Nonlinear Convection of Electrically Conducting Fluid in a Rotating Magnetic System

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Abstract

- Nonlinear analysis in a **rotating Rayleigh-Bernard system** of electrical conducting fluid is studied numerically in the presence of **externally applied horizontal magnetic field** with **rigid-rigid boundary conditions**.
- This research model is also studied for **stress free boundary conditions in the absence of Lorentz and Coriolis forces**.
- This **DNS approach** is carried near the onset of convection to study the flow behaviour in the limiting case of Prandtl number.
- The fluid flow is visualized in terms of **streamlines, limiting streamlines and isotherms**. The dependence of Nusselt number on the **Rayleigh number, Ekman number, Elasser number** is examined.

Outline

- Introduction
- Physical model
- Governing equations
- Methodology
- Validation
  - RBC – 2D
  - RBC – 3D
- Results
  - RBC
  - RBC with magnetic field (MC)
  - Plane layer dynamo (RMC)

Introduction

- Nonlinear interaction between convection and magnetic fields (Magnetoconvection) may explain certain prominent features on the solar surface.
- Yet we are far from a real understanding of the dynamical coupling between convection and magnetic fields in stars and magnetically confined high-temperature plasmas etc. Therefore it is of great importance to understand how energy transport and convection are affected by an imposed magnetic field: i.e., how the Lorentz force affects convection patterns in sunspots and magnetically confined, high-temperature plasmas.
- Magnetoconvection exhibits a rich variety of behavior when the magnetic Prandtl number (P_m) is small. This condition is satisfied in the solar convection zone near the sunspot and magnetically confined, high temperature plasmas.
- Convection in planetary cores and stellar interiors often occurs in the presence of strong rotational and magnetic constraints.

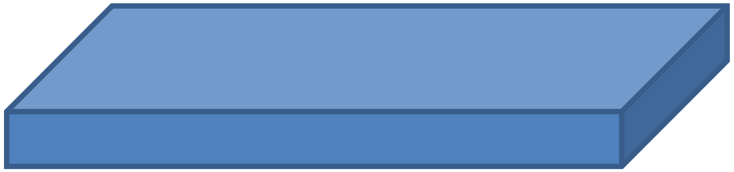
Motivation

- To link the dynamics of geophysical fluid flows with the structure of these fluid flows in physical space and the transitions of this structure arising in the system due to the following parameters:
  - ✓ **Rayleigh Number, Ra**
  - ✓ **Nusselt Number, Nu**
  - ✓ **Chandrasekhar Number, *Q***
  - ✓ **Taylor Number, *Ta***
  - ✓ **Thermal Prandtl Number, *Pr***
  - ✓ **Magnetic Prandtl Number, *Pm***

- *To reproduce "Experiments on Rayleigh-Benard convection, magnetoconvection and rotating magnetoconvection in liquid gallium", J. M. AURNOUy AND P. L. OLSON,* J. Fluid Mech. (2001), vol. 430, pp. 283-307.

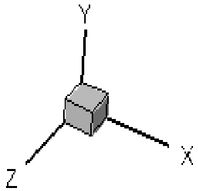
Physical Model

A horizontally stratified fluid layer of characteristic height d,



Cartesian coordinate system

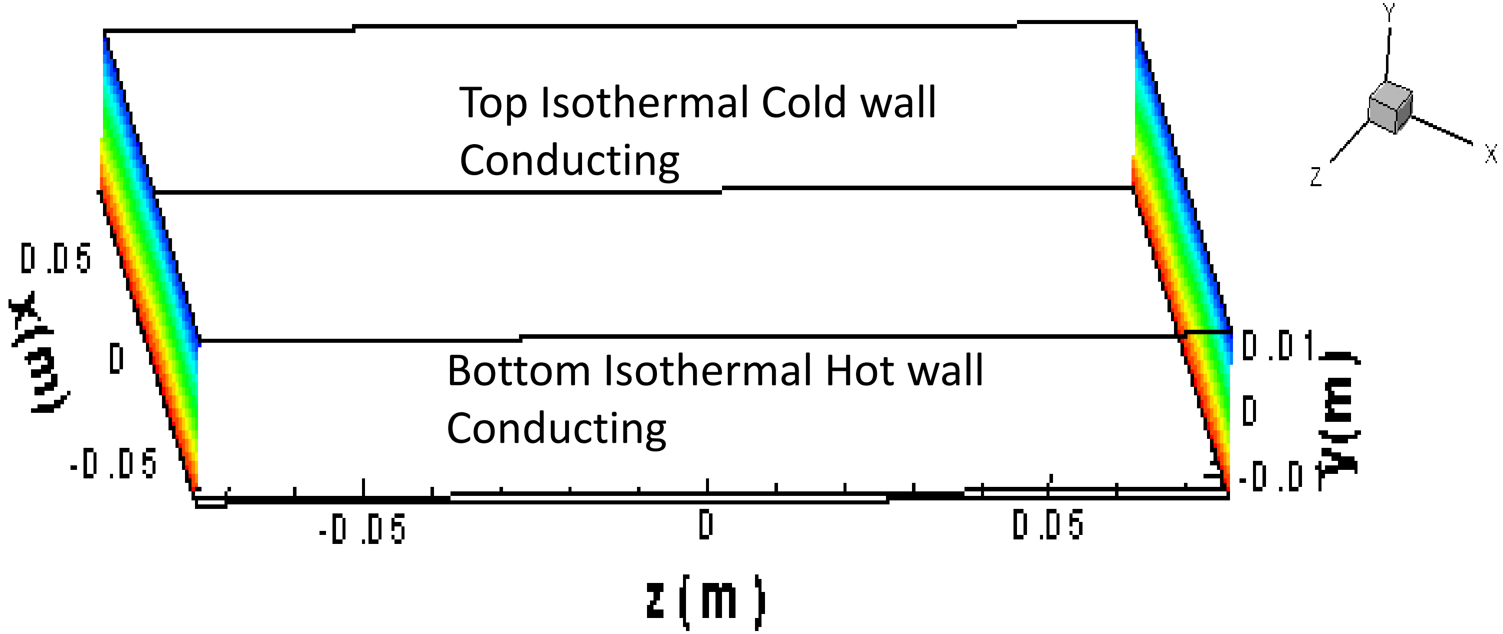
y axis pointing vertically upward



x, z axes in the horizontal direction for the three-dimensional model.

Liquid : Gallium
Walls : Copper

Externally applied vertical Magnetic field



Side walls : insulated walls,

horizontal periodic conditions

Physical properties of Gallium

Property	Units	Value
Density, ρ	$kg\ m^{-3}$	6.095×10^{3}
Melting temperature, T_0	$^{\circ}C$	29.7
Thermal expansion coefficient, β	K^{-1}	1.27×10^{-4}
Specific heat, C_P	$J\ kg^{-1}\ K^{-1}$	397.6
Kinematic viscosity, υ	$m^2 s^{-1}$	3.2×10^{-7}
Thermal diffusivity, $\left(\alpha = \frac{\kappa}{\rho C_P}\right)$	$m^2 s^{-1}$	1.27×10^{-5}
Thermal conductivity, κ	$W\ m^{-1}\ K^{-1}$	31
Magnetic diffusivity, η	$m^2 s^{-1}$	0.21
Electrical conductivity, σ	$(ohm\ m)^{-1}$	3.85×10^{6}

Thermal Prandtl Number = $Pr_1 = \frac{\nu}{\kappa} =$ 0.025

Magnetic Prandtl Number = $Pr_2 = \frac{\nu}{\eta}$ = 1.5E-6

Roberts Number $= \frac{Pr_2}{Pr_1} = 0.0006$

The considered RMC system is a straightforward extension of the Lorenz model for Boussinesq convection, with the Lorentz force taken into account.

$$\nabla.\bar{V}' = 0, \quad \nabla.\bar{B}' = 0, \quad \frac{\rho}{\rho_0} = 1 - \beta(T' - T'_0)$$

$$\frac{\partial \bar{V}'}{\partial t'} + (\bar{V}'.\nabla')\bar{V}' = -\frac{1}{\rho_0}\nabla p - \frac{\rho}{\rho_0}\mathrm{g}\widehat{e_y} + \nu\nabla^2\overline{V'} + \frac{1}{\mu_0\rho_0}(\nabla \times \overline{B'}) \times \overline{B'} - \bar{\Omega} \times (\bar{\Omega} \times \bar{r}) + 2(\bar{V} \times \bar{\Omega})$$

Transient term + Advection (Nonlinear) term =

Pressure force + buoyancy force +

Lorenz force + Coriolis force +centrifugal force

$$\frac{\partial \bar{B}'}{\partial t'} = \nabla \times \left(\bar{V}' \times \bar{B}'\right) + \eta\nabla^2\bar{B}'$$

$$\frac{\partial T'}{\partial t'} = -\nabla.(T'\overline{V'}) + \alpha\nabla^2 T'$$

Solutions at Conduction State

- $\bar{V}_s = 0,$
- $T'_s = T'_0 - \dot{\beta} y',$ where $\dot{\beta}$ is the adverse temperature gradient.
- $p'_s = p'_0 - g\rho_0 \left(y' + \frac{1}{2} \beta \dot{\beta} y y'^2 \right)$
- $B'_s = B'_0 \widehat{e_y}$

Consider small perturbations in the equilibrium solutions as $\bar{f} = \bar{f_s} + \bar{f_*}$, where f is the field variable.

Basic (Non-dimensional) equations

The quantities $\overline{V_*'}, T_*', t', P_*', \bar{B}'_*$ are made dimensionless by using the scales

$$\alpha/d\,, \Delta T', d^2/\alpha\,, \rho_0\alpha^{-2}/d^2,\ \alpha B'_0/\eta$$

$$\nabla.\bar{V} = 0, \nabla.\bar{B} = 0$$

$$\frac{1}{Pr_1}\left[\frac{\partial\bar{V}}{\partial t} + (\bar{V}.\nabla)\bar{V}\right] - Q\frac{Pr_2}{Pr_1}(\bar{B}.\nabla)\bar{B} =$$

$$= -\nabla\left(\frac{P}{Pr_1} + \frac{Q}{2}\frac{Pr_2}{P\,r_1}\quad|\bar{B}|^2 + QB_y - \frac{Ta\;Pr_1}{8}\left|\widehat{e_y}\times\bar{r}\right|^2\right) + Q\frac{\partial\bar{B}}{\partial z}$$

$$+Ra\;T\widehat{e_y} + Ta^{1/2}(\bar{V}\times\widehat{e_y}) + \nabla^2\bar{V}$$

$$\left(\frac{\partial}{\partial t} - \nabla^2\right)T = w - (\bar{V}.\nabla)T$$

$$\left(\frac{Pr_2}{Pr_1}\frac{\partial}{\partial t} - \nabla^2\right)\bar{B} = \nabla\times(\bar{V}\times\widehat{e_y}) + \frac{Pr_2}{Pr_1}\nabla\times(\bar{V}\times\bar{B})$$

Non-dimensional numbers

- Rayleigh number,

$$Ra = \frac{g\ \beta\ \Delta T\ d^3}{\alpha \nu} = \frac{\text{Buoyancy force}}{\text{visocous force}}$$

- Chandrasekhar number,

$$Q = \frac{\mu_0 B_0^2 d^2}{\rho_0 \nu \eta} = \frac{\textbf{Lorentz force}}{\text{viscous force}}$$

- Taylor number, $Ta = \frac{4\ \Omega^2 d^4}{\nu^2} = \frac{\textbf{Coriolis force}}{\text{Viscous force}}$

- Nusselt number, $Nu = \frac{h\ d}{\kappa\ \Delta T}$

Methodology

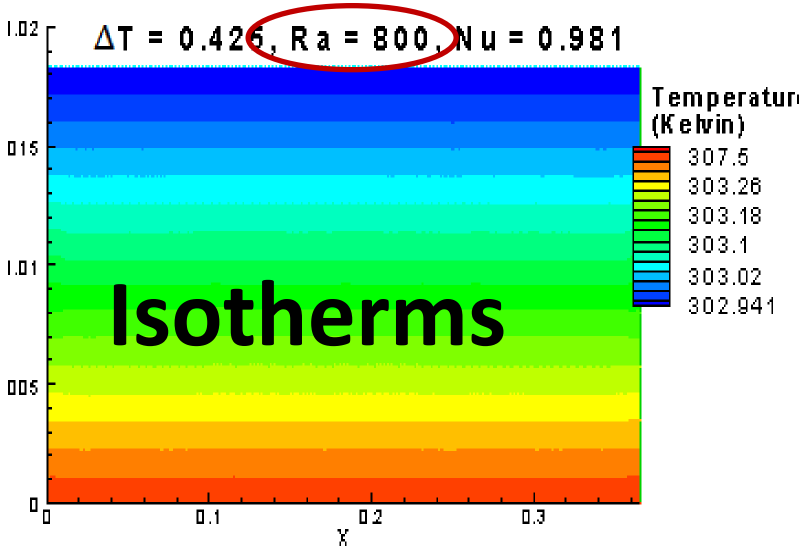
- Mesh : Uniform (Nodes : 80864)

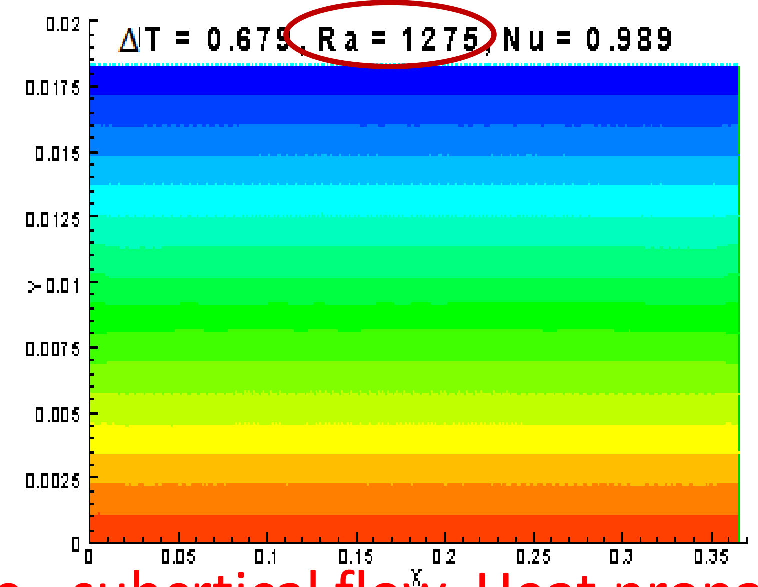
- **Finite Volume method**
- Advection terms: First order upwind
- Pressure : SIMPLE (Semi Implicit Pressure Linked Equations, *Patankar* 1979)
- Temperature : Second order upwind
- Magnetic field : First order upwind
- System of algebraic equations
  - AMG (Algebraic Multi Grid) Solver
- Convergence criteria < 1E-4

RBC 2D

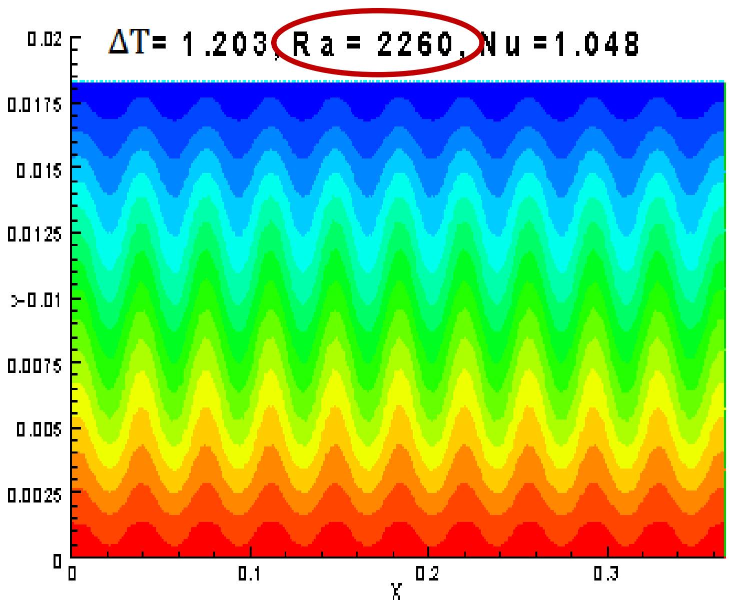
$Q = 0$, $Ta = 0$

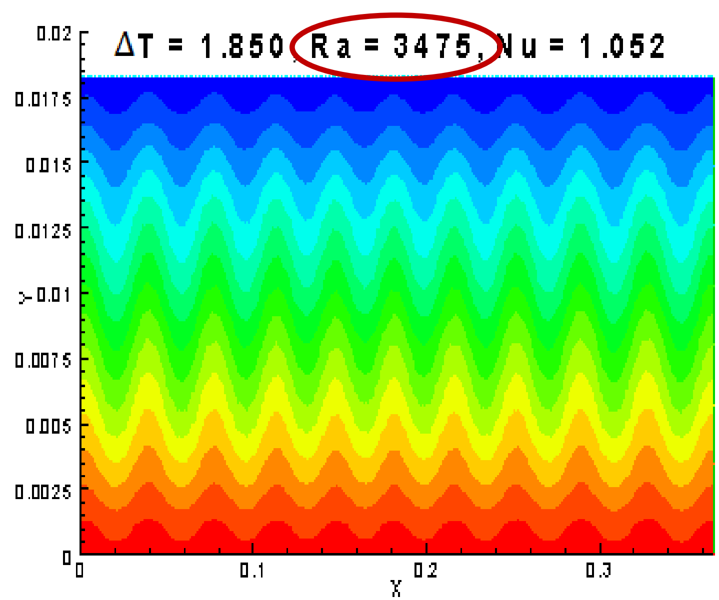
Note: Temperature is given in Kelvin and velocity is given in m/s.



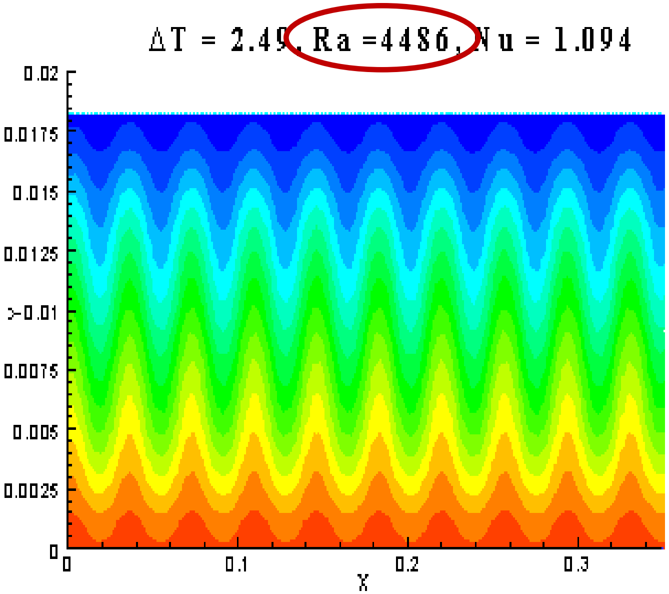


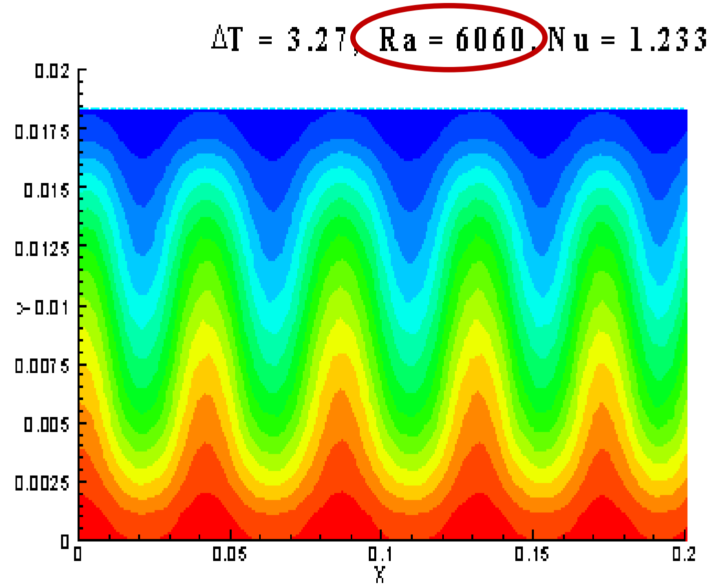
When Ra < 1708 we get Nu < 1, i.e., subcrtical flow. Heat propagates in the system as a conduction mode.

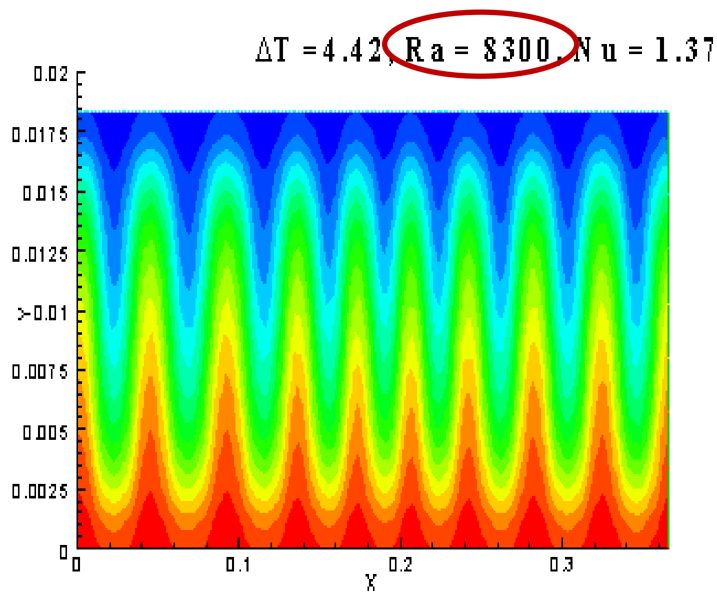




Isotherms



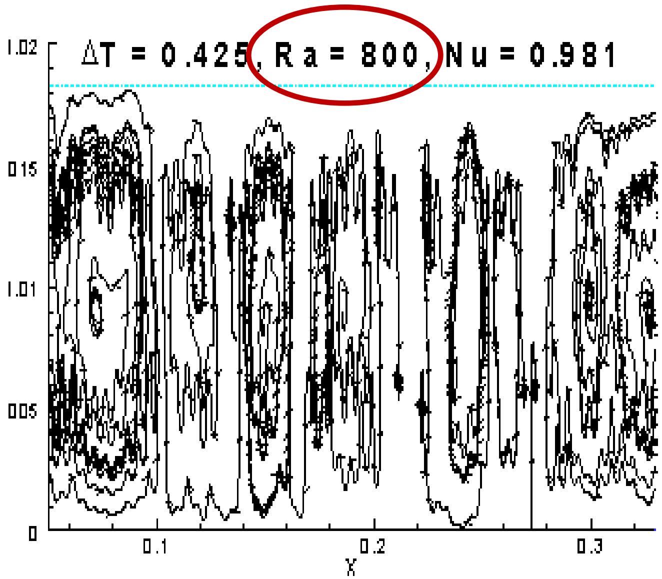


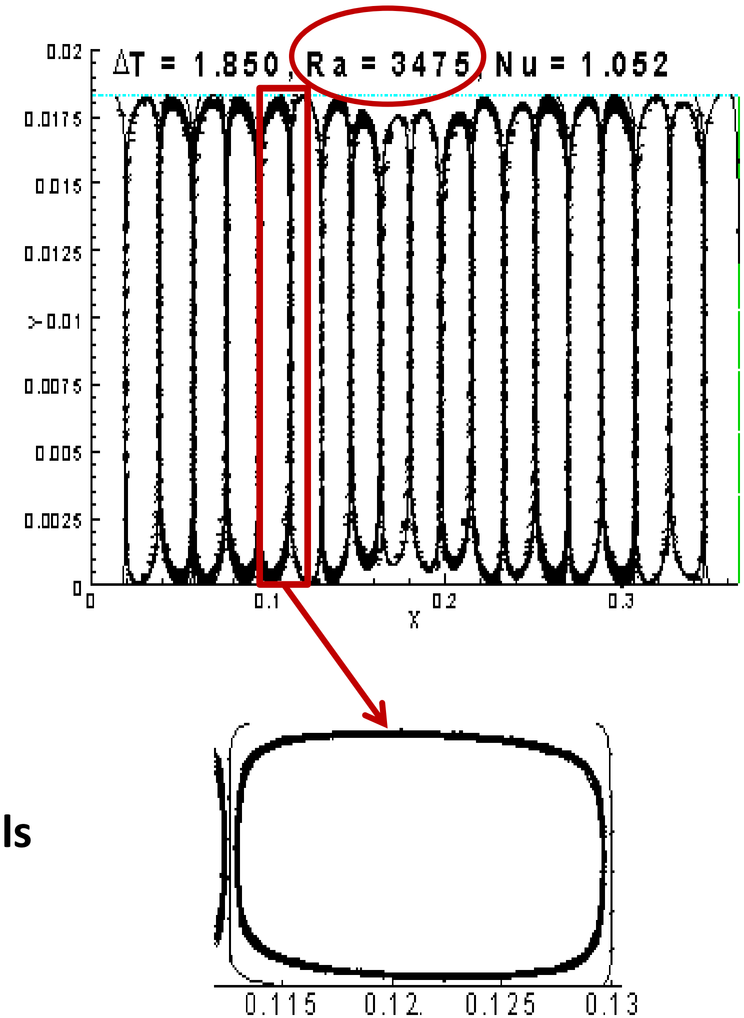


For Ra > 1708 we get supercritical flow.

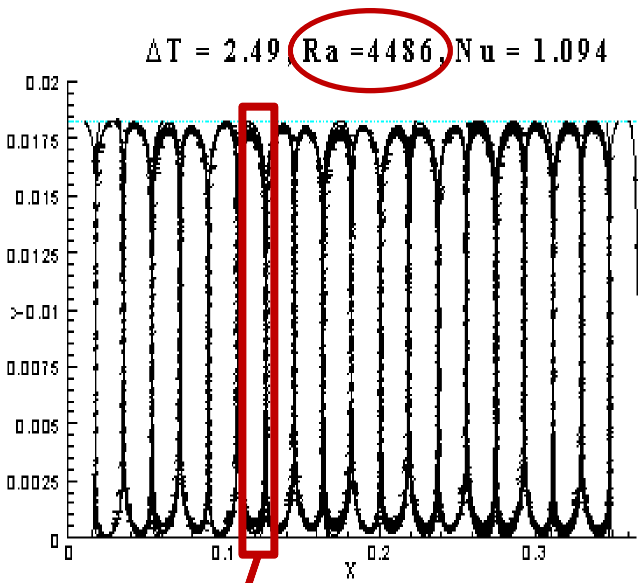
Theoretically Chandrasekar (1961) showed that for RBC only Stationary convection occur as a first instability at the onset.

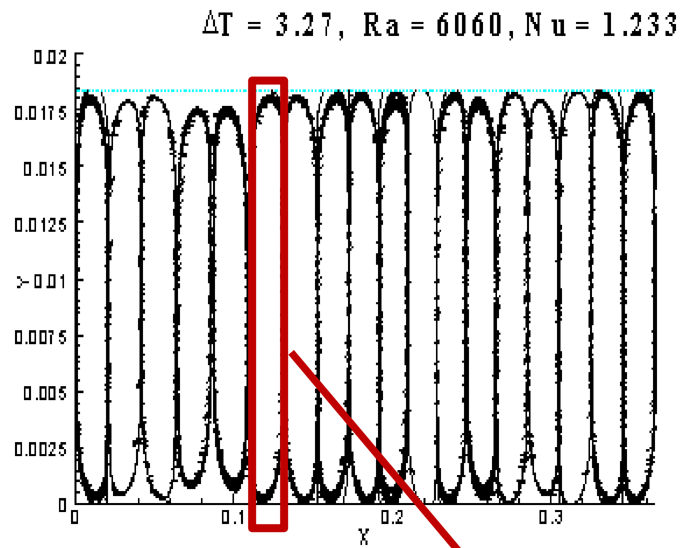
Streamlines

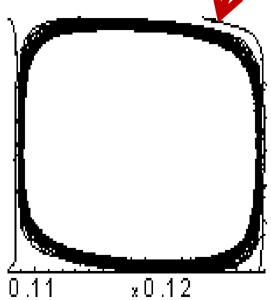


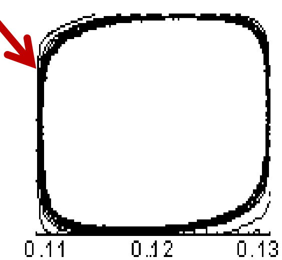


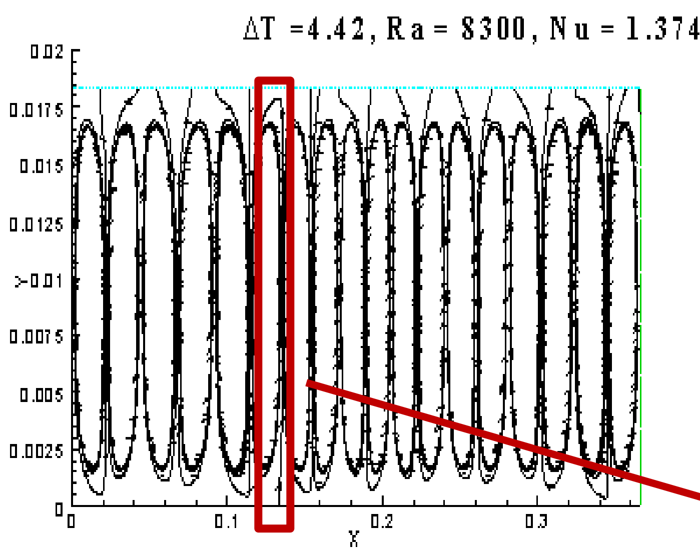
For Ra > 1708 we get symmetric rolls having y-axis as a axis of rotation.

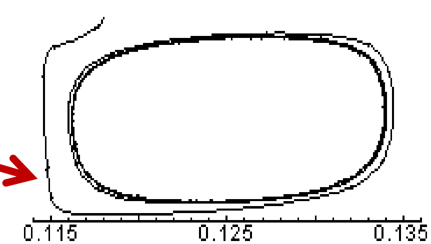










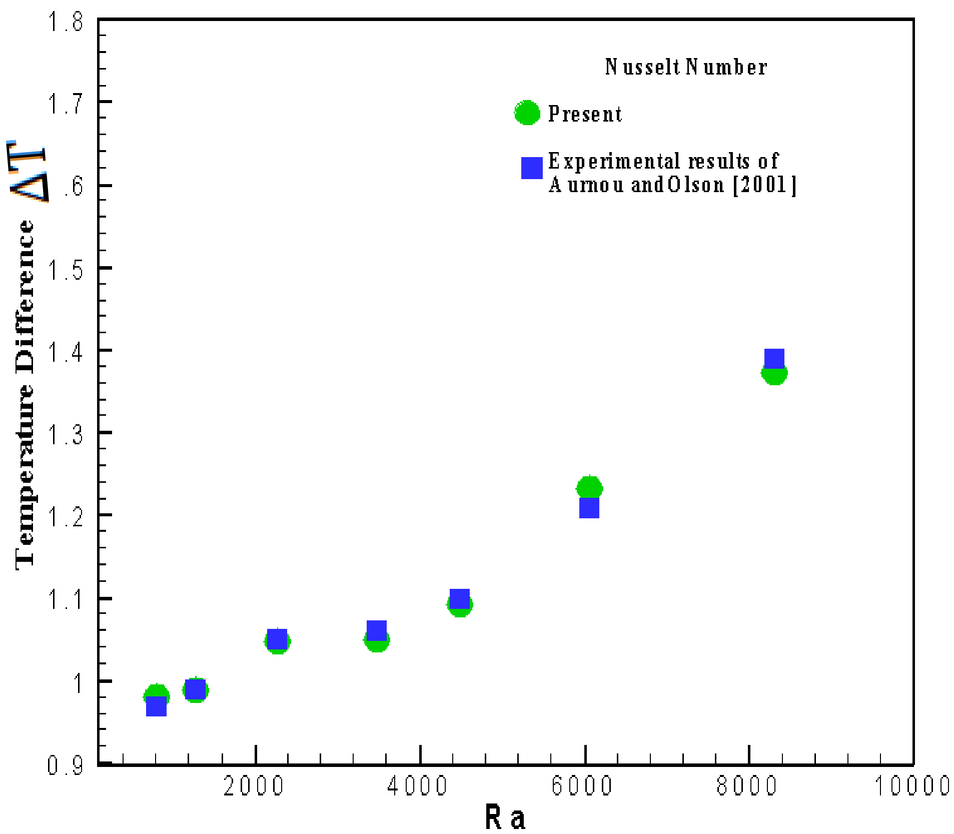


As Ra increases horizontal length increases.

Experiments on Rayleigh-Benard convection, magnetoconvection and rotating magnetoconvection in liquid gallium.

J. M. AURNOU AND P. L. OLSON.

J. Fluid Mech. (2001), vol. 430, pp. 283-307.

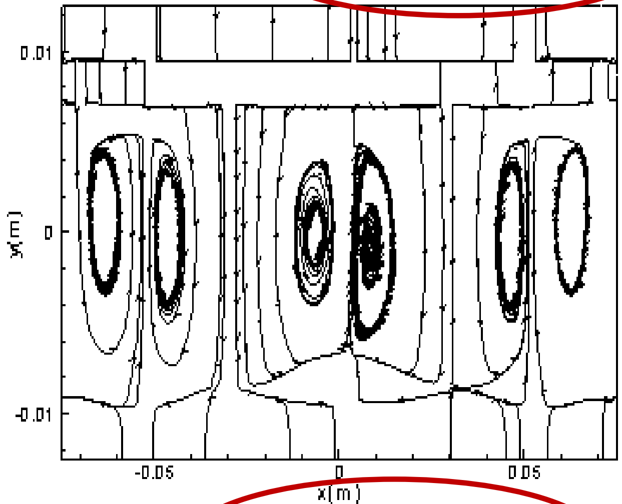


RBC 3D
Magnetoconvection (MC)

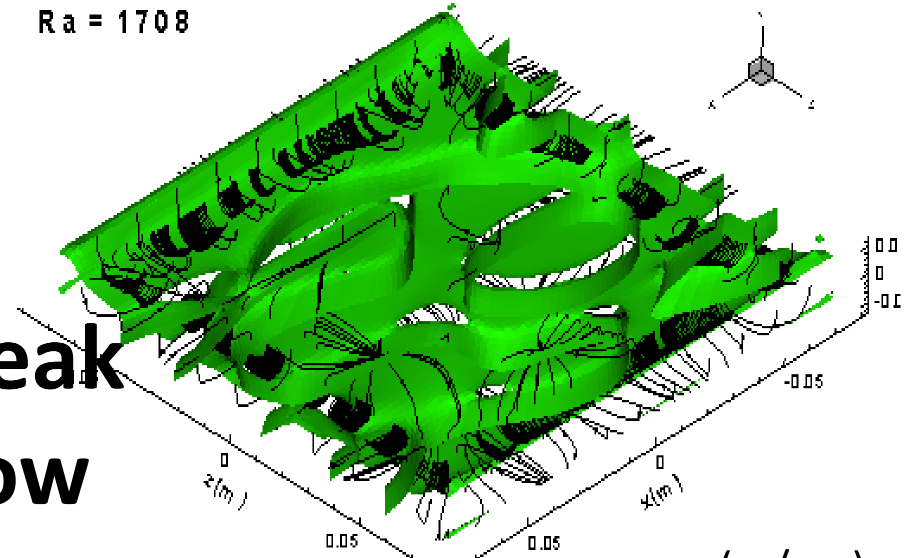
With Ta = 0

$Q = 0$

Ra = 1708, Nu = 0.98

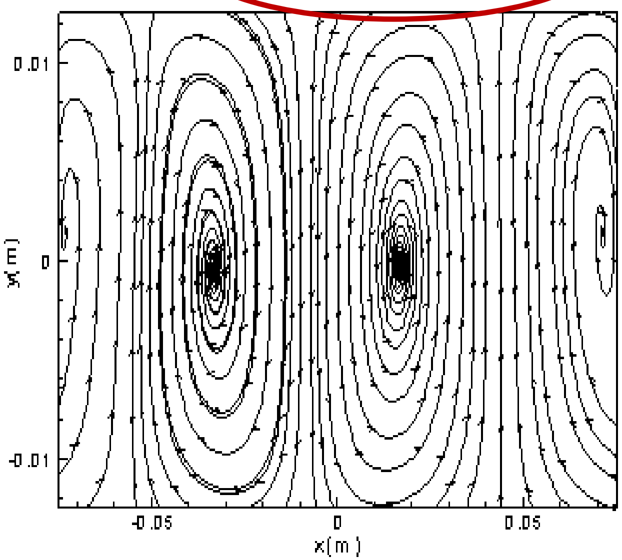


Ra = 1708

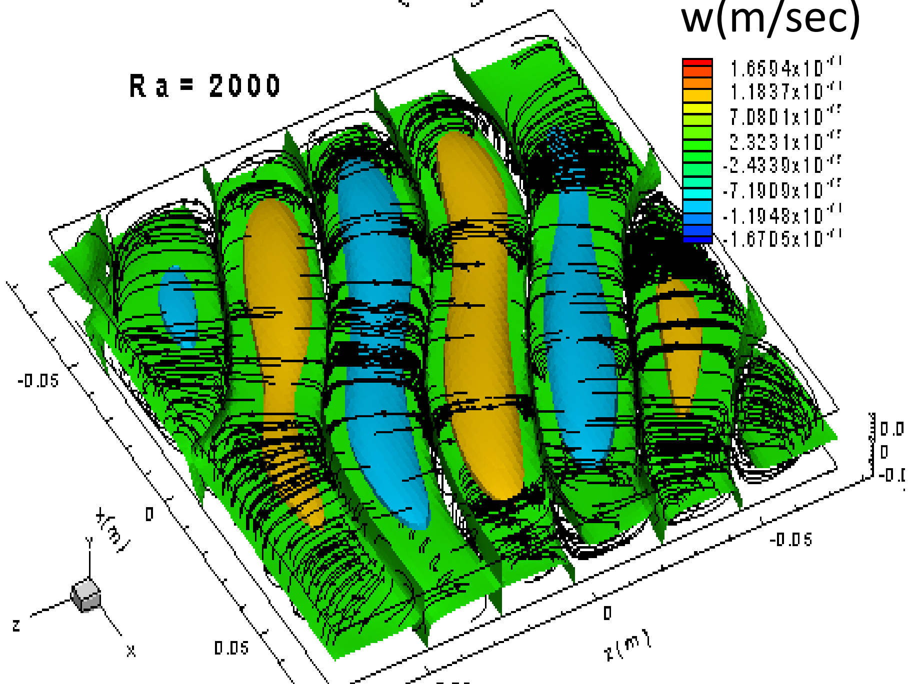


Weak Flow

Ra = 2000, Nu = 1.01



Ra = 2000

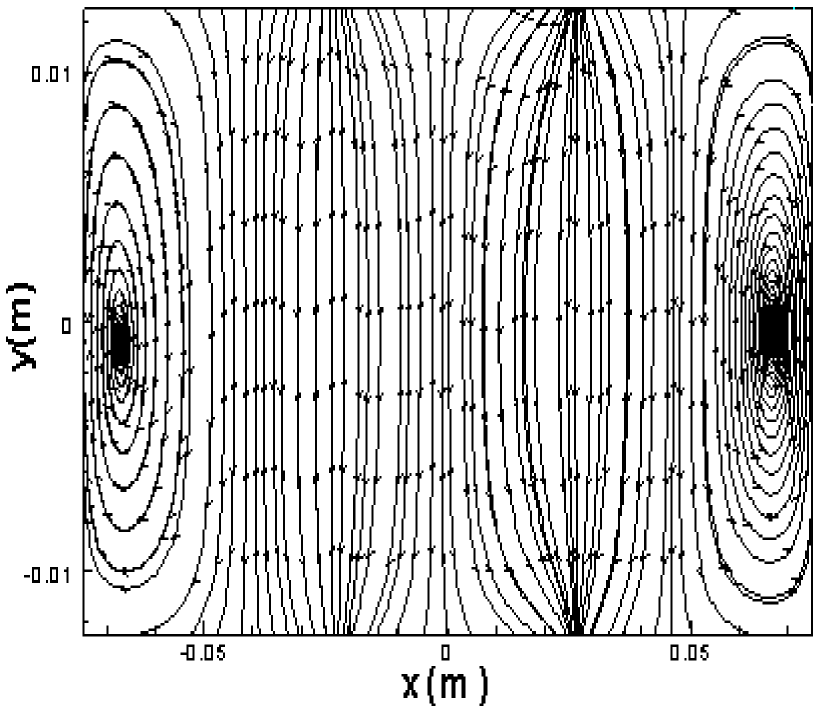


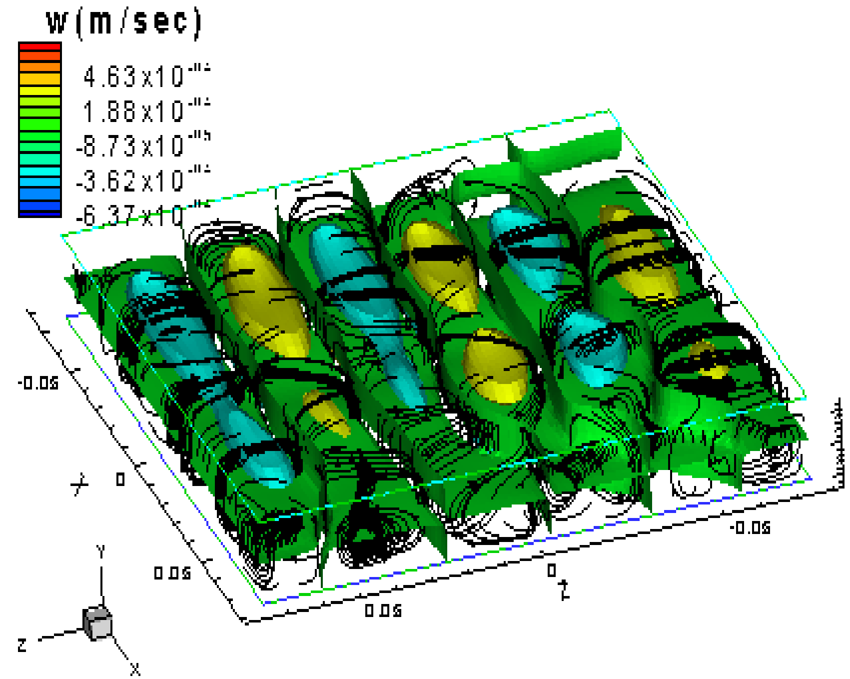
Cylindrical rolls with sneaking structures

Magnetoconvection

$Q = 1210$, $Ra = 27100$

$Nu = 1.02$

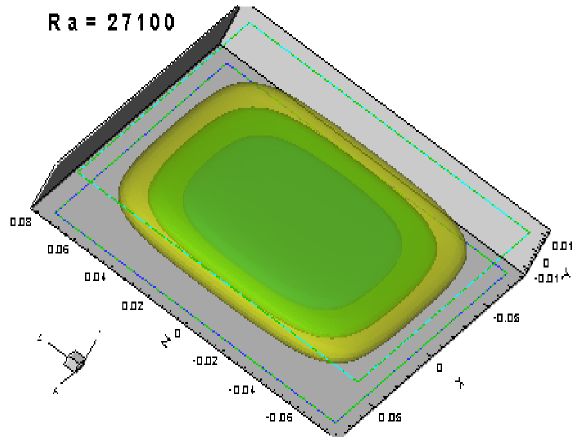


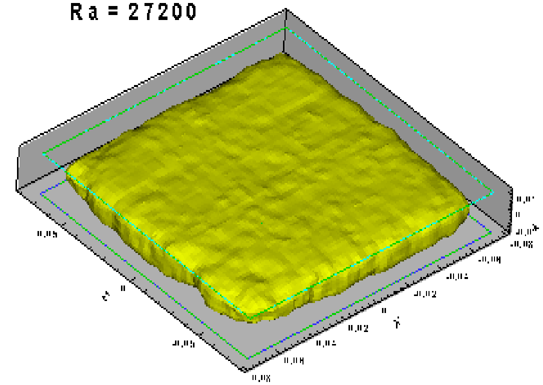


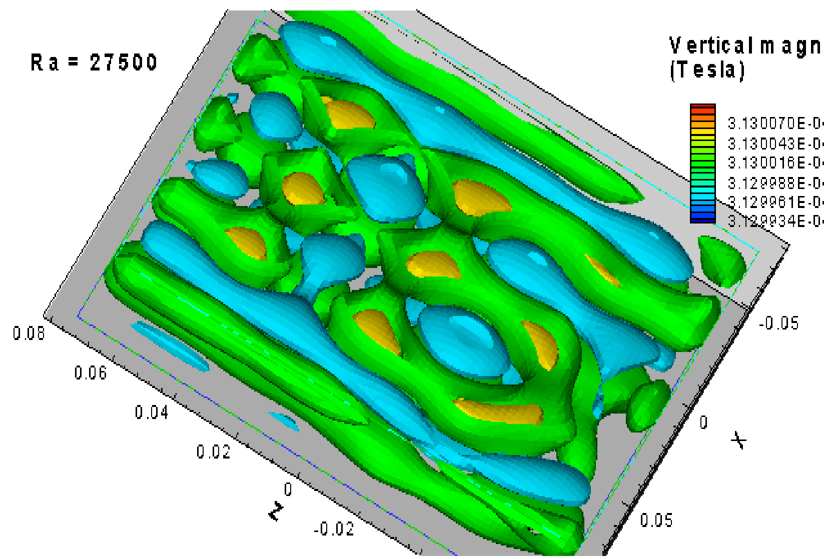
Onset of convection delayed due to the effect of Lorenz force on thermal buoyancy force

Induced Vertical Magnetic field

$Q = 1210$



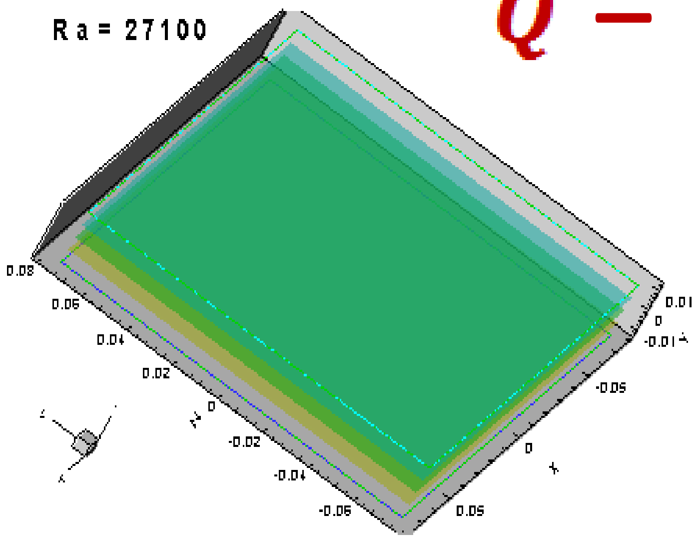


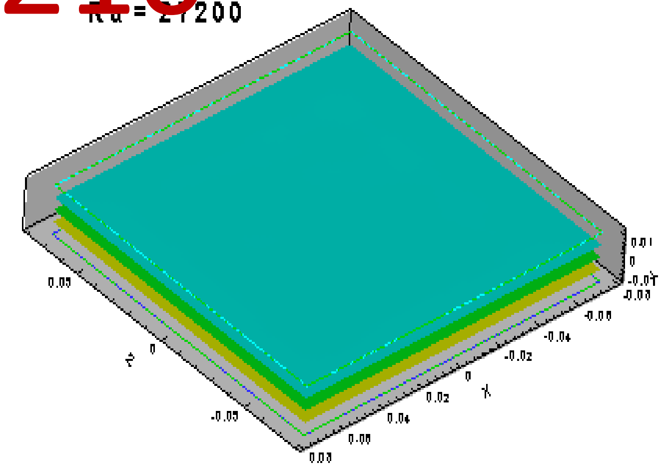


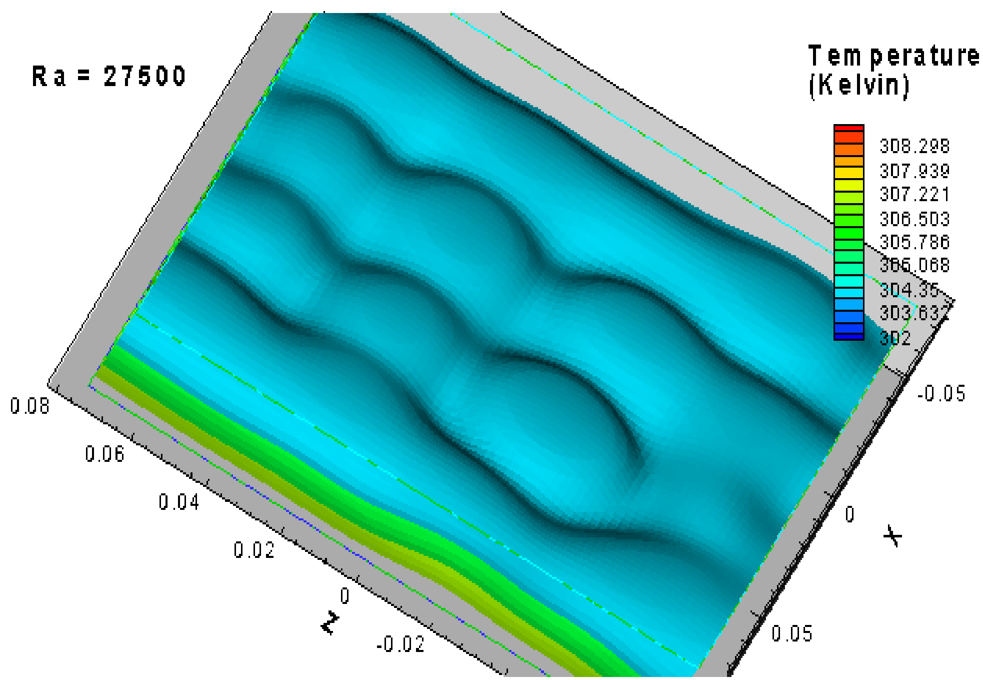
Onset of convection delayed due to the effect of Lorenz force on thermal buoyancy force.

Temperature Variation

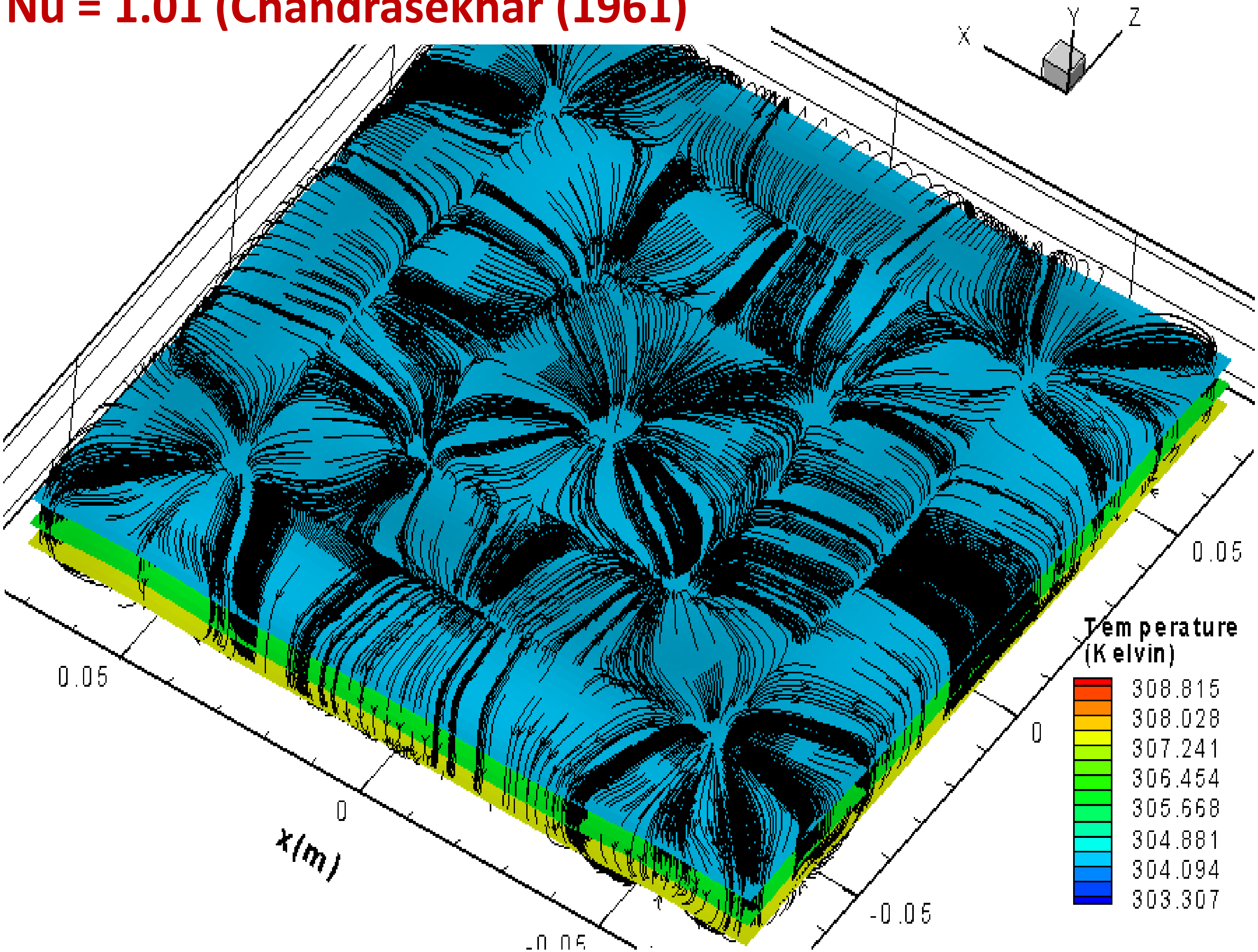
$Q = 1210$





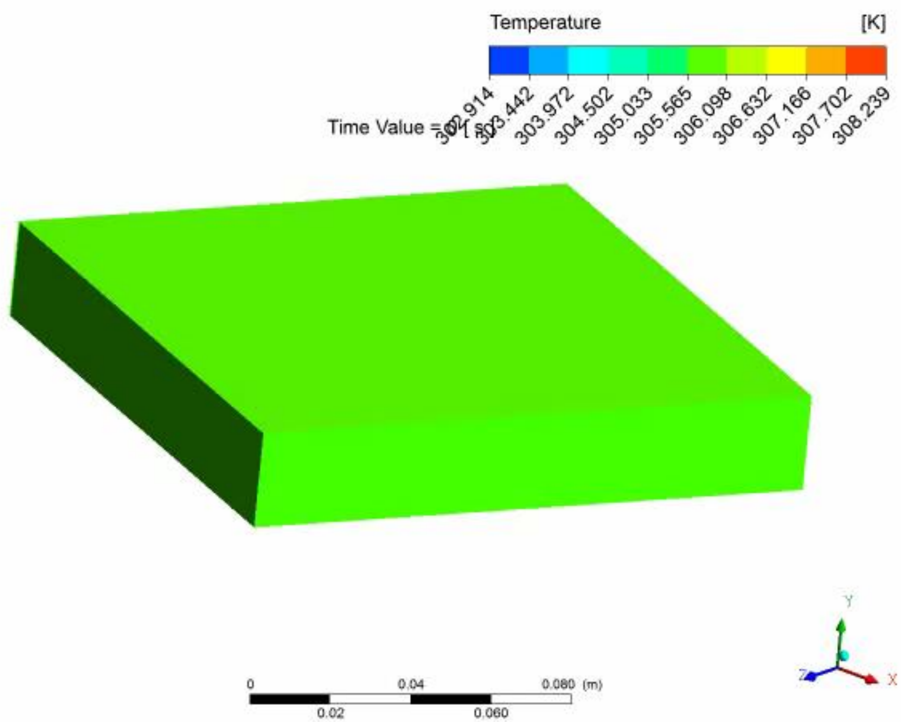


Q= 2000, Ra = 30139
Nu = 1.01 (Chandrasekhar (1961))



Plane layer Dynamo
RBC with magnetic field and rotation

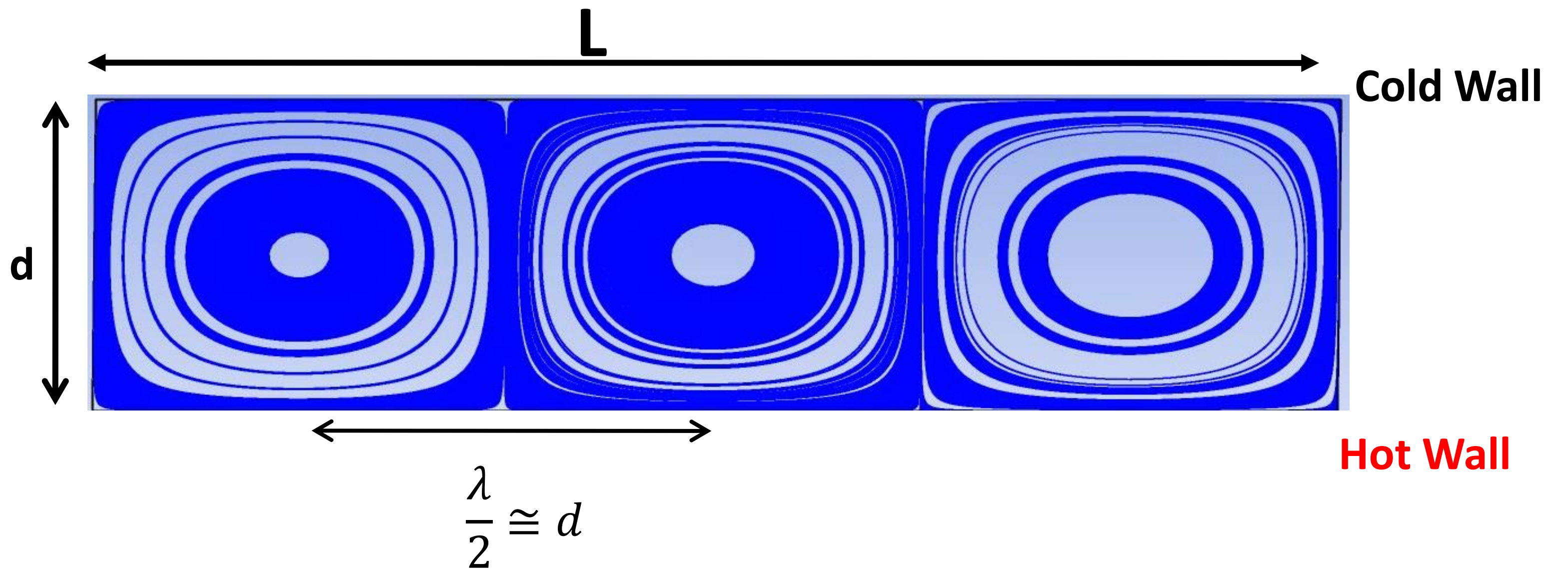
Ta = 970, *Q* = 1210 and *Ra* = 25500



Stress free boundary conditions

Ta = 0, *Q* = 0

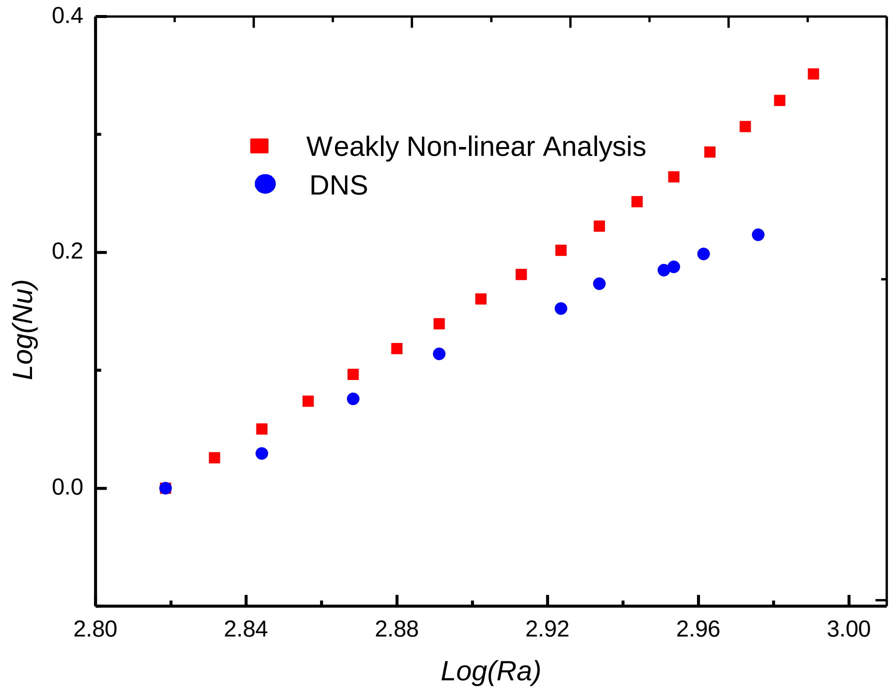
Aspect Ratio (=L/D)= 4



Diameter of the convection cells is half the critical wave-length $\lambda_c = \frac{2\pi}{k_c} = 2.82$, k_c is the critical wavenumber and $k_c = \pi/\sqrt{2}$.

Critical *Ra* = 657.25

Comparison



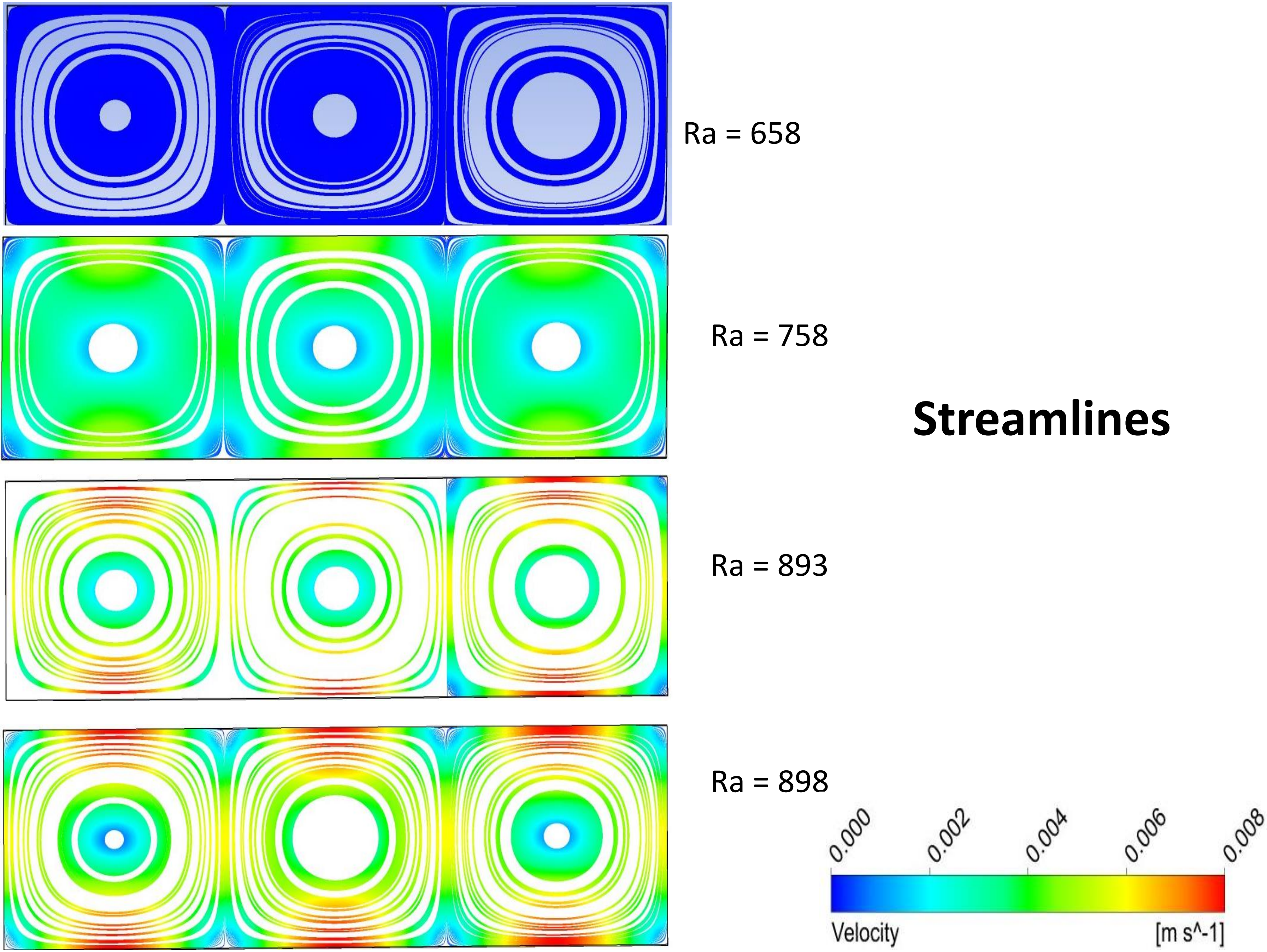
Weakly non-linear analysis – Amplitudes $A_i, i = 1, 2, 3, 4, 5, 6,$ [Kuo (1961)]

$$A_1 = \left[\frac{R_{os}}{R_o}\frac{2(\pi^2 + a^2)(4\pi^2)}{\pi^2}\right]^{\frac{1}{2}} \qquad A_{3\ at\ Q=0} = (0.906)\left(\frac{R_{os}}{R_o}\right)^{3/2} + (5.442)\left(\frac{R_{os}}{R_o}\right)^{1/2}$$

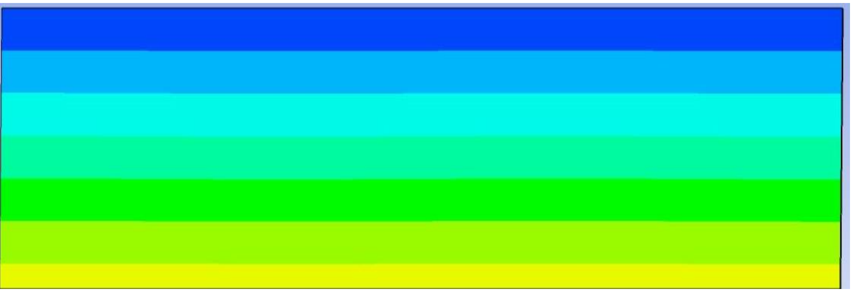
$$A_5 = \left(\frac{R_{os}}{R_o}\right)^{5/2}(S_3) + \left(\frac{R_{os}}{R_o}\right)^{3/2}(S_4) + \left(\frac{R_{os}}{R_o}\right)^{1/2}(S_5) \qquad A_2 = A_4 = A_6 = 0.$$

$$S_3 = \frac{0.0064}{Pr_1^2} + \frac{0.029}{Pr_1} - 0.22, \qquad S_4 = 1.36,\ S_5 = 10.23$$

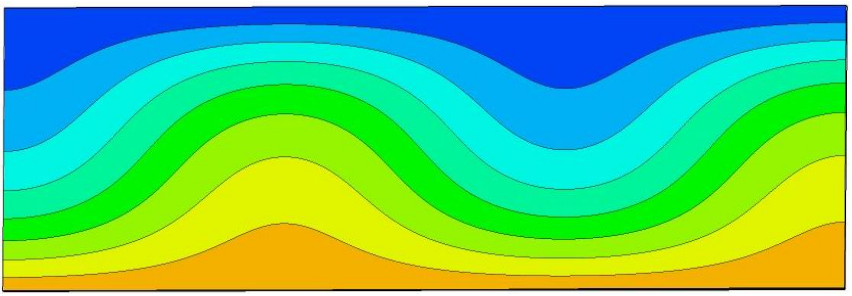
Streamlines



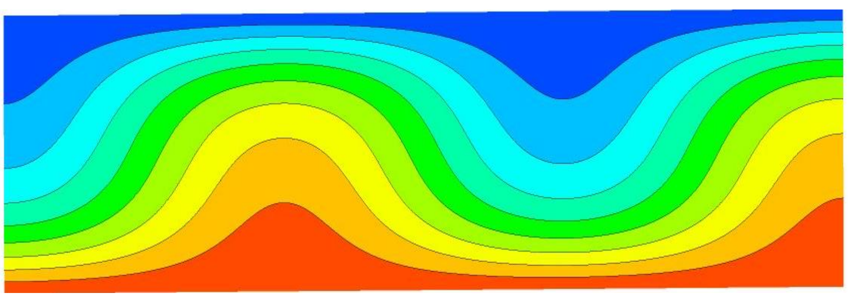
Isotherms



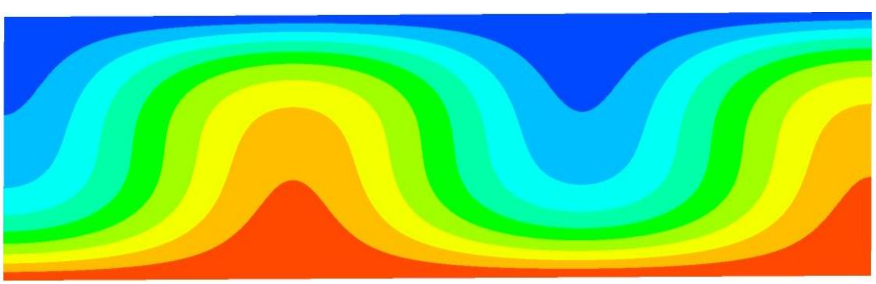
Ra = 658



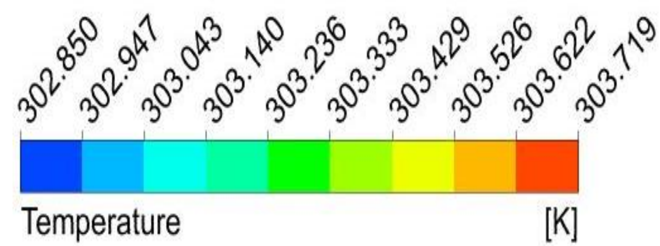
Ra = 758



Ra = 893

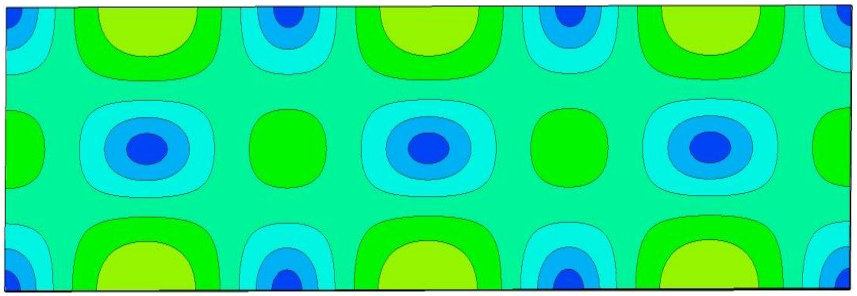


Ra = 1183

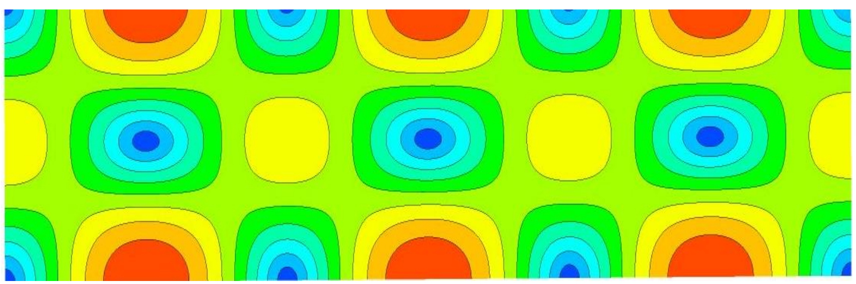


Velocity magnitude

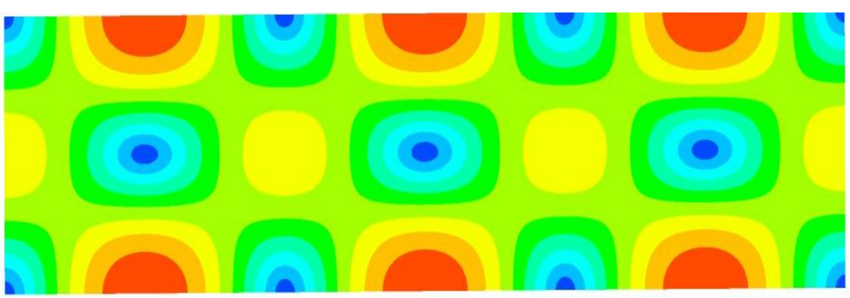
Ra = 658



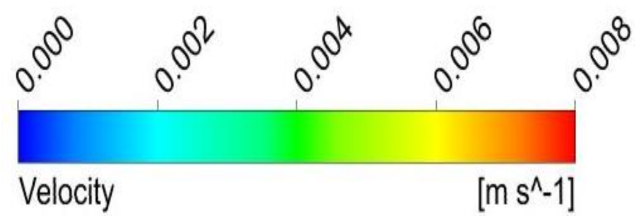
Ra = 758



Ra = 893



Ra = 898



Conclusions

- In RBC when Ra < 1708 we get Nu < 1, i.e. the existence of the sub-critical flow. This result implies that heat can propagate in the system due to a “conductive mode”.
- When Ra>1708 symmetric rolls perpendicular to vertical Y -axis are observed.
- As Ra further increases from 1708, the horizontal width of the rolls increases.
- In the presence of an applied magnetic field, the MC shows the cylindrical rolls. For the Chandrasekhar number 2000, a unique pattern of 3D rectangular vortical structures is noticed. The onset of convection is delayed due to the effect of Lorentz force on thermal buoyancy force.

- In **RMC** at low rotation rates horizontally stretched multi-cellular rolls perpendicular to the gravity axis again arise.
- Simulations show the oscillatory nature of thermal convection with the formation of thin Hartmann layers close to boundaries due to strong damping effect of the magnetic field on flow velocities and heat transfer.
- For stress-free boundary conditions
  - Nu = 1.00 for Ra = 658
  - the plumes increases in vertical direction with Ra.

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Thank you