Self-similar solution analysis of hydraulic fracture growth with bottom hole pressure restriction

Grigory V. Paderin
Gazpromneft Science & Technology Centre
Paderin.GV@gazpromneft-ntc.ru

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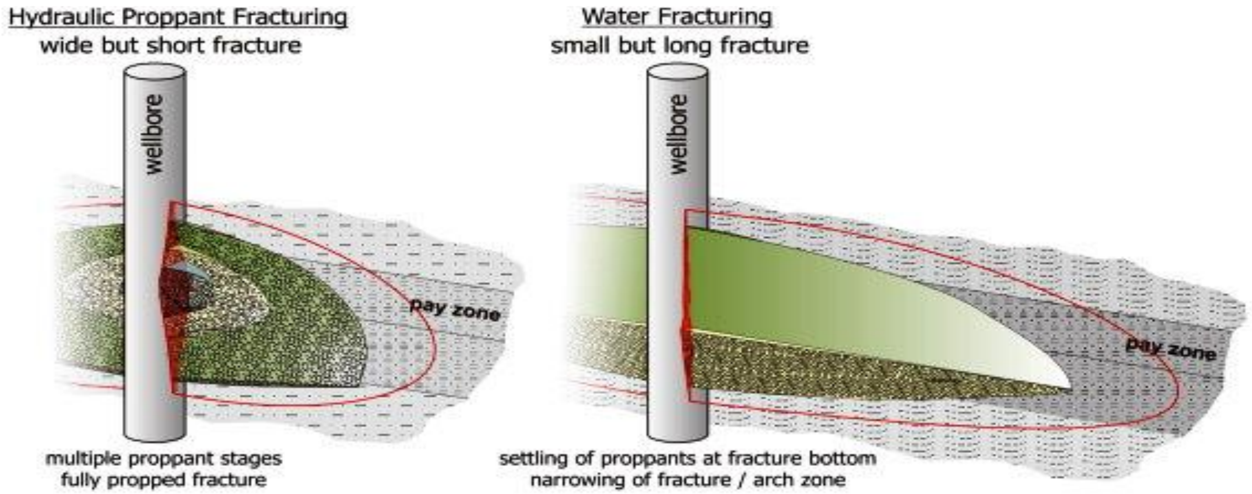
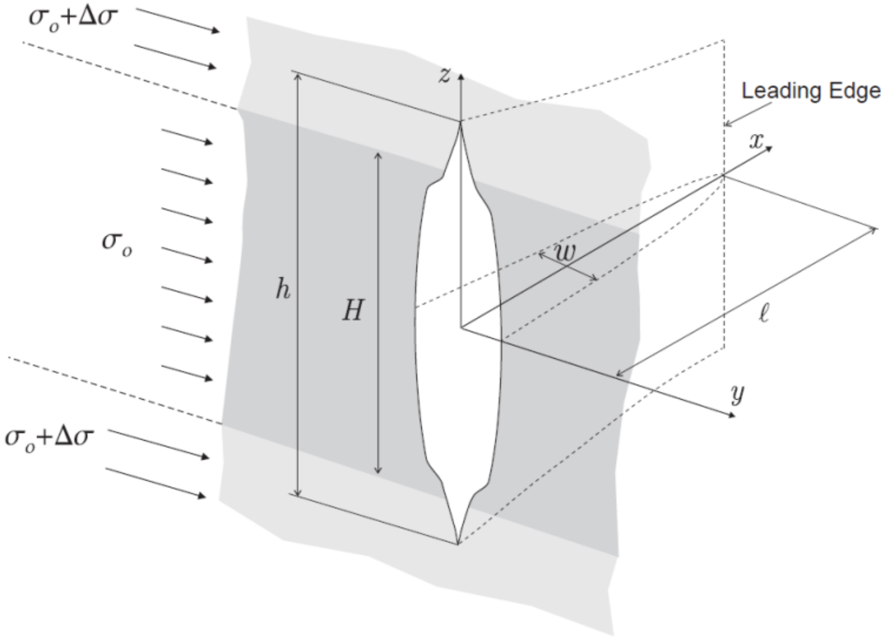
Introduction

Hydraulic fracturing -
is the creation of cracks (L≈100-500m) in the rock by high pressure injection of liquid with sand (proppant) into the reservoir.

Hydraulic fracturing is the main method of efficient recovering hydrocarbons in low-permeability reservoirs.



Pseudo3D crack model

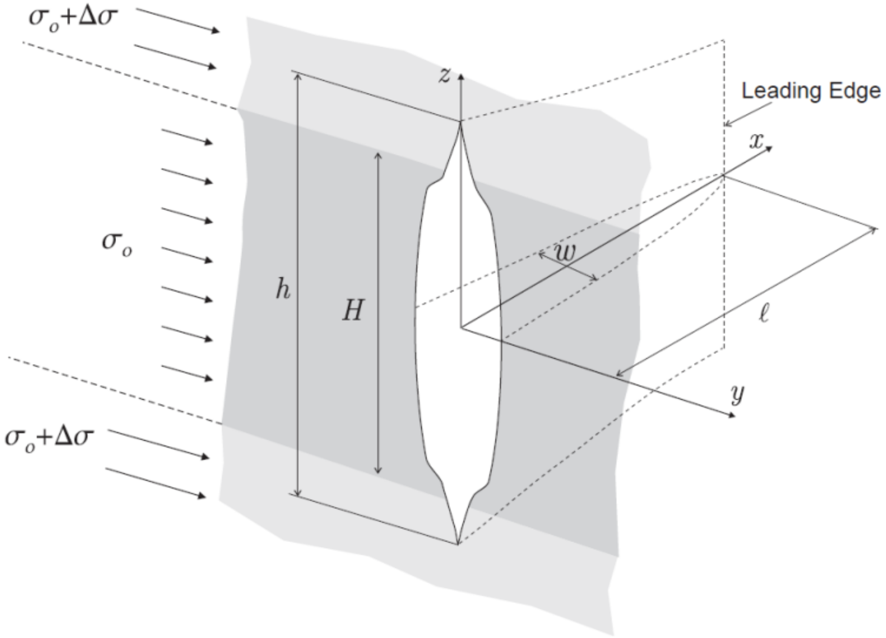


Main assumptions of the Pseudo3D model:

- A crack develops in a vertical plane.
- The length is much greater than the height of the crack.
- The fluid flow along the crack is quasi-one-dimensional and directed horizontally.
- Crack growth in height is determined by the mechanics of brittle fracture
- Leak-off described by Carter's law $u_L = \frac{C_l H}{\sqrt{t - t_0(x)}}$, C_l - Carter leak-off coefficient

*Adachi, José I., E. Detournay, and A. P. Peirce. 2010. Analysis of the classical pseudo-3D model for hydraulic fracture with equilibrium height growth across stress barriers. International Journal of Rock Mechanics and Mining Sciences 47 (4): 625–639.

Main equations of Pseudo3D model



Rock Deformation – Linear elasticity in 2D:

$$w = \frac{4}{\pi E'}[p\int_0^{\frac{H}{2}} B(s,z)ds + (p-\Delta\sigma)\int_{\frac{H}{2}}^{\frac{h}{2}} B(s,z)ds\,]$$

$p = p_f - \sigma_0$ is the net pressure, $B(s,z)$ - the elasticity kernel, $E' = E/(1-\upsilon^2)$ is the plane strain modulus

Crack growth in height (Irwin Criterion):

$$K_I = K_{Ic}, K_{Ic} - \text{fracture toughness}$$

$$K_I = \sqrt{\frac{8h}{\pi}}\left[p\int_0^{\frac{H}{2}}\frac{ds}{\sqrt{h^2-4s^2}} + (p-\Delta\sigma)\int_{\frac{H}{2}}^{\frac{h}{2}}\frac{ds}{\sqrt{h^2-4s^2}}\right]$$

Fluid Flow in Crack:

- Continuity equation (the law of local volume balance)

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} + \frac{2C_l H}{\sqrt{t - t_0(x)}} = 0, A = \int_h w(z)dz - \text{cross} - \text{sectional area}$$

- Lubrication flow

$$Q(x) = -\frac{\partial p}{\partial x}\frac{1}{12\mu}\int_{-\frac{h}{2}}^{\frac{h}{2}} w^3 dz$$

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Self-similar solution analysis for low-permeable reservoir

The Pseudo3D model reduces to the nonlinear second-order partial derivative equation

$$\frac{\partial}{\partial x}\left[K(p)\left(\frac{\partial p}{\partial x}\right)^{\frac{1}{n}}\right]+\frac{\partial A(p)}{\partial t}+\cancel{\frac{2C_l H}{\sqrt{t-\ t_0(x)}}}=0$$

0 - low-permeable reservoir

$$K(p)=\left(\frac{1}{\phi K_f}\right)^{\frac{1}{n}}\int\limits_{h(p)} w(p,z)^{\frac{2n+1}{n}}dz\ ,\phi=2^{n+1}(2n+1)^n n^{-n},$$

were K_f is the flow consistency index and n is the flow behavior index of the power-law fluid

In the case without leak-off:

$$\frac{\partial K(p)}{\partial p}\left(\frac{\partial p}{\partial x}\right)^{\frac{1}{n}+1}+\frac{1}{n}K(p)\left(\frac{\partial p}{\partial x}\right)^{\frac{1}{n}-1}\frac{\partial^2 p}{\partial x^2}+\frac{\partial A(p)}{\partial p}\frac{\partial p}{\partial t}=0$$

We are looking for a solution in the form of:

$$p=p(z),\qquad z=Cxt^{\gamma}$$

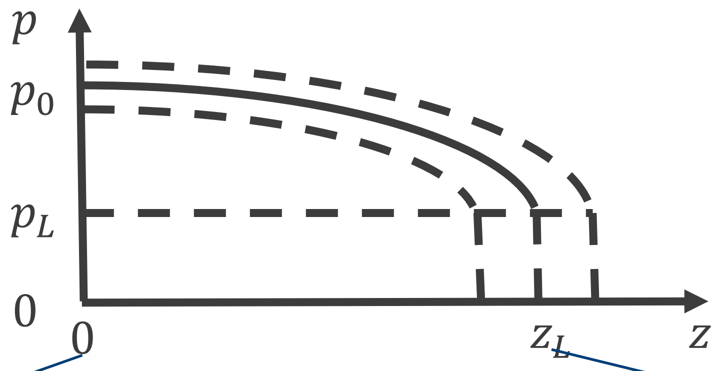
Assuming $\gamma=-\frac{n}{n+1}, C=1$ leads to non-linear second-order ODE with the unknown variable $p(z)$ of one argument z:

$$\boxed{\frac{1}{n}K(p)\big(p'(z)\big)^{\frac{1}{n}-1}p''(z)+\frac{\partial K(p)}{\partial p}\big(p'(z)\big)^{\frac{1}{n}+1}-\frac{n}{n+1}\frac{\partial A(p)}{\partial p}\,z\,p'(z)=0} \qquad (*)$$

In the case of Newtonian fluid $n=1$ and the PKN model

$$C_1p^2\big(p'(z)\big)^2+C_2p^3(p''(z))-\frac{1}{2}C_3zp'(z)=0$$

Boundary conditions



The position of crack edge z_L is unknown.

3 conditions is required

1. Pressure at bottom hole $z = 0$

$$p(0) = p_0$$

2. Pressure at crack edge $z = z_L$

$$p(z_L) = p_L = \sigma_0 + \sqrt{\frac{2}{\pi H}} K_{IC_0}$$

3. No leakage condition at crack edge $z = z_L$ – the velocity of the crack edge and the liquid are equal:

$$v_e = v_f$$

$v_e = \frac{\partial x_e}{\partial t} = \frac{n}{n+1} z_e \, t^{-\frac{1}{n+1}}$ – velocity of the crack edge

$v_f = \frac{Q}{A} = \frac{K(p_e)}{A(p_e)} \left(\left| \frac{\partial p}{\partial z} \right| \right)^{1/n} t^{-\frac{1}{n+1}}$ – velocity of the liquid

Numerical solution

The substitution $p'(z) = \xi(z), p''(z) = \xi'(z)$, reduces equation (*) to the solution of the system

$$\begin{cases} \xi'(z) = f(\xi, p, z), \\ p'(z) = \xi \end{cases}$$

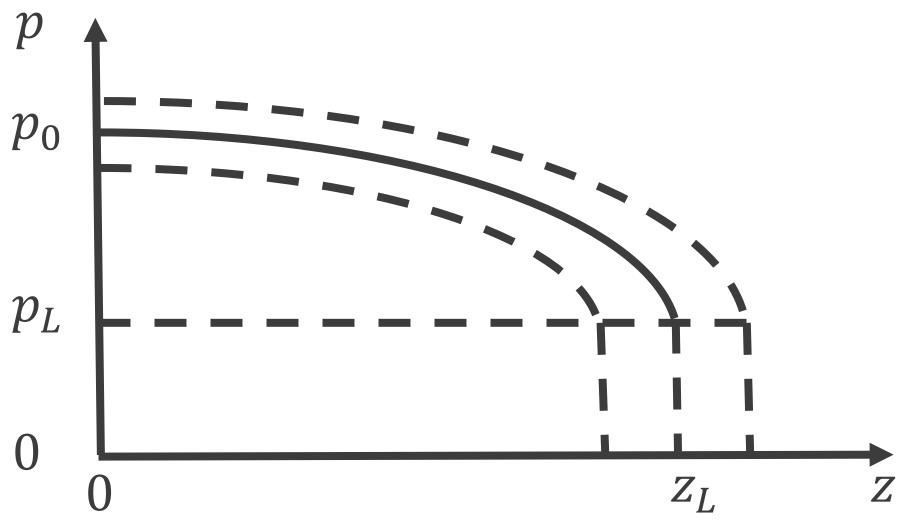
Discretization of the equation

If N numerical nodes $n = 1 \dots N$ with step $z_n - z_{n-1} = h$, then the numerical discretization of this system has the form:

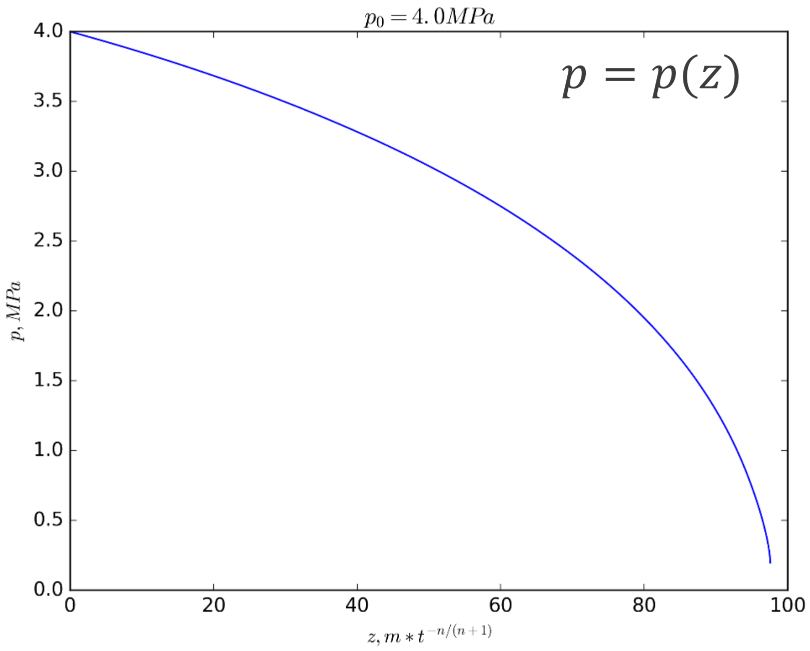
$$\begin{cases} \frac{\xi_n - \xi_{n-1}}{h} = f(\xi_n, p_n, z_n), \\ \frac{p_n - p_{n-1}}{h} = \xi_n \\ p_N = p(z_e),\ \xi_N = p'(z_e) \end{cases} \qquad (**)$$

Algorithm:

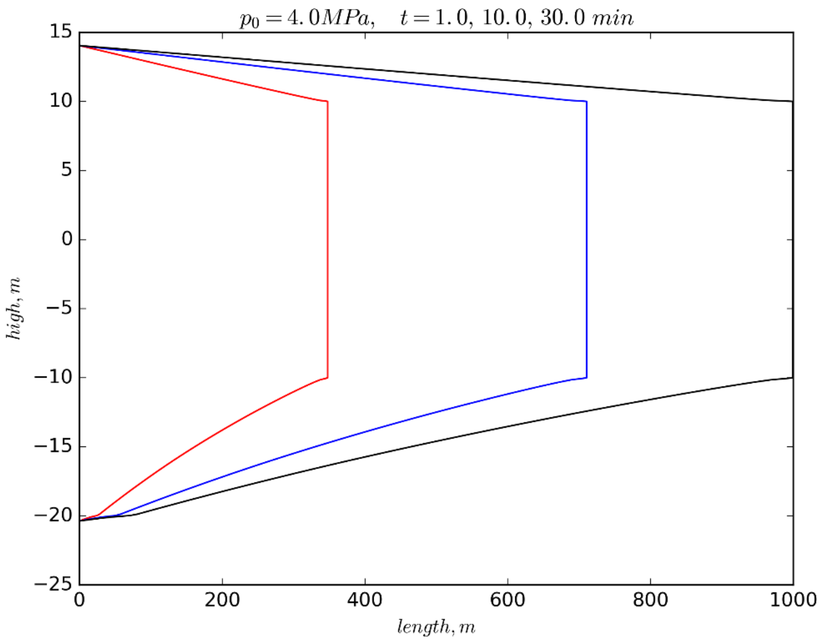
1. Set the approximation z_L
2. Solve eq. (**) using the BC at the crack edge of the hydraulic fracture $p(z_L),\ p'(z_L)$
3. Check the found solution $p(z)$ to the condition $p(0) = p_0$
4. If the condition is not satisfied within the specified accuracy, change approximation z_L and go to step 2.



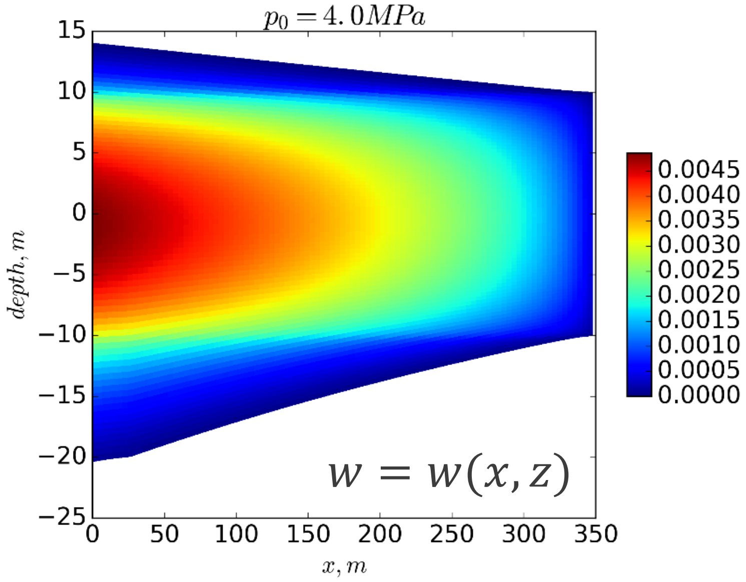
Self-similar solution



Pressure distribution along crack length



Crack contours on time $t = 1, 10, 30\ min$



Crack width distribution for $t = 1\ min$

Input Data:

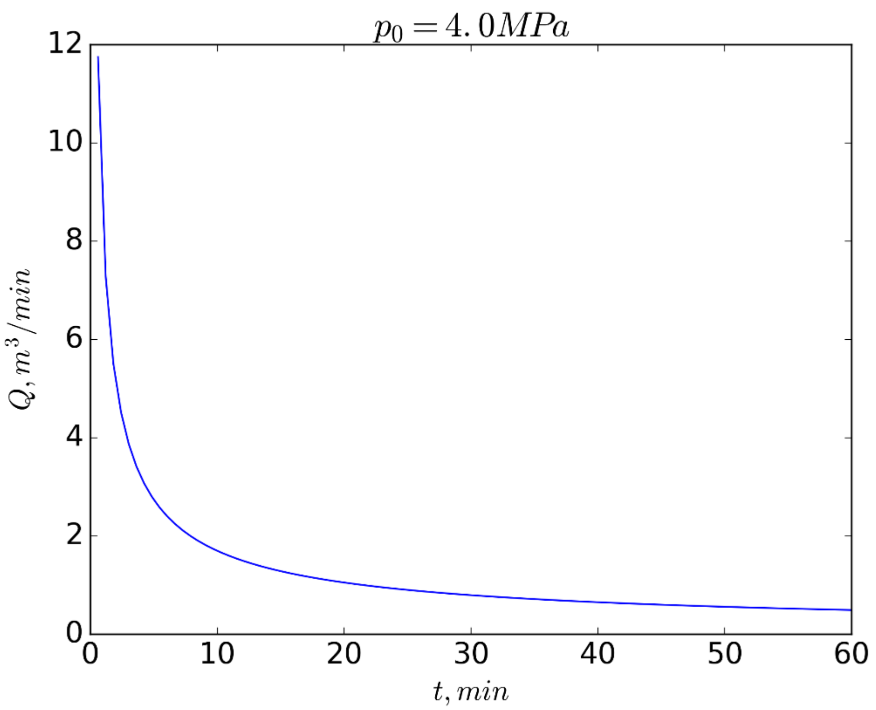
$$n = 0.45$$
$$K_f = 0.85\ Pa * sec^n$$
$$E' = 20\ GPa$$

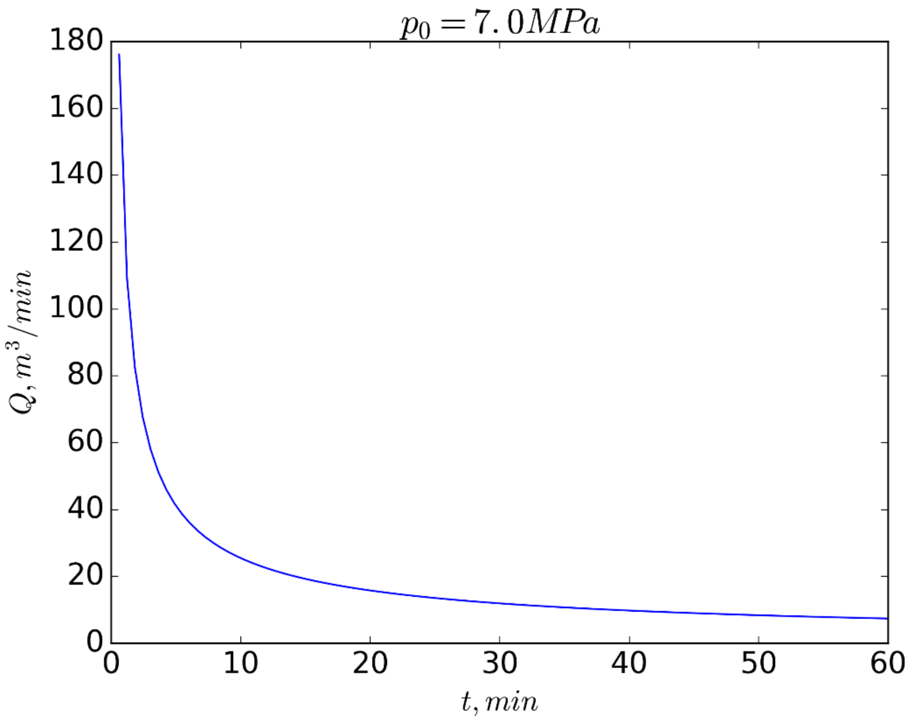
Layer	z_{bot}, m	$\Delta\sigma, MPa$	$K_{IC}, MPa \cdot m^{0.5}$
1	20	8.0	1.1
2	10	0	1.1
3	-10	5.5	1.1
4	-20	6.0	1.1
5	-30	10	1.2

It the case of constant bottom-hole pressure the solution scales, the crack length L and surface area S depends on time t in flowing form:

- $L \sim t^{\frac{n}{n+1}}, S \sim t^{\frac{n}{n+1}}$

Injection rate





The obtained self-similar solution in the case of constant bottomhole pressure leads to the following expression for injection rate into the crack

- $$Q(t) = C_q\, t^{-\frac{1}{n+1}},\ C_q = \left[K\big(p(z)\big)\left(\frac{dp}{dz}(z)\right)^{\frac{1}{n}}\right]_{z=0}$$

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Results

1. Self-similar solution analysis of hydraulic fracture growth with constant bottom hole pressure was presented, which allows to simulate the hydraulic fractures growth with restricted height. Due to the reformulation of the problem, instead of the non-linear partial differential equation of the second order, it was possible to reduce the problem to a non-linear second-order ODE, which positively affects the complexity of the calculations.

2. It was shown that in the case of a constant bottomhole pressure, the crack length and surface area increase according to a power law $L \sim t^{\frac{n}{n+1}}$, $S \sim t^{\frac{n}{n+1}}$

3. It was found that the flow rate must be reduced to maintain a constant bottomhole pressure and fracture height by following law $Q(t) = C_q\, t^{-\frac{1}{n+1}},\ C_q = \left[K\big(p(z)\big)\left(\frac{dp}{dz}(z)\right)^{\frac{1}{n}}\right]_{z=0}$