Eigenvector Model Descriptors for the Seismic Inverse Problem

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Scope

Scope of this work:

- iterative minimization to recover sub-surface Earth's models;
- the models are represented using a basis of eigenvectors to reduce the number of unknowns;
- i.e., Regularization by discretization to reduce ill-posedness;
- it allows to overcome the missing low-frequency data, hence it is used to build initial models.

Overview

1. Introduction
2. Regularization by discretization
   - Adaptive eigenspace model representation
   - Illustration of representation
3. Numerical experiments of reconstruction
4. Conclusion

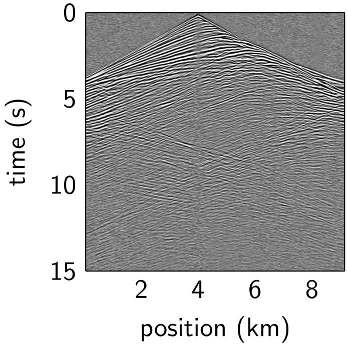
Plan

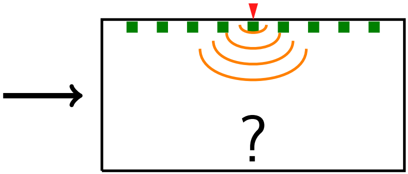
1. Introduction

Seismic inverse problem

Reconstruction of subsurface Earth properties from seismic campaign: **mechanical wave** propagation data recorded at the surface.

From surface waves observation to parameter identification.





nonlinear, ill-posed inverse problem.

Full Waveform Inversion (FWI)

FWI provides a **quantitative reconstruction** of the subsurface parameters with an **iterative minimization of the cost function**,

$$\mathcal{J}(m) = \frac{1}{2}\|\mathcal{F}(m) - d\|^2.$$

- d are the data measured by n_{rcv} receivers,
- $\mathcal{F}(m)$ represents the simulation, here we consider the propagation of time-harmonic acoustic waves solution to

$$\Big(-\frac{\omega^2}{m^2} - \Delta\Big)p = f, \qquad \mathcal{F}(m) = \{p(\boldsymbol{x}_1), \ldots, p(\boldsymbol{x}_{n_{\text{rcv}}})\},$$

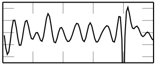
from n_{rcv} surface receivers for the identification of the wave speed m.

P. Lailly
The seismic inverse problem as a sequence of before stack migrations
Conference on Inverse Scattering: Theory and Application, SIAM, 1983

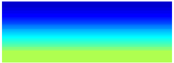
A. Tarantola
Inversion of seismic reflection data in the acoustic approximation
Geophysics, 1984

FWI, iterative minimization

Observations

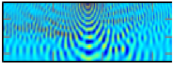


Initial model m_0



$k = 0$

Forward problem $F_\omega(m_k)$

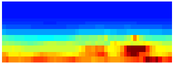


Misfit functional $\mathcal{J}$

Optimization procedure

1. Gradient
2. Search direction s_k
3. Line search α_k

$k = k + 1$



update model

$m_{k+1} = m_k + \alpha_k s_k$

- large-scale problem, we use HDG discretization;
- adjoint-state method to compute the gradient;

How to mitigate the ill-posedness of the problem?

Plan

2. Regularization by discretization
   - Adaptive eigenspace model representation
   - Illustration of representation

Regularization by discretization

$$\min_m \mathcal{J}(m)\,, \qquad \text{with} \quad \mathcal{J}(m) = \frac{1}{2}\|\mathcal{F}(m) - d\|^2.$$

To mitigate the ill-posedness of non-linear optimization: **Regularization**,

- e.g., by adding constraints: $\mathcal{J}_+(m) = \mathcal{J}(m) + \text{constraints on } m.$
  e.g., Total Variation (TV): $\mathcal{J}_{\text{TV}}(m) = \mathcal{J}(m) + \int |\nabla m|.$

Regularization by discretization

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- **Regularization by discretization** approach: The model m is represented in a specific basis to reduce the number of unknowns:

  - less unknowns $\Rightarrow$ better stability, [1];
  - **Need a compromise between the number of unknowns and the resolution.**

[1] E. Beretta, M. V. de Hoop, F. Faucher, O. Scherzer
Inverse boundary value problem for the Helmholtz equation: quantitative conditional Lipschitz stability estimates
SIAM Journal on Mathematical Analysis, 2016.

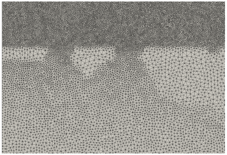
Regularization by discretization

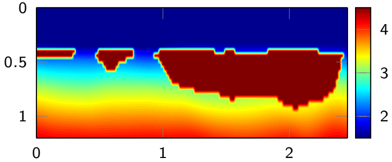
Usual representation of the wave-speed model: from the discretization.

- Piecewise constant (one value per cell)

$$m(\boldsymbol{x}) = \sum_{j=1}^{N} m_j \; \chi_{D_j}(\boldsymbol{x})$$

But it leads to a large number of coefficients to represent the model: below right, more than 250 000.





Eigenspace decomposition

Efficient model representation using a basis of eigenvectors,

1. find m_0 the solves the linear PDE, $\quad -\nabla \cdot \big(\eta(m)\,\nabla\big)\, m_0 = 0\,.$
2. compute the eigenvectors ψ_k of $\quad -\nabla \cdot \big(\eta(m)\,\nabla\big)\,.$
3. represent the model with N_{ev} eigenvectors,

$$\mathfrak{m} = m_0 + \sum_{k=1}^{N_{\mathrm{ev}}} \alpha_k \psi_k(\boldsymbol{x})\,.$$

Several choices for η from image processing, can relate to 'usual' regularization (TV, Tikhonov, etc.).

M. Grote, M. Kray, U. Nahum
Adaptive eigenspace method for inverse scattering problems in the frequency domain
Inverse Problems, 2017.

F. Faucher, O. Scherzer and H. Barucq
Eigenvector Model Descriptors for Solving an Inverse Problem of Helmholtz Equation
Geophysical J. International (2020).

Illustration

Piecewise-constant,

$$m = \sum_{k=1}^{N_c} m_k \; \chi_{D_k} \, .$$

Adaptive eigenspace,

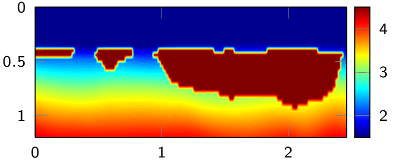
$\mathfrak{m} = m_0 + \sum_{k=1}^{N_{\text{ev}}} \alpha_k \psi_k,$
ψ_k eigenvectors of $-\nabla \cdot (\eta(m)\, \nabla)$,
with $\eta = \dfrac{2\epsilon}{(\epsilon + |m|^2)^2}$.

Illustration

Piecewise-constant,

$$m = \sum_{k=1}^{N_c} m_k \; \chi_{D_k} \, .$$

$N_c > 150\,000$



Adaptive eigenspace,

$$\mathfrak{m} = m_0 + \sum_{k=1}^{N_{ev}} \alpha_k \psi_k ,$$
ψ_k eigenvectors of $-\nabla \cdot (\eta(m) \, \nabla)$,
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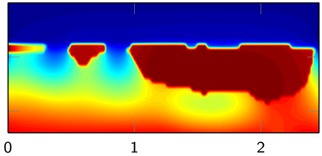
$N_{ev} = 10$



Comparison of approximations with η in [2].

[2] F. Faucher, O. Scherzer and H. Barucq
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Geophysical J. International (2020).

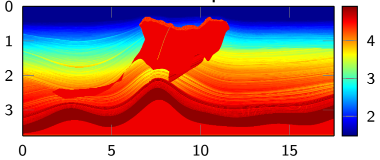
Plan

3 Numerical experiments of reconstruction

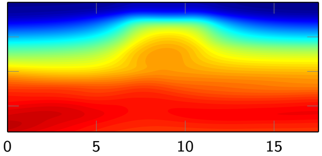
FWI: SEAM benchmark

FWI for reconstruction, using frequency from 2 Hz to 10 Hz.

True wave speed



Initial guess



Iterative minimization with respect to

v1 the N_c coefficients m_k of the piecewise-constant representation: $m = \sum_{k=1}^{N_c} m_k \, \chi_{D_k}$,

v2 the N_{ev} weights α_k of the eigenspace decomposition $m = m_0 + \sum_{k=1}^{N_{ev}} \alpha_k \psi_k$, here, the ψ_k are computed from the initial model.

$$N_{ev} << N_c.$$

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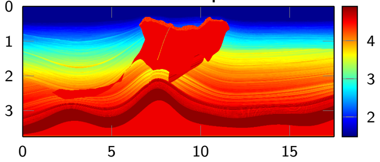
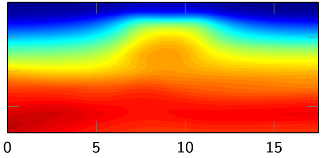
True wave speed

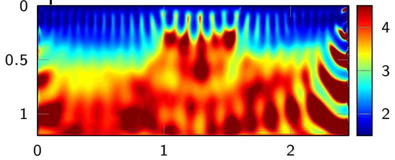


Initial guess

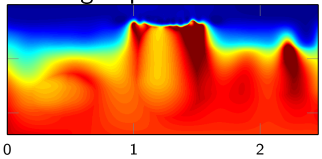


low-frequency reconstruction (2 to 4 Hz)

piecewise-constant inversion



eigenspace inversion

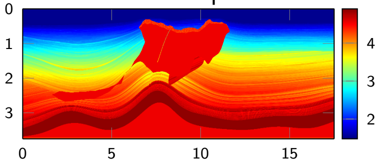


The eigenspace representation provides the appropriate regularization for low-frequency reconstruction

FWI: SEAM benchmark

FWI for reconstruction, using frequency from 2 Hz to 10 Hz.

True wave speed



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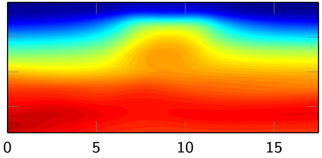
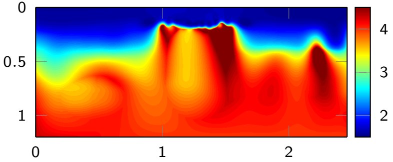


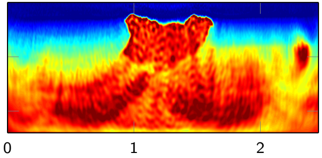
step 1:

- low-frequency;
- adaptive eigenspace for convergence;
- recovery of smooth models.

step 2:

- from model built in step 1;
- higher frequency for resolution;
- increased number of unknowns (possibly piecewise constant).





Plan

4 Conclusion

Conclusion

Iterative minimization for sub-surface Earth parameter reconstruction (FWI)

- Regularization by discretization to reduce ill-posedness,
- compromise between number of unknowns and resolution,
- adaptive eigenspace for low-frequency reconstruction,
- **perspective:** multi-parameter reconstructions, the basis of each models must be connected.

Conclusion

Iterative minimization for sub-surface Earth parameter reconstruction (FWI)

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- compromise between number of unknowns and resolution,
- adaptive eigenspace for low-frequency reconstruction,
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THANK YOU FOR TAKING A LOOK!

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