Unique solvability of a warm cloud parcel model, avoiding saturation adjustment

Nikolas Porz, P. Spichtinger, M. Hanke-Bourgeois

Johannes Gutenberg-Universität, Mainz, Germany

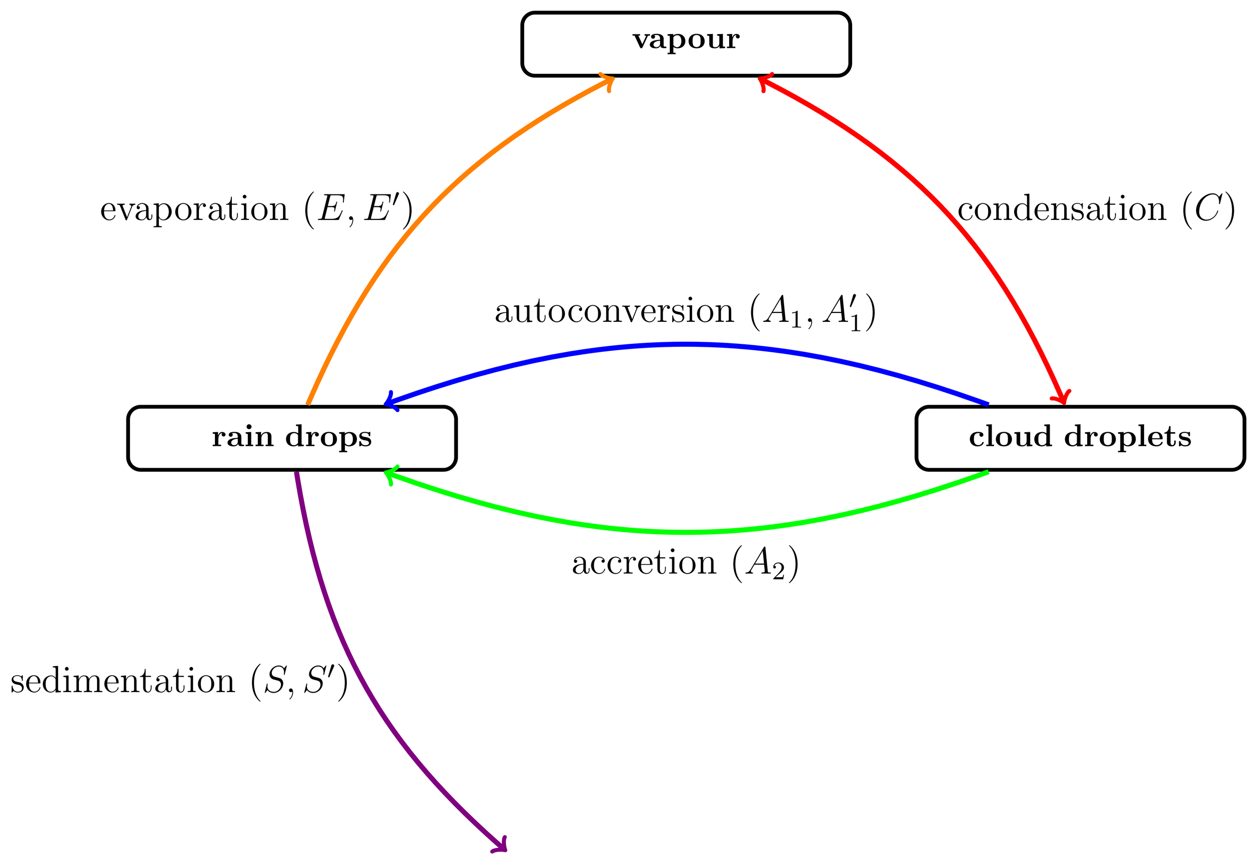
porz@uni-mainz.de

Introduction

Numerical cloud models consist of differential equations which describe micro physical processes. These equations come with parameters, some of which cannot be measured, but need to be determined using inverse methods. For this we develop efficient algorithms and a benchmark cloud model for testing these algorithms.

Cloud Model

We consider a vertical stack of volume boxes (a column) with specific quantities of air and water in different phases. The column is considered from a Lagrangian point of view while moving with the surrounding flow. During elevations each box may change its height so that its total mass of air is conserved at all times. The column extends up to a height where precipitation inflow from the top can be ignored. The loss of rain drops at the bottom of the column (precipitation on the ground) is our reference observational data. Within each box



- water mass occurs as vapor, small droplets, and rain drops
- we keep book of mass *and* number of rain drops in contrast to, e.g., ICON or IFS
- thermodynamics is affected by vertical movement and latent heat production
- precipitation may enter from the top and drop out at the bottom.

Model Equations

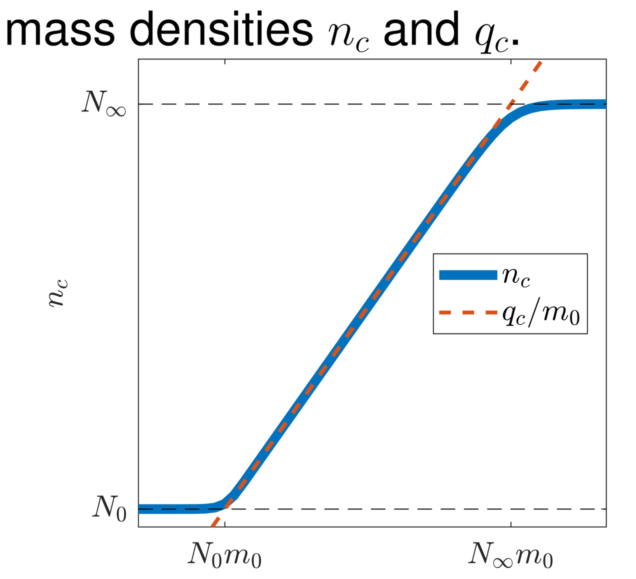
$$
\begin{aligned}
\dot{q}_v &= -\, c q_s n_c^{2/3} q_c^{1/3} && - q_{s-}\left(e_1 q_r^{1/3} n_r^{2/3} + e_2 v_t^{1/2} q_r^{1/2} n_r^{1/2}\right) \\
\dot{q}_c &= c q_s n_c^{2/3} q_c^{1/3} - a_1 q_c^2 - a_2 v_t q_c q_r^{2/3} n_r^{1/3} && \\
\dot{q}_r &= a_1 q_c^2 + a_2 v_t q_c q_r^{2/3} n_r^{1/3} && + q_{s-}\left(e_1 q_r^{1/3} n_r^{2/3} + e_1 v_t^{1/2} q_r^{1/2} n_r^{1/2}\right) - v_q q_r \\
\dot{n}_r &= a_1' n_c q_c && + q_{s-}\left(e_1' q_r^{1/3} n_r^{2/3} + e_2' v_t^{1/2} q_r^{1/2} n_r^{1/2}\right) - v_n n_r \\
\dot{p} &= -\, g \rho w && \\
\dot{T} &= -\, \gamma w + \frac{L}{c_p}\left(c q_s n_c^{2/3} q_c^{1/3} - q_{s-}\left(e_1 q_r^{1/3} n_r^{2/3} + e_2 v_t^{1/2} q_r^{1/2} n_r^{1/2}\right)\right) &&
\end{aligned}
$$

with the saturation $q_s = q_v - q_{vs}$ and $q_{s-} = \min\{0, q_v - q_{vs}\}$

Cloud droplets

A key aspect is a novel treatment of droplet activation without artificial switches and without saturation adjustment. For this we have introduced the following nonlinear relation (1) between the droplet count and mass densities n_c and q_c.

(1) $$n_c = \frac{q_c N_\infty}{q_c + N_\infty m_0} \coth \frac{q_c}{N_0 m_0}$$



Droplet count as a function of droplet mass

With a proper choice of the associated parameters m_0, N_0, and N_∞ this provides a means to emulate Köhler theory of droplet activation.

Cloud process terms

- Autoconversion $(A_1,\ A_1')$: two cloud droplets collide and form a rain drop
- Accretion (A_2): a cloud droplet and a rain drop collide and form a larger rain drop
- Condensation (C): cloud droplets form from water vapor and CCN
- Evaporation $(E,\ E')$: drops grow or shrink due to diffusion of water vapor
- Sedimentation $(S_{\text{out}},\ S_{\text{out}}')$: droplets fall due to gravity; frictional forces act against the gravity force

Cloud Model Implementation

For the droplet mass density q_c we use a differential equation which is similar to those of other cloud models, and which assumes the form

(2) $$\dot{q}_c = c n_c^{2/3} q_c^{1/3} - a_1 q_c - a_2 q_c^2$$

where c, a_1, and a_2 are continuous expressions of the other model variables. The physical solution of (2) can be solved numerically with an implicit Euler type scheme.

Numerical computation order

To be specific, starting from the current values $q_{v,i}$, $q_{c,i}$, $q_{r,i}$, $n_{r,i}$, $n_{c,i}$, p_i, T_i, and ρ_i in some given box at time $t_i = i\tau$, we first solve

(3) implicit $$q_{r,i+1/2} = q_{r,i} - \tau\big(E(q_{r,i+1/2}, n_{r,i+1/2}) + S_{\text{out}}(q_{r,i+1/2})\big),$$

(4) $$n_{r,i+1/2} = n_{r,i} - \tau\big(E'(q_{r,i+1/2}, n_{r,i+1/2}) + S_{\text{out}}'(n_{r,i+1/2})\big),$$

(5) $$q_{c,i+1} = q_{c,i} + \tau\big(C(q_{c,i+1}) - A_1(q_{c,i+1}) - A_2(q_{c,i+1}, q_{r,i+1/2}, n_{r,i+1/2})\big),$$

and finally, we update the new values of q_r and n_r as

(6) explicit $$q_{r,i+1} = q_{r,i+1/2} + \tau\big(A_1(q_{c,i+1}) + A_2(q_{c,i+1}, q_{r,i+1/2}, n_{r,i+1/2}) + S_{\text{in}}\big),$$

(7) $$n_{r,i+1} = n_{r,i+1/2} + \tau(A_1'(q_{c,i+1}) + S_{\text{in}}').$$

while in (6) and (7) the inflows S_{in} and S_{in}' are given by the corresponding outflows of the neighboring box, which have been determined in steps (3) and (4). Note that this allows for a straightforward SIMD parallelization (single instruction, multiple data) of the column model.

Analytical Solution Properties

The model equations with the non-Lipschitz right-hand side allow for nontrivial smooth solutions. We proved under mild assumptions on the external forcing that this system of equations has a unique physically consistent solution, i.e., a solution with a nonzero droplet population in the supersaturated regime. To this end we rewrite the system to a Fuchs-type equation

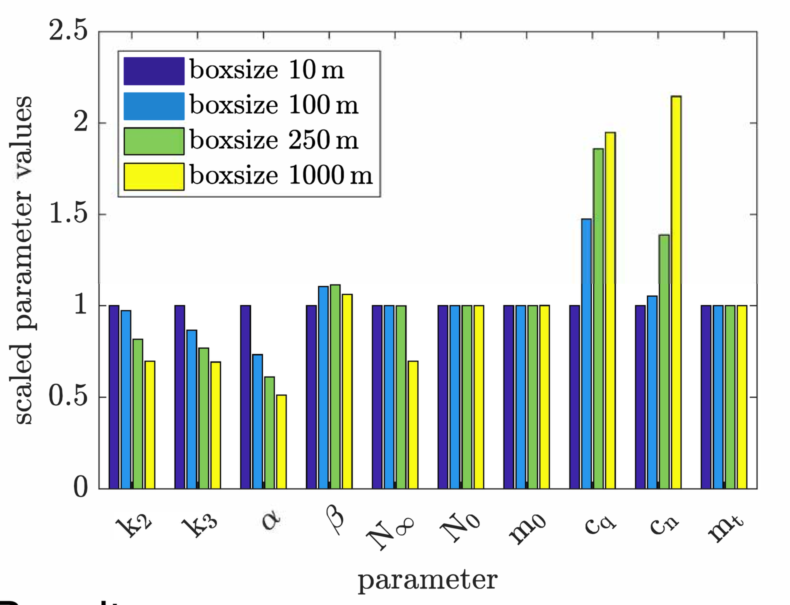
$$t\frac{\mathrm{d}y}{\mathrm{d}t} + Ay = tF(y,t)$$

via a substitution with leading order time dependencies, which is a tested approach for spherically symmetric static fluid bodies with a given equation of state, cf. [3].

Inverse Methods

The determination of model parameters, e.g. N_∞ in (1), is an ill-conditioned or even ill-posed inverse problem which requires sophisticated numerical algorithms, e.g., Levenberg-Marquardt regularization. These methods minimize the error between model output and observations (e.g., precipitation over time). Currently our model comes with ten parameters to be estimated from these data.

Parameter dependence on the numerical resolution



In a numerical study we have investigated how model parameters depend on the resolution of the model. Therefore we have

1. set up a column with constant updraft and 1 km vertical extension,
2. generated data with a high resolution run,
3. fitted the parameters in lower vertical resolution runs to the data.

Results:

- the parameters α, k_2 and k_3 decrease with coarser resolution
- the sedimentation parameters c_q and c_n increase with coarser resolution
- some parameters are unaffected by the resolution

Outlook

- implementation as a micro physics routine in ICON
- parameter estimations on a multitude of meteorological conditions (e.g. slow/fast updrafts) and with external data sets
- extension of the model to the ice phase

References

[1] M. Hanke and N. Porz. "Unique Solvability of a System of Ordinary Differential Equations Modeling a Warm Cloud Parcel". In: *SIAM Journal on Applied Mathematics* 80.2 (2020).

[2] N. Porz, M. Hanke, M. Baumgartner, and P. Spichtinger. "A model for warm clouds with implicit droplet activation, avoiding saturation adjustment". In: *Mathematics of Climate and Weather Forecasting* 4 (2018).

[3] A. D. Rendall and B. G. Schmidt. "Existence and properties of spherically symmetric static fluid bodies with a given equation of state". In: *Classical and Quantum Gravity* 8.5 (1991).