analyzed.

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