

ASYMMETRIC FRICTION EFFECTS ON SURFACE INTERACTION

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PRESENTATION OVERVIEW

Background (slide 3)

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Conclusion (slide 33)

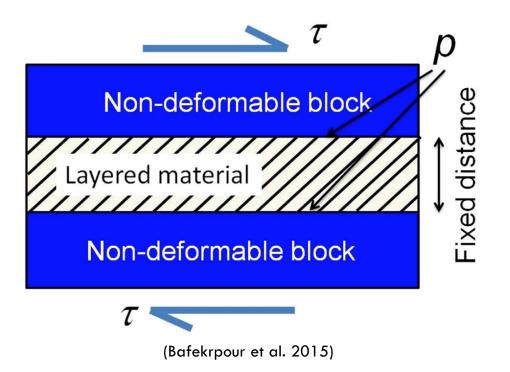
BACKGROUND

In 2015, Arcady Dyskin, Elena Pasternak and colleagues investigated an interesting **unidirectional asymmetric friction** that is created in a constraint environment by **material anisotropy**.

A block with **inclined ribs in a constraint environment moves** such that different normal stress is produced when the block moves in different directions.

Indeed, consider the figure on the right. When shear stress is in the direction shown (along the inclination of the ribs), the normal stress will be less than the normal stress in a case where shear stress is in the opposite direction (going against the inclined ribs).

It has been shown that the **presence of asymmetric friction** can cause **instability** at lower magnitudes of vibration.



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MOTIVATION

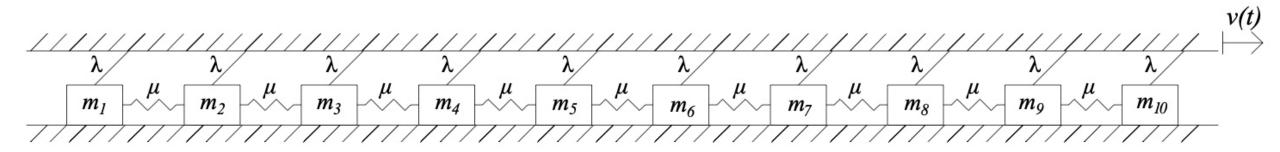
Consider the case when the surface contact contains regions where the symmetric axis is inclined to the contact area. This will mean there is a combination of asymmetric friction regions and regions with symmetric friction. In this presentation, we will explore the effect on the dynamics of surface

interaction when asymmetric friction is present.

DYNAMIC MODEL

To investigate the dynamics during surface interaction, we consider a springblocks model of the type proposed by Burridge and Knopoff (1967).

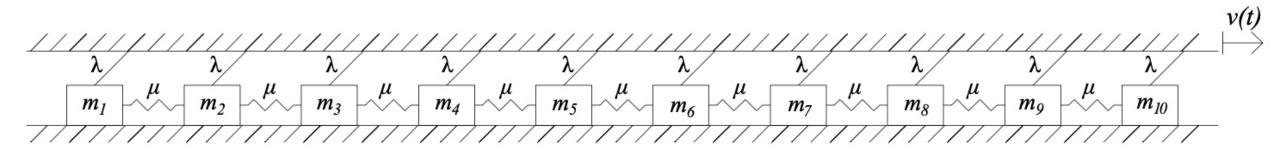
The model consist of multiple blocks – some blocks with asymmetric friction property and others with symmetric friction property – connected by springs. Each of these blocks are connected to the driving block (shown as the top surface in the diagram below) and can slide on the bottom surface.



DYNAMIC MODEL

This model contains three mechanisms:

- 1. A **loading mechanism** (driving block) for the blocks by applying shear stress through the flat springs.
- 2. A mechanism (coiled and flat springs) to store elastic potential energy.
- 3. Stick-slip friction interaction between the blocks and the bottom surface.



EQUATION OF MOTION

$$m_{j}\ddot{x}_{j} = \mu_{j}(x_{j+1} - x_{j}) - \mu_{j-1}(x_{j} - x_{j-1}) + \lambda_{j}(v_{j,0} + v(t) - x_{j}) + \phi_{j}(\dot{x}_{j}, f_{j})$$

$$f_{j}$$

Where

j = 1, 2, ..., N; $x_0 = x_1 + a$ and $x_{N+1} = x_N - a$ a = Interblock distance $m_i = Mass of j^{th} block$ x_i = Displacement of jth block μ_i = Coiled spring stiffness connecting $(j + 1)^{th}$ to j^{th} block $\lambda_i = Flat \ spring \ stiffness \ connected \ to \ j^{th} \ block$ v(t) = Displacement of driving block as a function of time t $v_{i,0} = Initial position of flatspring$ f_i = Sum of forces from the coiled springs and flat spring $\phi_i(\dot{x}_i, f_i) = Frictional force model for j^{th} block_{\odot RUI XIANG}$

SIMPLIFICATION FOR THIS PRESENTATION

In this presentation, we shall consider the following simplification:

Mass of all the blocks being equal. $(m_1 = m_2 = \cdots m_N = m)$

All coiled spring stiffness are equal. $(\mu_1 = \dots = \mu_N = \mu)$

All flat spring stiffness are equal. $(\lambda_1 = \cdots = \lambda_N = \lambda)$

The displacement of driving block has a linear and oscillating term.

$$v(t) = vd * t + \frac{G}{\omega_d^2}\sin(\omega_d t)$$

NORMALISED EQUATION OF MOTION

$$X_{j}^{\prime\prime} = K^{2} (X_{j+1} - 2X_{j} + X_{j-1}) + V_{j,0} + V(T) - X_{j} + \Phi(X_{j}^{\prime}, F_{j})$$

$$F_{j}$$

Where

j = 1, 2, ..., N; $X_0 = X_1 + A$ and $X_{N+1} = X_N - A$ $X_j = \left(\frac{\lambda}{f_c}\right) x_j$; Normalised displacement f_c = Reference force (used to normalise friction model) $T = \omega_p t$; Normalised time $\omega_p = \sqrt{\frac{\lambda}{m}}$; Characteristic frequency $K = \frac{\mu}{2}$; Ratio of coiled spring stiffness to flat spring stiffness $V_{i,0}$ = Initial normalised position of flat spring $V(T) = Vd * T + \alpha \sin(\beta T)$; Normalised displacement of driving block $Vd = \frac{\omega_p \lambda}{f_c} vd$; $\alpha = \frac{G\lambda}{\omega_d^2 f_c}$; $\beta = \frac{\omega_d}{\omega_p}$ $F_i = Sum \ of \ normalised \ acceleration \ due \ to \ coiled \ spring \ and \ flat \ spring$ $\Phi(X'_i, F_i) = Normalised friction force$

*Prime indicates derivative with respect to normalised time.

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FRICTION CONSIDERATION

• Static and kinetic friction forces

 \circ Opposes direction of forces F_j when the block is static, and opposes direction of velocity

Symmetric Friction

• Equal friction forces in opposing directions

Asymmetric Friction

• Frictional force in the 'hard' direction is greater than the 'easy' direction.

FRICTION MODEL

$$\phi_{j}(\dot{x}_{j}, f_{j}) = \begin{cases} -min(f_{s}^{+}, |f_{j}|), & \dot{x}_{j} = 0 \text{ and } f_{j} > 0 \\ -f_{d}^{+} - e \dot{x}_{j}, & \dot{x}_{j} > 0 \\ min(f_{s}^{-}, |f_{j}|), & \dot{x}_{j} = 0 \text{ and } f_{j} < 0 \\ f_{d}^{-} - e \dot{x}_{j}, & \dot{x}_{j} < 0 \end{cases}$$

Where

 $f_s^+ = Static friction force in the positive direction$

- $f_d^+ = Dynamic friction force in the positive direction$
- $f_s^- = Static friction force in the negative direction$
- $f_d^- = Dynamic friction force in the negative direction$
- *e* = *Damping coefficient*

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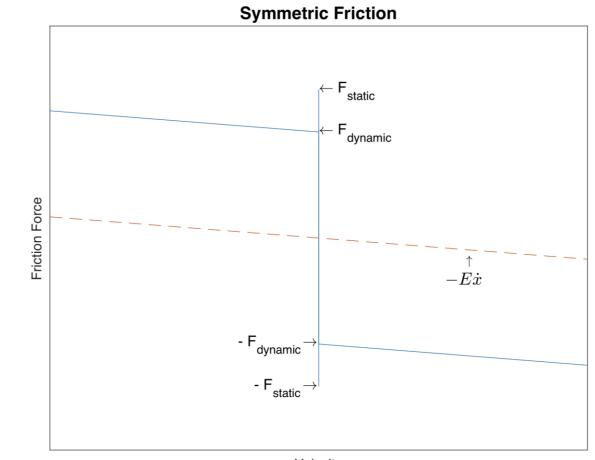
DIMENSIONLESS FRICTION MODEL

$$\Phi_{j}(X'_{j}, F_{j}) = \begin{cases} -\min(\eta_{s}^{+}, |F_{j}|), & X'_{j} = 0 \text{ and } F_{j} > 0 \\ -\eta_{d}^{+} - E X'_{j}, & X'_{j} > 0 \\ \min(\eta_{s}^{-}, |F_{j}|), & X'_{j} = 0 \text{ and } F_{j} < 0 \\ \eta_{d}^{-} - E X'_{j}, & X'_{j} < 0 \end{cases}$$
Where
$$\eta_{s}^{+} = \frac{f_{s}^{+}}{f_{c}}; \text{ Static friction ratio in the positive direction} \\ \eta_{d}^{+} = \frac{f_{d}^{+}}{f_{c}}; \text{ Dynamic friction ratio in the positive direction} \\ \eta_{s}^{-} = \frac{f_{s}^{-}}{f_{c}}; \text{ Static friction ratio in the negative direction} \\ \eta_{d}^{-} = \frac{f_{d}^{-}}{f_{c}}; \text{ Dynamic friction ratio in the negative direction} \\ E = \frac{e\omega_{p}}{f_{c}}; \text{ Normalised damping coefficient} \end{cases}$$

SYMMETRIC FRICTION FORCE

Here we have a plot of the model for a symmetric friction block.

The plot shows the friction force (y-axis) over velocity (x-axis; center being when velocity is zero).

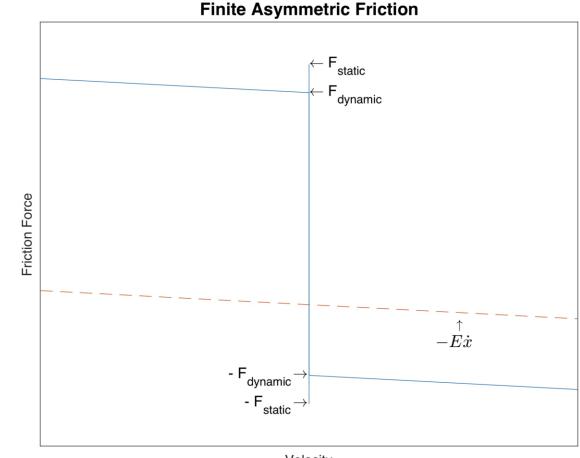


Velocity

ASYMMETRIC FRICTION FORCE

Here we have a plot of the model for an asymmetric friction block, where the friction force is greater in the negative direction when compared to the positive direction.

The plot shows the friction force (y-axis) over velocity (x-axis; center being when velocity is zero).



Velocity

PARAMETRIC ANALYSIS 1

> N = 4

 $\succ \alpha = \frac{G\lambda}{\omega_d^2 f_c} = 1$

 $> \beta = \frac{\omega_d}{\omega_n} = 1$

 $\succ Vd = \frac{\omega_p \lambda}{f_c} vd = 0$

 $\succ K = \frac{\mu}{\lambda} = 1$

 $\succ E = \frac{e\omega_p}{f_c} = 0$

We will consider the case with the following parameters being kept constant:

Number of blocks

Normalised amplitude of driving block's oscillation

Normalised driving block's frequency

Normalised translational velocity of driving block

Ratio of coiled spring stiffness to flat spring stiffness

Normalised damping coefficient

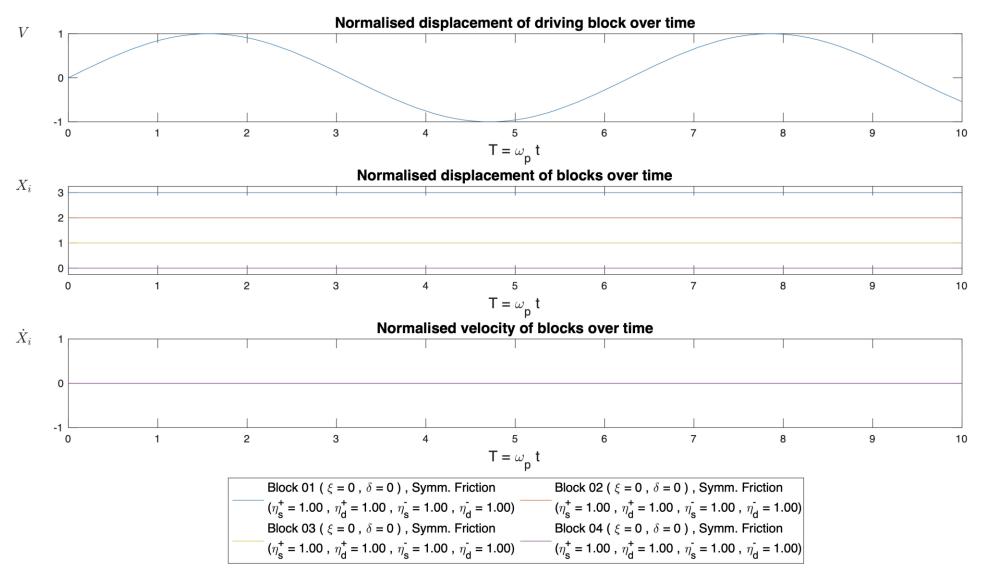
We will also assume that static and dynamic friction are of the same magnitude in this analysis ($\eta_s = \eta_d$).

PARAMETRIC ANALYSIS 1

Varied parameters:

- > (Slide 17) All blocks having symmetric friction properties with $\eta^+ = \eta^- = 1$.
- > (Slide 19) Alternating symmetric and asymmetric friction blocks. The symmetric friction mass has the same properties as before, while the asymmetric friction mass has $\eta^+ = 0.5$ (easy direction; along inclined ribs) and $\eta^- = 2$ (hard direction; against the inclined ribs).

(Slide 21) First half of the blocks have asymmetric friction property, while the second half of the blocks have symmetric friction property. (N = 4 , K = 1.00 , Vd = 0 , α = 1.00 , β = 1.00 , E = 0.00)



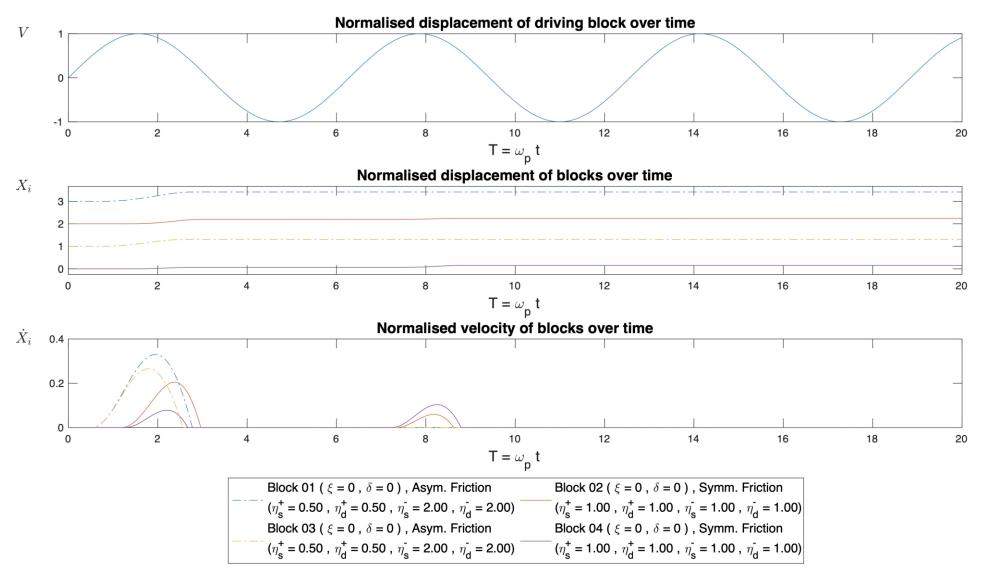
PARAMETRIC ANALYSIS 1: DISCUSSION (SLIDE 17)

Due to the definition of $\alpha = \frac{G\lambda}{\omega_d^2 f_c}$, $\alpha = 1$ will ensure that mass with friction ratio of $\eta \ge 1$ will remain stationary

For this reason, it makes sense that in slide 17, none of the symmetric friction block ($\eta^+ = \eta^- = 1$) moved.

In the next slide, we will introduce asymmetric friction properties for alternate blocks (i = 1, 3).

(N = 4 , K = 1.00 , Vd = 0 , α = 1.00 , β = 1.00 , E = 0.00)



PARAMETRIC ANALYSIS 1: DISCUSSION (SLIDE 19)

In slide 19, asymmetric friction was introduced as a property for alternate blocks (i = 1, 3).

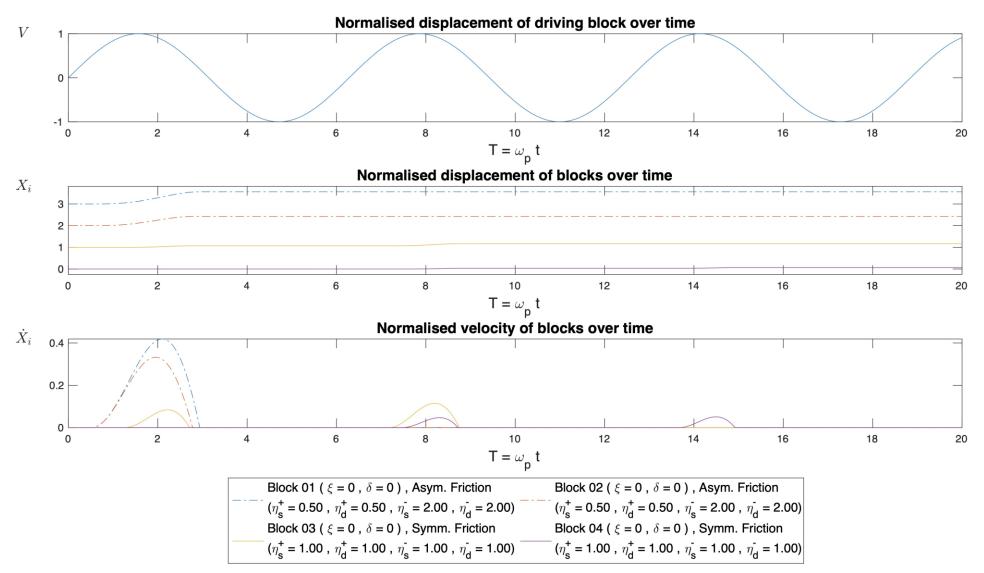
Here, the asymmetric friction block has $\eta^+ = 0.5$ and $\eta^- = 2$, enabling the asymmetric friction block to move in the positive direction but not in the negative direction.

As the asymmetric block moves in the positive direction, the coiled spring connecting it to the symmetric friction block loads up. The force by the loaded coiled spring in combination with the force by the driving block enables the symmetric friction block to overcome friction and slip.

System can be seen to reach stability after some time; the loaded springs are able to keep the asymmetric friction block from displacing.

Now consider when first half of the blocks have asymmetric friction (i = 1, 2), while the next half have symmetric friction (i = 3, 4).

(N = 4 , K = 1.00 , Vd = 0 , α = 1.00 , β = 1.00 , E = 0.00)



PARAMETRIC ANALYSIS 1: DISCUSSION (SLIDE 21)

Comparing slides 19 and 21, we can see the asymmetric blocks that are connected together are able to displace further.

As the asymmetric block 2 moves, this loads the coiled spring connected to symmetric friction block 3.

Symmetric friction block 3 can be seen to overcome friction, while block 4 remains stationary during the first instance of instability.

During the second instance of instability block 3 and 4 can be seen to displace.

After some time, the system once again reaches stability

This shows that the instability caused by asymmetric friction regions of the surface can propagate to neighbouring regions.

PARAMETRIC ANALYSIS 2

In parametric analysis 1, we considered when static and kinetic friction are equal in magnitude ($\eta_s = \eta_d$). Now, we will consider the case when dynamic friction is smaller than static friction ($\eta_s > \eta_d$); specifically, we will set $\eta_d = 0.8\eta_s$.

We will consider the same constant parameters as in parametric analysis 1:

- > N = 4 Number of blocks
 - Normalised amplitude of driving block's oscillation
 - Normalised driving block's frequency
- $Vd = \frac{\omega_p \lambda}{f_c} vd = 0$ $K = \frac{\mu}{\lambda} = 1$

 $\succ \alpha = \frac{G\lambda}{\omega_d^2 f_c} = 1$

 $> \beta = \frac{\omega_d}{\omega_n} = 1$

 $\succ E = \frac{e\omega_p}{f_c} = 0$

- Normalised translational velocity of driving block
 - Ratio of coiled spring stiffness to flat spring stiffness
 - Normalised damping coefficient

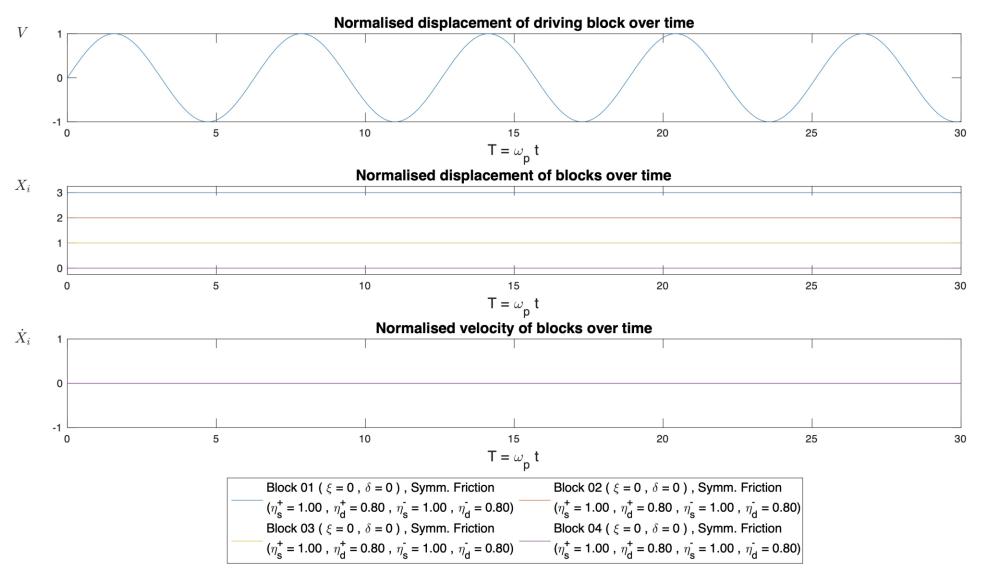
PARAMETRIC ANALYSIS 2

Varied parameters:

> (Slide 25) All blocks having symmetric friction properties with $\eta_s^+ = \eta_s^- = 1$ and $\eta_d^+ = \eta_d^- = 0.8$.

> (Slide 27) Alternating symmetric and asymmetric friction blocks. The symmetric friction mass has the same properties as before, while the asymmetric friction mass has $\eta_s^+ = 0.5$, $\eta_d^+ = 0.4$, $\eta_s^- = 2$ and $\eta_d^- = 1.6$.

(Slide 29) First half of the blocks have asymmetric friction property, while the second half of the blocks have symmetric friction property. (N = 4 , K = 1.00 , Vd = 0 , α = 1.00 , β = 1.00 , E = 0.00)

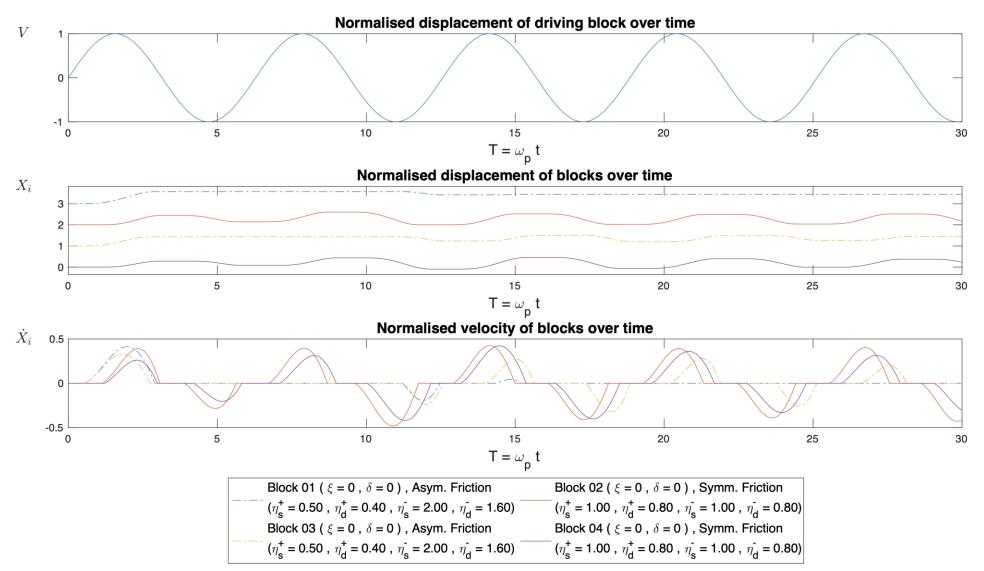


PARAMETRIC ANALYSIS 2: DISCUSSION (SLIDE 25)

As expected, the blocks remain stable when $\alpha = 1$ and $\eta_s = 1$.

The force on the block cannot overcome the static friction, and so dynamic friction has no impact on this system.

(N = 4 , K = 1.00 , Vd = 0 , α = 1.00 , β = 1.00 , E = 0.00)



PARAMETRIC ANALYSIS 2: DISCUSSION (SLIDE 27)

In slide 27, the asymmetric friction block can overcome friction in the easy direction $(\eta_s^+ = 0.5 < 1)$.

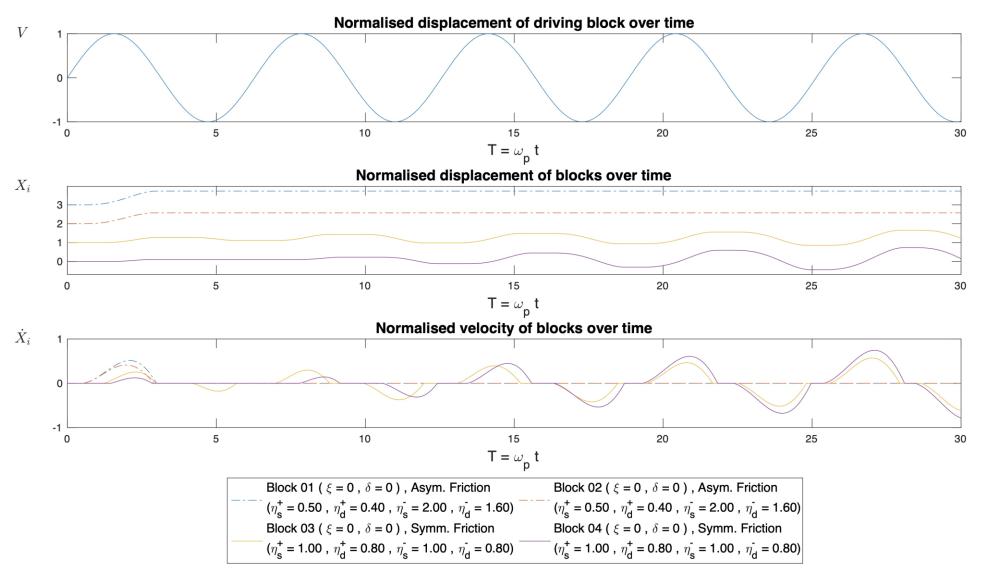
When the asymmetric friction block displaces, it again loads up the coiled spring.

The force from the loaded coiled spring with the force from the driving block allows the symmetric friction block to overcome static friction.

When the symmetric friction block moves, it is subjected to dynamic friction, and so will reach a stable state when it can no longer overcome this dynamic friction.

Since dynamic friction is smaller than static friction, the symmetric friction block displaces further than it did in slide 19.

This causes the springs to be loaded such that the symmetric friction blocks can be seen to have reoccurring displacement overtime. (N = 4 , K = 1.00 , Vd = 0 , α = 1.00 , β = 1.00 , E = 0.00)



PARAMETRIC ANALYSIS 2: DISCUSSION (SLIDE 29)

In slide 29, the asymmetric friction blocks that are connected together (i = 1, 2), can displace further when compared to slide 27.

This loads the spring connecting asymmetric friction block 2 to symmetric friction block 3.

This loaded spring in combination with the force from the driving block causes the symmetric friction block 3 to displace.

As the dynamic friction is smaller than the static friction, block 3 can displace further, which in turn causes block 4 to displace during the first occurrence of instability (as oppose to the second instance of instability observed in slide 21).

Again, due to the loaded springs, the symmetric friction blocks can be seen to continually displace over time.

PARAMETRIC ANALYSIS 3

It follows from parametric analysis 2, that if the force by the driving block is not able to overcome the dynamic friction of the symmetric friction blocks, the system will eventually reach stability as it did in parametric analysis 1.

To test this, we shall set $\alpha = \frac{G\lambda}{\omega_d^2 f_c} = 0.8$, which ensures that mass with friction ratio $\eta_d = 0.8$ will remain stationary.

We will again consider the same constant parameters as the previous analysis (with exception of α):

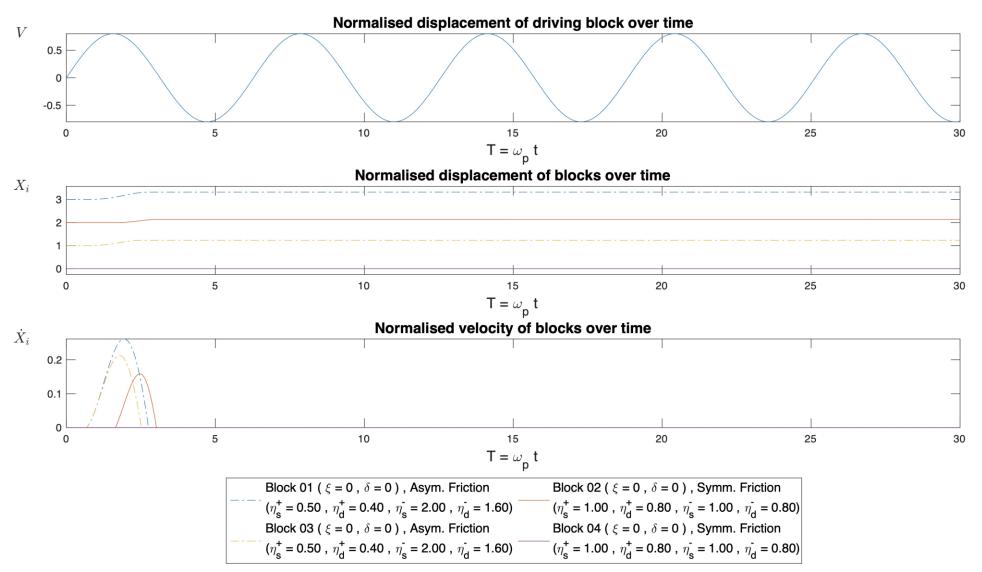
N = 4Number of blocks $\alpha = \frac{G\lambda}{\omega_d^2 f_c} = 0.8$ Normalised amplitude of driving block's oscillation $\beta = \frac{\omega_d}{\omega_p} = 1$ Normalised driving block's frequency $Vd = \frac{\omega_p\lambda}{f_c}vd = 0$ Normalised translational velocity of driving block $K = \frac{\mu}{\lambda} = 1$ Ratio of coiled spring stiffness to flat spring stiffness $E = \frac{e\omega_p}{f_c} = 0$ Normalised damping coefficient

PARAMETRIC ANALYSIS 3

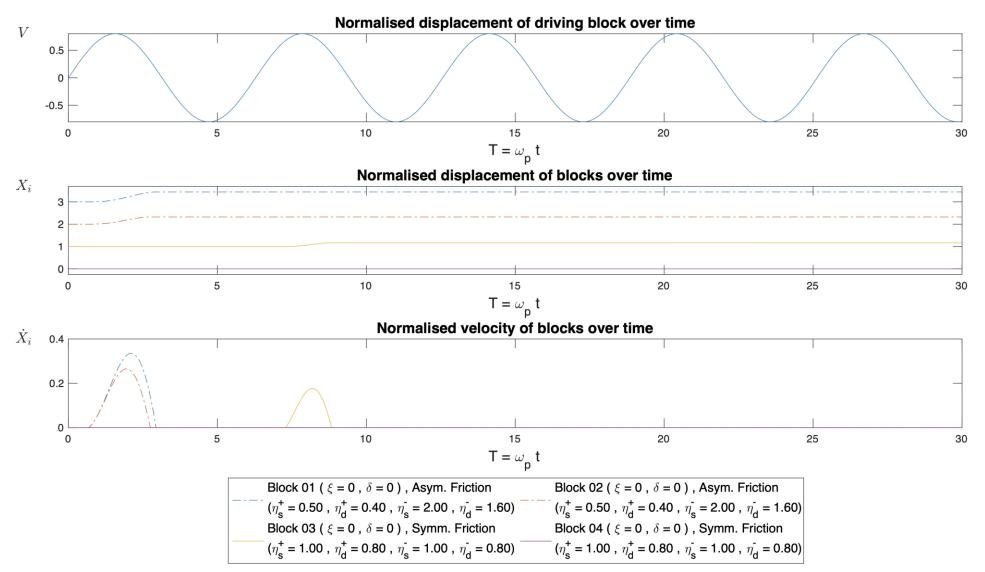
Varied parameters:

> (Slide 33) Alternating symmetric and asymmetric friction blocks. The symmetric ($\eta_s^+ = \eta_s^- = 1$ and $\eta_d^+ = \eta_d^- = 0.8$) and asymmetric ($\eta_s^+ = 0.5$, $\eta_d^+ = 0.4$, $\eta_s^- = 2$ and $\eta_d^- = 1.6$) friction mass has the same properties as in parametric analysis 2.

(Slide 34) First half of the blocks have asymmetric friction property, while the second half of the blocks have symmetric friction property. (N = 4 , K = 1.00 , Vd = 0 , α = 0.80 , β = 1.00 , E = 0.00)



(N = 4 , K = 1.00 , Vd = 0 , α = 0.80 , β = 1.00 , E = 0.00)



PARAMETRIC ANALYSIS 3: DISCUSSION (SLIDES 33 AND 34)

The asymmetric friction blocks can be seen to displace $(\eta_s^+ = 0.5 < 0.8)$ in the positive direction, loading the coiled spring.

In slide 33, the symmetric block 2 can be seen to displace during the first occurrence of instability. The system can be seen to achieve stability after the first occurrence of instability.

In slide 34, the symmetric block 3 can be seen to displace during the second instance of instability, and the system achieve stability after this displacement.

SUMMARY

Threshold of stability can be lowered if asymmetric friction regions are introduced through material anisotropy.

When the driving force is not able to overcome static friction but sufficient enough to exceed dynamic friction, introducing asymmetric friction can cause the system to be in an unstable state with reoccurring stick slip.

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