



# Partitioning uncertainty components of an incomplete ensemble of climate projections using data augmentation

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**EGU2020: Sharing Geoscience Online**

<https://meetingorganizer.copernicus.org/EGU2020/displays/36913>



- Adaptation planners need
  - on regional scales and for the next few decades –  
climatic projections but associated uncertainties are large
- To reduce uncertainties, climate scientists need to know  
where they mostly come from, and then  
where allocation of funds / researches has to be concentrated
- Questions: in climatic projections...
  - How to obtain a robust partition of uncertainty sources ?
  - What is the main effect of each model (each GCM, each RCM, each HM)
  - What estimation method is to be used for unbalanced ensembles ?
  - What estimation method is to be used for incomplete ensembles ?
  - What are the largest uncertainty sources ?
  - Is it possible to narrow total uncertainty ?



- Uncertainty Sources in Multimodel Ensembles (MMEs)  
& The Hawkins and Sutton, 2009 heuristic partitioning approach
- Uncertainty Sources in MMEs of regional projections
  - Partitioning Uncertainty with the Quasi-Ergodic ANOVA approach
  - Ensemble Mean, Main Effects, Uncertainty estimates
  - An illustration from ADAMONT projections
- Partitioning Uncertainty in incomplete ensembles with QUALYPSO
- Uncertainty in precip. and temp. EUROCORDEX projections
- Supplementary Material
  - SM1 : More on the Quasi-Ergodic Assumption
  - SM2 : Large Scale and Small Scale Components of Internal Variability
  - SM3 : Comparing the precision of estimates in the Time Series ANOVA and the Single Time ANOVA approaches
  - SM4 : More on QUALYPSO



## Different uncertainty sources in climate projections

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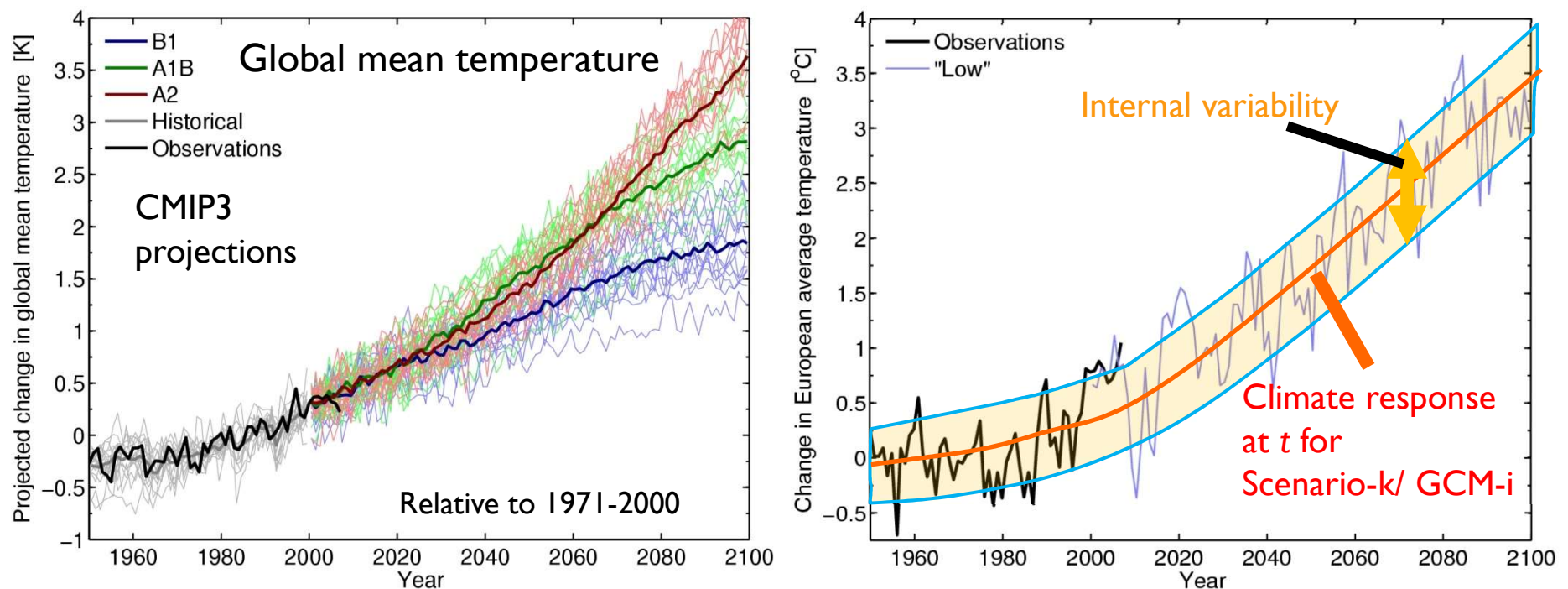
- **scenario uncertainty**, obtained from the different scenarios of future socio-economic development retained by IPCC
- **model uncertainty**, resulting from imperfections in representations of the earth system and of socio-hydro-systems targeted by the climate impact analysis
- **internal variability**, irreducible and resulting from the chaotic nature of the climate system

# Mean trends and uncertainty sources in climate projections



are quantified from multi-scenario multi-model multi-member ensembles.

The heuristic partition of Hawkins and Sutton, 2009, 2011 ...



**Internal variability** – spread in residuals from **climate responses** (quasi-ergodic assumption)

**Scenario uncertainty** – spread between multi-model means of **climate responses**

**Model uncertainty** – spread between multi-scenario means of **climate responses**

**Climate response** of each simulation chain is the long term trend of the variable over CTL+FUT

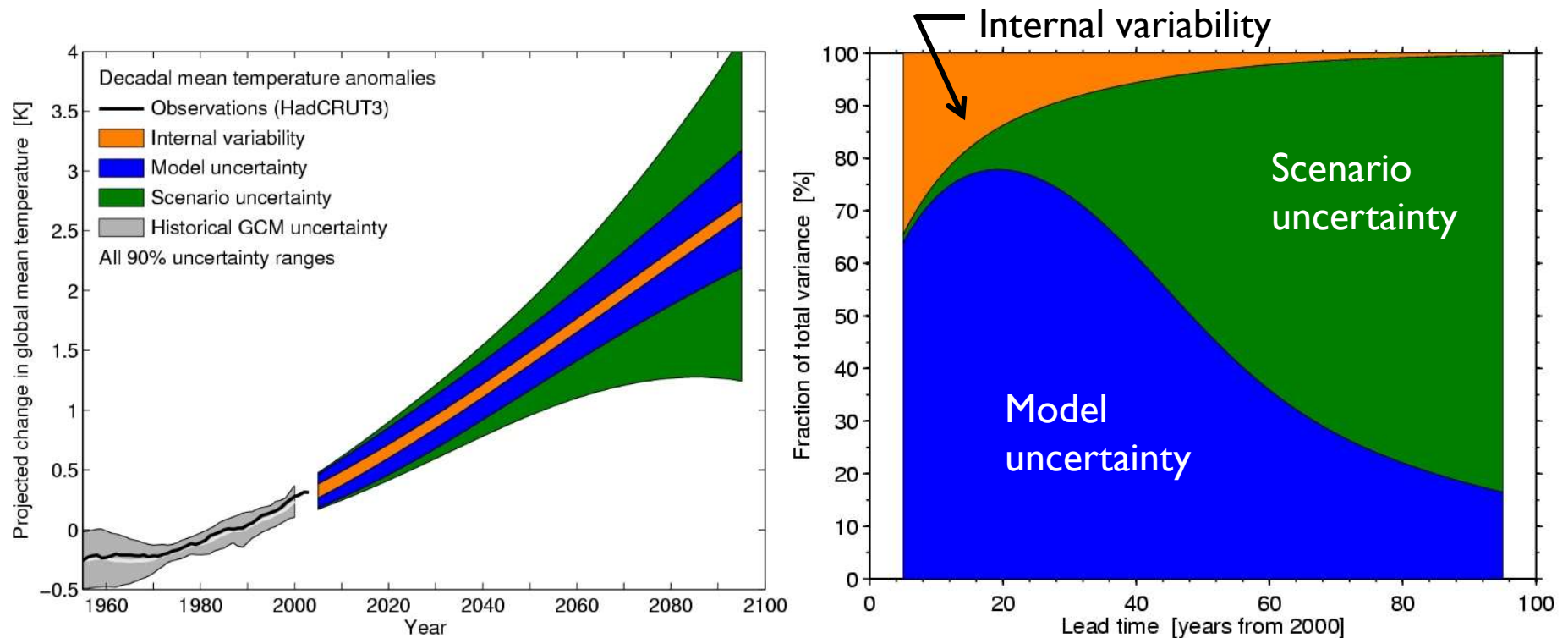
Hawkins and Sutton, 2009, 2011 : heuristic partition

# Mean trends and uncertainty sources in climate projections



are quantified from multi-scenario multi-model multi-member ensembles.

The heuristic partition of Hawkins and Sutton, 2009, 2011 leads to the iconic figures



**Internal variability** – spread in residuals from **climate responses** (quasi-ergodic assumption)

**Scenario uncertainty** – spread between multi-model means of **climate responses**

**Model uncertainty** – spread between multi-scenario means of **climate responses**

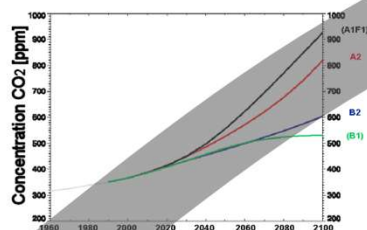
**Climate response** of each simulation chain estimated from smooth fits to CTL+FUT simulations



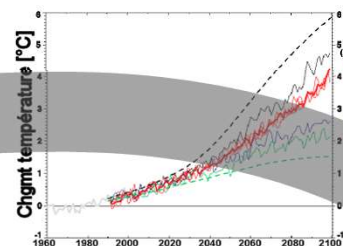
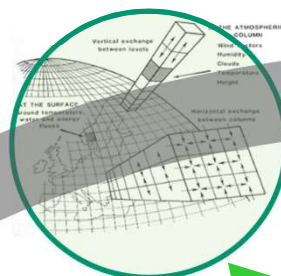
For regional climate projections in broad sense (e.g. hydrological) ...

The **simulation chain**  
(GCM+RCM+HM...)  
typically includes  
different types of models

GHG emission  
scenario

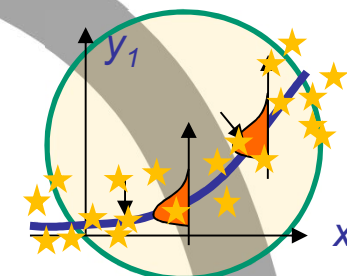


Global Climate  
Model (GCM)



Global / regional  
Climate Scenarios

Regional Downscaling Model  
(RCM or SDM)



Time series of  
Local (spatial)  
Weather Scenarios

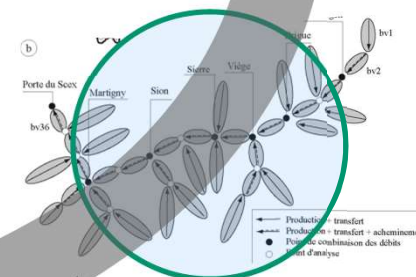
And then gives other  
components of Model Uncertainty :  
RCM uncertainty, HM uncertainty...

Impact Model  
(e.g. Water Resource  
Managt.)



Times series of performance /  
impact criteria for different  
related eco-socio-systems

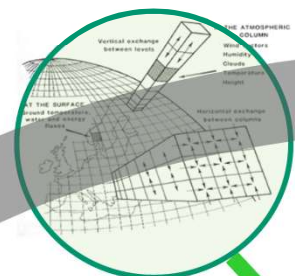
Hydrological  
Model (HM)



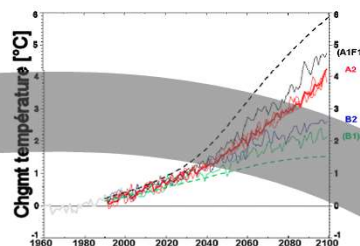
Times series of discharges at  
different locations of the river  
basin

For regional climate projections in broad sense (e.g. hydrological) ...

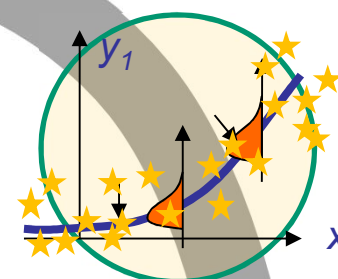
The **simulation chain** may additionally include a small scale component of Internal Variability (disregarded in the following)



**Global Climate Model (GCM)**



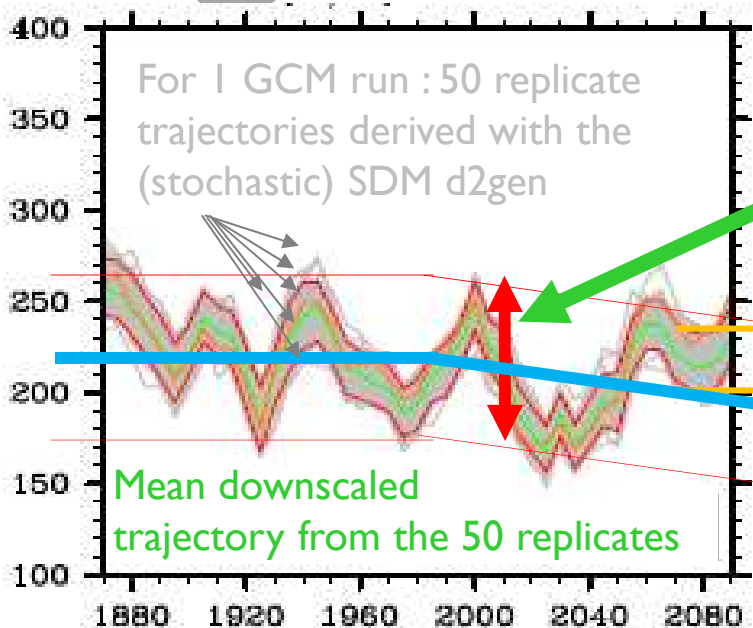
Global / regional Climate Scenarios



**Regional Downscaling Model (RCM or SDM)**

Time series of Local (spatial) Weather Scenarios

P DJF Durance River



**Large Scale Internal Variability**  
Derived from GCM internal variability

**Small Scale Internal Variability**  
Derived from the uncertain statistical downscaling link

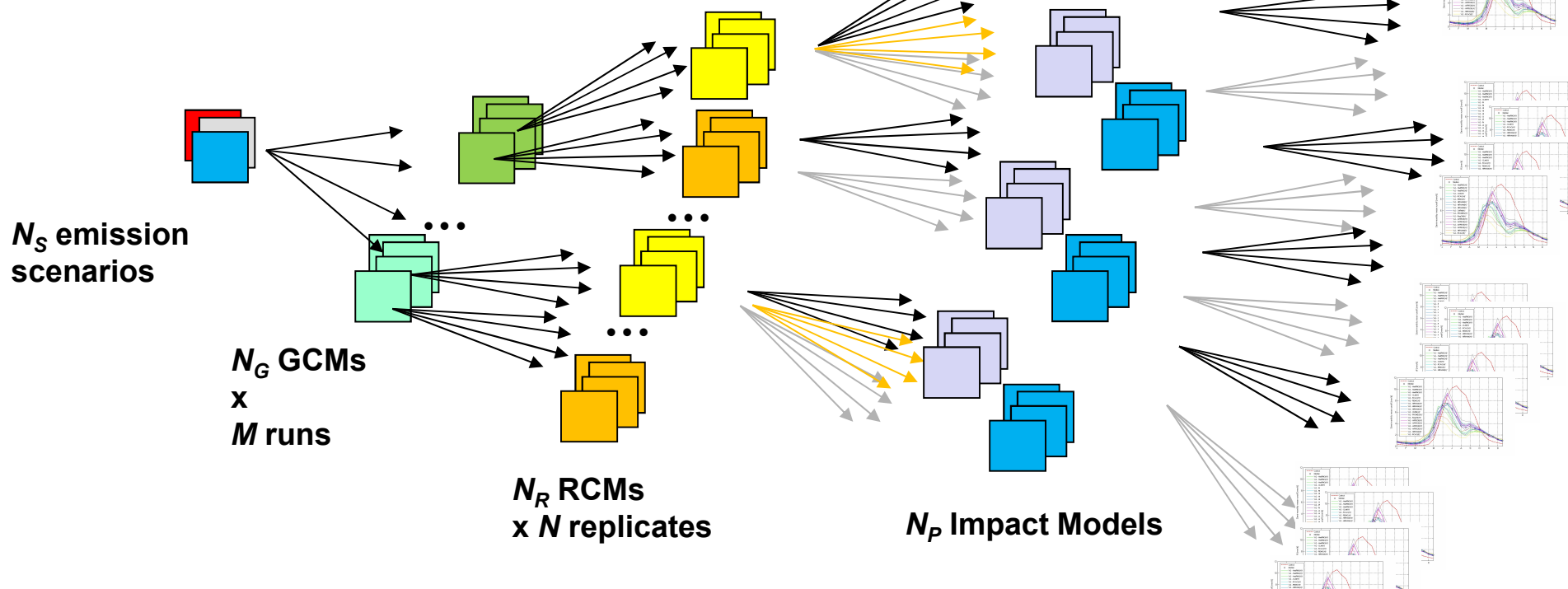
>> see Supplementary SM2 for more and (Lafaysse&al, 2014,Vidal&al2016)



... uncertainty sources are then estimated with ensembles of

- multiple scenarios
- multiple GCMs (the norm)
- multiple RCMs / Impact Models (also almost the norm)
- multiple GCM runs (to be perhaps a also norm)
- multiple replicates of the downscaling model (unusual yet)

➔ The « cascade » of uncertainty  
(Scenarios/Models + Internal variability)



# Which method for a Hawkins and Sutton like analysis For ensembles with multiple types of models (GCM, RCM, HM...)



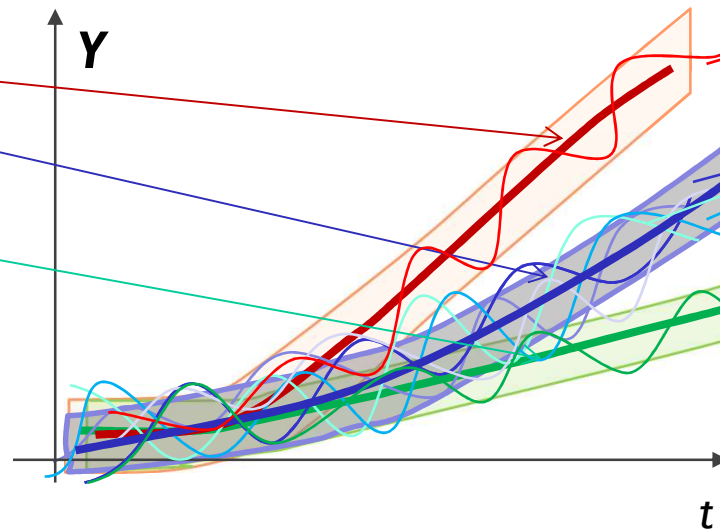
**Times Series ANOVA** approach with the **Quasi-Ergodic assumption** (Hingray&Said, 2014)

## STEP 1: For each GCM/RCM/HM... simulation chain

- Estimation of the climate response
- Estimation of **internal variability** from the deviations from the climate response
- Estimation of the **climate change response** :  $\text{response(FUT)} - \text{response(REF)}$

Climate Response

at  $t$   
of **chain 1**  
of **chain 2**  
...  
of **chain N**



One experiment with chain  
GCM « g » / RCM « r » / HM « h »...

Or multiple experiments  
if multiple runs of GCM « g »  
are available

Internal variability band  
of the chain around its  
climate response

**Quasi-Ergodic assumption** (see Supplementary SMI for more), in short :  
Sample variance of  $Y$  at  $t \approx$  temporal variance of  $Y$  for one run (or multiple runs if any)

## Which method for a Hawkins and Sutton like analysis For ensembles with multiple types of models (GCM, RCM, HM...)

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### STEP 2: ANOVA : from the climate change responses of all chains in the ensemble

The change response  $X_{s,g,r}(t)$  at  $t$  for chain = [scen. «  $s$  », GCM «  $g$  », RCM «  $r$  », ... ]  
is assumed to be the sum of the main effects of scenario «  $s$  », GCM «  $g$  », RCM «  $r$  », ...:

$$x_{s,g,r}(t) = \mu(t) + S_s(t) + G_g(t) + R_r(t) + \dots + \varepsilon(s,g,r,\dots,t)$$

$\mu(t)$  : ensemble mean change response for  $t$

$S_s(t)$  : main effect of scenario «  $s$  » for  $t, s = 1..N_s$

$G_g(t)$  : main effect of GCM «  $g$  » for  $t, g = 1..N_G$

$R_r(t)$  : main effect of RCM «  $r$  » for time  $t, r = 1..N_R$

$\varepsilon(s,g,r,\dots,t)$  : model residuals (including interactions (scenario/GCM, GCM/RCM...))

*With constraints : sum of scenario effects = 0, sum of GCM effects = 0 ....*

# Which method for a Hawkins and Sutton like analysis For ensembles with multiple types of models (GCM, RCM, HM...)

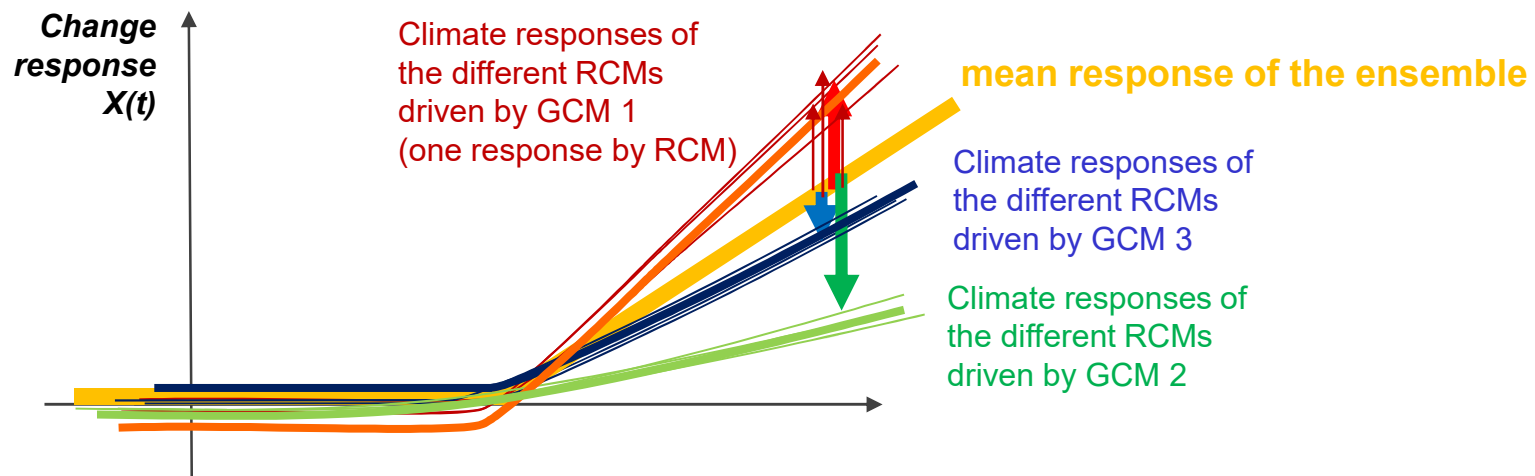


## STEP 2: ANOVA : from the climate change responses of all chains in the ensemble

The ANOVA gives

- An estimate of the ensemble mean, of the main effect of each scenario, of each model and of interactions between scenario/GCM, GCM/RCM, ....

The main effect of GCM1 for  $t$  is the mean deviation (vertical bold red arrow) of GCM1 climate response (bold red line) over all scenarios/all RCMs.... from the multimodel mean response (orange)



Nb : Vertical red, green, blue bold arrows are the main effects  $G_1(t)$ ,  $G_2(t)$  and  $G_3(t)$  of GCM 1, 2 and 3 respectively  
This scheme is for a MME configuration where only one scenario is available. Same principle for multiscenario MMEs

A similar representation would highlight the main effects  $R_1(t)$ ,  $R_2(t)$  ... of the different RCMs (or scenarios, or HMs...)

# Which method for a Hawkins and Sutton like analysis For ensembles with multiple types of models (GCM, RCM, HM...)



## STEP 2: ANOVA : from the climate change responses of all chains in the ensemble

The ANOVA gives next

b. Estimates of scenario uncertainty and of the model uncertainty components

Scenario uncertainty	– spread	between the main effects	of the different scenarios
GCM uncertainty	– spread	....	of the different GCMs
RCM uncertainty	– spread	....	of the different RCMs
HM uncertainty	– spread	....	of the different HMs
...	...		

c. An estimate of total uncertainty variance :  $T(t) = S(t) + G(t) + R(t) + RV(t)$

Where :  $S(t)$  : Scenario Uncertainty variance  
 $G(t)$  : GCM Model Uncertainty variance  
 $R(t)$  : RCM Model Uncertainty variance  
...  
 $RV(t)$  : Residuals variance

Important Note : in most cases, this « Time Series ANOVA » gives a more precise and robust estimate of all uncertainty components than the « Single Time ANOVA » (Hingray et al. 2019).  
See Supplementary material SM3 at the end of the ppt for more



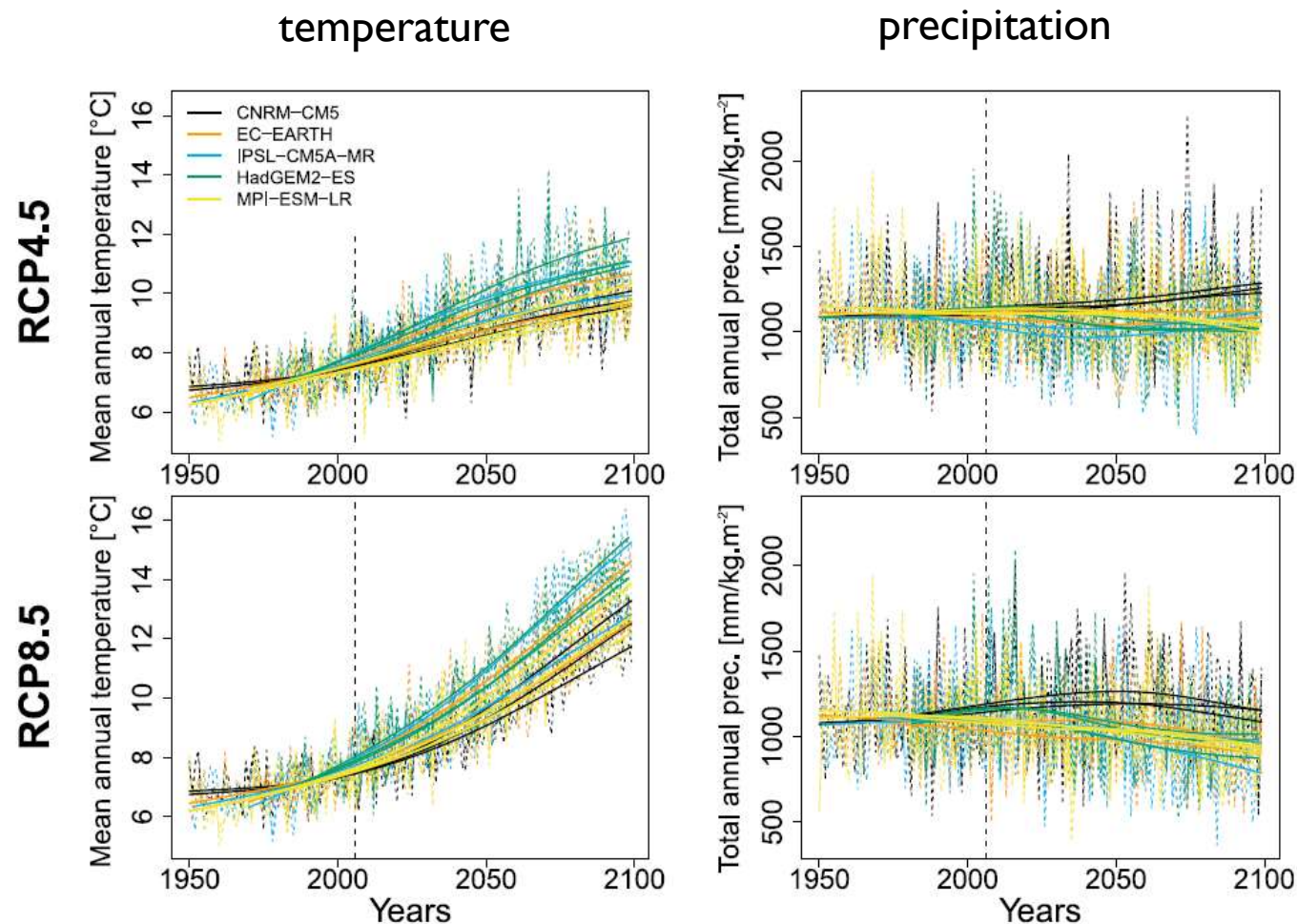
# Illustration : ADAMONT projections: Haut Verdon, French Alps



## STEP I: a. Estimation of climate responses from times series of raw projections

19 ADAMONT  
experiments  
(Verfaillie et al .  
2018)  
available  
from  
5 GCMs  
6 RCMs  
From  
EuroCordex

(Evin et al. 2019)



- Notes :
1. the different RCMs driven by the same GCM share the same color
  2. The figures of this illustration are extracted from Evin&al 2019

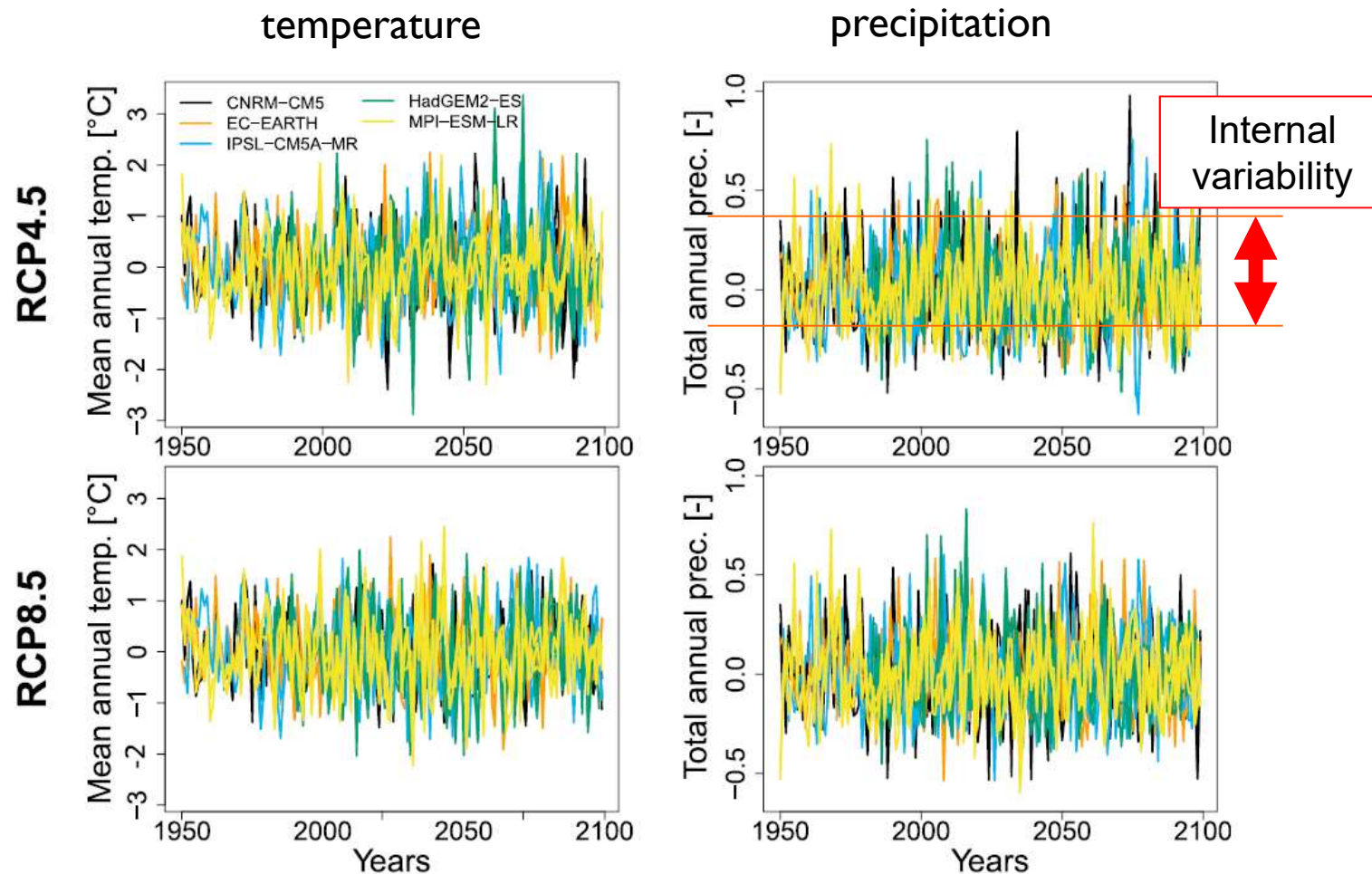
## Illustration : ADAMONT projections: Haut Verdon, French Alps



### STEP I: b. Estimation of internal variability from the deviations from the climate response

19 ADAMONT  
experiments  
(Verfaillie et al .  
2018)  
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# Illustration : ADAMONT projections: Haut Verdon, French Alps



## STEP 2 : ANOVA for each t gives

### >> a. estimates of Main Effects

#### How to read it ?

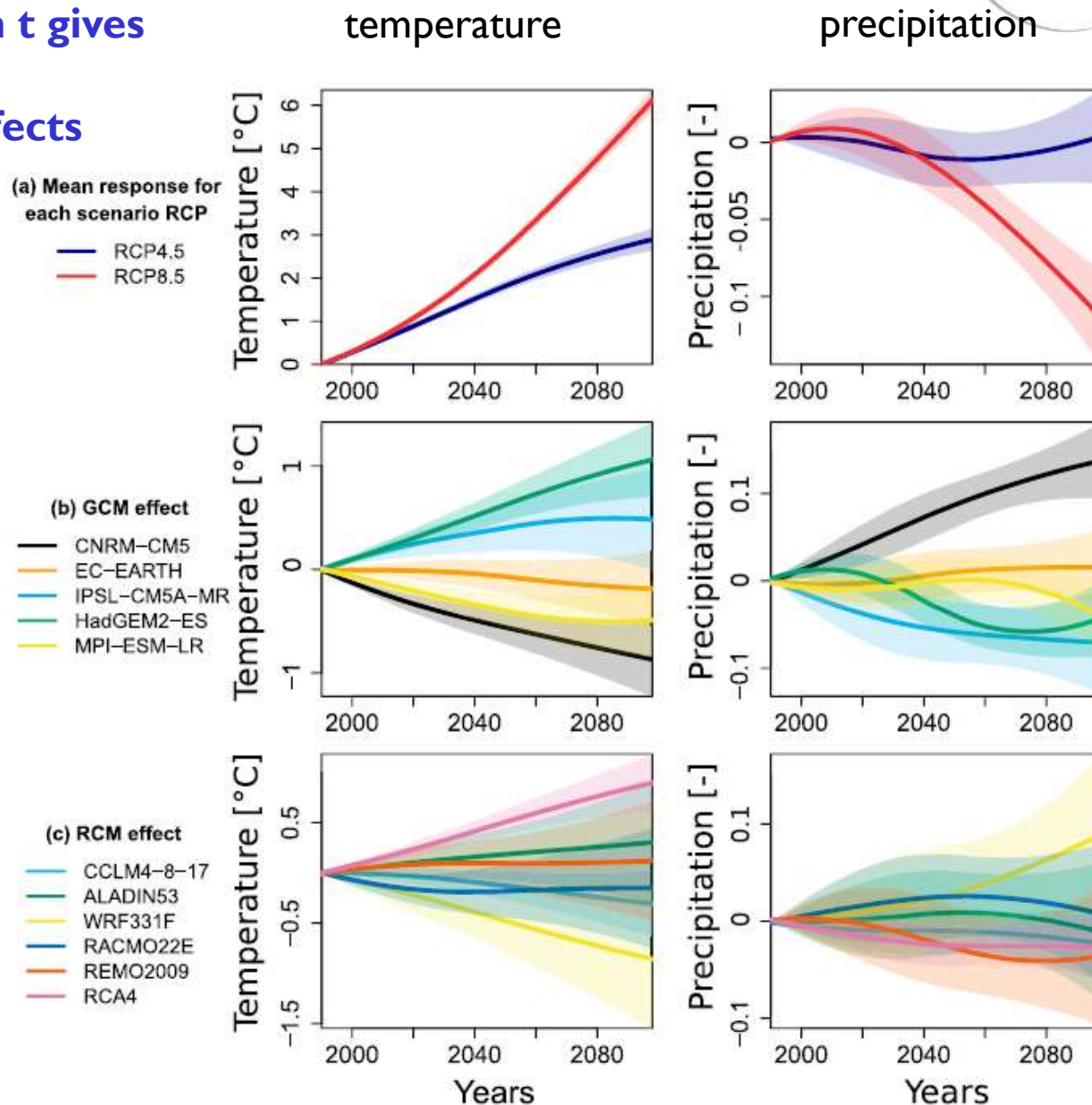
For instance : in 2080...

- HadGEM2 warms 1.1°C more than the mean
- RCA4 warms 0.7°C more than the mean

The change in prec. is

- 13% higher in CNRM-CM5 than the mean
- 10% higher in VVRF than the mean

Note : the colored bounds give the uncertainty of the estimation of each effect





# Illustration : ADAMONT projections: Haut Verdon, French Alps



## STEP 2 : ANOVA for each t gives

>> b. estimates of :

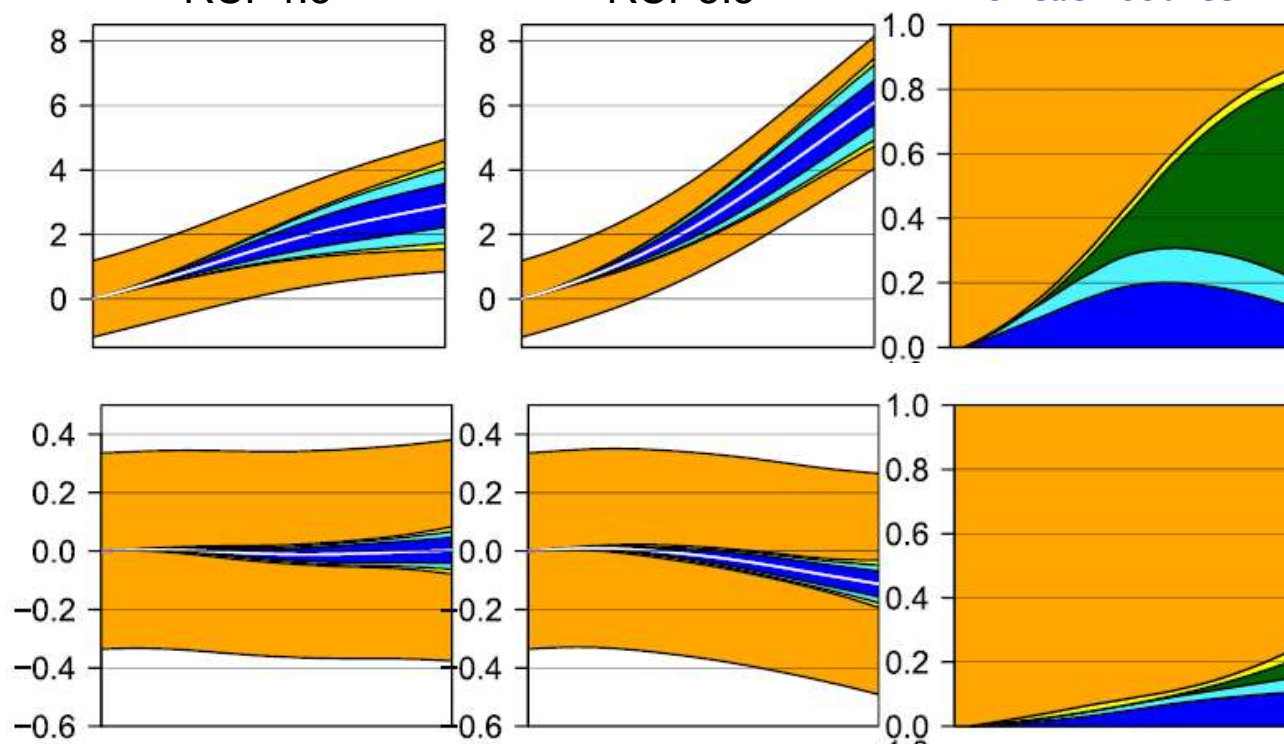
>> Mean trend + total uncertainty  
(without RCP uncertainty)

>> Contribution  
(% Total variance)  
of each source

temperature

RCP4.5

RCP8.5



(a) Trends + uncertainty  
sources with RCP4.5

■ GCM  
■ RCM  
■ Res. Var.  
■ Int. Variab.

(b) Trends + uncertainty  
sources with RCP8.5

■ GCM  
■ RCM  
■ Res. Var.  
■ Int. Variab.

(c) Variance  
decomposition

■ GCM  
■ RCM  
■ RCP  
■ Res. Var.  
■ Int. Variab.

# Partitioning uncertainty sources in incomplete ensembles



EUROCORDEX projections, at least 87 experiments available today :

- Transient : +1980-2100+
- Resolution : 12.5km
- RCP2.5, 4.5, 8.5
- 9+ GCMs
- 13+ RCMs

GCM/RCM	CanESM2	CNRM-CM5	EC-EARTH	HadGEM2-ES	IPSL-CM5A-MR	MIROC5	MPI-ESM-LR	NorESM1-M	GFDL-ESM2G
REMO	○	⊕	⊗	⊗		⊗	⊗	⊗	⊗
CCLM4-8-17	○	⊕	⊗	⊕		⊗	⊗		
ALADIN63		⊗		○					
RACMO22E		⊗	⊗	⊗	○		○	○	
HIRHAM5		○	○	⊕			○	⊕	
WRF381P		○		○	⊕			○	
RCA4		⊕	⊗	⊗	⊕		⊗	⊗	
WRF361H			○	○		○	⊗		
RegCM4-6				⊗			○		
COSMO-CLIM							○	○	
ALARO-0		⊗	○						
ALADIN53		⊗	○						
HadREM3-GA7				○					

But... EUROCORDEX is an incomplete ensemble :  
A lot of missing GCM x RCM combinations ...

This is a very usual configuration...

Here, simple ANOVA approaches are not suited.

They often lead to biased estimates of uncertainty components.

○ RCP8.5  
⊕ RCP4.5  
⊗ RCP2.6

They often require to drop a large number of experiments in order to have a complete or almost complete ensemble (waste of information) ....

> e.g. 5x4 GCMs x RCMs from EUROCORDEX in Christensen & al. ClimDyn2020

Models allowing for a almost complete MME

RCM GCM



# Partitioning uncertainty sources in **incomplete ensembles**

## QUALYPSO : Quasi-ergodic AnaLYsis of climate ProjectionS using data augmentatiOn (Evin et al. 2019)

A Bayesian estimation approach  
based on data augmentation  
techniques which allows for :

- The reconstruction of all missing GCM x RCM combinations
- The estimation of main effects for all scenarios, GCMs, RCMs
- Provides in addition the uncertainty of estimates

**The same 2 steps estimation:**

- Step 1 :**
- a. Climate response** of each chain estimated with a trend model (cubic splines)  
> **inference with Bayesian Methods**
  - b. Climate change response (absolute or relative) / reference period (e.g. 1980-2010)
- Step 2 :** **ANOVA on climate responses** > main effects + uncertainty components  
> **inference with Bayesian Methods and Data Augmentation**



Evin, G., B. Hingray, J. Blanchet, N. Eckert, S. Morin, and D. Verfaillie. 2019. "Partitioning Uncertainty Components of an Incomplete Ensemble of Climate Projections Using Data Augmentation." *Journal of Climate*.

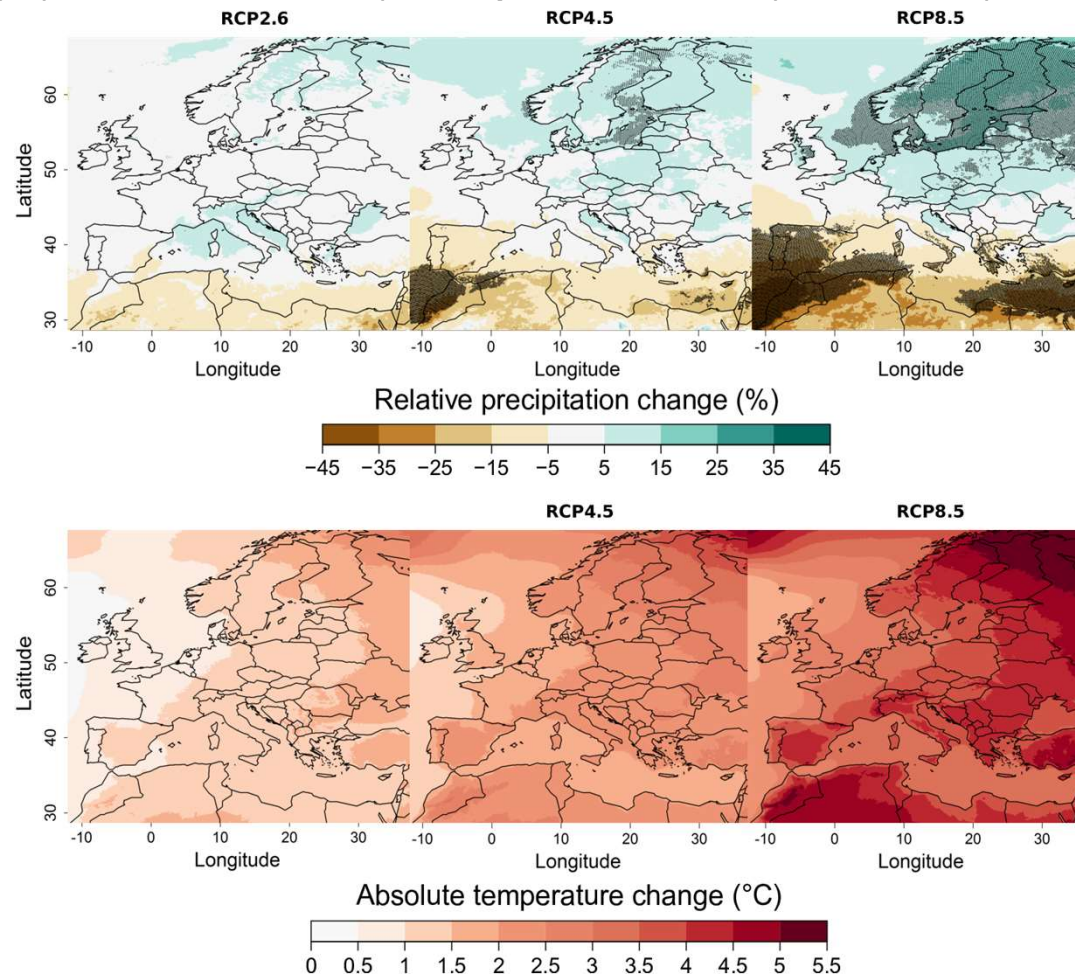


# Results from the 87 projections of EUROCORDEX

## Mean projected changes for annual precip. and temperature



End of Century (EoC - 2071-2099) compared to REF (1981-2010) for 30-year averages.



Extracted from : Evin, G., Hingray, B., Somot, S., (in prep) Uncertainty sources in Eurocordex Precipitation and Temperature projections : internal variability, scenario and model uncertainty, GCM and RCM effects.

Note : following results have been obtained "using smoothing splines (extended QUALYPSO method):"

<https://cran.r-project.org/web/packages/qualypsoss/index.html>

# Results from the 87 projections of EUROCORDEX

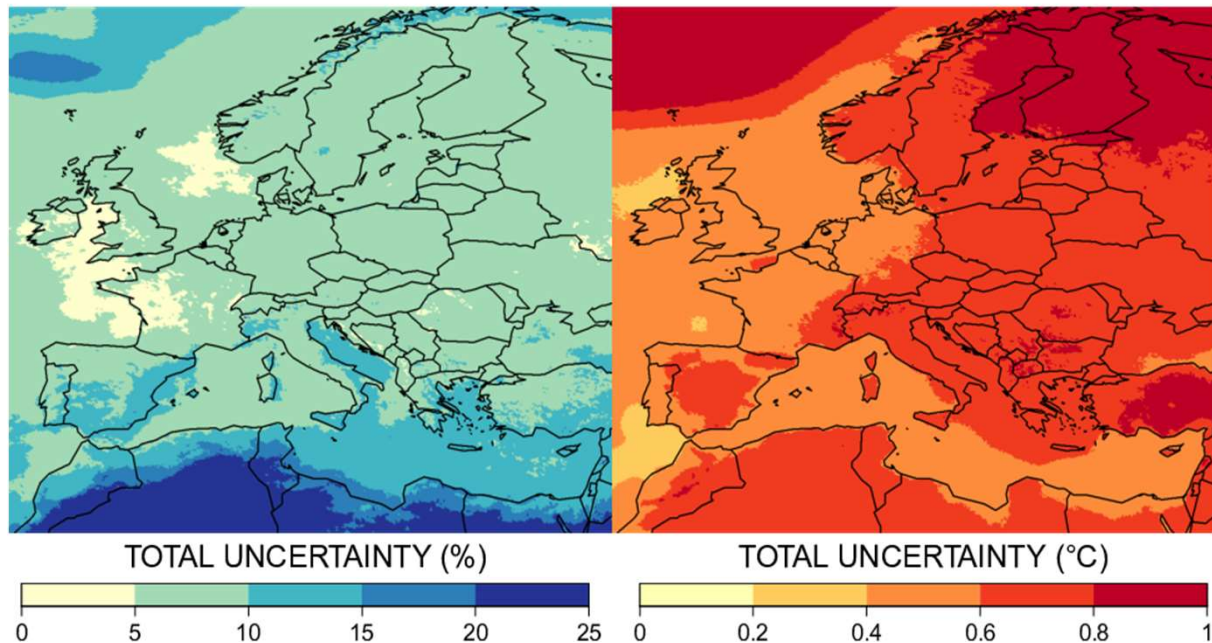
## Mean projected changes (EoC) for annual precip. and temperature



Total Uncertainty (without RCP uncertainty)

Relative precipitation change [%]

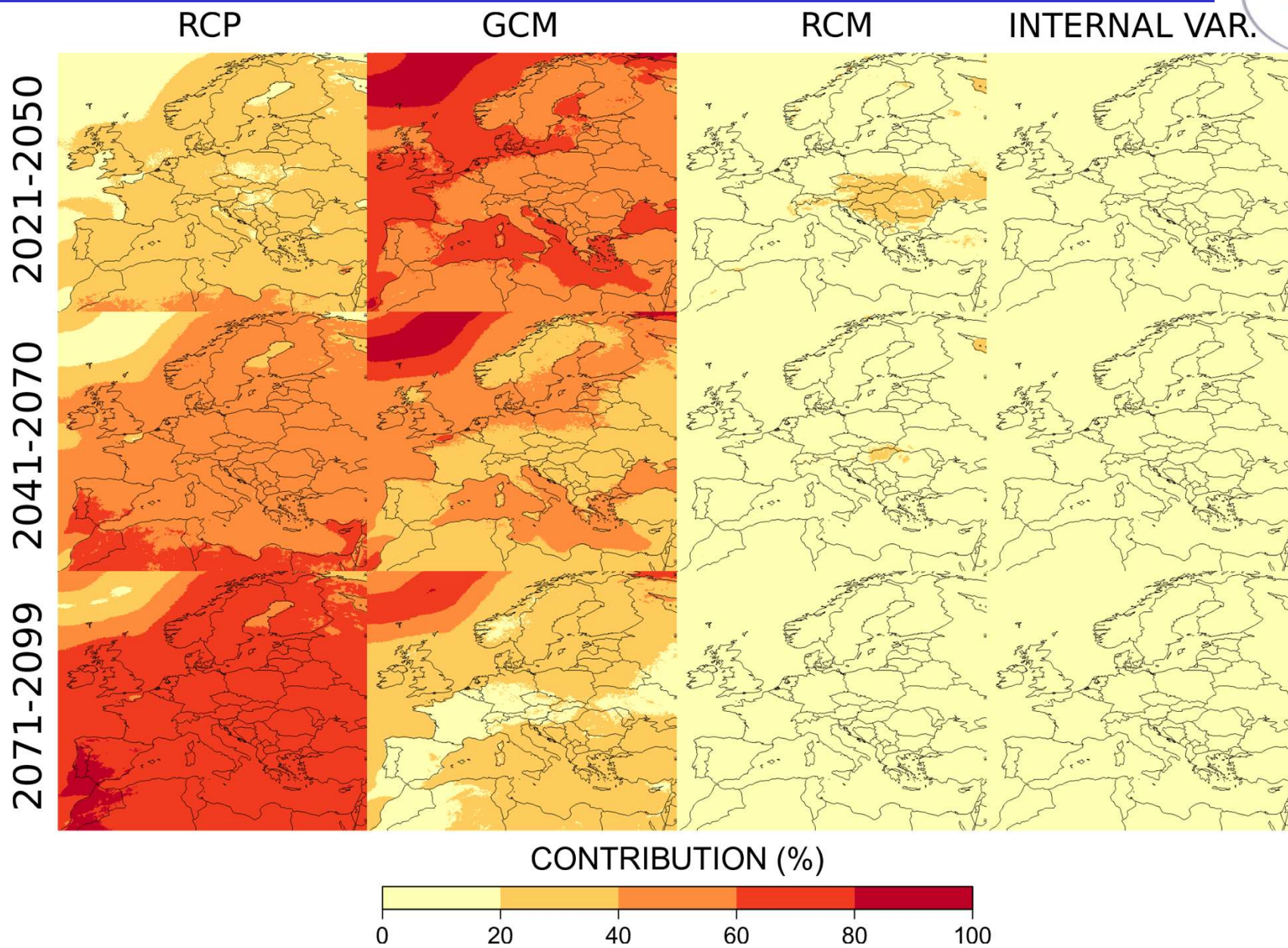
Absolute temperature change [°C]





# Annual temperature change (EoC)

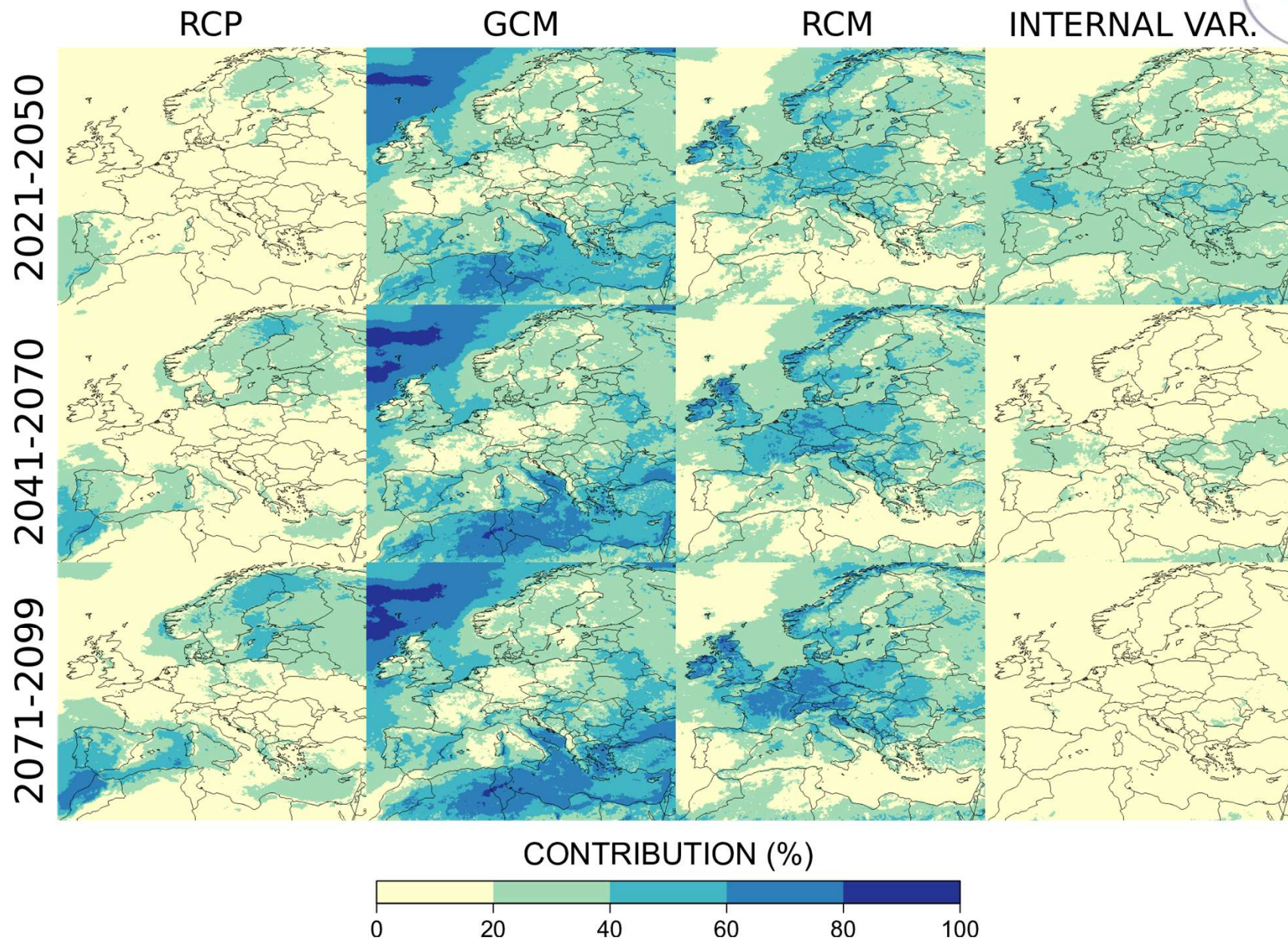
## Fractional variance for each uncertainty source





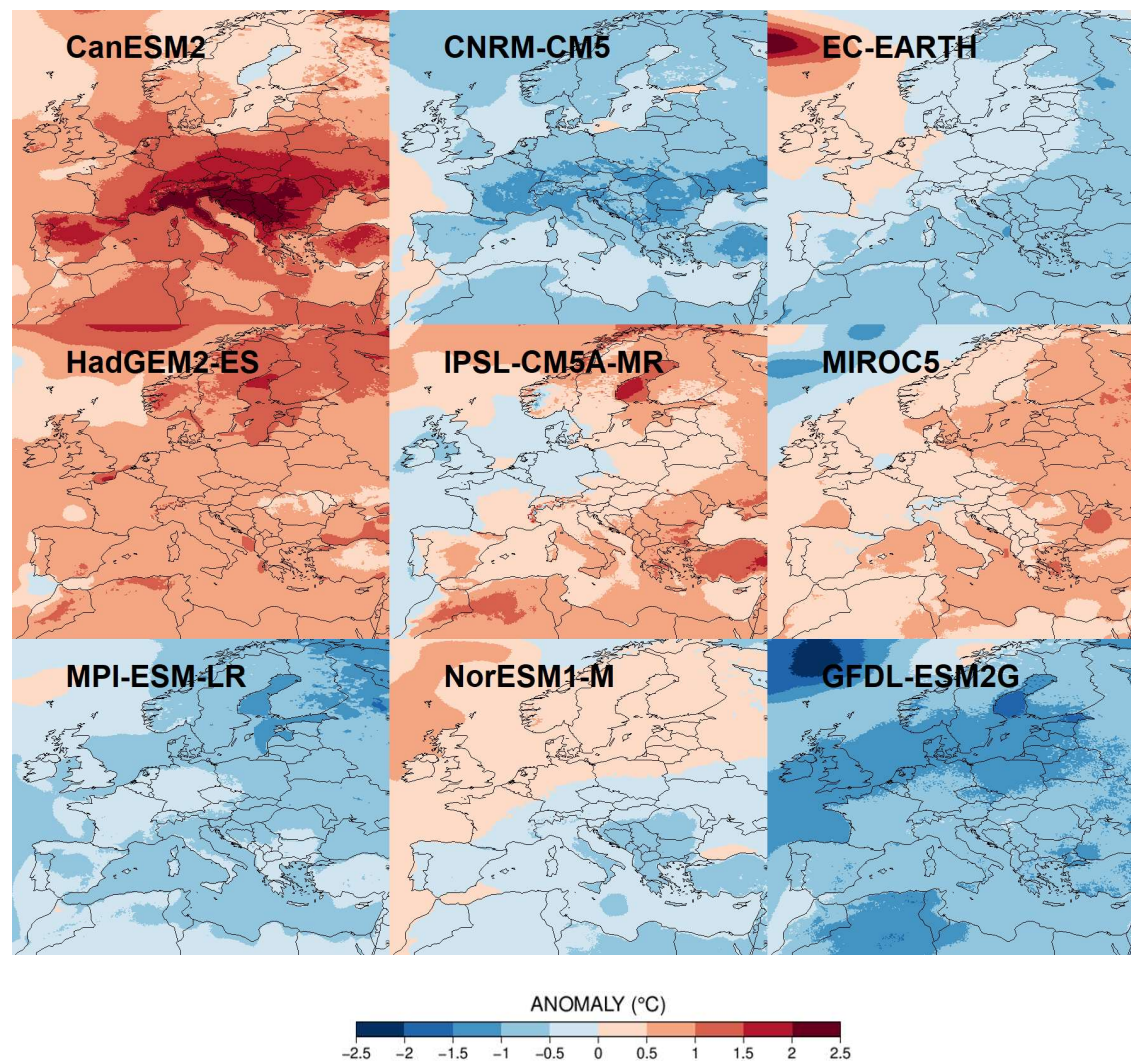
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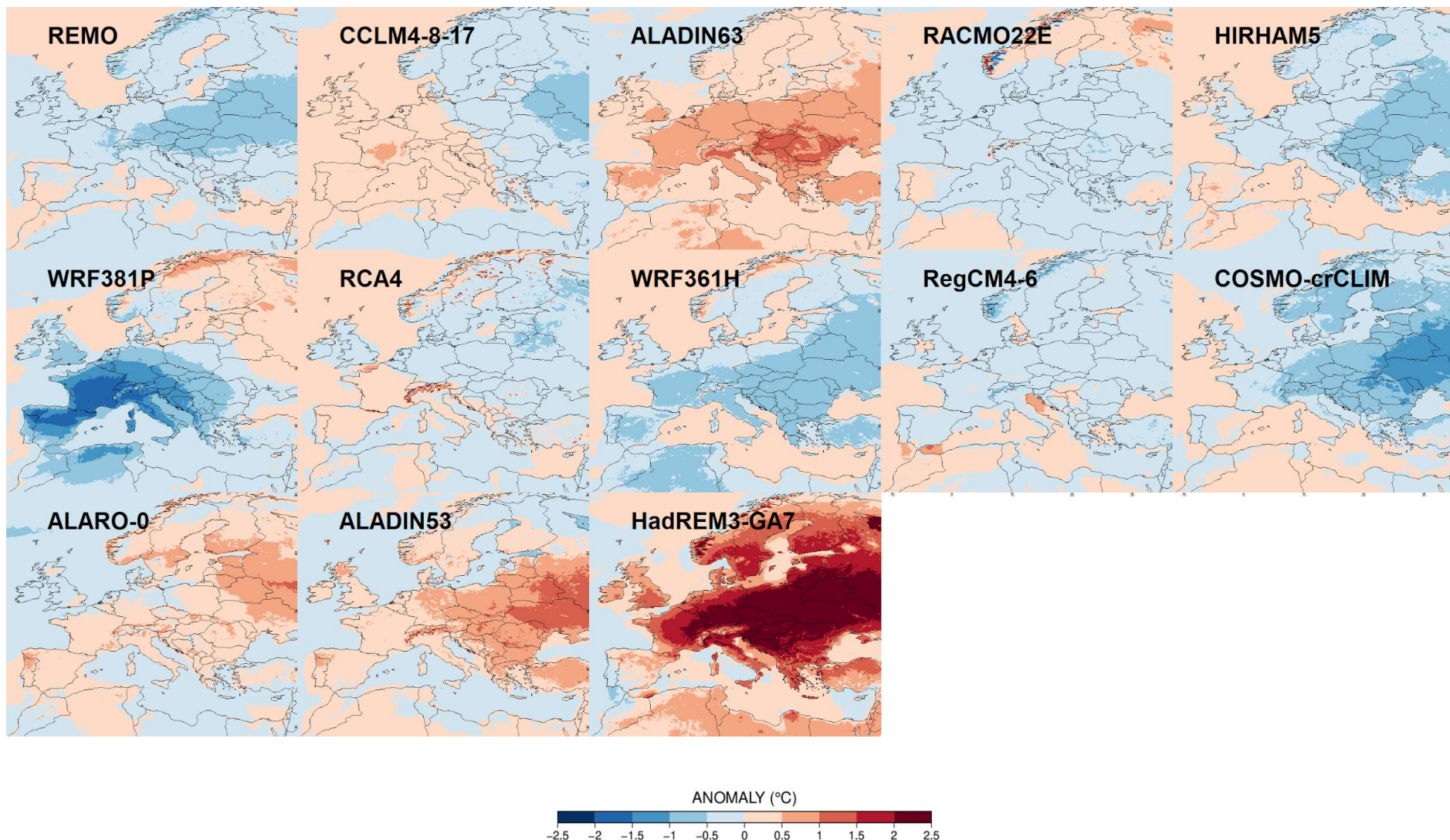




## JJA Temperatures changes (EoC) : GCM main effects



## JJA Temperatures changes (EoC) : RCM main effects







## Conclusions and perspectives

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- Most ensembles of projections are incomplete. This requires an appropriate statistical framework to estimate mean trends and to quantify / partition uncertainty
- QUALYPSO strengths :
  - It uses transient simulations and a time series ANOVA approach. This makes estimates more robust. It can deal with unbalanced MME (i.e. different #runs btw GCMs)
  - It uses data augmentation approaches. Missing chains are reconstructed. It can then be used as a scenario emulator.
  - It exploits all available projections (all chains, all runs). This allows avoiding wasting data and allows for more robust estimates
  - It is applicable to all climate projections (hydrology, agriculture, renewable energy) and can account for additional Model Uncertainty sources (Hydro. Model, ...)
- Results for P/T Projections from 87 EUROCORDEX GCM/RCM chains
  - Trends are coherent with previous studies
  - Precipitation : Largest contribution is RCM uncertainty in many places
  - Large land/sea contrasts are obtained for many RCMs, especially for T projections
  - A few GCM/RCM have a major contribution to Model Uncertainty in specific areas
- Package available on CRAN : <https://CRAN.R-project.org/package=QUALYPSO>
- Coming soon : extended QUALYPSO method with Smoothing Splines:  
<https://cran.r-project.org/web/packages/qualypsoss/index.html>



## Some references...

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### Partitioning model uncertainty and internal variability components

Hawkins and Sutton, 2009. The potential to narrow uncertainty in regional climate predictions.  
BAMS doi:10.1175/2009BAMS2607.1

Hingray et Saïd 2014. Partitioning internal variability and model uncertainty components  
in multimodel multimember ensembles of projections  
J.Climate. doi:10.1175/JCLI-D-13-00629.1  
**QEANOVA matlab package** : [http://www.lthe.fr/RIWER2030/download\\_fr.html](http://www.lthe.fr/RIWER2030/download_fr.html)

### Precision of uncertainty estimates : QEANOVA versus Single Time ANOVA

Hingray et al. 2019. Precision of uncertainty components estimates in climate projections  
Clim.Dyn. <https://doi.org/10.1007/s00382-019-04635-1>

### Incomplete ensembles

Evin et al. 2019. Partitioning uncertainty components with data augmentation  
J.Climate. <https://journals.ametsoc.org/doi/pdf/10.1175/JCLI-D-18-0606.1>  
**R package on CRAN** : <https://CRAN.R-project.org/package=QUALYPSO>  
**R package with smoothing splines** : <https://cran.r-project.org/web/packages/qualypsoss/index.html>

### Exemples of applications of a Time Series ANOVA, QEANOVA or QUALYPSO in impact studies

Vidal et al., 2016. Hydrol. Earth Syst. Sci. <https://www.hydrol-earth-syst-sci.net/20/3651/2016/>

Giuntoli et al. 2018. Climatic Change. <https://doi.org/10.1007/s10584-018-2280-5>

Alder and Hostetler 2018. WRR. <https://doi.org/10.1029/2018WRR023458>

Bichet et al. 2019. ERL <https://iopscience.iop.org/article/10.1088/1748-9326/ab500a>

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## Some Supplementary Material

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- SM1 : Some more on the QEANOVA approach
- SM2 : Large Scale and Small Scale Components of Internal Variability  
An illustration
- SM2 : Precision of a time serie and a single time ANOVA  
in synthetic ensembles of experiments
- SM3 : Some more on QUALYPSO





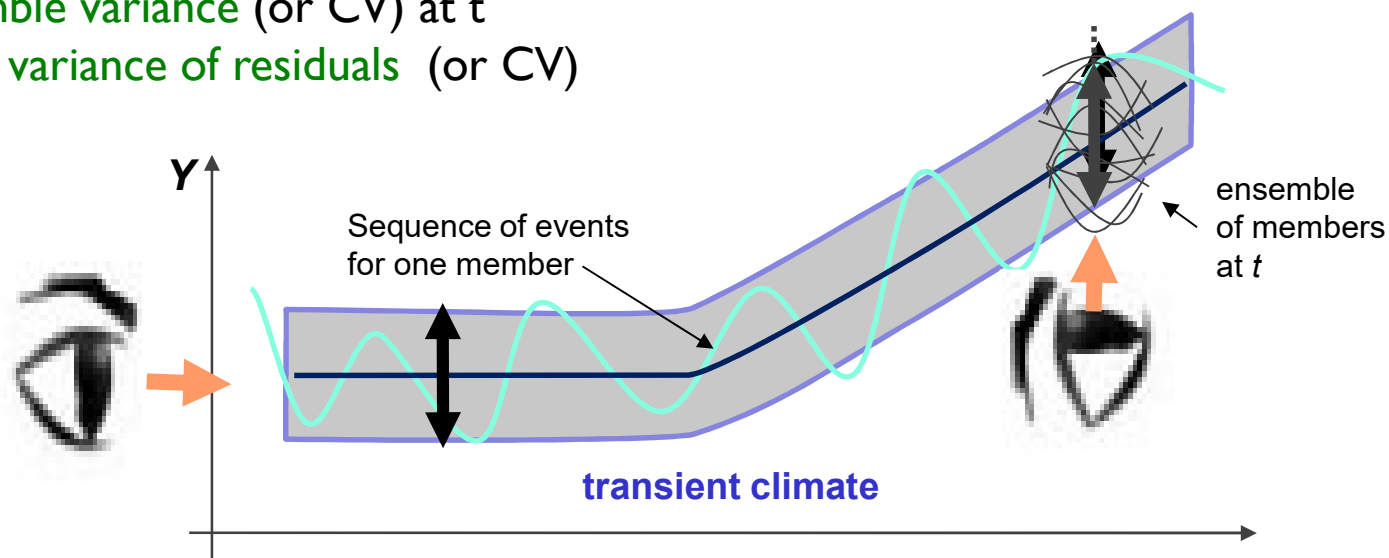
From

Hingray, B., Saïd, M., 2014. Partitioning internal variability and model uncertainty components in a multimodel multireplicate ensemble of climate projections. J.Climate. 27(17); pp. 6779-6798. <https://doi.org/10.1175/JCLI-D-13-00629.1>

## The quasi-ergodic assumption for transient climate simulations

For a **quasi-ergodic** system (stationary + transient state) : Hingray&Said, 2014

- **ensemble average** at a given time  $t$   
= trend at  $t$  of the **mean** for one sequence of events
- **ensemble variance** (or CV) at  $t$   
= **time variance of residuals** (or CV)



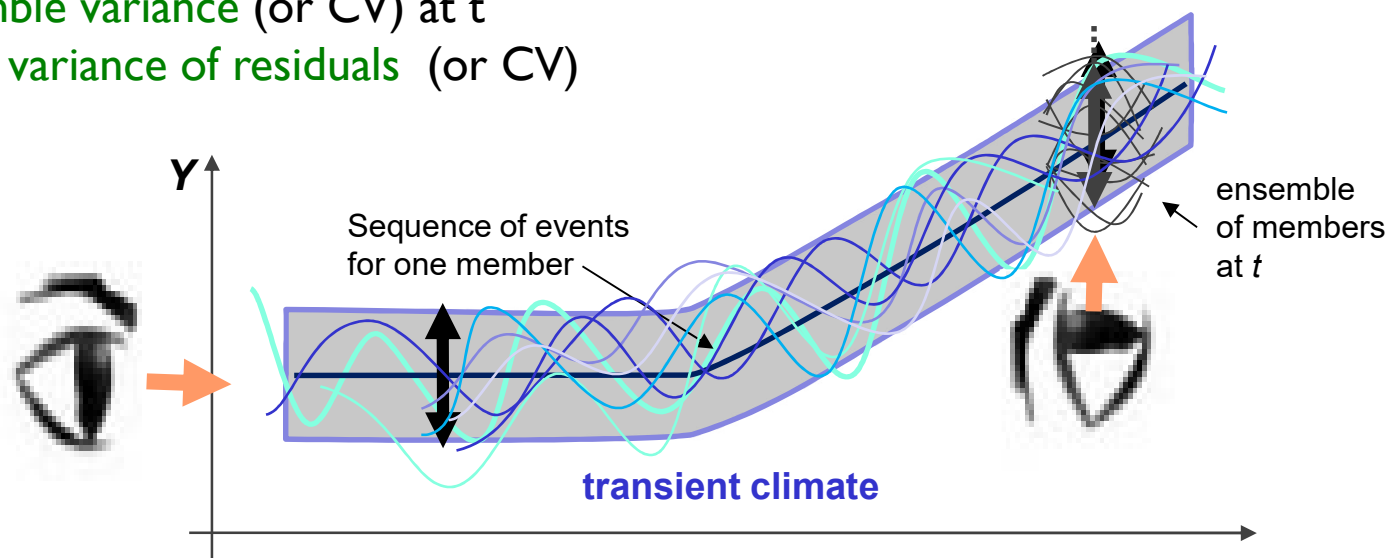
THUS, the Climate Response and Internal Variability of a given simulation chain

- **can be estimated even with a single member**

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= **trend at  $t$  of the mean** for one sequence of events
- **ensemble variance** (or CV) at  $t$   
= **time variance of residuals** (or CV)



THUS, the Climate Response and Internal Variability of a given simulation chain

- **can be estimated even with a single member**
- **can ALSO be estimated with the multiple members available for the chain**

>> The QEANOVA method can make use of all available data for the uncertainty estimation

>> The QEANOVA method can be applied on unbalanced MMEs

(i.e. MMEs where the number of runs differ from one chain to the other).



From

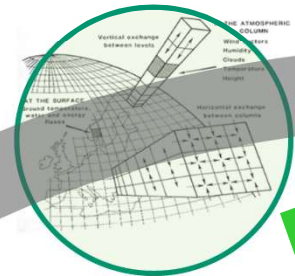
Hingray, B., Saïd, M., 2014. Partitioning internal variability and model uncertainty components in a multimodel multireplicate ensemble of climate projections. *J.Climate*. 27(17); pp. 6779-6798. <https://doi.org/10.1175/JCLI-D-13-00629.1>

Vidal, J.P., Hingray, B., Magand, C., Sauquet, E., Ducharne, A., 2016. Hierarchy of climate and hydrological uncertainties in transient low flow projections. *Hydrol. Earth Syst. Sci.* 20, 3651–3672,. doi:10.5194/hess-20-3651-2016; <https://www.hydrol-earth-syst-sci.net/20/3651/2016/>

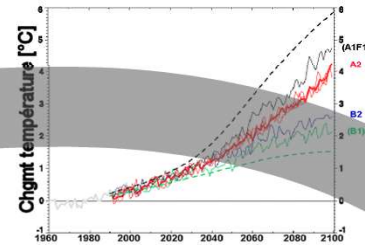
## SM2.Large and Small Scale Components of Internal Variability : Illustration



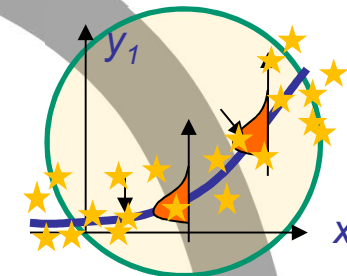
The **simulation chain** (GCM+RCM+HM...) may additionally include different components of Internal Variability



**Global Climate Model (GCM)**



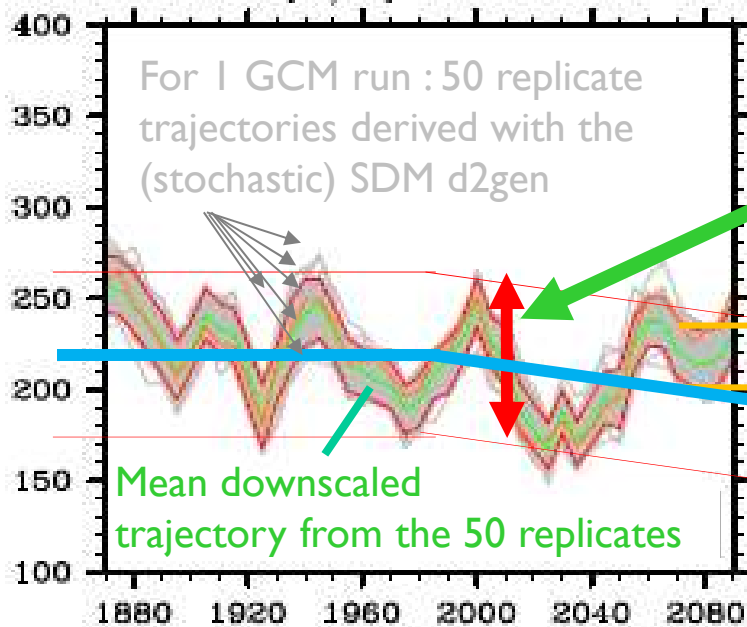
**Global / regional Climate Scenarios**



**Regional Downscaling Model (RCM or SDM)**

Time series of Local (spatial) Weather Scenarios

**P DJF Durance River**



For 1 GCM run : 50 replicate trajectories derived with the (stochastic) SDM d2gen

Mean downscaled trajectory from the 50 replicates

**Large Scale Internal Variability**  
Derived from GCM internal variability

**Small Scale Internal Variability**  
Derived from the uncertain statistical downscaling link

Results (see illustrations in the 2 next slides) :  
The small scale IV can be as large as the large scale IV  
See (Lafaysse&al 2014,Vidal&al2016). for more

## SM2. Illustration : Uncertainty in downscaled precipitation

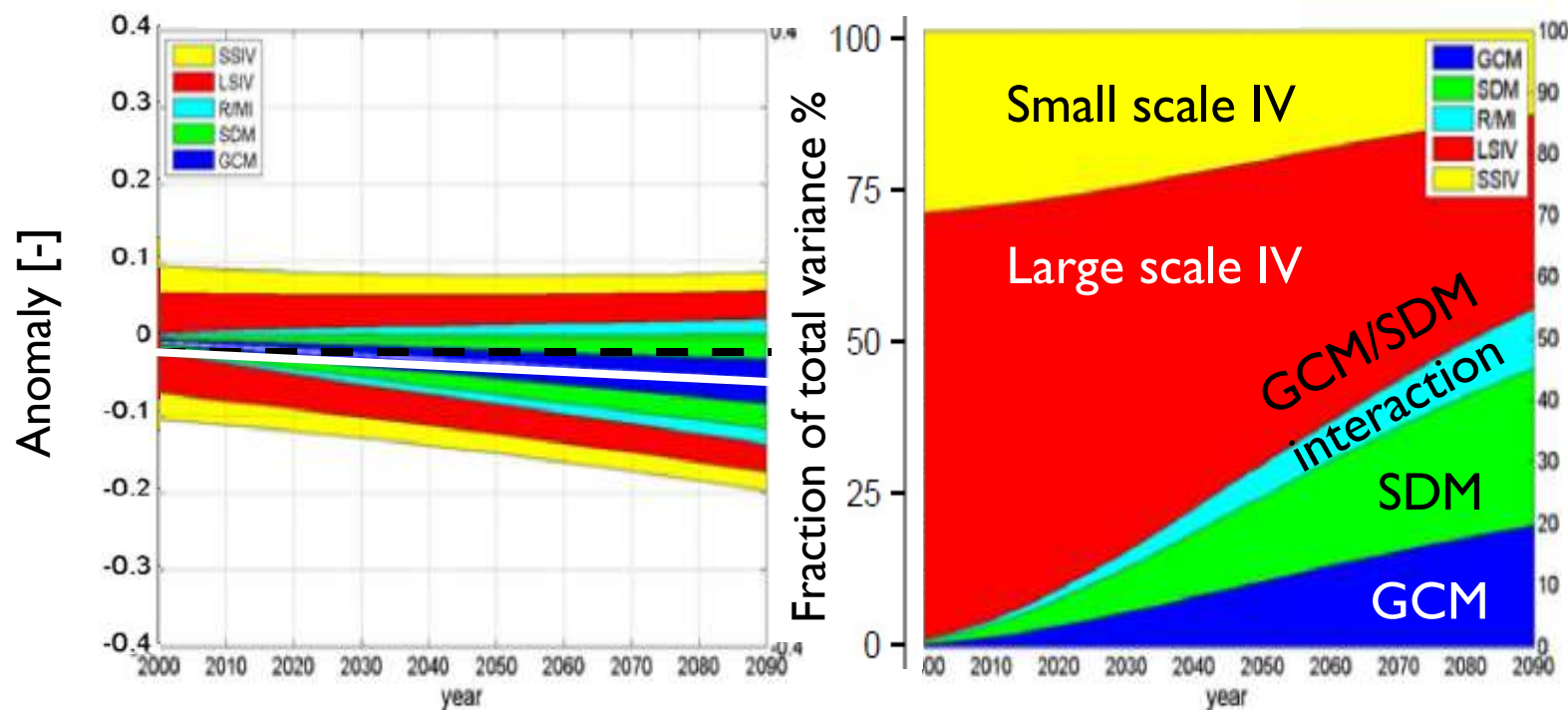


Durance River  
French Alps  
12'000 km<sup>2</sup>  
Lafayssse et al. VRR 2014

1 SRES scenario  
6 GCM (CMIP3)  
5 SDMs (TFs, WT, Analogs)

50 replicate trajectories have been derived with each SDM for each GCM run >>  
This resulted in a (non-negligible) Small Scale Component of Internal Variability (SSIV below)

20yrs mean, annual precip.





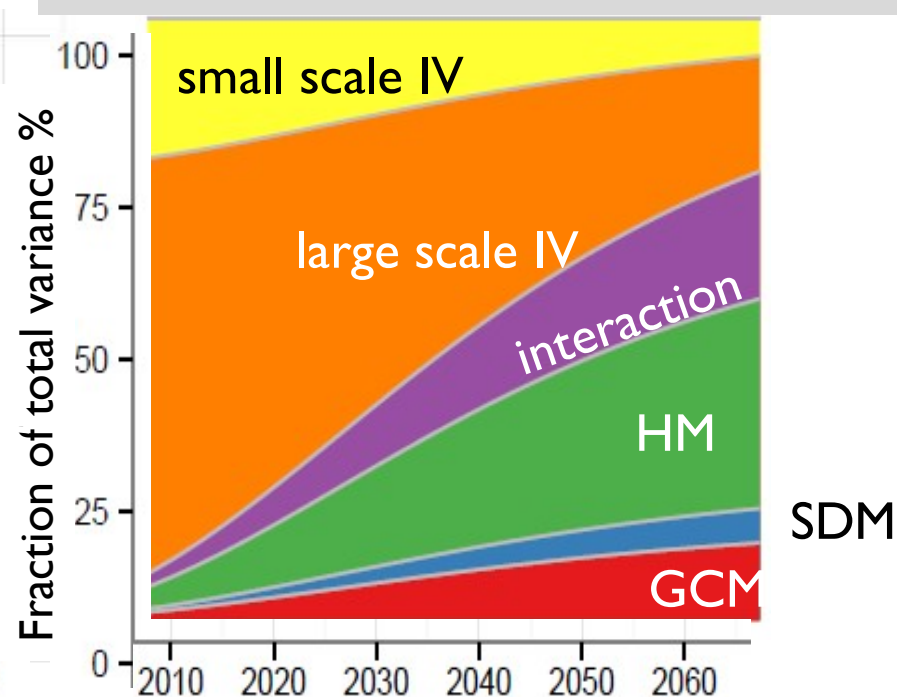
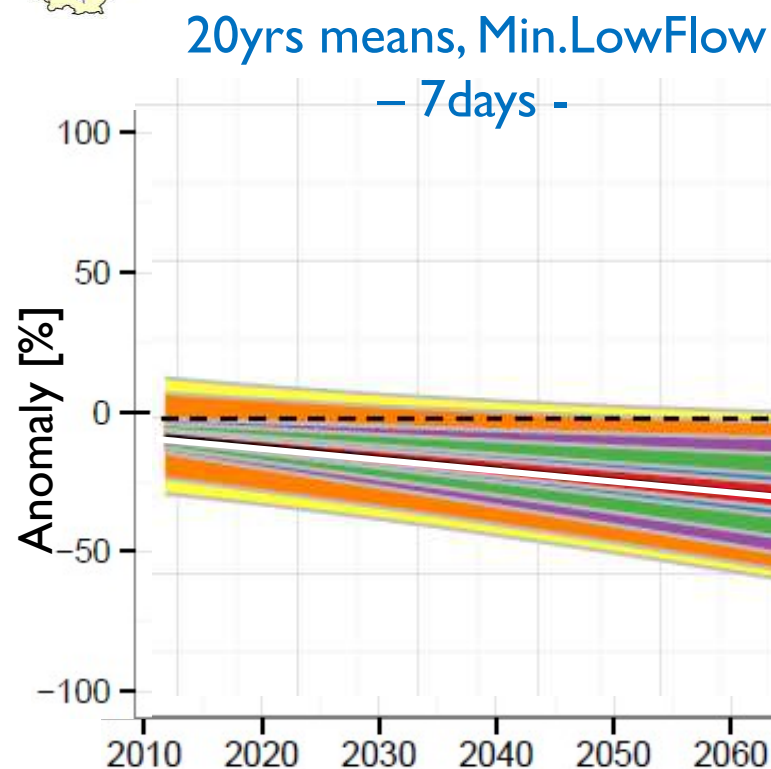
## SM2. Illustration : Uncertainty in low flow discharges



Durance River  
French Alps  
12'000 km<sup>2</sup>  
Vidal et al. HESS, 2014

1 SRES scenario  
6 GCM (CMIP3)  
5 SDMs (TFs, WT, Analogs)  
6 HMs (Mordor, Cequeau, GR, CLSM, J2000, Orchidee)

The same for hydro projections...50 replicate trajectories of discharges have been derived for each GCM/DSM/HM chain...



Note also the large contribution of Hydrological Model (HM)

Vidal et al, HESS, 2016

# Supplementary SM3. Single Time versus Time Series ANOVA



Climate Dynamics (2019) 53:2501–2516  
<https://doi.org/10.1007/s00382-019-04635-1>



## Uncertainty component estimates in transient climate projections

### Precision of estimators in a single time or time series approach

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### Abstract

Quantifying model uncertainty and internal variability components in climate projections has been paid a great attention in the recent years. For multiple synthetic ensembles of climate projections, we compare the precision of uncertainty component estimates obtained respectively with the two Analysis of Variance (ANOVA) approaches mostly used in recent works: the popular Single Time approach (STANOVA), based on the data available for the considered projection lead time and a time series based approach (QEANOVA), which assumes quasi-ergodicity of climate outputs over the available simulation period. We show that the precision of all uncertainty estimates is higher when more members are used, when internal variability is smaller and/or the response-to-uncertainty ratio is higher. QEANOVA estimates are much more precise than STANOVA ones: QEANOVA simulated confidence intervals are roughly 3–5 times smaller than STANOVA ones. Except for STANOVA when less than three members is available, the precision is rather high for total uncertainty and moderate for internal variability estimates. For model uncertainty or response-to-uncertainty ratio estimates, the precision is low for QEANOVA to very low for STANOVA. In the most unfavorable configurations (small number of members, large internal variability), large over- or underestimation of uncertainty components is thus very likely. In a number of cases, the uncertainty analysis should thus be preferentially carried out with a time series approach or with a local-time series approach, applied to all predictions available in the temporal neighborhood of the target prediction lead time.

## SM3. Two types of ANOVA approaches for partitioning uncertainty

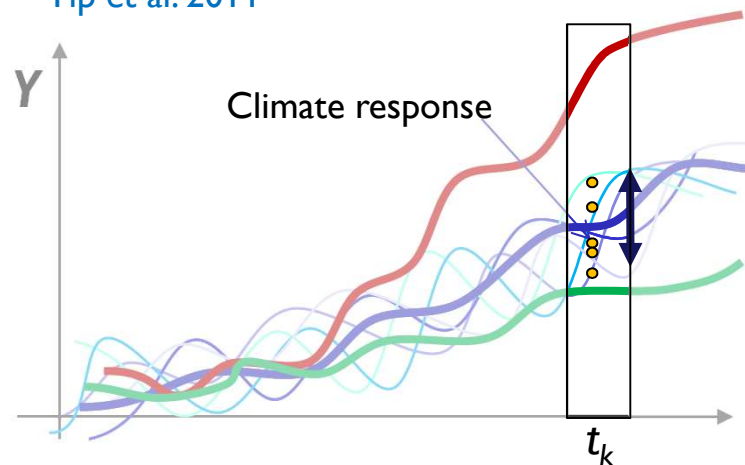
### The Single Time ANOVA and Time Series ANOVA approaches



One major difference : the way the climate response of each simulation chain is estimated ...

#### Single Time ANOVA approach STANOVA

Yip et al. 2011



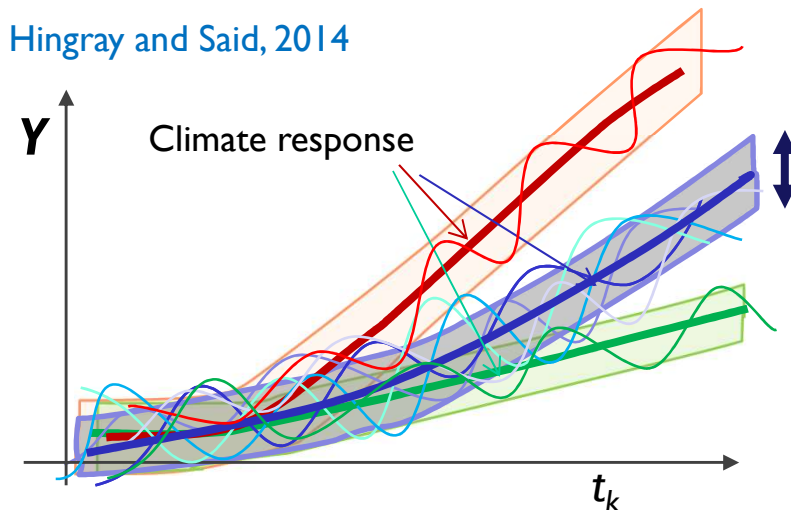
For each model  $g$ , at  $t_k$  :  
climate response = **multimember mean** at  $t_k$   
internal variability = **inter-member** variance

Requires : multiple runs of each GCM

Limitation : This constraint typically leads to  
drop many GCMs  
(waste of information)

#### Quasi-Ergodic ANOVA approach QEANOVA

Hingray and Said, 2014



For each model  $g$ , at  $t_k$  :  
climate response = **trend estimate** at  $t_k$   
internal variability = **time variance** of residuals

Requires : Quasi-Ergodicity Hypothesis

Limitation : IV has to be assumed constant or  
roughly constant over time. This assumption can  
be relaxed (cf; Hingray Said, 2014 and the  
**LocalQEANOVA** approach in Hingray&al.2019)

## SM3. Synthetic MMEs to compare the precision of The Single Time ANOVA and Time Series ANOVA approaches

---



To compare both ANOVA estimation methods

- We generated a large number of Multimodel Ensembles (MMEs) for different configurations :
  - different numbers of GCMs ( $N_G$ )
  - different numbers of GCM runs (with a same number  $M$  for each GCM),
  - different contributions of internal variability to total uncertainty variance ( $F_n$  in %)
  - MMEs with different « Mean Response – To – Total Uncertainty » ratios (R2U)
- For each configuration (e.g.  $N_G = 5$ ,  $M = 3$  runs/GCM,  $R2U = 1$ ,  $F_n = 80\%$ ):
  - we generated (10'000) synthetic MMEs (sharing the same statistical characteristics (same Mean Response, GCM effects, Internal Variability) and features (same number of GCMs; number of runs / GCM...))
  - we then estimated the ability of each method to obtain the true values of the prescribed uncertainty components (Internal Variability, Model Uncertainty, Total Uncertainty, GrandEnsemble Mean).
- The precision is estimated with SD, the standard deviation of the 10'000 estimates of the considered feature obtained from the 10'000 synthetic MMEs generated for the considered configuration
- Next Slide : precision (SD, Y-Axis) as a function of fractional variance due to internal variability ( $F_n$ , X-axis) for different numbers of runs ( $M$ ) available for each GCM (the different lines)

nb : QEANOVA can be applied even if only one run is available, STANOVA not

$F_n = 1 \gg$  Total Uncertainty is fully due to internal variability (no uncertainty due to the GCM)

$F_n = 0 \gg$  Total Uncertainty is fully due to the spread between GCM responses (no internal variability)

The gain in precision obtained with a QEANOVA is given in the 3rd column



## SM3. Single Time versus Time Series ANOVA



**Fig. 2** Precision of QEANOVA (left) and STANOVA (middle) estimates for uncertainty variance components, and gain in precision between STANOVA SD values and QEANOVA SD values (right) (gain = ratio between STANOVA SD values and QEANOVA SD values). Top: internal variability, middle: model uncertainty, bottom: total uncertainty. SD values are given as a function of the fraction of

total variance explained by internal variability ( $F_\eta$ ) for a few representative values of number of members  $M$ : (1), 2, 3, 5, 10, 20. Results are presented for a theoretical ratio  $R2U = 1$ . For the sake of clarity, the upper limit of the figures for  $s_\alpha^2$  is truncated to 2. The highest values are greater than 10. Figures of the ratios between SD values obtained with both approaches are truncated to 5

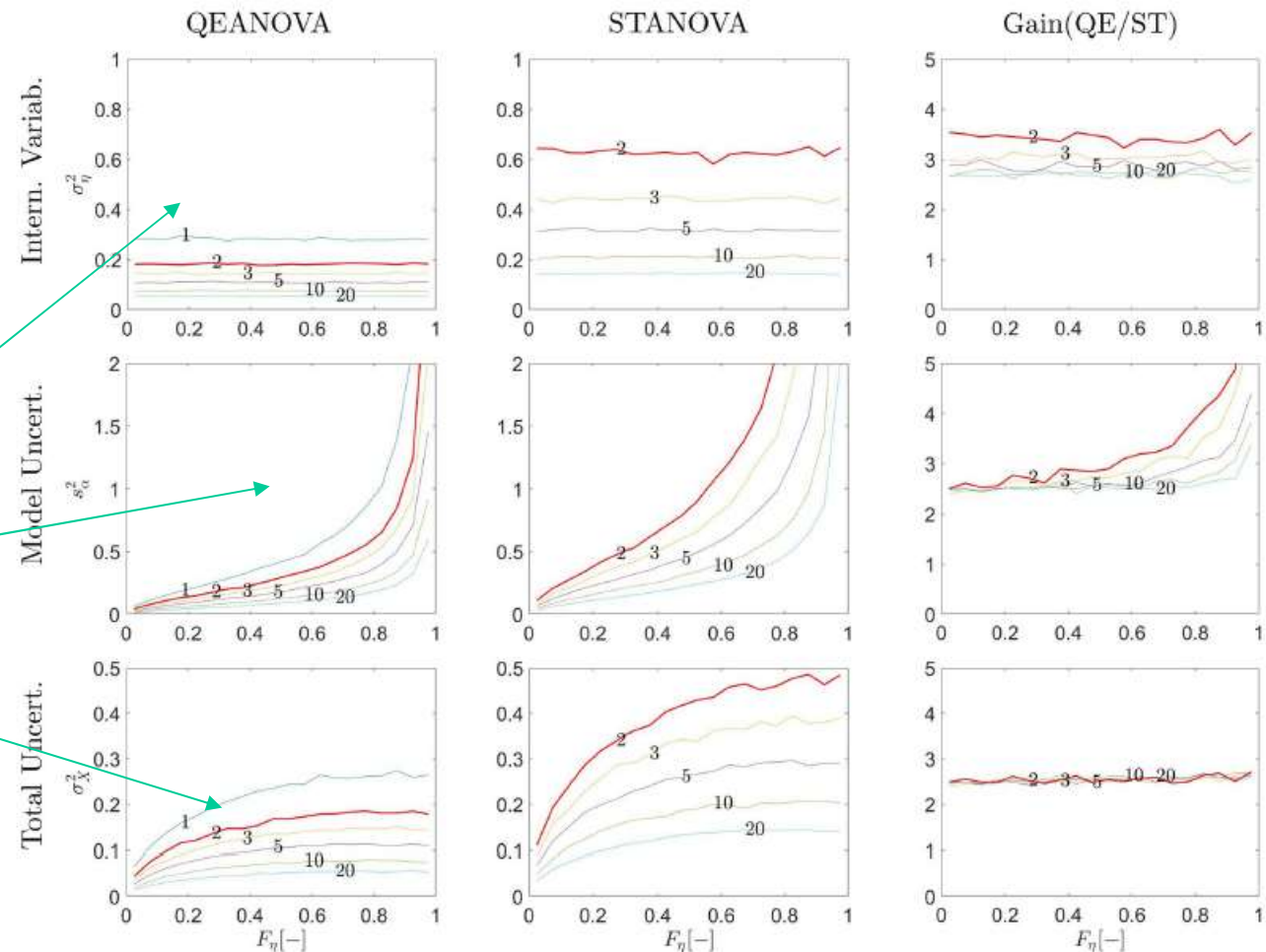
The smaller the SD value, the better the precision of the estimate

Estimates of

Internal Variability

Model Uncertainty

Total Uncertainty



Extracted from  
Hingray et al. 2019

# SM3. Two flaws of a single time ANOVA



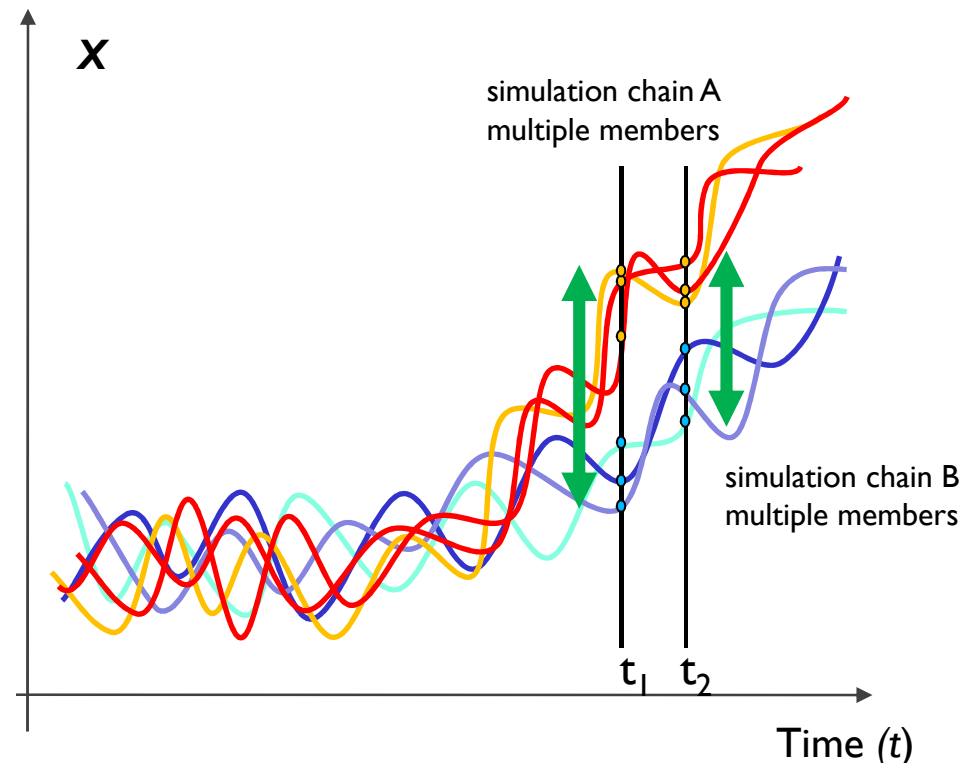
Single Time ANOVA : uncertainty analyse based on  
projections available at a single projection lead time

1. The **STANOVA** thus requires multiple runs but those are rarely available for all chains in today's MMEs

2. In case of a small number of runs and / or in case of a variable with a large internal variability (e.g. precipitation and all precipitation related variables (discharges)), the estimation of the climate response of a given scenario/GCM/RCM/.... Chain is likely poor.

## Consequences :

- This leads to **poor estimates of effects** of the different models and poor robustness of all uncertainty estimates
- This poor robustness is typically expressed as **a low temporal coherency of estimates** from one projection lead time to the other
- This leads also to a **likely high sample dependency** (dependency to the run(s) available)



See Illustration next slide for 3 different synthetic ensembles. The precision of estimates obtained for the 2nd synthetic MME is very low. This is obviously to be related also with the very poor temporal robustness of estimates

Note : those MMEs are unrealistic as we considered that we have 3 or 9 runs available for each GCM)

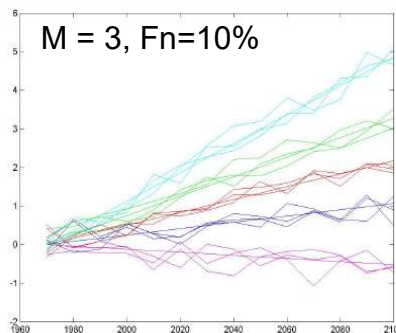


# SM3.90% Confidence Intervals of Internal Variability ( $\sigma_\eta$ ) and GCM Model Uncertainty ( $s_\alpha$ ) estimates for 3 different synthetic ensembles

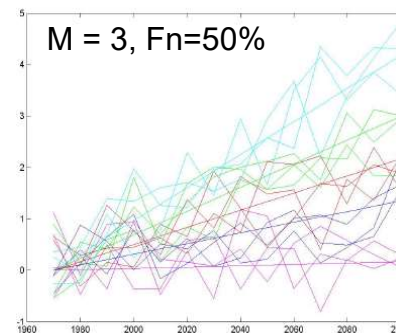
Extracted from  
Hingray et al. 2019

- 5 GCMs
- M: Number of runs available for each GCM
- Fn : Fraction of total uncertainty explained by Internal Variability

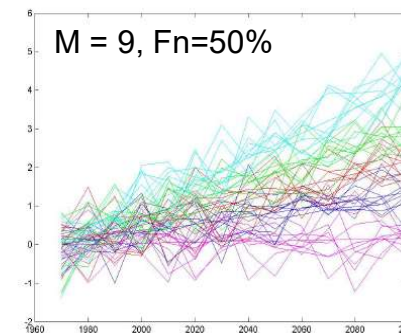
Ensemble MME#1



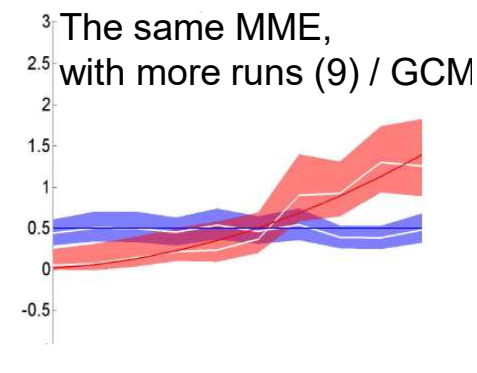
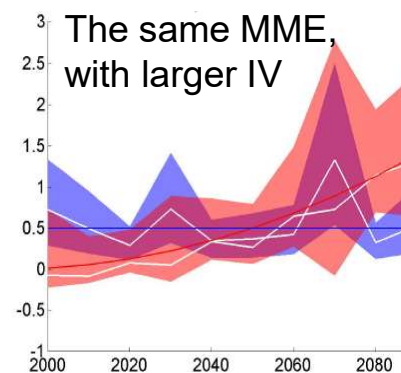
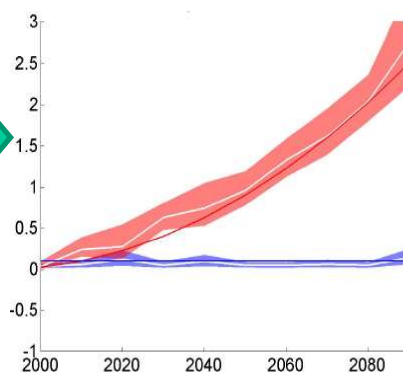
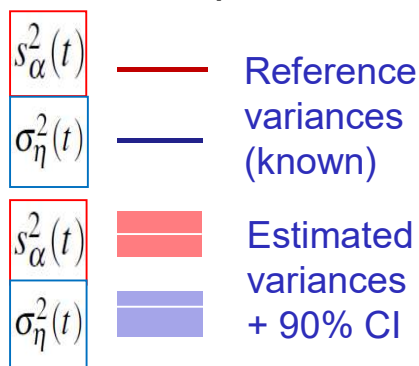
Ensemble MME#2



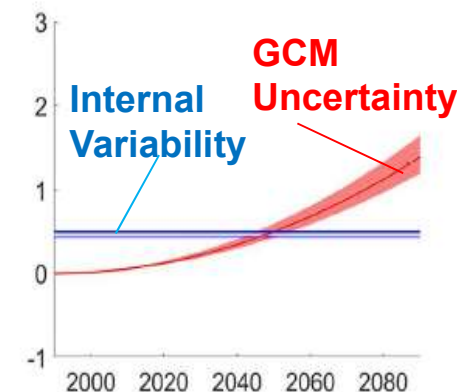
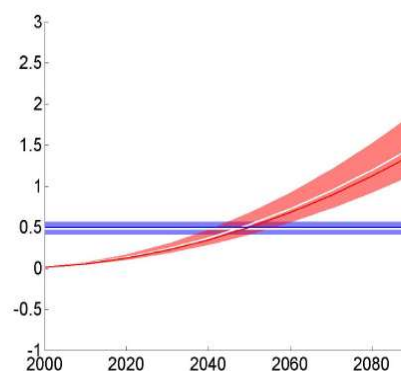
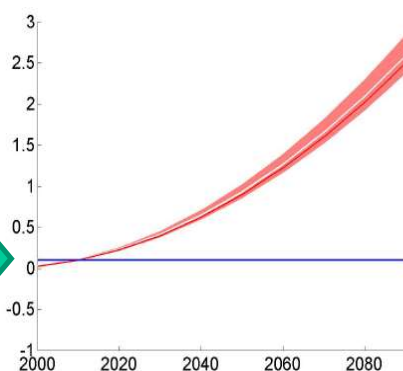
Ensemble MME#3



Estimates with a  
**Single Time ANOVA  
(STANOVA)**



Estimates with a  
**Quasi-Ergodic ANOVA  
(QEANOVA)**



From

Evin, G., Hingray, B., Blanchet, J., Eckert, N., Morin, S., Verfaillie, D. 2019.  
Partitioning uncertainty components of an incomplete ensemble of climate  
projections using data augmentation. J.Climate.  
<https://doi.org/10.1175/JCLI-D-18-0606.1>

### Bayesian Inference

**Goal:** Obtain posterior distributions of all unknown quantities  $\theta$   
(ANOVA parameters  $\mu, \alpha, \beta, \gamma, \sigma^2$  + missing combinations  $\phi^{*m}$ )

$$\begin{aligned} P(\theta | \phi^{*o}) &\propto P(\phi^{*o} | \theta) \times P(\theta) \\ &= \underbrace{P(\phi^{*o} | \theta)}_{\text{LIKELIHOOD}} \times \underbrace{P(\phi^{*m} | \mu, \alpha, \beta, \gamma, \sigma^2, \phi^{*o})}_{\text{MISSING COMB.}} \times \underbrace{P(\mu, \alpha, \beta, \gamma, \sigma^2)}_{\text{JOINT PRIOR}}. \end{aligned}$$