Instabilities in extensional flows and the dynamics of rifts in ice shelves

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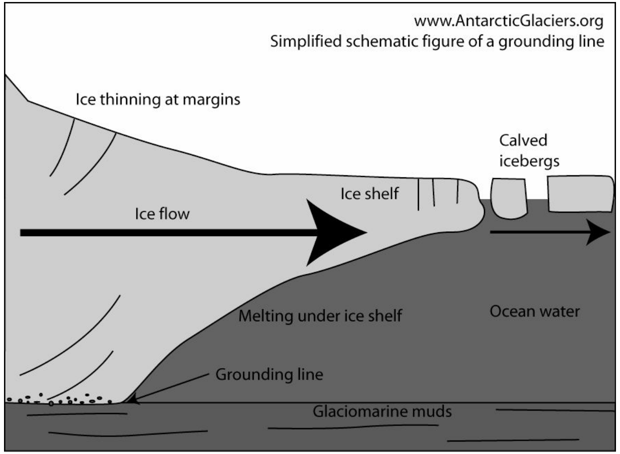
Motivation: Antarctic Ice Shelves

Ice shelves: parts of ice that are still connected to the main ice sheet, but are floating on top of the ocean.

Ice Rifts: full-thickness fractures or cravasses in the ice shelves.May trigger calving processes.



This simplified diagram shows how an ice shelf propagates onto the open ocean, crosses the grounding line and possibly later breaks up, in a calving process to form ice bergs.

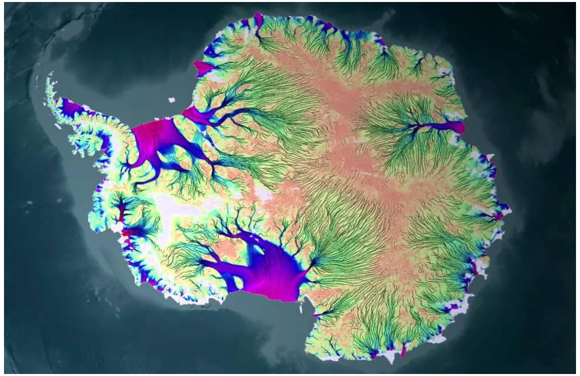


Figure: *The new Nasa/UCI map of ice velocity in Antarctica. Credit: Jeremie Mouginot , UCI*

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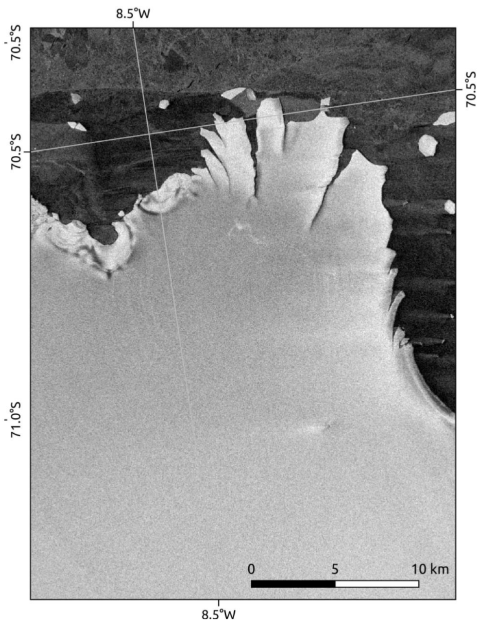
knowing the mass balance of ice sheets is important to predicting the rise of sea level. Calving processes account for about 50% of the mass loss in Antarctica (Rignot, 2013). Some of these processes are believed to be triggered by rifts.

Closer look: Ice Rifts

Our ongoing research focuses on **modeling** the evolution of the interface of one fluid that displaces another fluid in a quasi 2D geometry.Patterns of tongues may emerge and evolve along such interfaces.

- ➔ What is the **mechanism** behind ice rift propagation and the tongue formation in the flow model?
- ➔ Floating ice shelves experience negligible traction along their boundaries with the atmosphere and the ocean, implying that their flow is **extensionally** dominated.



Left: Image of Ekstroemisen ice shelf in Antarctica, where rifts can be recognized normal to the edge of the moving front (Courtesy A. Humbert). *Right:* a snapshot from a laboratory experiment showing a blue polymer solution displacing a denser transparent solution (*Sayag, 2019*), demonstrating the formation of rifts.

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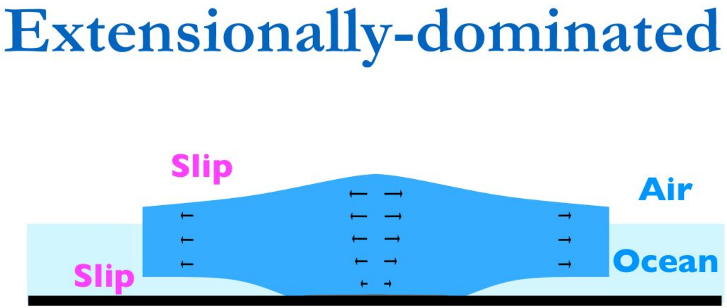
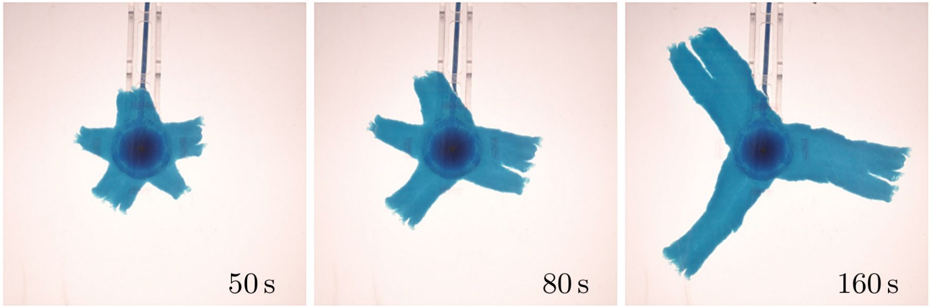
The similarity of the two patterns is reflected in the direction of the rifts with respect to the advancing fronts.

Lab Experiment (Sayag & Worster, 2019)

- ➔ Extensionally dominated flow
- ➔ Strain-rate softening fluid

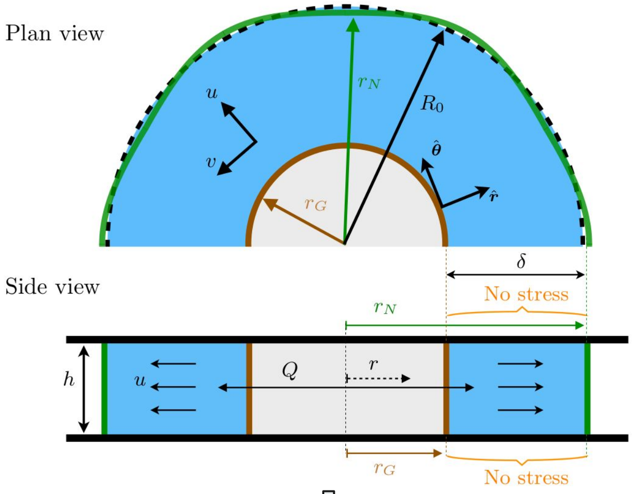




- ➔ Experiments show that finger emerge
- ➔ Over time, the number of tongues\rifts declines

Linear Stability Analysis

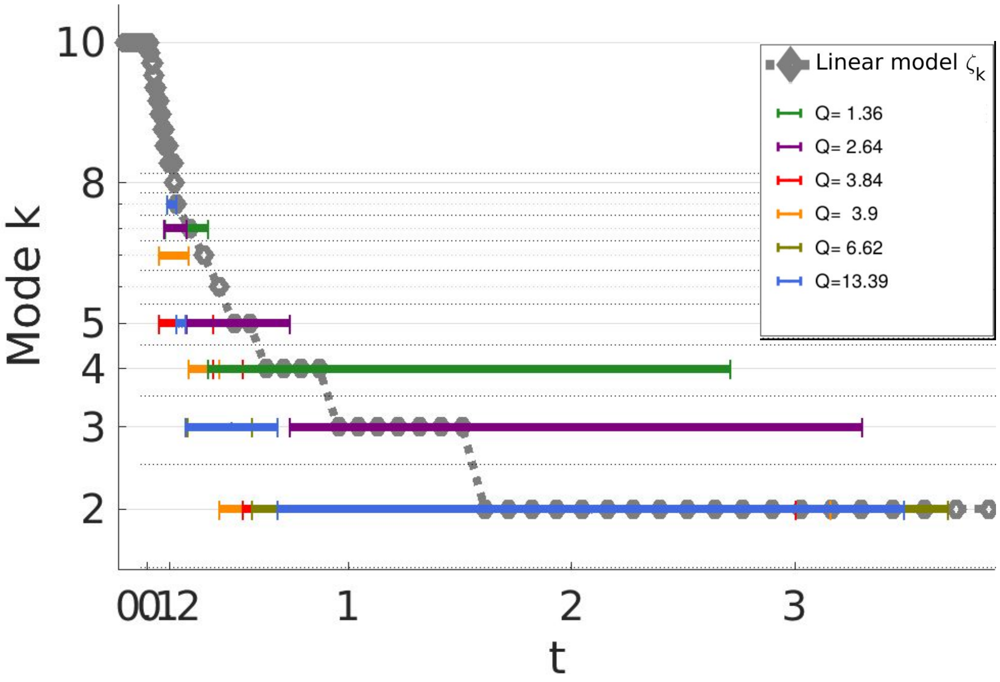
- $\nabla\cdot(2\mu\mathbf{e})=\nabla p$
- $\nabla\cdot u=0$
- $\iint r\,dr\,d\theta=Qt/h$ → constant flux
- $\mu=\mu_0\left(\frac{1}{2}\boldsymbol{e}:\boldsymbol{e}\right)^{\frac{1}{2}\left(\frac{1}{n}-1\right)}$ Power-law viscosity
- Axisymmetric base flow with slip conditions
- $R=R_0(t)+\zeta(\theta,t)$ → adding perturbation to the circular front
- Resulted growth rate **G** varies with the wavenumber k which represents the number of radial rifts. It depends on the radial velocity of the interface and a stabilising geometric stretching term



the only dimensionless parameter of the model is the **fluid exponent n** . This implies that the present system is dynamically similar to ice shelves of a geophysical scale

We study the evolution of the perturbation amplitude $\zeta(\theta,t)$ to determine the dominant mode, k, of the perturbation

Time evolution of perturbation amplitude



- ➔ Comparison of time evolution of the dominant wavenumber in laboratory experiment (color) and the linear time evolution for the perturbation amplitude of the mode with the maximal perturbation amplitude. Results are in log scale as a function of time. The viscosity exponent is similar to the experimental value n = 6 , which is similar to the value used for ice flow.

- ➔ Model versus experiment: The cascade of modes is not continuous but rather jumps on between modes. This discrepancy may be resolved through the more advanced methods including the weakly nonlinear analysis or full solution using numerical simulations.

Key elements to the instability (Sayag & Worster, 2019)

➔ The source of the instability is the base-flow hoop stress along the leading front. This flow configuration (viscous fluid that displaces a denser lower viscosity fluid) appears ***stable*** in the classical shear-dominated instability (such as Saffman-Taylor fingering).

Is linear analysis consistent?

➔ In early stage in the experiments the shape of the front quickly diverges from circular. Therefore, it is **inconsistent** to use the results of linear stability analysis to predict the late time evolution of the front. Even though the prediction of the linear analysis is coincidentally consistent with some of the experimental results, there are observations that the linear theory does not predict. For example, the measured transition time between one mode wavenumber to another is much longer then the predicted one. In addition, at relatively low flux the late-time behavior consists of stochastic transitions between modes which are not predicted by the model. Thus, the **linear model** need to be extended in order to explain the late-time nonlinear behavior.

Weakly nonlinear Stability Analysis

- First, we use linear stability analysis to calculate the growth rate and the linear perturbation amplitude ζ_k^{linear}
- We expand the flow equations up to **second** order in the perturbation amplitude in order to include second order terms and the interaction between the modes （denoted by $H(k,m)$ ）:

$$\dot{\zeta}_k = G(k)\zeta_k + \sum_m \zeta_k \zeta_m H(k,m)$$

- We **linearise** the nonlinear terms by using the linear solution:

$$\zeta_k \zeta_m \rightarrow \zeta_k^{\text{linear}} \zeta_m^{\text{linear}}$$

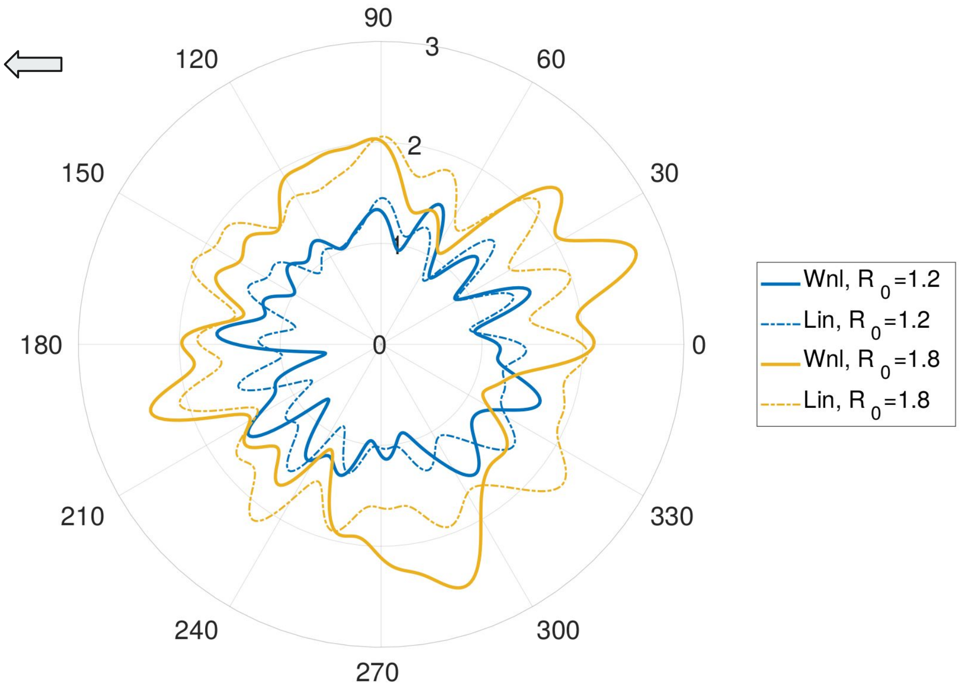
- Then, weakly **nonlinear** amplitude equation becomes:

$$\zeta_k(t) = \zeta_k^{\text{linear}}(t) + \zeta_k^{\text{linear}}(t) \int_{t_c}^{t} \frac{\sum_m \zeta_k^{\text{linear}} \zeta_m^{\text{linear}} H(k,m)}{\zeta_k^{\text{linear}}(t')} dt'$$

➔ ***What controls the decline in the number of tongues? Could weakly nonlinear analysis be enough?***

Weakly nonlinear analysis: Interface evolution

The evolving interface at two interface locations, predicted by the linear analysis (dash) and by the weakly nonlinear analysis (solid). The fluid is strain-rate softening with n = 6



➔ The decreasing number of fingers is predicted both for the linear and the weakly nonlinear analysis. However the weakly nonlinear method is consistent and contributes mainly in later times, when the impact of the mode-coupling terms becomes dominant.

Next focus: Compare to geophysical observations

Morphological similarity: Patterns of rifts

- Independency on spatial and temporal scales
- Only one dimensionless parameter n
- Geophysical scale: Ice shelves

Next steps

➔ Collect data on ice-rift propagation
  - Long time span
  - Radial rifts:grows perpendicular to the moving interface

➔ Compare observations and analysis: Diminishing number of rifts, measure the transition time

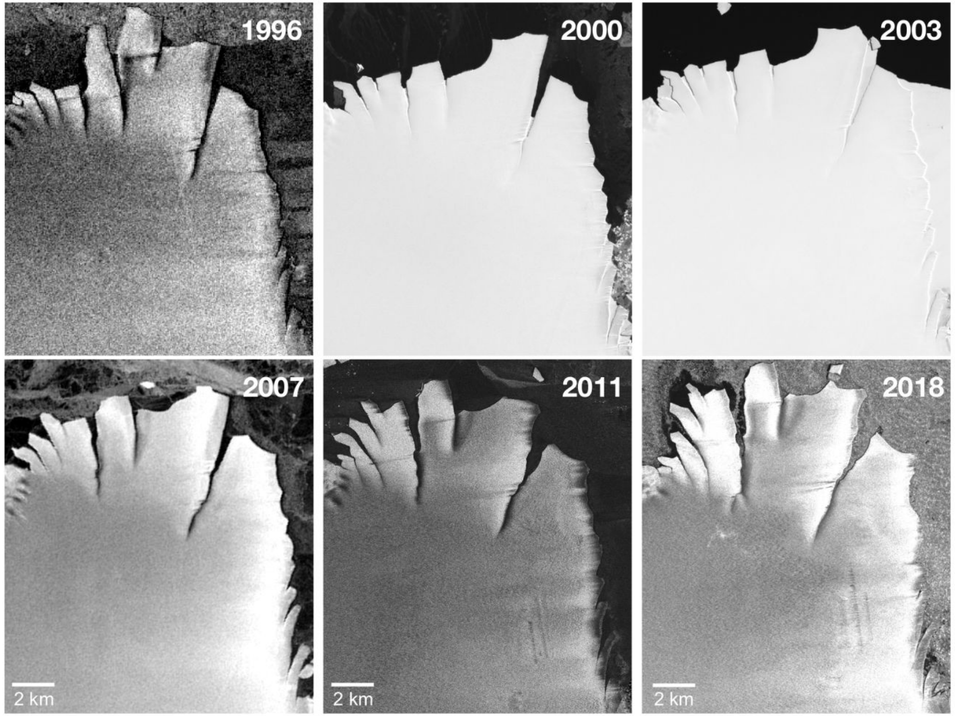
*Another important future focus is a **strongly nonlinear** analysis:*
*A numerical solution to the **full** flow equations !*

Could the model explain the behavior of an ice rift? Can it helps in understanding its mechanism?

Example: Possible Ice-rift observations

The rifts on Ekstroemisen ice-shelf in Antarctica are growing perpendicular to the moving ice front. By tracing their time evolution (allowed by distanced time span) we test the consistency of the theory on ice front flow behavior.

Other possible ice-rift data to consider?



Time evolution of the front of Ekstroemisen, Antarctica
(Courtesy of A. Humbert)