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Test of gravitational redshift based on tri-frequency combination of frequency links between Atomic Clock Ensemble in Space and a ground station

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Presented at EGU2020, 3-8 May 2020

1 Introduction

Gravitational redshift

Testing gravitational redshift is one of key issues in testing Einstein's equivalence principle (EEP, Will 2014), which has been confirmed by various experiments (Pound and Rebka 1959, 1960a, b, 1965; Hafele and Keating 1972; Alley 1979; Vessot et al. 1980; Turneaure et al. 1983; Krisher el al. 1993).

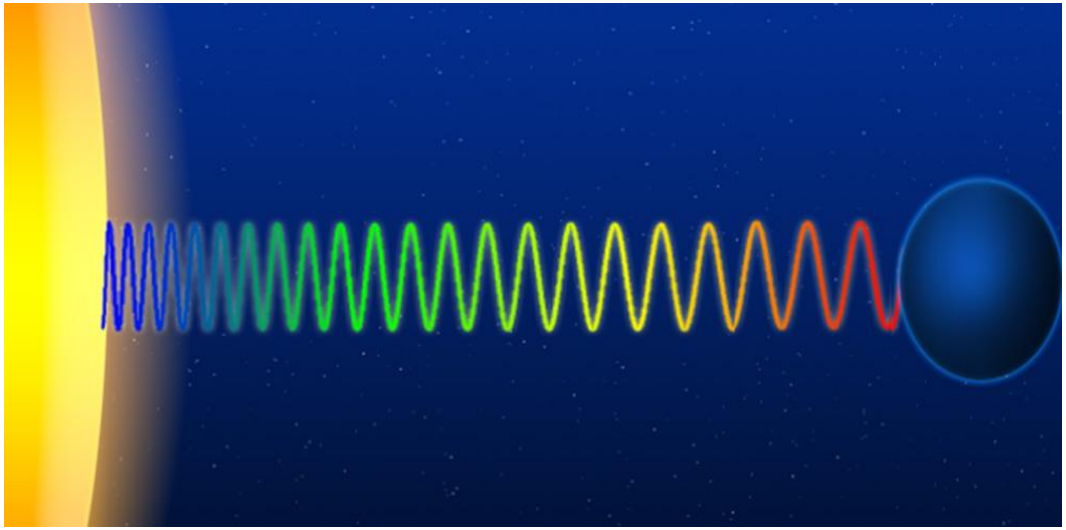


Fig1. There is a gravitational redshift when observing light signals coming from a massive star

1 Introduction

The most precise experiment was performed by two Galileo satellites (**~ 2×10^{-5}**) (Delva et al. 2018; Herrmann et al. 2018).

ACES mission expected: **~ 2×10^{-6}**.

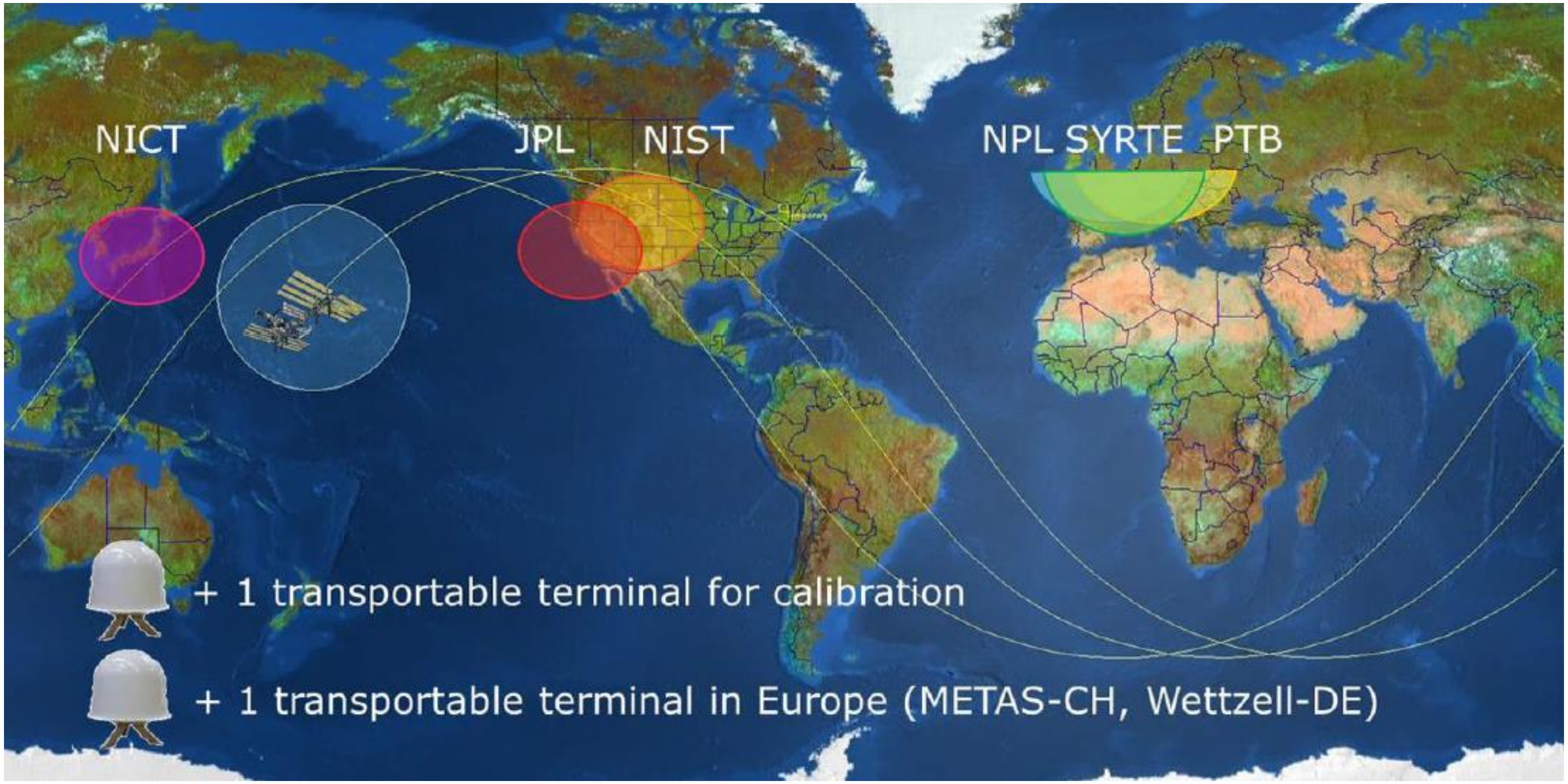
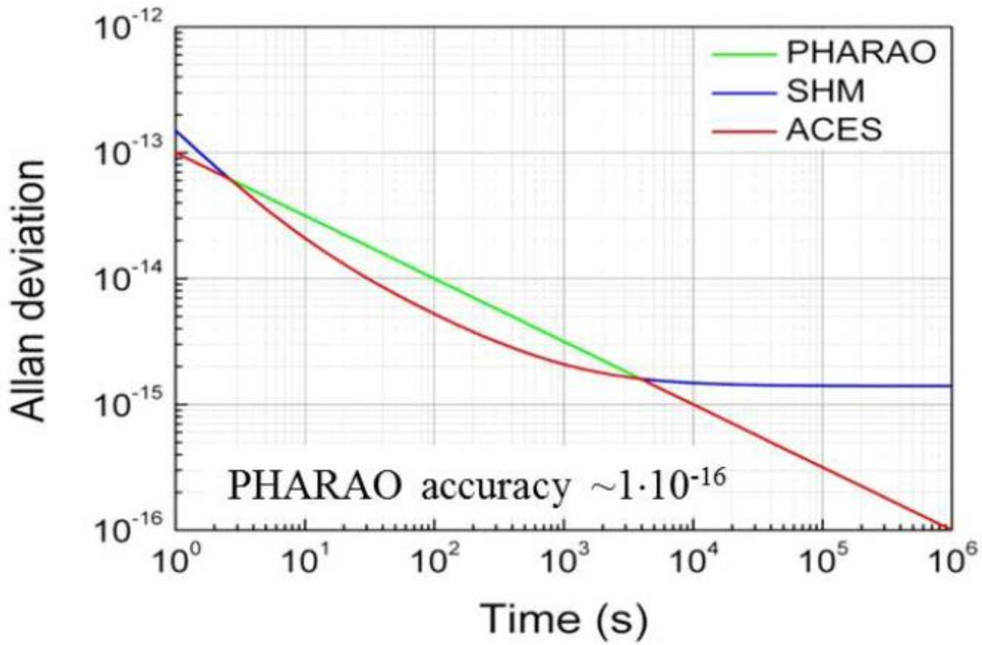


Fig2. Cacciapuoti et al. 2017

1 Introduction

ACES payloads

A hydrogen maser (SHM) with medium-term frequency stability;
A cold cesium atoms (PHARAO) with long-term frequency stability;
Frequency stability: $1 \times 10^{-13}\tau^{-1/2}$, with accuracy 2×10^{-16}.



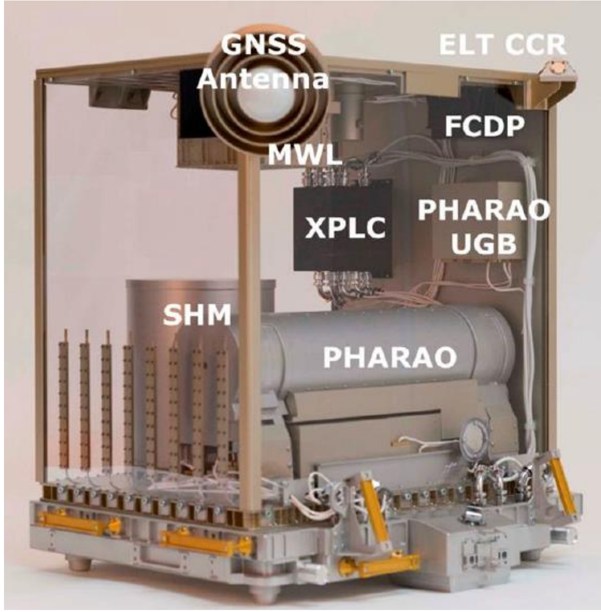


Fig3. Cacciapuoti et al. 2017

1 Introduction

Two-way MWLs and ELT

Time signals on board are compared to ground clocks using two-way microwave links (MWL) for time-transfer operating in the Ku band as well as a laser time-transfer system ELT (European Laser Timing).

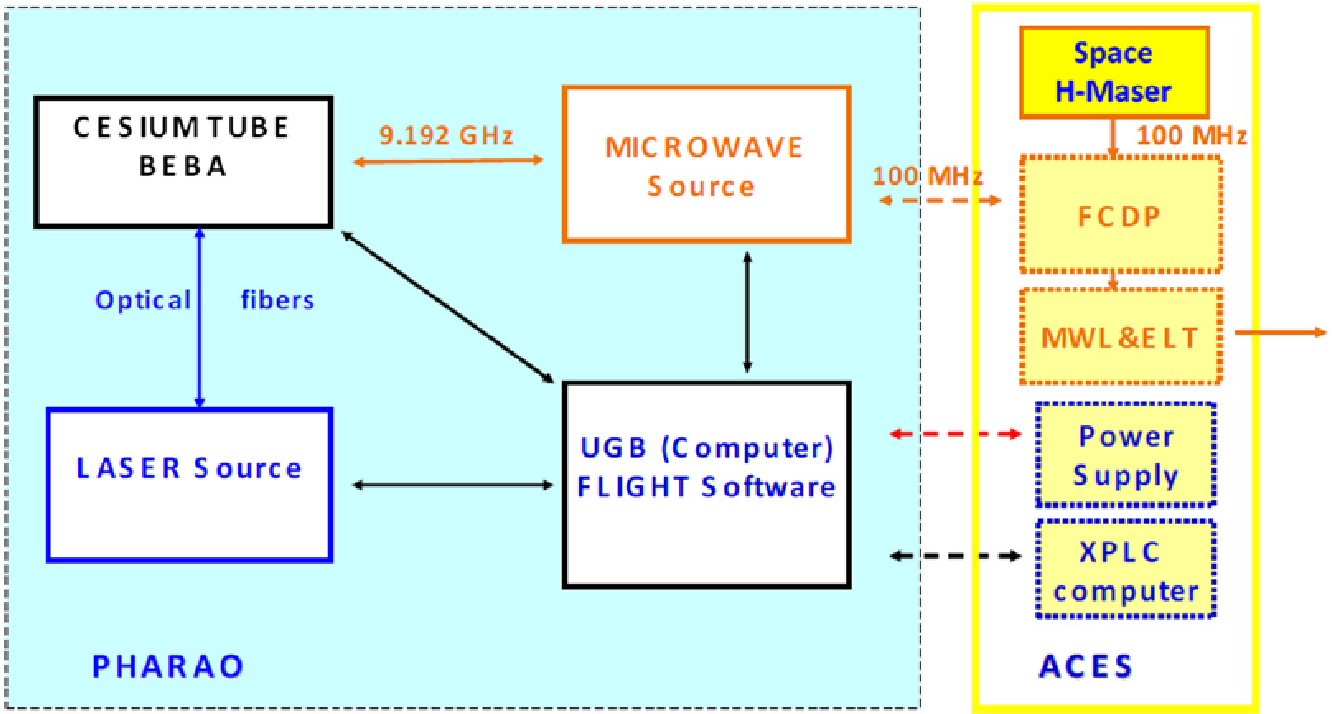


Fig4. Laurent et al. 2015

1 Introduction

Frequency comparison VS time comparison

There are literatures of testing gravitational redshift using time comparison (Cacciapuoti and Salomon 2009; Duchayne et al. 2009; Meynadier et al. 2018), but short of publications related to frequency comparison.

Frequency comparison has several strengths:

1. Influence of phase ambiguity is greatly reduced;
2. Instant gravitational potential can be determined

Here we use two-way MWLs to compare frequency signals

2 One-way frequency transfer

We consider a downlink from Space Station A to ground station B. The proper frequency shift of the photon from A to B is expressed as (Blanchet et al. 2001) (accurate to $1/c^3$ **throughout this study**)

$$\frac{\nu_B}{\nu_A}=\frac{1-\frac{1}{c^2}\left[U_E\left(r_A\right)+\frac{v_A^2}{2}\right]}{1-\frac{1}{c^2}\left[U_E\left(r_B\right)+\frac{v_B^2}{2}\right]}\frac{q_B}{q_A} \qquad (1)$$

Gravitational redshift and transverse Doppler effects

whrer

$$q_A=1-\frac{\mathbf{N}_{AB}\cdot\mathbf{v}_A}{c}-\frac{4GM_E}{c^3}\frac{\left(r_A+r_B\right)\mathbf{N}_{AB}\cdot\mathbf{v}_A+R_{AB}\frac{\mathbf{r}_A\cdot\mathbf{v}_A}{r_A}}{\left(r_A+r_B\right)^2-R_{AB}^2} \qquad (2)$$

$$q_B=1-\frac{\mathbf{N}_{AB}\cdot\mathbf{v}_B}{c}-\frac{4GM_E}{c^3}\frac{\left(r_A+r_B\right)\mathbf{N}_{AB}\cdot\mathbf{v}_B-R_{AB}\frac{\mathbf{r}_B\cdot\mathbf{v}_B}{r_B}}{\left(r_A+r_B\right)^2-R_{AB}^2} \qquad (3)$$

Doppler effect

Shapiro effect

2 One-way frequency transfer

Set simplified notation

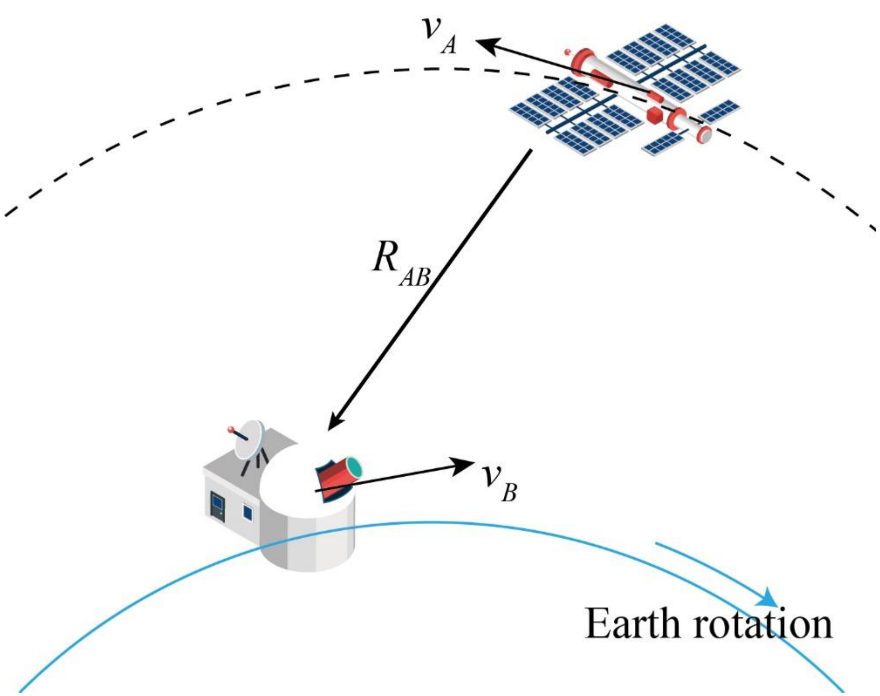


Fig5. One-way frequency transfer

$$
\begin{cases}
G_A = \dfrac{4GM_E}{c^3}\dfrac{\left(r_A + r_B\right)\mathbf{N}_{AB}\cdot\mathbf{v}_A + R_{AB}\dfrac{\mathbf{r}_A\cdot\mathbf{v}_A}{r_A}}{\left(r_A + r_B\right)^2 - R_{AB}^2} \\
G_B = \dfrac{4GM_E}{c^3}\dfrac{\left(r_A + r_B\right)\mathbf{N}_{AB}\cdot\mathbf{v}_B - R_{AB}\dfrac{\mathbf{r}_B\cdot\mathbf{v}_B}{r_B}}{\left(r_A + r_B\right)^2 - R_{AB}^2} \\
A_{rel} = \dfrac{1 - \dfrac{1}{c^2}\left[U_E\left(r_A\right) + \dfrac{v_A^2}{2}\right]}{1 - \dfrac{1}{c^2}\left[U_E\left(r_B\right) + \dfrac{v_B^2}{2}\right]} \\
A_{dop} = \dfrac{1 - \dfrac{\mathbf{N}_{AB}\cdot\mathbf{v}_B}{c}}{1 - \dfrac{\mathbf{N}_{AB}\cdot\mathbf{v}_A}{c}}
\end{cases}
\tag{4}
$$

we have

$$
\frac{\nu_B}{\nu_A} = A_{rel}\left(A_{dop} + G_A - G_B\right) \tag{5}
$$

2 One-way frequency transfer

Models (4)-(5) hold only in free space. In a real space with medium, we have

$$\frac{\nu_B}{\nu_A} = A_{rel}\left(\bar{A}_{dop} + G_A - G_B\right) \tag{6}$$

where $\bar{A}_{dop}$ is modified Doppler frequency shift with considering atmospheric contributions.

Considering refraction effect, Doppler effects could be rewritten as (Bennett 1968)

$$\bar{A}_{dop} = \frac{1 - \frac{n_B \mathbf{T}_B \cdot \mathbf{v}_B}{c}}{1 - \frac{n_A \mathbf{T}_A \cdot \mathbf{v}_A}{c}} + \frac{40.3}{cf^2}\frac{d}{dt}\int_{Li}\rho(t)ds - \frac{1}{c}\frac{d}{dt}\int_{Lt}\left(M_1 + M_2\right)ds \tag{7}$$

Caused by the velocity of both ends

Caused by the time varying index

2 One-way frequency transfer

Taking into account the influences caused by the refractive index variation and wave path (Sun et al. 2020), eq. (6) can be written as

$$\frac{\nu_B}{\nu_A} = A_{rel}\left(\boxed{A_{dop} + \delta f_{refr} + \delta f_{ion} + \delta f_{trop}} + G_A - G_B\right) \tag{8}$$

where

$$\begin{aligned}
\delta f_{refr} &= \frac{\left(v_{Ax}\delta_A + v_{Bx}\delta_B\right)\sin\gamma_B - \left(v_{Ay}\delta_A + v_{By}\delta_B\right)\cos\gamma_B}{c} - \frac{\left(M_1 + M_2\right)\mathbf{N}_{AB}\cdot\mathbf{v}_B}{c} - \frac{40.3 n_e \mathbf{N}_{AB}\cdot\mathbf{v}_A}{cf^2} \\
\delta f_{ion} &= \frac{40.3}{cf^2}\frac{d}{dt}\int_{Li}\frac{dn_e}{dt}ds \\
\delta f_{trop} &= -\frac{1}{c}\frac{d}{dt}\int_{Lt}\frac{d\left(M_1 + M_2\right)}{dt}ds
\end{aligned} \tag{9}$$

δf_{refr} is the bending effects acting on Doppler frequency shift, caused by refraction, δf_{ion} and δf_{trop} are atmospheric effects cause by time-varying refractive index.

3 Formulation for testing gravitational redshift

According to ISS ACES mission, information of frequency links are as follows:

- Ku band uplink, with carrier frequency 13.475 GHz, and frequency shift is known afterwards;
- Ku band downlink, with carrier frequency 14.70333 GHz；
- S band downlink, with carrier frequency 2248 MHz.

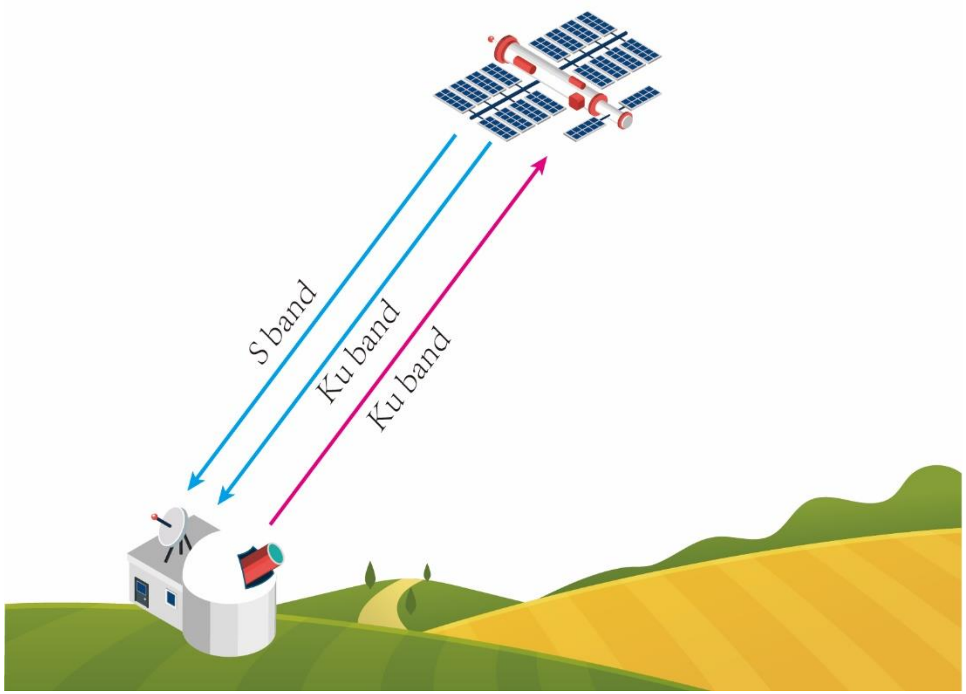


Fig 6. three links of ACES mission

We denoted respectively the frequencies of these three links as f_1, f_2, f_3.

3 Formulation for testing gravitational redshift

◻ Procedure

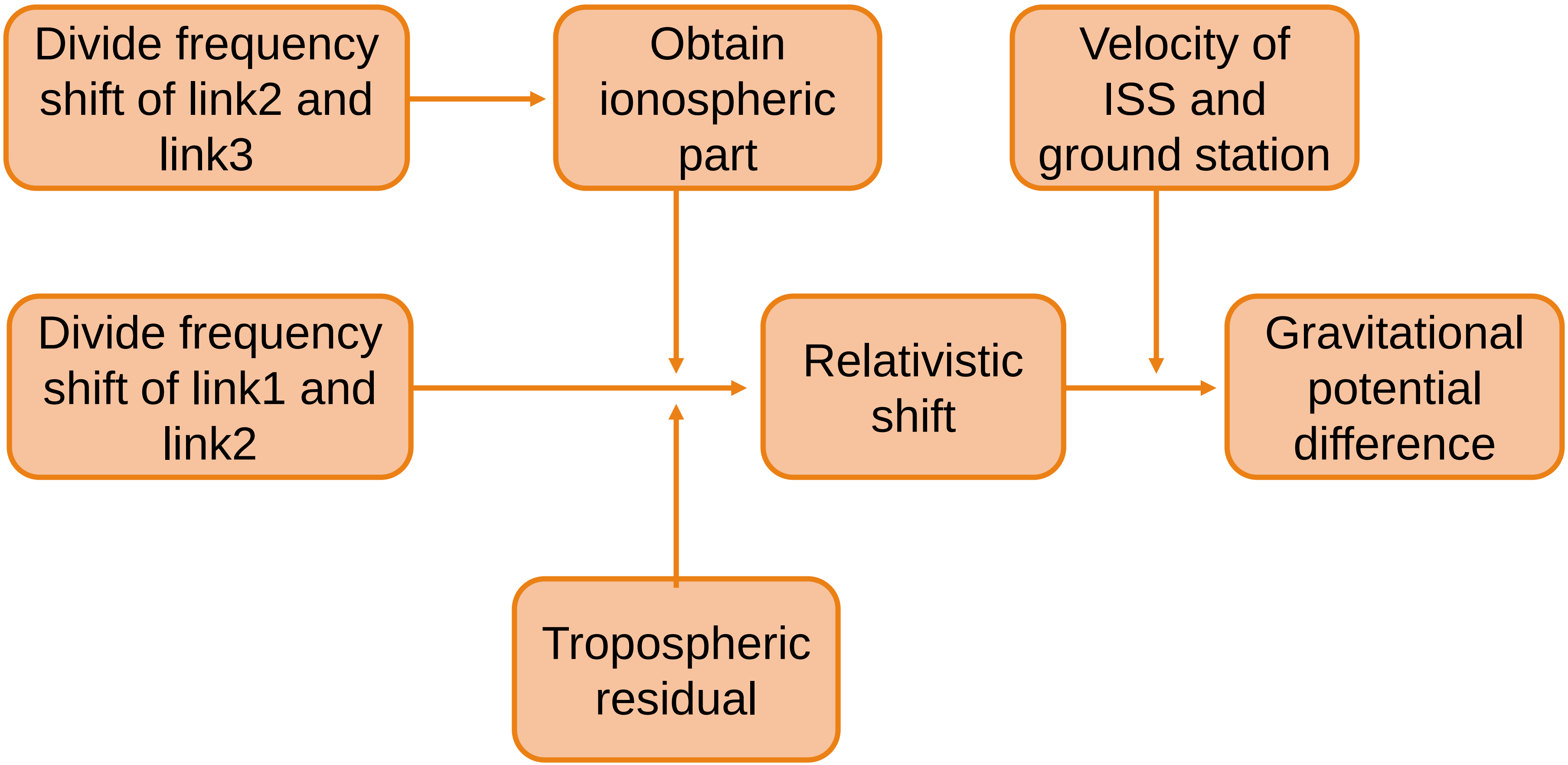


Fig 7. diagram of procedure

3 Formulation for testing gravitational redshift

We define $T_{23}=t_3-t_2$ and $T_{35}=t_5-t_3$, according to Meynadier et al. (2018), T_{23} is about *1us*, and T_{35} is about *100ns*.

Based on position relation of ISS and ground station, T_{35} and T_{46} are in the same level, but T_{14} is much larger, about *2ms*.

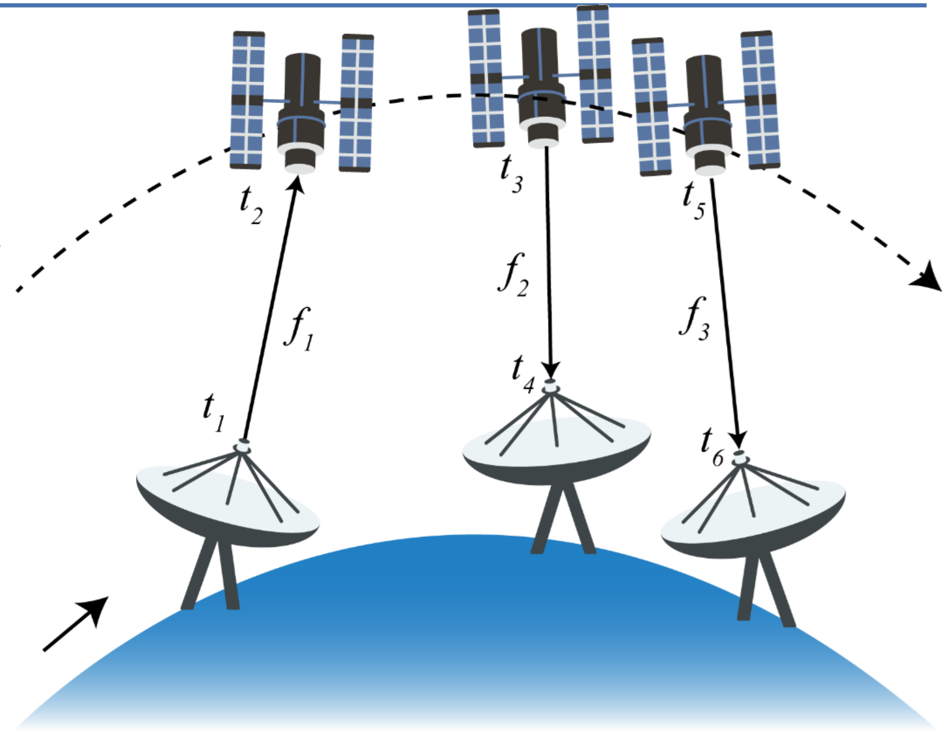


Fig 8. Observations of ACES mission

By combining three frequencies, we obtain frequency gravitational red shift (Sun et al. 2020).

3 Formulation for testing gravitational redshift

☐ Residual errors

Because we did some approximation and ignored some effects such as tidal effects, some residuals exist, mainly on Doppler effect and ionosphere.

Table 1 Frequency shift residuals of Tri-frequency comparison

Type	Residual	Source
Doppler shift	$<1.5\times10^{-16}$	Velocity and position difference of link 2 and link 3
Atmospheric part	$\sim1\times10^{-15}$	High order ionospheric part
Gravitational part	$<4.2\times10^{-17}$	Tidel effects acting on ground geopotential
Transverse Doppler	$<1\times10^{-18}$	Velocity difference of link 1 and link 2

3 Formulation for testing gravitational redshift

After the gravitational potential (GP) difference $\Delta U = U_B - U_A$ is determined by frequency comparison as given by this study, it is compared to the same entity determined conventional method, namely, we need to examine the following equation：

$$z = \Delta U_m = \left(1 + \beta\right) \Delta U \qquad (18)$$

If GRT is correct, coefficient β should be zero.

4 Numerical analysis

Procedure of our experiments

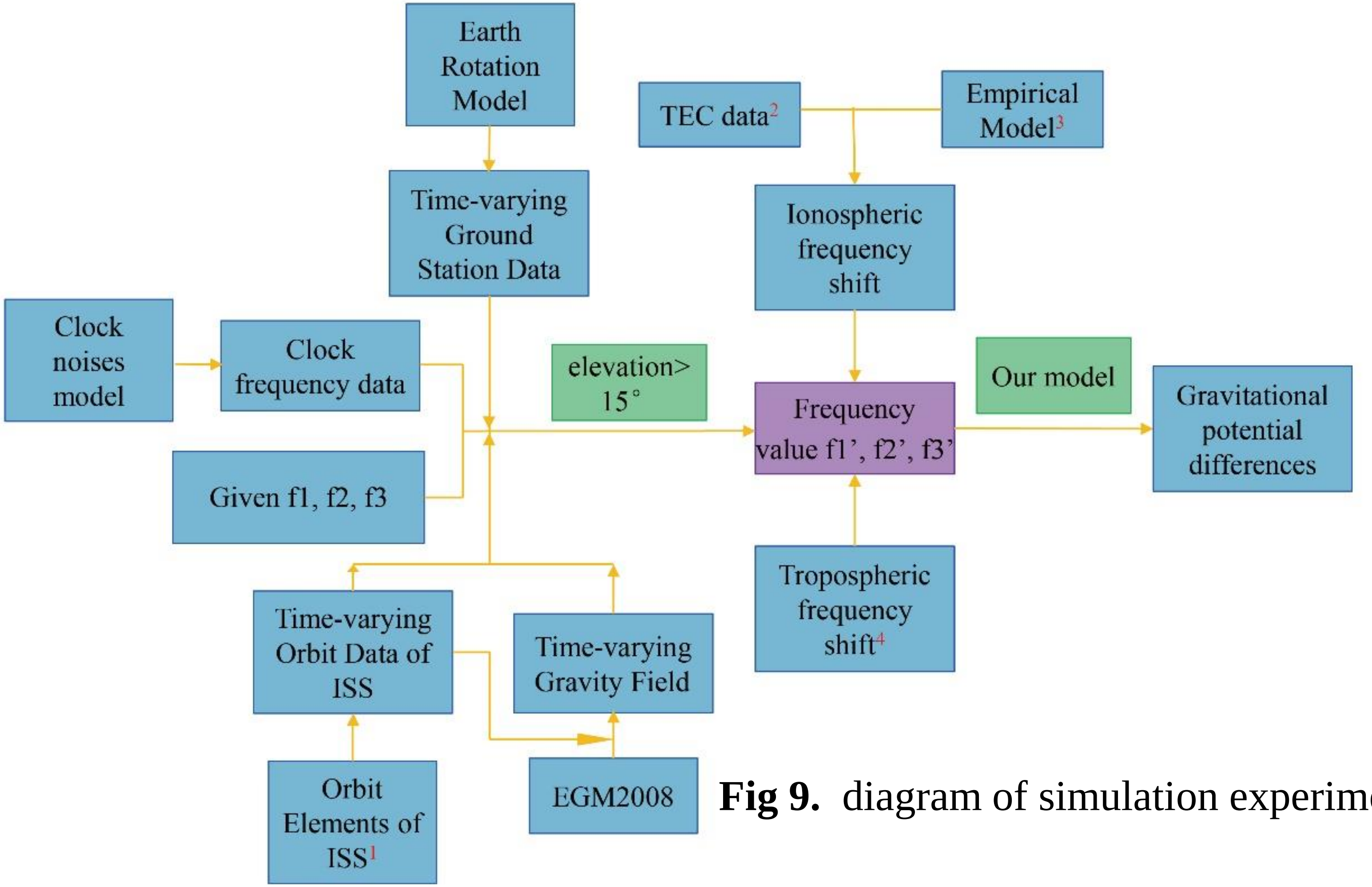


Fig 9. diagram of simulation experiments

4 Numerical analysis

Parameters of Ground station and ISS

For our simulation, we choose the station Observatoire de Paris (OP).

Table 1 OP information

Parameters	**latitude**	**longitude**	**height**	**geopotential**
Value	48.836°N	2.336°E	124.2 m	62636077.171 $m^2\ s^{-2}$

Orbit data of ISS are calculated based on real orbit elements.

Table 2 Orbit and data information

Data source	orbit elements
Inclination	51.6°
Height	400-430km
Sampling rate	1s
Data length	29 days
Time	2019June 9-July 7

4 Numerical analysis

Clock simulations

When simulating clock frequency data, we considered five kinds of noises.

Random walk FM (RW FM)

Flicker FM (F FM)

White FM (W FM)

Flicker PM (F PM)

White PM (W PM)

Where W FM is the largest, and this figure shows Allan deviation curve is close to the curve told by ACES organizer.

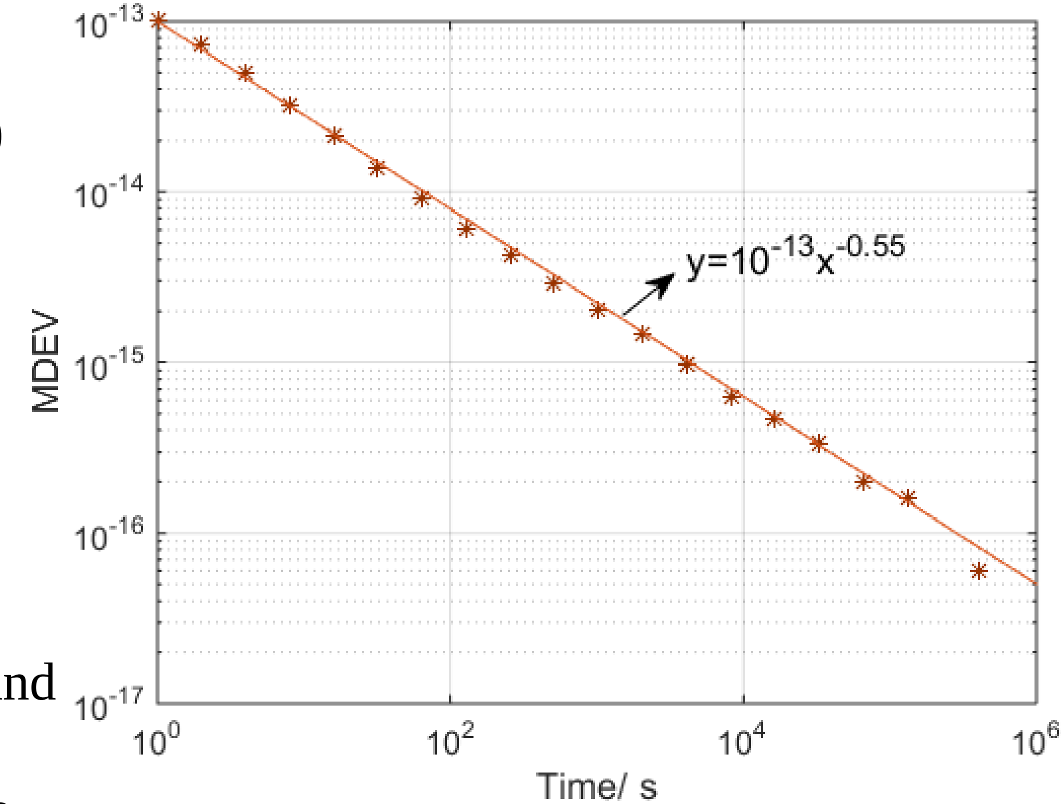


Fig 11. Modified Allan Deviation of our simulation

4 Numerical analysis

Various errors

Here we show some frequency shifts of link 3 in one epoch.

Frequency shift	Magnitude
Doppler shift	10^{-5}~10^{-6}
Ionospheric shift	~10^{-12}
Tropospheric shift	10^{-13}~10^{-14}
Atmospheric shift cause by refraction	~10^{-10}
Relavistic effects	10^{-9}~10^{-10}
Shapiro frequency shift	~10^{-14}

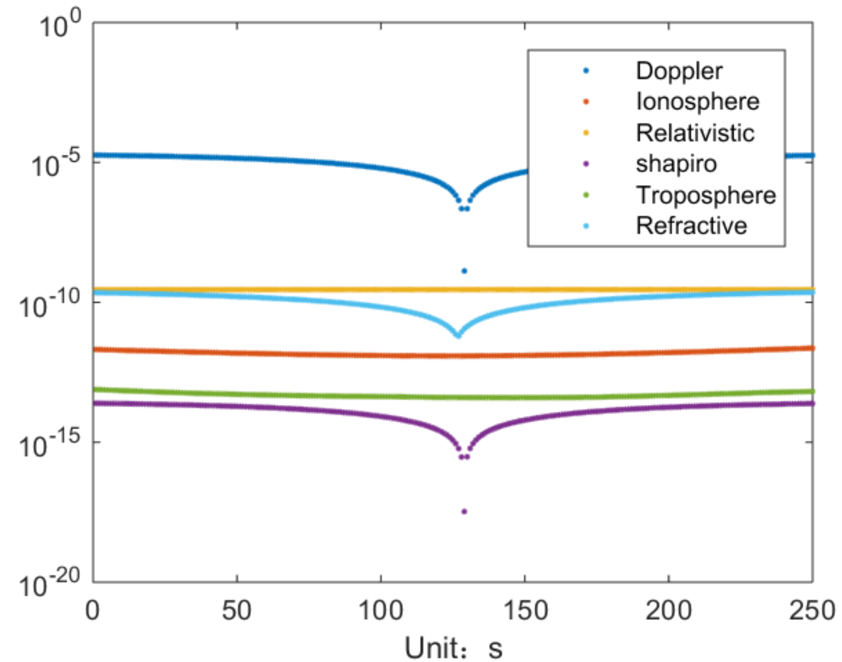
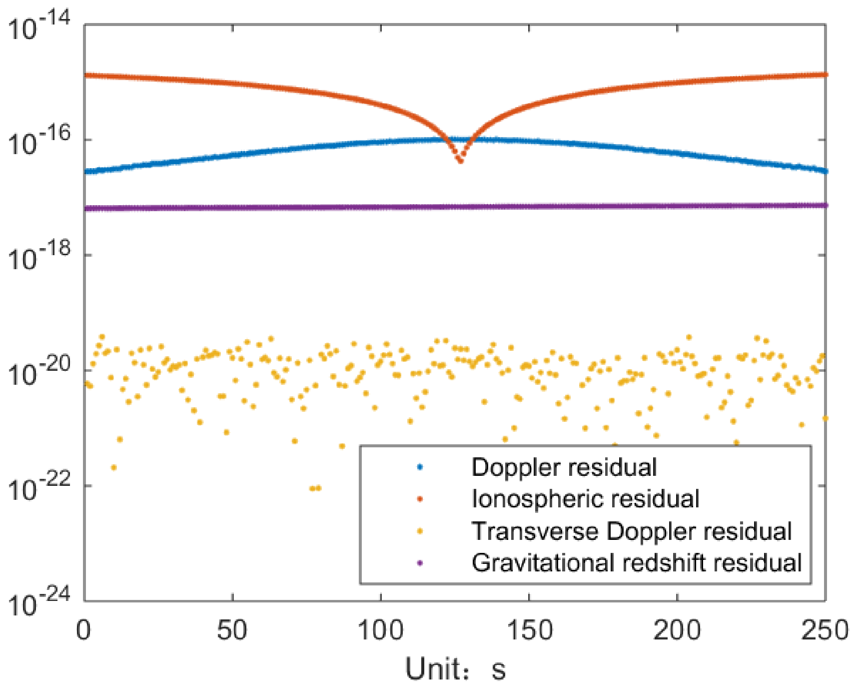


Fig 12 Various errors in one epoch

4 Numerical analysis

And we show frequency shift in figure 14.

Final geopotential results are shown in figure 15, where (a) is during one epoch and (b) shows averaging data within epochs.



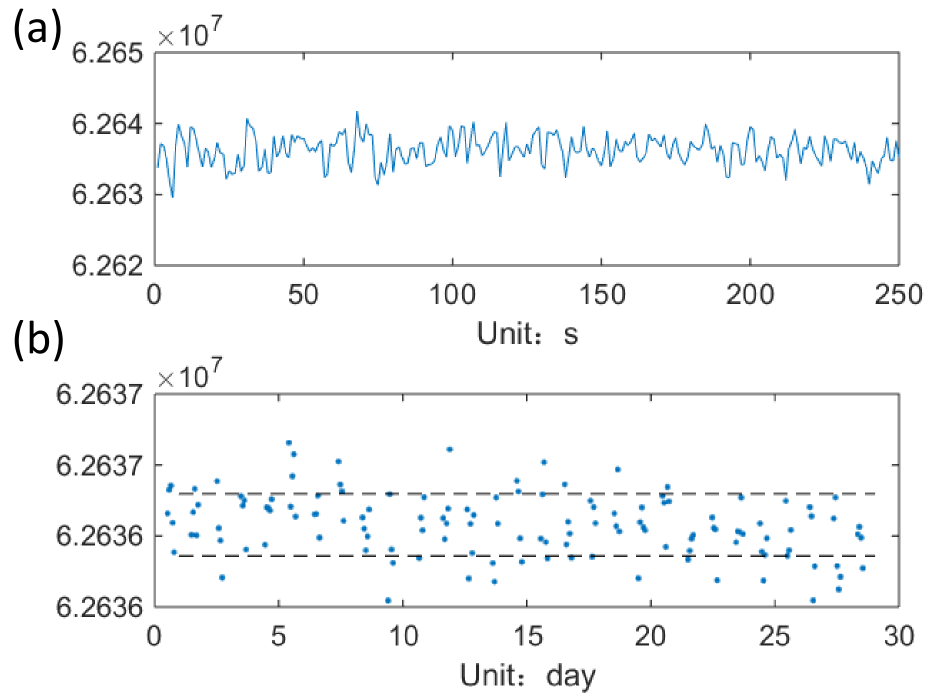


Fig 14 After 29 days averaging, final results are compared with standard geopotential, and the error is 0.4 m.

4 Numerical analysis

Table 5 Results of experiments

Parameters	GP difference	Measured GP difference	GP bias	STD of GP	Testing level
Value	3789481.6 $m^2 \cdot s^{-2}$	3789485.6 $m^2 \cdot s^{-2}$	$-4 m^2 \cdot s^{-2}$	$-19 m^2 \cdot s^{-2}$	1×10^{-6}

We calculated demanded precision of some parameters. It is easy for ISS to satisfy these demands.

Table 6 The required precision of parameters

Parameters	Demand along the rail	Horizontal demand	Radial demand
r_A	240 m	689 m	465 m
v_A	1.23 m/s	3.26 m/s	2.48 m/s
a_A	69.6 m/s^2	80.0 m/s^2	65.3 m/s^2
T_{23}	5.5×10^{-7} s		

5 Conclusions

- With ACES payloads (SHM and PHARAO) in frequency stability of 10^{-13}/s, the gravitational redshift could be tested at a level of 2×10^{-6}, which is one and half order higher than the result of Vessot et al (1980).

Thank you for your attention!

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