Short-term retrospective forecasting of earthquakes based on temporal variations of the b-value of the magnitude-frequency distribution

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Introduction and purposes

Earthquake forecasting has proved to be a perfidiously complex problem for Seismology and will probably take a long time to be resolved. In order to reach the final solution to the problem, it is important that researches to continues and progresses in this field.

- In this work we tried to understand whether **b-value temporal variations** of the magnitude-frequency relationship can be considered as **precursor signals** of potentially destructive **seismic events**.

- The idea of testing the b-value of the magnitude-frequency relationship as a precursor of seismic events has already been proposed in literature (Gulia et al. 2016, GRL, 43, 1001-1008; Smith, W. D., 1986, GJRAS, 86, 815-838;).

- **Laboratory experiments** seem to show that this parameter is **inversely related** to the accumulation of differential **stress in crustal rocks** (Scholz, C. H. 1968, BSSA, 58, 399-415). Specifically, when the parameter decreases, the differential stress increases until the fracture happens.

Introduction and purposes

- These observations provide the basis for the definition of an **alarm-based retrospective forecasting algorithm** based on the **temporal variations of b-value**.

- The **forecasting model** has been tested in **Italy** and **Southern California** and compared with another alarm-based model that uses the occurrence of foreshocks (magnitude range $4.4 \leq M \leq 4.6$ for Italy and $4.0 \leq M \leq 4.2$ for southern California) as a precursor (Gasperini et al. 2020, submitted).

- The models are evaluated using **Molchan's test**, which is specific for **assessing the predictive ability** of alarm-based prediction models (Zechar, J. D. et al. 2008, GJI, 172, 715-724).

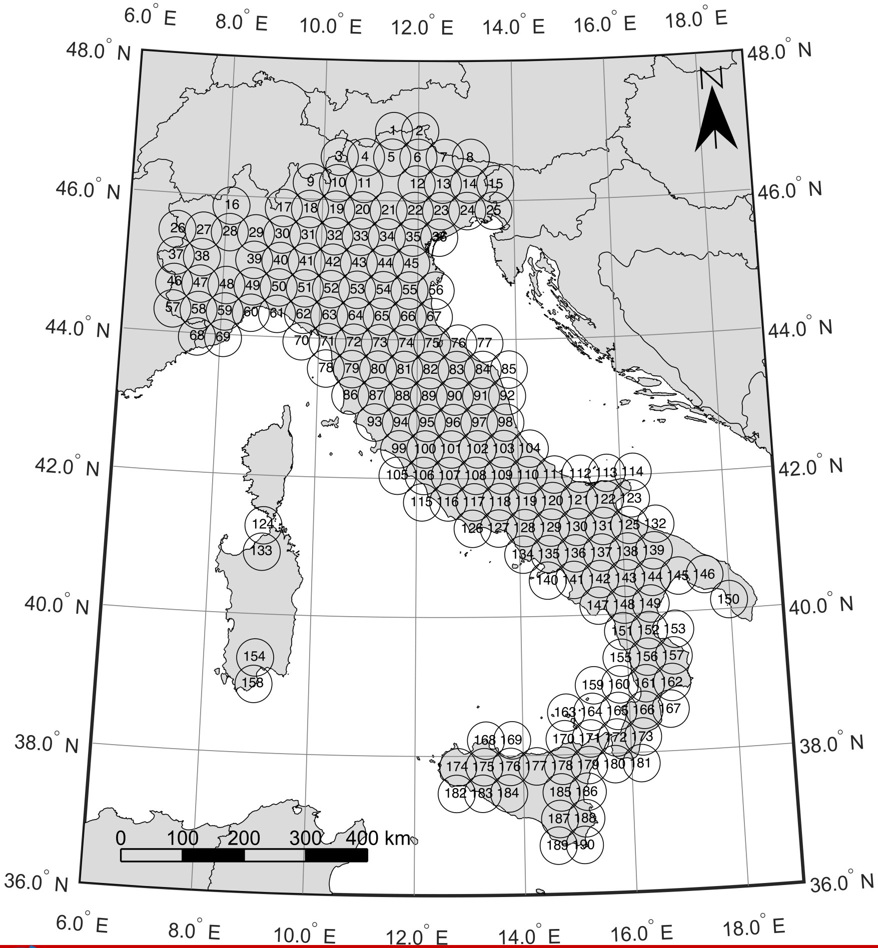
Setting up the forecasting hypothesis: definition of test space domains

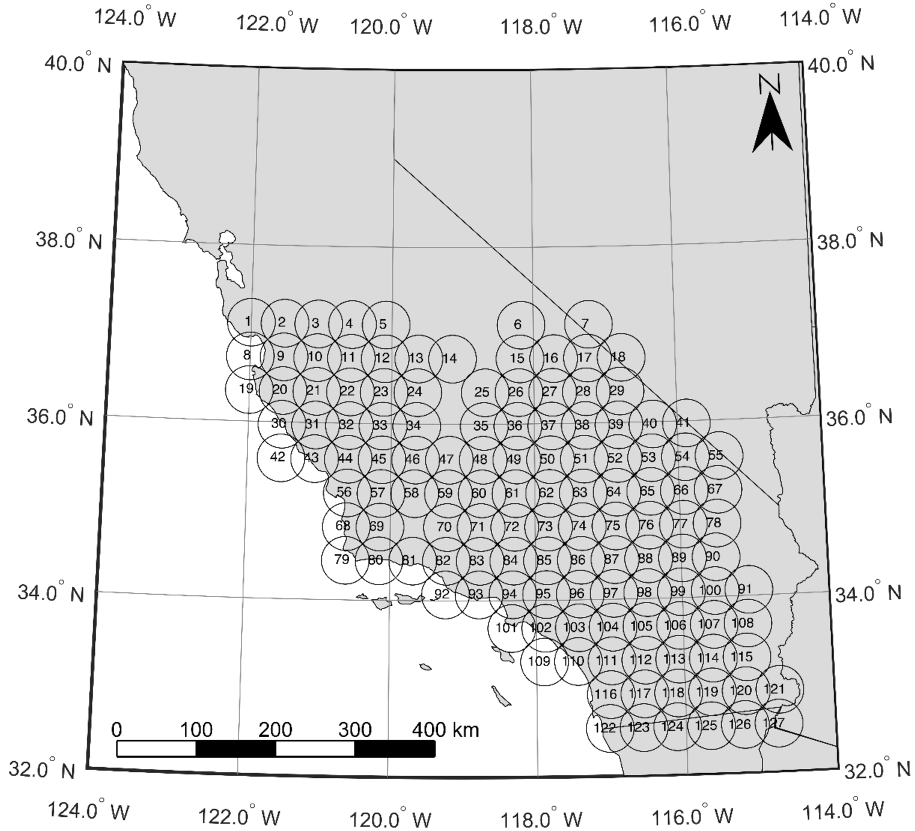
- As **reference area** we consider a regular tessellation of both territories made of partially **overlapping circles (CA)** with fixed radius $\boldsymbol{R = 30\ km}$

- The centers of the circles are spaced by $\mathbf{D = 30\sqrt{2}}$ both in longitude and in latitude

Italy	Southern California
• Initial CA, centered at Lat 47º and Lon 77º • Only circles in which at least one Mw≥4.0 earthquake occurred between 1600 and 1959 are selected (from CPTI15 catalog). • **190 CAs considered**	• Initial CA, centered at Lat 37,5º and Lon -122º • Only circles in which at least one Mw≥3.5 earthquake occurred between 1932 and 1989 are selected (from SCSN earthquake catalog). • **127 CAs considered**

Setting up the forecasting hypothesis: definition of test space domains





(Gasperini et al. 2020, submitted)

Seismic catalogues for analysis:

For Italy

- HOmogenized instRUmental Seismic catalog (HORUS) (Gasperini et al., 2013, BSSA, 103/4, 2227-2246)
- **The forecasting experiment was conducted considering earthquakes from 1995 to 2020**
- We estimated an average magnitude of completeness of the catalogue from 1995 to 2020 of **Mc=1.7**

For Southern California

- Southern California Seismic Network earthquake catalog (Hutton et al., 2010, BSSA, 100, 423-446)
- **The forecasting experiment was conducted considering earthquakes from 1990 to 2020**
- We estimated an average magnitude of completeness of the catalogue from 1990 to 2020 of **Mc=1.5**

Setting up the forecasting hypothesis: target events to forecast

Target events to forecast

- **All earthquakes (not declustered)** → **M≥5.0, 5.5, 6.0**
- **First earthquakes** of each **seismic sequence (declustered)** → **M≥5.0, 5.5, 6.0**

- We checked the **model ability** to predict only the **first shocks (M≥5.0, 5.5, 6.0)** of the sequences by considering a declustered set of **target events**.

- The **declustering** was made by eliminating those events occurred within a spatial distance R=30 km and a time window of six months (0.5 years) after another **target event**, even if they are larger than the **first** main **shock of the sequence**.

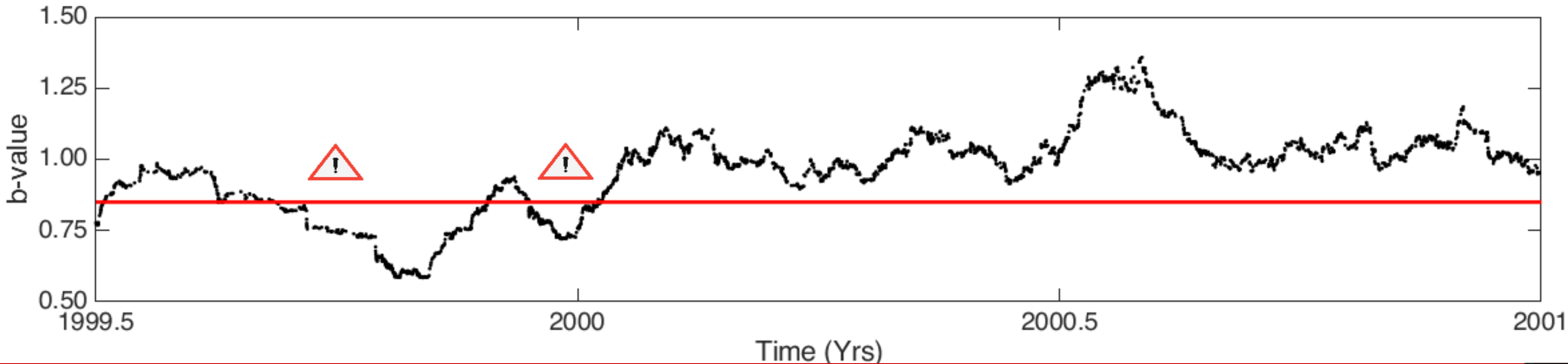
Setting up the forecasting hypothesis: definition of alarms

- For each CA the b-value temporal behavior was analyzed considering events windows of 100 events (Marzocchi and Sandri, 2003, AoG, 46, 1271-1282)

$$b = \frac{Log(e)}{\bar{M} - (M_c - \Delta M/2)}$$

$\bar{M}$: average magnitude; ***Mc:*** *completeness magnitude; ΔM: magnitude binning (typically 0.1)*

- We issue an **alarm** of duration **Δt** within a circular area (CA) of radius R every time the **b-value falls below** the threshold value (chosen through an **optimization procedure**) of **b=0.85**.

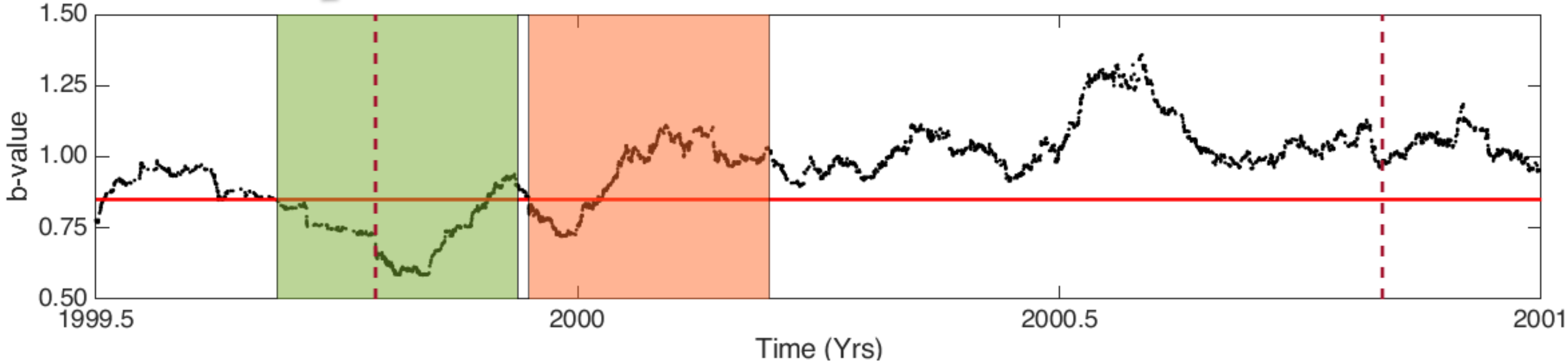


Setting up the forecasting hypothesis: definition of "hit" or "miss" target

A **target earthquake** is considered **"successfully predicted"** if it occurs in a CA **during an alarm window**

A **false alarm** is the case in which **no target earthquake** occurs in an **alarm time window**.

A **target earthquake** is considered **"missed"** if it occurs **outside** any **alarm window**.



Retrospective evaluations: Molchan test

(Molchan, 1990, 1991)

- The experiments are conducted by varying the alarm window Δt from 1 second to the total duration of the seismic catalogue.

- We computed the **miss rate** $\boldsymbol{\nu}$:

$$\boldsymbol{\nu = (N - h)/N}$$

*(where **h** is the numb. of target events successfully forecasted and **N:** total n. of target events)*

- For each CA we computed the **fraction of time occupied by alarms** $\boldsymbol{\tau_c}$:

$$\boldsymbol{\tau_c = \frac{d_c}{T}} ; \quad d_c = \bigcup \Delta t = n\Delta t - \sum \cap t_s$$

*(where **dc** is temporal duration of alarms within one circular area; **T** is total duration of the forecasting experiment and* $\cap \boldsymbol{t_s}$ *is time intersections between alarm windows)*

- We computed the **overall fraction of space-time occupied by alarms** as the average of $\boldsymbol{\tau_c}$ of all CAs:

$$\boldsymbol{\tau_u = \sum_{1}^{M} \tau_c \Big/ M}$$

*(where **M** is the n. of Circular Areas CA)*

Retrospective evaluations: Molchan test

(Molchan, 1990, 1991)

- Following Shebalin et al. (2011) we also computed the so-called **fraction of space-time occupied by alarms** by **weighting** each **alarm** with the long-term seismicity (λ_{ave}) within each CA.

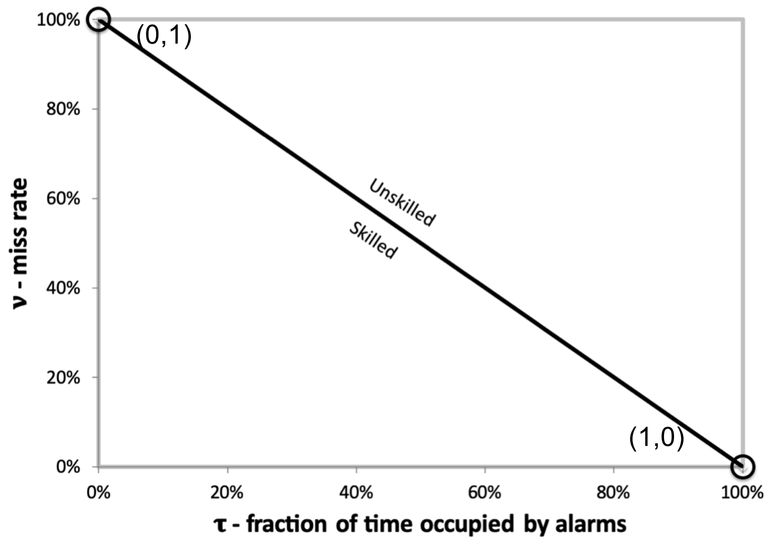
$$\tau_w = \frac{\sum \lambda_{ave} \tau_c}{\sum \lambda_{ave}}$$

- The **Molchan error diagram** consists of a plot of the miss rate ν as a function of the fractions of space-time occupied by alarms τ (τ_u or τ_w).

- If $\tau = 0$ then $\nu = 1$ while if $\tau = 1$ then $\nu = 0$. A score on **the diagonal line** connects the points (0,1) and (1,0) indicates a purely random forecasting method.

- **Confidence limits** of the score can be computed by the binomial distribution : $p = \binom{N}{h} (\tau)^h (1-\tau)^{N-h}$

- If the **score curve** (τ, ν) is well **below the diagonal line**, the forecasting **model** has some **forecasting ability**.



Retrospective evaluations: Area Skill score
(Zechar and Jordan, 2008, 2010)

- The **Area Skill score (AS)** is an **index** of the **performance** of an alarm-based forecasting method.
- The AS is calculated as the integral of the success rate function $1 - \nu_f(\tau)$ normalized to the alarm space-time coverage τ so that its value ranges between 0 and 1:

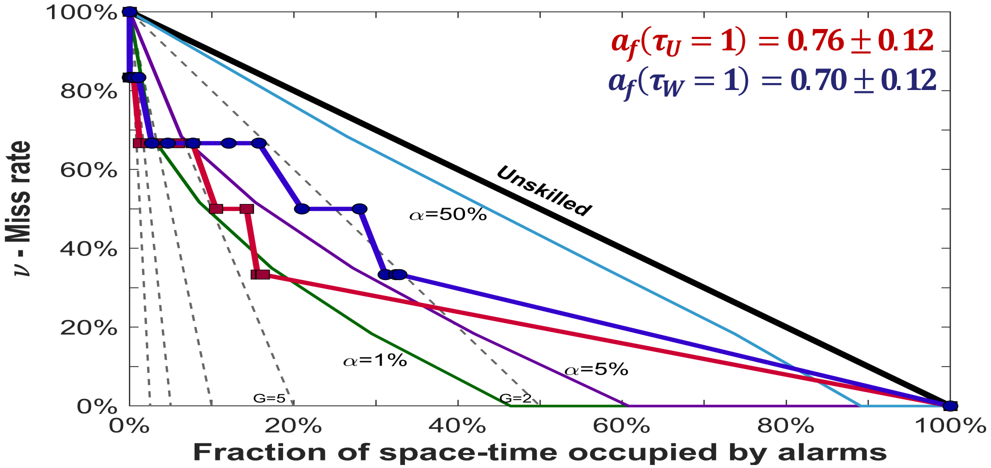
$$a_f(\tau) = \frac{1}{\tau}\int_0^{\tau} [1 - \nu_f(t)]\, dt$$

- The **better the statistics**, the **better the model's** forecasting **performance**.
- The expected value of the AS score for a purely random method is given by:

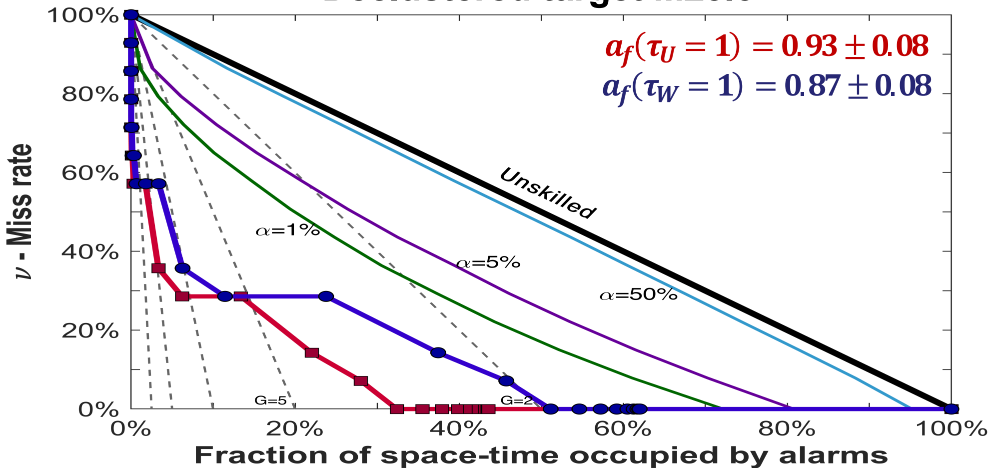
$$\langle a_f(\tau) \rangle = \frac{1}{\tau}\int_0^{\tau} [1 - (1 - t)]\, dt = \frac{1}{\tau}\frac{\tau^2}{2} = \frac{\tau}{2} = 0.5$$

Molchan error diagrams for Italian test region

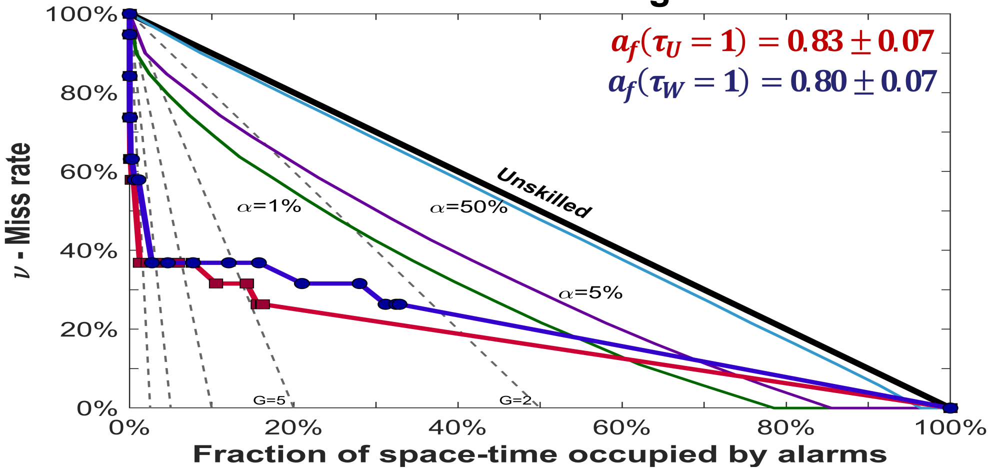
b-value temporal variation model
Declustered target M≥5.5



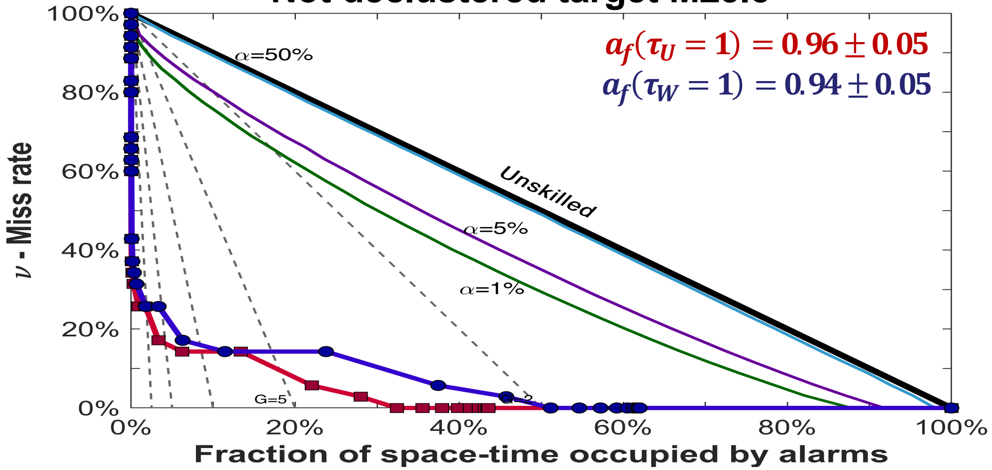
Foreshock model
Declustered target M≥5.5



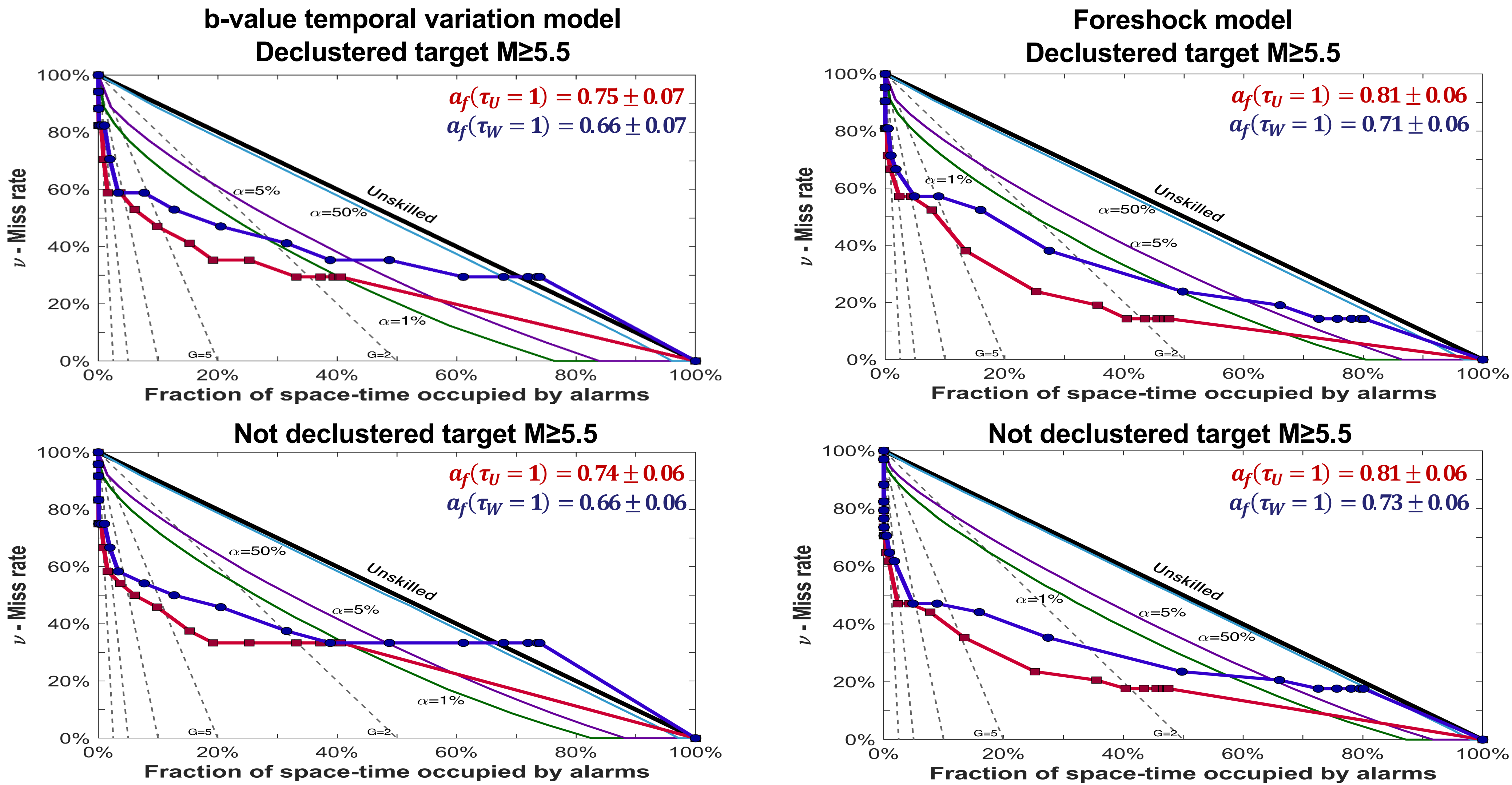
Not declustered target M≥5.5



Not declustered target M≥5.5



Molchan error diagrams for Southern California test region



Outcomes of the retrospective evaluations

- Both models evaluated and compared are characterized by scores well below the diagonal line.

- The **Area Skill score** (Zechar and Jordan, 2008) methods would **confirm** that such approaches clearly **overperform** a purely random method with high or very **high confidence**.

- We found that the forecasting ability remains high even if results are lower than considering all main shocks.

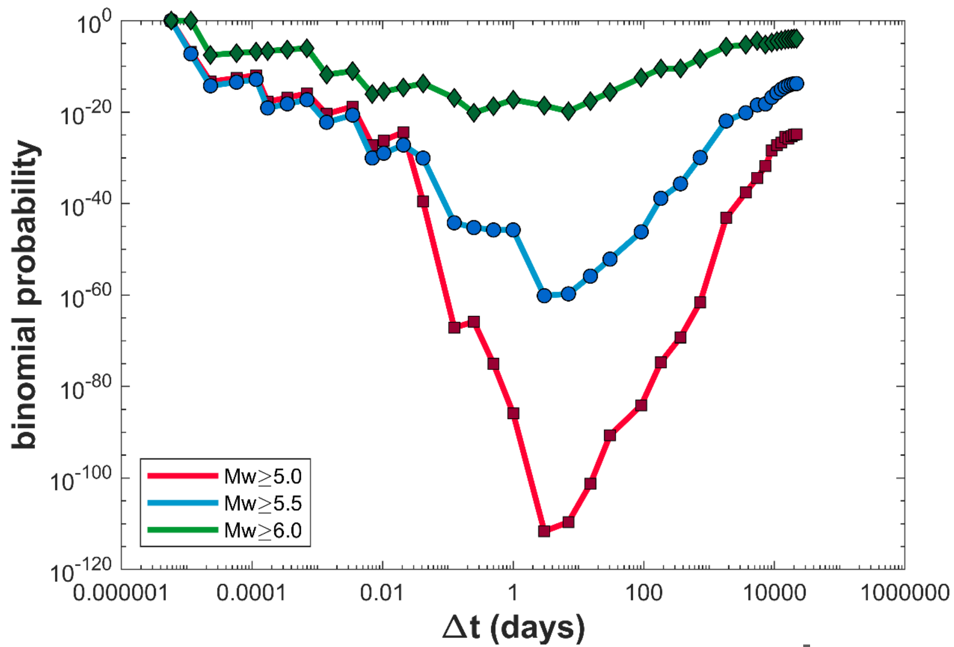
- Overall we can conclude that **both approaches** have **fair performances**, in particular the forecasting **model based** on the occurrence of **foreshock as a precursor**.

- The results are also similar for the other target magnitudes (M≥5.0, 6.0) .

Operational application of forecasting models

- For an **operational application** of forecasting models it is appropriate to determine the **alarm Δt** for which the models are **most efficient** in terms of **seismic risk mitigation**.

- The trend of binomial probability as the alarm time (Δt) changes, expresses the probability that the number of successful forecasts is due to pure chance.



- The **lower the probability**, the **higher the strength** of the **forecast**.
- In general, probabilities are relatively low within a wide range going from a one day to some months.
- We examine here as an example the choice of **Δt = 3 months** (0.25 years). This choice, in most cases, results in a fairly trade-off between a good efficiency in most cases and a narrow space-time fraction covered by alarms **τ ≈1-3%,** which might even be considered acceptable by the involved population.

Retrospective forecast results considering Δt =3 months

Foreshock model

	Italy Not declustered						Southern California Not declustered					
Target Magnitude	**≥5**		**≥5.5**		**≥6**		**≥5**		**≥5.5**		**≥6**	
Forecasted/total shocks	55/98	56%	26/35	74%	7/10	70%	54/105	51%	18/34	53%	6/13	46%
Successful/total alarms	115/617	18.60%	72/617	11.70%	30/617	4.90%	286/916	31.22%	120/916	13.10%	34/916	3.71%
Space-time fraction occupied by the alarms τ_u, τ_w	0.91%	1.79%	0.91%	1.79%	0.91%	1.79%	2.34%	4.88%	2.34%	4.88%	2.34%	4.88%
	Declustered						**Declustered**					
Forecasted/total shocks	11/48	23%	6/14	43%	4/7	57%	17/51	33%	9/21	43%	3/10	30%
Successful /total alarms	44/617	7.1%	9/617	1.46%	8/617	1.30%	69/916	7.53%	62/916	6.77%	7/916	0.76%
Space-time fraction occupied by the alarms τ_u, τ_w	0.91%	1.79%	0.91%	1.79%	0.91%	1.79%	2.34%	4.88%	2.34%	4.88%	2.34%	4.88%

b-value model

	Italy Not declustered						Southern California Not declustered					
Target Magnitude	**≥5**		**≥5.5**		**≥6**		**≥5**		**≥5.5**		**≥6**	
Forecasted/total shocks	19/51	37%	12/19	63%	4/6	67%	32/85	38%	11/24	46%	5/9	56%
Successful/total alarms	81/448	18.08%	35/448	7.81%	18/448	4.02%	332/1664	19.95%	190/1664	11.42%	53/1664	3.19%
Space-time fraction occupied by the alarms τ_u, τ_w	1.25%	2.71%	1.25%	2.71%	1.25%	2.71%	3.66%	7.68%	3.66%	7.68%	3.66%	7.68%
	Declustered						**Declustered**					
Forecasted/total shocks	5/19	26%	2/6	33%	2/4	50%	12/54	22.22%	7/17	41%	3/8	37.50%
Successful /total alarms	46/448	10.27%	2/448	0.45%	2/448	0.45%	129/1664	7.75%	122/1664	7.33%	4/1664	0.24%
Space-time fraction occupied by the alarms τ_u, τ_w	1.25%	2.71%	1.25%	2.71%	1.25%	2.71%	3.66%	7.68%	3.66%	7.68%	3.66%	7.68%

Retrospective forecast results considering Δt =3 months for Mw≥5.5 in Italy

- This table reports the **earthquakes successfully retrospectively forecast** with **success** with both models analyzed:

Year	Month	Day	Lat	Lon	Mw	Epicentral area	Foreshock	b-value
1962	**8**	**21**	**41.233**	**14.933**	**5.7**	**Irpinia**	***Success***	**-**
1968	**1**	**15**	**37.7**	**13.1**	**5.7**	**Valle del Belice**	***Success***	**-**
1976	**5**	**6**	**46.25**	**13.25**	**6.5**	**Friuli**	***Success***	**-**
1979	*9*	*19*	*42.717*	*12.95*	*5.8*	*Valnerina*	*Miss*	-
1980	*11*	*23*	*40.8*	*15.367*	*6.8*	*Irpinia-Basilicata*	*Miss*	-
1984	*4*	*29*	*43.204*	*12.585*	*5.6*	*Umbria settentrionale*	*Miss*	-
1984	*5*	*7*	*41.666*	*13.82*	*5.9*	*Monti della Meta*	*Miss*	-
1990	**5**	**5**	**40.65**	**15.882**	**5.8**	**Potentino**	***Success***	**-**
1997	**9**	**26**	**43.023**	**12.891**	**5.7**	**Appennino umbro-marchigiano**	***Success***	***Success***
1998	*9*	*9*	*40.06*	*15.949*	*5.5*	*Appennino lucano*	*Miss*	*Miss*
2002	*10*	*31*	*41.717*	*14.893*	*5.7*	*Molise*	*Miss*	*Miss*
2009	**4**	**6**	**42.342**	**13.38**	**6.3**	**Aquilano**	***Success***	***Success***
2012	*5*	*20*	*44.896*	*11.264*	*6.1*	*Pianura Emiliana*	*Miss*	*Miss*
2016	*8*	*24*	*42.698*	*13.234*	*6.2*	*Monti della Laga*	*Miss*	*Miss*

Conclusions

- We have developed a **forecasting algorithm** for **potentially destructive earthquakes** (M≥ 5.0, 5.5, 6.0) occurring on the continental territory of **Italy** and **Southern California** based on the **temporal variations of b-value** within each **circular area** of the tessellation.
- The retrospective testing of such hypothesis based on the analysis of the **HOmogenized instRUmental Seismic catalog (HORUS)** and **Southern California Seismic Network earthquake catalog (SCSN)** show **promising results**:
  - **Molchan diagram** and **area skill score** criteria indicate a very high performance.
  - With a **Δt = 3 months**, retrospectively **70%** of the **not-declustered target earthquakes** in **Italy** and **50%** of the earthquakes in **Southern California** are forecast with an **alarm space-time coverage** between **1-3%.**
  - The first main shocks of each seismic sequence (**declustered**) are forecasted with rates **25-43%** with larger frequencies for larger magnitudes.

Conclusive observations and future developments

- The **temporal variations of b-value** of the magnitude-frequency relationship seems to have **good qualities** as **a precursor** of potentially destructive **seismic events**.
- The **forecasting experiment** based on the **occurrence of strong foreshock** ,described in detail in Gasperini et. al., 2020 (submitted), provide better results.
- Both **retrospective forecasting** models **perform better for Italy**.
- Given the promising results of this simple experiment, in the near future we will try to optimize the forecasting approach based on temporal variations of b-value considering the percentage variations of the parameter and also the temporal variability of the completeness magnitude.

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Thank you for your attention!

Questions, comments and suggestions are welcome!

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