A NEW AND EFFICIENT PROCEDURE FOR DISPERSIVE TSUNAMI SIMULATIONS ON SPHERICAL COORDINATES BASED ON A HYPERBOLIC APPROACH

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Goal: Efficient model implementations for tsunami propagation and coastal-type computations.

Requirements:

- **Simple** models as the hydrostatic Shallow water equations (SW).
- Accounting for **dispersive** effects: Stokes - Airy theory

$$\boxed{Celerity^2_{Airy} = gH\frac{\tanh(kH)}{kH}.}$$

Hydrostatic Shallow Water equations:

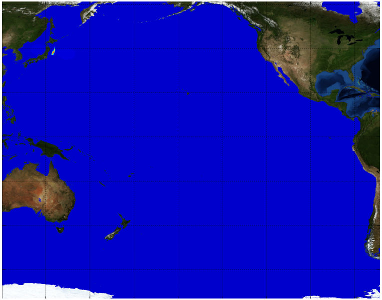
$$\boxed{Celerity^2_{SW} = gH - \text{Inaccurate.}}$$

k : wave-number, H : typical depth.

- *Faster Than Real Time* (***FTRT***):

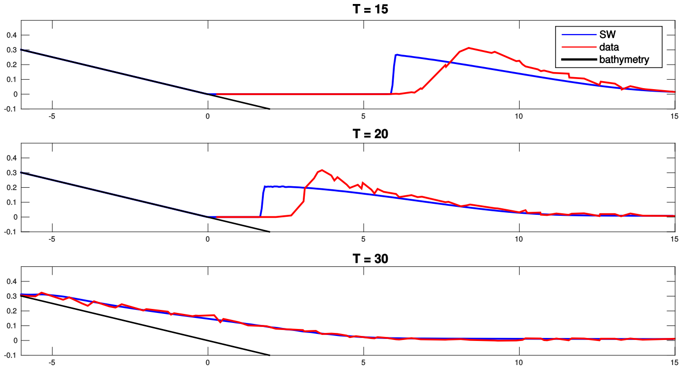
  Tsunami-HySEA: Link

  - FTRT multi-GPU SW solver
  - Hydrostatic - non dispersive
  - Aim: include dispersive effects



Shallow water -non dispersive- results[1]

Solitary wave on a beach, Synolakis, 1987.



▷ Hydrostatic models, such as SW, do not take into account dispersive effects:
Wrong propagation speed. Inaccurate arrival amplitude and time of the wave

[1] J. Macías, M.J. Castro, S. Ortega, C. Escalante, and J.M. González-Vida. "Performance Benchmarking of Tsunami–HySEA Model for NTHMP's Inundation Mapping Activities". In: *Pure and Applied Geophysics* 174.8 (2017), pp. 3147–3183.

Dispersive models

▷ Two prominent families of systems for the simulation of dispersive water waves:

Boussinesq type:

- Boussinesq (1872)
- Peregrine (1967)
- Madsen (1992)
- Lynett (2002,2006,2019)
- Usually contains **high-order derivatives** in the final model
- Unaffordable complexity for 2D domains.

Non-hydrostatic pressure:

- Casulli *et al* (1995)
- **Bristeau** *et al* (2008)
- **Yamazaki** *et al* (2009) (NEOWAVE code)
- Fernández-Nieto *et al* (2017)
- First order systems
- Simple systems

Non-hydrostatic solvers are capable of solving many relevant features of coastal water waves such as:

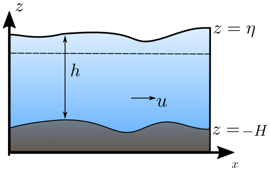
- Dispersive water waves **propagation**
- **Shoaling**
- Non-linearities
- Refraction
- **Run-up**
- **Run-down**

Non-Hydrostatic pressure model

1D domain equations. Bristeau *et al*, 2008.

$$\begin{cases} \partial_t h + \partial_x(hu) = 0, \\ \partial_t(hu) + \partial_x\left(hu^2 + \frac{1}{2}gh^2 + hp\right) = (gh + \gamma p)\,\partial_x H, \\ \partial_t(hw) + \partial_x(uhw) = \gamma p, \\ \partial_x u + \frac{w - w_b}{h/2} = 0, \quad w_b = -\partial_t H - u\partial_x H \end{cases}$$

- Improved **dispersion** relation valid from **intermediate** to **shallow-waters**
- For $\gamma = 2$, the model is similar to Yamazaki *et al* system (NEOWAVE code)
- For $\gamma = 3/2$, the model reduces to the Green-Naghdi system



$$C_{NH}^2 = gH\frac{1}{1 + \frac{1}{2\gamma}(kH)^2}.$$

We choose $\gamma = 2$.

Non-Hydrostatic implementation. Escalante, Castro and Morales[1]

▷ Numerical implementation of the previous system.

Pros:

- Efficiently solved for 2D domains on **GPU** (Escalante *et al* 2018[1]). Hybrid finite-volume finite-difference 2nd order scheme.
- **Computational cost**: 2 times more expensive than a one-GPU hydrostatic Shallow Water solver.

Cons:

- The system is **not hyperbolic** and the numerical scheme involves the solution of **linear systems**
- The computational cost increases for **high-order** schemes
- Not amenable for simple **multi-GPU** implementations

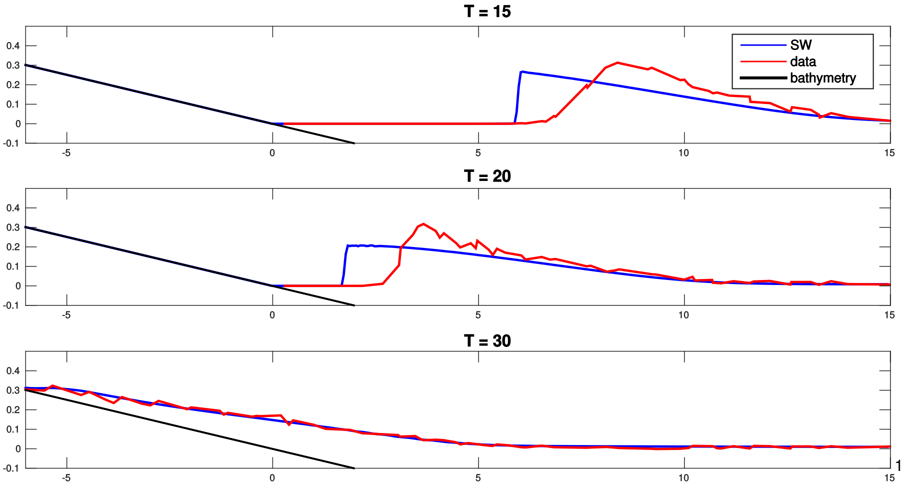
▷ The **difficulties** arise from the imposition of the divergence condition:

$$\partial_x u + \frac{w - w_b}{h/2} = 0, \quad w_b = -\partial_t H - u\partial_x H.$$

▷ That will lead to developing implicit numerical schemes. Otherwise, a quite restrictive time-step CFL condition must be considered. The same difficulties arise for Boussinesq-type systems.

[1] C. Escalante, T. Morales, and M.J. Castro. "Non–Hydrostatic Pressure Shallow Flows: GPU Implementation Using Finite Volume and Finite Difference Scheme". In: *Applied Mathematics and Computation* 338 (2018), pp. 631–659.

Solitary wave on a plane beach. Hydrostatic approach - SW



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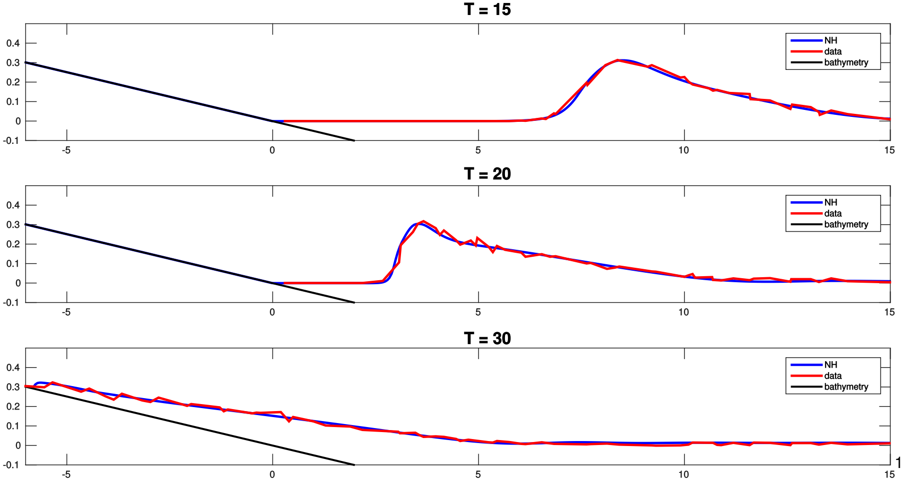
[1] “Performance assessment of the Tsunami-HySEA model for NTHMP tsunami currents benchmarking. Laboratory data”. In: *Coastal Engineering* 158 (2020), p. 103667. ISSN: 0378-3839. DOI: https://doi.org/10.1016/j.coastaleng.2020.103667.

Solitary wave on a plane beach. Non-hydrostatic approach



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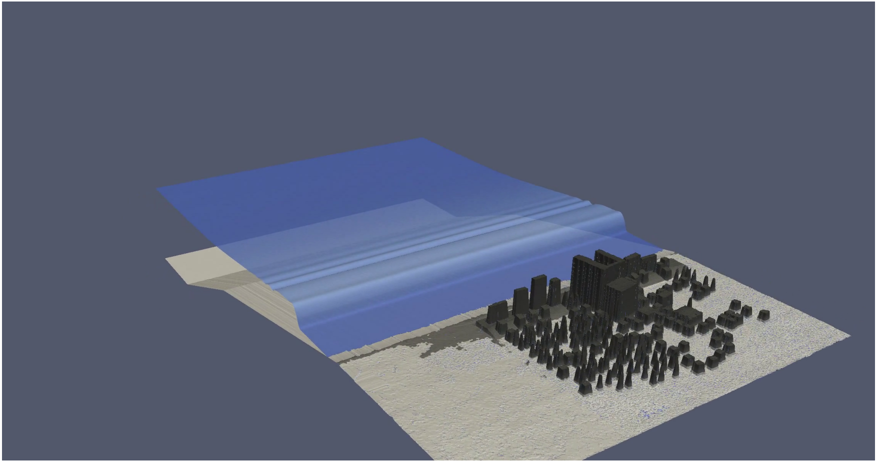
Solitary wave impinging on a small-scale model of Seaside, Oregon[1]

- ▷ Benchmark problem. 2^{nd} order hybrid finite-volume finite-difference scheme



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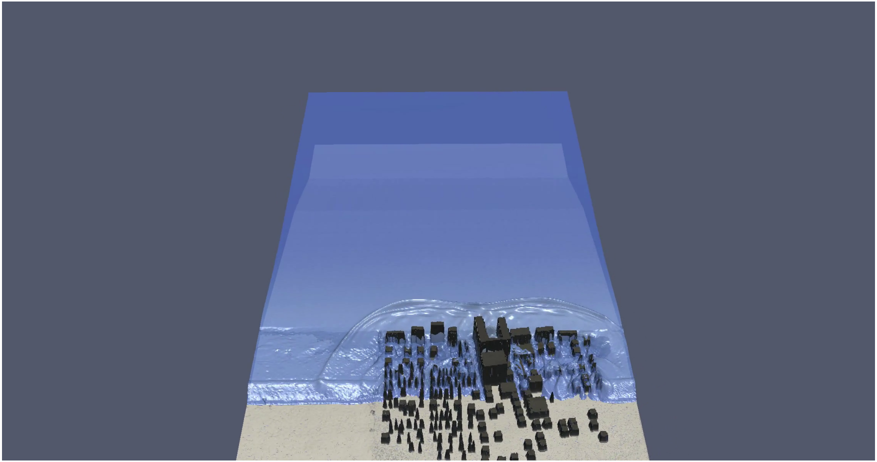

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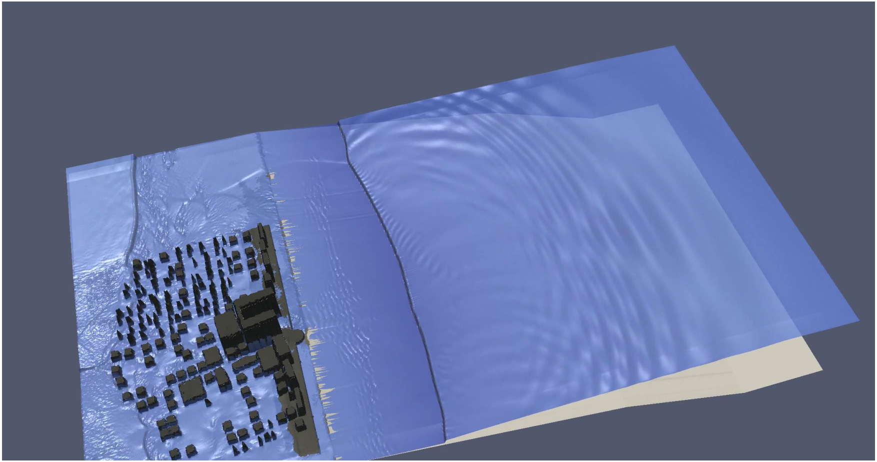

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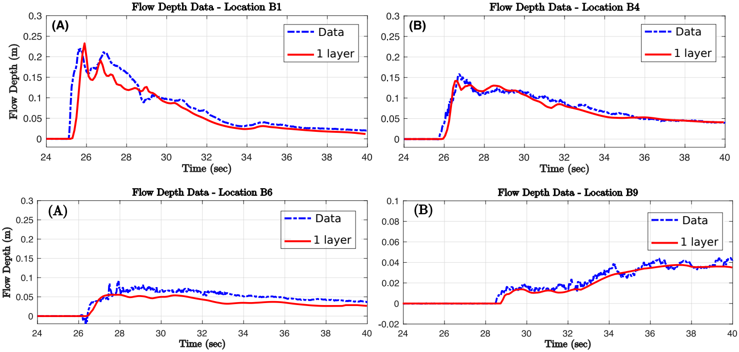

▷ **Computational time**: 2-3 times slower than a one-GPU SW simulation.

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A novel hyperbolic relaxation non-hydrostatic pressure system[1]

▷ A more efficient approach is developed in Escalante, Castro and Dumbser, 2019[1]. The **key idea** is to replace the divergence condition:

$$\partial_x u + \frac{w - w_b}{h/2} = 0, \quad w_b = -\partial_t H - u\partial_x H.$$

by the relaxed equation:

$$\partial_t(hp) + \partial_x(uhp) + hc^2 \left(\partial_x u + \frac{w - w_b}{h/2} \right) = 0, \quad w_b = -\partial_t H - u\partial_x H.$$

▷ Here, divergence errors are transported with a wave speed $c = \alpha\sqrt{gH_0}$, H_0 being the typical depth.

▷ The resulting system is **hyperbolic** and the time-step CFL condition is similar to the SW system:

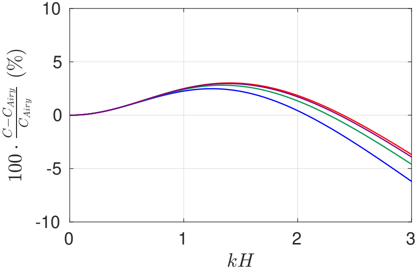
$$\Delta t \le \frac{\Delta x}{\lambda_{\max}}, \qquad \lambda_{\max} = |u| + \sqrt{gh + p + c^2}.$$

[1] C. Escalante, M. Dumbser, and M.J. Castro. "An efficient hyperbolic relaxation system for dispersive non-hydrostatic water waves and its solution with high order discontinuous Galerkin schemes". In: *Journal of Computational Physics* 394 (2019), pp. 385 –416. ISSN: 0021-9991.

A novel hyperbolic relaxation non-hydrostatic pressure system[1]

Pros:

- The system is **hyperbolic**: No need of solving linear systems
- Can be discretized by any efficient **high-order** and **explicit** numerical scheme.
- Simple and straightforwards **multi-GPU** implementations
- Preserves the **dispersion relation** from the original non-hyperbolic system
- The same model recovers hydrostatic results (**classical SW equations**) by setting $\boldsymbol{\alpha = 0}$**.**



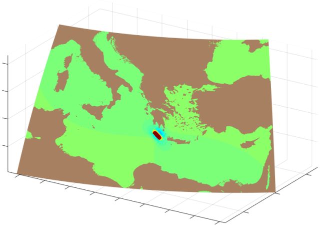
Relative error of the phase velocity with respect to the Airy theory for different values of the relaxation parameter: $\alpha = 3, 5, 10$ given in blue, green and magenta respectively. **In practice we choose** $\alpha = 3$.

[1] C. Escalante, M. Dumbser, and M.J. Castro. “An efficient hyperbolic relaxation system for dispersive non-hydrostatic water waves and its solution with high order discontinuous Galerkin schemes”. In: *Journal of Computational Physics* 394 (2019), pp. 385 –416. ISSN: 0021-9991.

Real-world Tsunami simulations

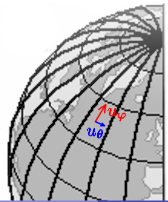
- The 2D implementation of the non-hyperbolic non-hydrostatic pressure system,
- and the 2D implementation of the hyperbolic non-hydrostatic pressure system,

may suffer the lack of **Earth-curvature** effects for simulations on bigger scenarios.



To account for curvature effects:

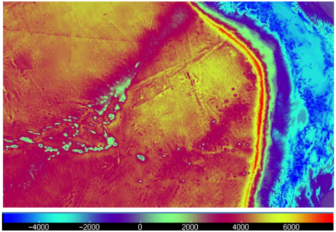
- We consider the non-hydrostatic pressure formulation for spherical coordinates,
- similar to the one given for the NEOWAVE code (Yamazaki *et al*, 2010).
- Then, we propose a similar hyperbolic approach technique to relax the divergence constraint, and obtaining a **hyperbolic model**.



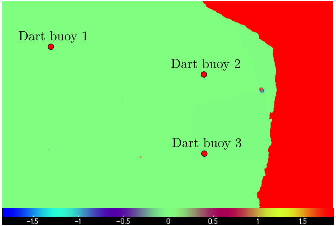
The 2014 Iquique earthquake. Comparison with NEOWAVE

The real test case to solve using a one-GPU implementation

- The domain: west coast of northern Chile. Grid resolution: 1 arc-min.
- Size: $2880 \times 1800 = 5184000$ cells.
- Wall clock to be simulated: 10000 seconds $\approx$ 2 hours 45 minutes.
- We are interested in compare the Green's funtions obtained in a subfault of 20×20 *km* size (corresponding to the 2014 Iquique earthquake) time series provided by **Dart buoys** in 3 locations against the computed numerical simulations from the new hyperbolic **non-hydrostatic model**.



Bathymetry

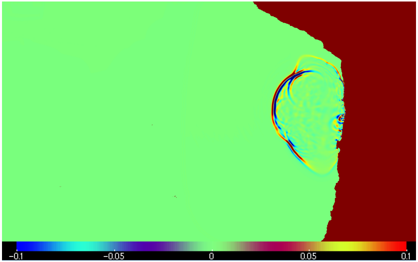


Free-surface initial cond. Dart buoys loc.

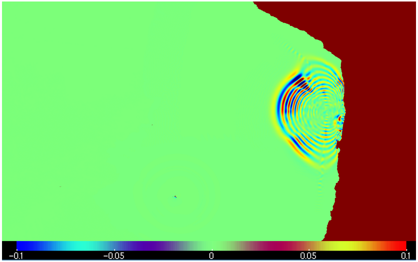
The 2014 Iquique earthquake. Comparison with NEOWAVE

The real test case to solve using a one-GPU implementation

- Hydrostatic SW simulation: $\alpha = 0$
- Non-hydrostatic simulation: $\alpha = 3$
- Numerical scheme: Finite-Volume
  3^{rd} order TVD Runge-Kutta in time and 3^{rd} order CWENO in space.



Free-surface at time 1500 *s*.
Hydrostatic **SW** simulation ($\alpha = 0$.)



Free-surface at time 1500 *s*.
Non-Hydrostatic hyperbolic simulation ($\alpha = 3$.)

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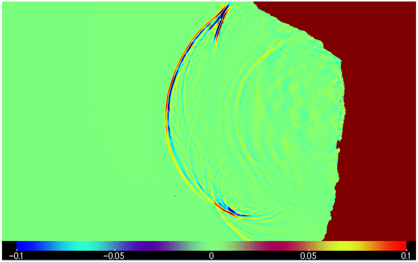
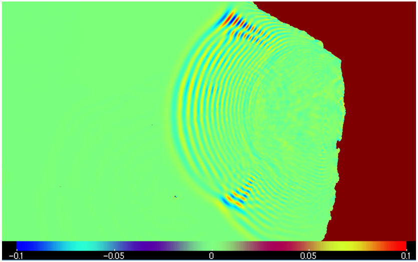
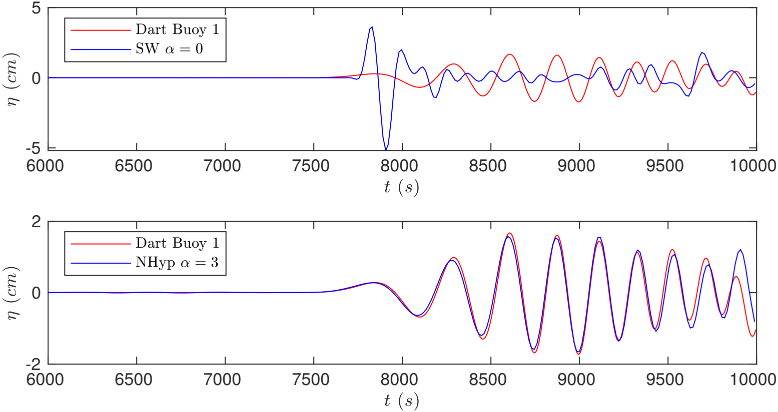


Free-surface at time 4500 s.
Hydrostatic **SW** simulation ($\alpha = 0$.)



Free-surface at time 4500 s.
Non-Hydrostatic hyperbolic simulation ($\alpha = 3$.)

The 2014 Iquique earthquake Time series. Comparison with NEOWAVE



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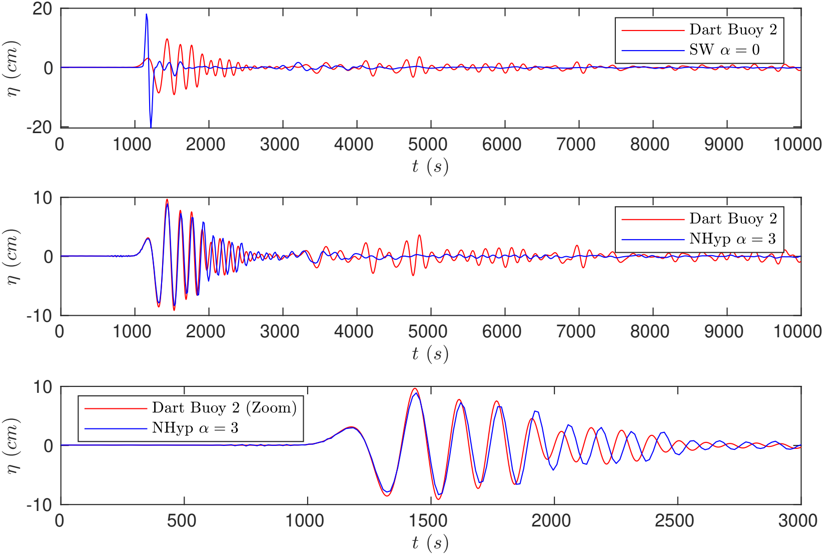

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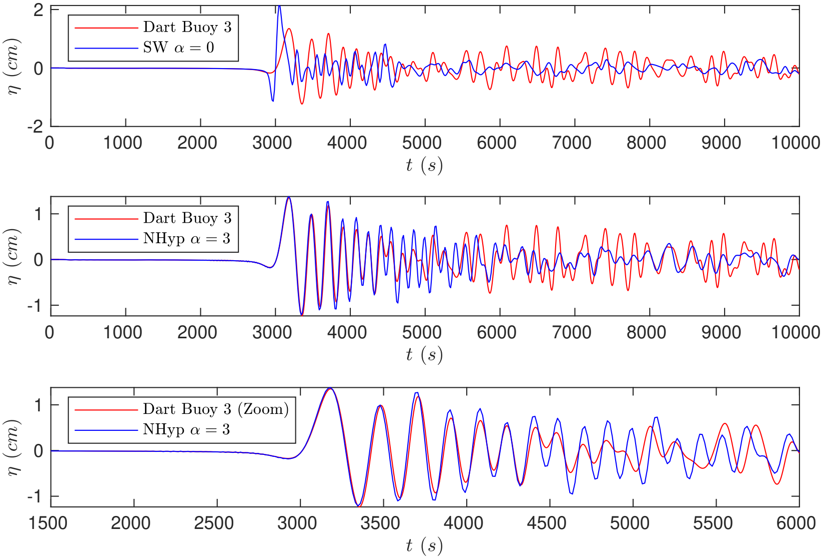

The 2014 Iquique earthquake. Computational effort

Simulated time: 10000 s. Third order scheme

Model	Comput. time	$\#$ times FTRT
Hydrostatic SW ($\alpha = 0$)	659.29	15.17
Non-hydrostatic ($\alpha = 3$)	1271.92	7.86

▷ Ratio computational times Non-Hydrostatic/SW: 1.93.

▷ Computations performed with nVIDIA TESLA V100.

Final comments

Conclusions and perspectives

- Towards an operational **FTRT** dispersive approach.
- Model includes **dispersive** effects and **curvature** effects,
- and efficient numerical implementations can be proposed due to its **hyperbolic** nature.
- Dispersive effects without solving linear systems.
- Validation: simple geometries (lab) and complex real cases.
- Computational times: around **two times** slower than a **SW** model. Single-GPU.
- Nested meshes and multi-**GPU** implementations can be straightforwardly implemented (Future work).
- Coupling with **landslides** models (Ongoing work).