Dictionary learning algorithms for the downward continuation of the gravitational potential

Naomi Schneider and Volker Michel

Geomathematics Group
Department of Mathematics
University of Siegen, Germany

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Go to overview →

Starting point

Introducing the
Learning Inverse Problem Matching Pursuit (LIPMP)
algorithms

Basics

Algorithms

These are

- advancements of the IPMP algorithms for learning a dictionary as well as
- approximation algorithms for, e. g., the downward continuation of the gravitational potential.

Results

Summary

They

- have less storage demand and (often) less runtime
- while producing at least equally good approximations as the IPMP algorithms.

For further interest

(Click buttons for more information on the topics)

The task under investigation

- The downward continuation of satellite data of the gravitational potential is important in order to monitor the system Earth (e. g. the climate change). However, mathematically speaking, it is an ill-posed inverse problem and, thus, demands sophisticated mathematical methods.
- Here, we are interested in matching pursuits as our choice of method: the gravitational potential is then approximated by a mixture of different types of trial functions from a so-called dictionary. Note that other methods usually represent the potential with only one type of trial functions.
- Generally, a dictionary is a set of trial functions. It usually contains different types of them like spherical harmonics, radial basis functions and wavelets (i. e. low and band pass filters) as well as Slepian functions.

Previously, the approximation obtained from a matching pursuit can only be built from a-priori chosen dictionary elements.
$\Rightarrow$ Which trial functions should the dictionary contain, e. g., for the downward continuation of satellite data?

- For traditional matching pursuits (used for interpolation tasks), dictionary learning approaches were developed. There, the evaluated dictionary elements were manipulated at grid points in order to determine optimal ones.

Due to their strategic aims and mathematical differences in the underlying problems, these methods cannot be transferred straightforwardly to the matching pursuits for inverse problems.
$\Rightarrow$ A dictionary learning technique for the Inverse Problem Matching Pursuit (IPMP) algorithms is needed.

← Back to overview

Necessary basics

Downward continuation

The IPMP algorithms

Trial functions

← Back to overview

← Back to LIPMPs

Data of the gravitational potential can be retrieved from the EGM2008 as well as the GRACE and GRACE-FO satellite missions. In contrast to the EGM2008, the GRACE data also yields time-dependent information. In particular, we are interested in its values at the Earth's surface when we are given data on a satellite orbit.

Mathematically, on a satellite orbit, the potential can be represented pointwise by

$$V(\sigma\eta) = (\mathcal{T}f)(\sigma\eta) = \sum_{n=0}^{\infty}\sum_{j=-n}^{n} \langle f, Y_{n,j}\rangle_{\mathrm{L}^2(\mathbb{S}^2)}\, \sigma^{-n-1} Y_{n,j}(\eta), \qquad \sigma > 1,\ \eta \in \mathbb{S}^2,$$

for the unit sphere $\mathbb{S}^2$ and with spherical harmonics $Y_{n,j},\ n \in \mathbb{N}_0,\ j = -n, \ldots, n$.

$\mathcal{T}$ is then called the upward continuation operator. Due to σ^{-n-1} for $\sigma > 1$ and $n \in \mathbb{N}_0$, we see that $\mathcal{T}$ has exponentially decreasing singular values.

Thus, the inverse downward continuation operator, has exponentially increasing singular values: that means, it cannot be continuous.

Hence, the downward continuation of the gravitational potential from satellite altitude to the Earth's surface is an ill-posed inverse problem: a challenging task in the geosciences!

Necessary basics

Downward continuation

The IPMP algorithms

Trial functions

← Back to overview

← Back to LIPMPs

The term Inverse Problem Matching Pursuit (IPMP) algorithm summarizes the

- Regularized Functional Matching Pursuit (RFMP) algorithm and the
- Regularized Orthogonal Functional Matching Pursuit (ROFMP) algorithm.

The IPMP algorithms solve (ill-posed) inverse problem using a Tikhonov regularization.

In particular, the RFMP algorithm starts with an initial approximation f_0 and iteratively adds weighted dictionary elements (=trial functions): $f_{N+1} := f_N + \alpha_{N+1} d_{N+1}$ where

$$(\alpha_{N+1}, d_{N+1}) := \operatorname*{arg\,min}_{(\alpha,d)\in\mathbb{R}\times\mathcal{D}} \left(\|y - \mathcal{T}_\daleth (f_N + \alpha d)\|^2_{\mathbb{R}^\ell} + \|f_N + \alpha d\|^2_{\mathcal{H}_2} \right)$$

for the finite dictionary $\mathcal{D}$, the Sobolev space $\mathcal{H}_2$ and the discretized upward continuation operator $(\mathcal{T}_\daleth \cdot) = \left((\mathcal{T} \cdot) \left(\sigma \eta^i \right) \right)_{i=1,\ldots,\ell}$, $\eta \in \mathbb{S}^2$, $\sigma > 1$.

The ROFMP algorithm follows a similar routine but chooses α_{N+1} and d_{N+1} in an orthogonal fashion.

Hence, the algorithms support accuracy, are flexible with respect to the task and the data-sources and are stable as well as yield a continuous function as their result. Moreover, they proved their applicability in a wide range of applications like downward continuation, inverse gravimetry and medical imaging.

Necessary basics

Downward continuation
The IPMP algorithms
Trial functions
← Back to overview
← Back to LIPMPs

We consider the following trial functions as dictionary elements:

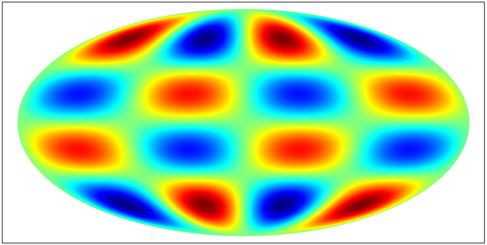
spherical harmonics	Slepian functions
Abel–Poisson low pass filters	Abel–Poisson band pass filters

Then

- an approximation (e. g. of the gravitational potential) will most likely be built from a mixture of these functions.
- the (in-)finite dictionary for an IPMP algorithm is the union of corresponding trial function classes:
$$\mathcal{D} = [N]_{\mathrm{SH}} \cup [S]_{\mathrm{SL}} \cup [B_K]_{\mathrm{APK}} \cup [B_W]_{\mathrm{APW}} \,,$$
with $N \subset \{(n,j) \mid n \in \mathbb{N}_0,\ j \in \{-n, \ldots, n\}\},\ S \subset [-1,1] \times \mathrm{SO}(3)$ and $B_K,\ B_W \subset \mathbb{B}_1(0)$.
- all types of trial functions are represented by characteristic parameters: spherical harmonics by their degree n and order j, Slepian functions by their localization region $R(c, A(\alpha,\beta,\gamma))$, Abel–Poisson low and band pass filters by their centre $x/|x|$ and scale $|x|$.

✗ Close figure

Fully normalized spherical harmonics are global polynomials on the sphere: e. g.

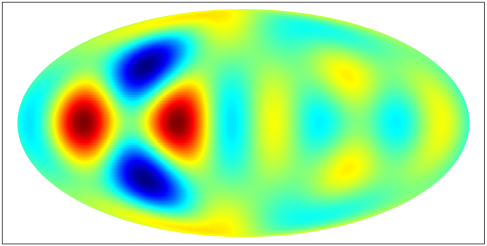


$$Y_{n,j}\left(\eta(\varphi,t)\right) := \sqrt{\frac{2n+1}{4\pi}\frac{(n-|j|)!}{(n+|j|)!}}P_{n,|j|}(t)\begin{cases}\sqrt{2}\cos(j\varphi), & j<0,\\ 1, & j=0,\\ \sqrt{2}\sin(j\varphi), & j>0,\end{cases}$$

for $\eta(\varphi,t)\in\mathbb{S}^2$.

✗ Close figure

Slepian functions are band-limited and optimally localized in a localization region (here a spherical cap): e. g.

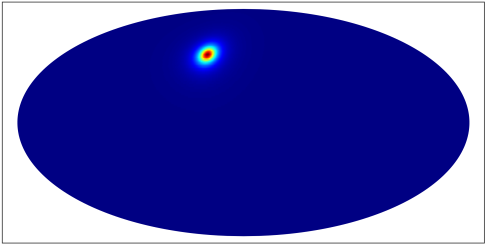


$$g^{(k,N)}\left(\left(c, A\left(\alpha, \beta, \gamma\right)\varepsilon^3\right), \eta\right) \coloneqq \sum_{n=0}^{N}\sum_{j=-n}^{n} g_{n,j}^{(k,N)}\left(c, A\left(\alpha, \beta, \gamma\right)\varepsilon^3\right) Y_{n,j}\left(\eta\right)$$

for $\eta \in \mathbb{S}^2,\ \varepsilon^3 = (0,0,1)^{\mathrm{T}},\ c \in [-1,1]$ and $A(\alpha,\beta,\gamma) \in \mathrm{SO}(3)$.

✗ Close figure

Abel–Poisson kernels are local functions and, in particular, low pass filters: e. g.

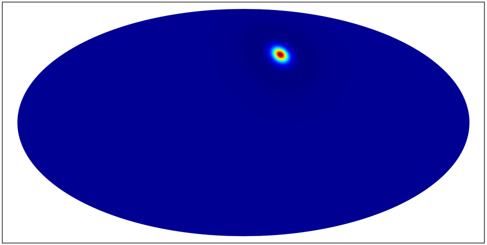


$$K(x,\eta) := \frac{1-|x|^2}{4\pi(1+|x|^2-2x\cdot\eta)^{3/2}}$$

for $\eta \in \mathbb{S}^2$ and $x \in \mathring{\mathbb{B}}_1(0)$.

✗ Close figure

Abel–Poisson wavelets are also local functions and, in particular, band pass filters: e. g.



$$W(x, \eta) := K(x, \eta) - K(|x|x, \eta)$$

for $\eta \in \mathbb{S}^2$ and $x \in \mathring{\mathbb{B}}_1(0)$.

About the learning algorithms

The Learning Inverse Problem Matching Pursuit (LIPMP) algorithms shall provide a strategy to avoid choosing manually and a-priori a dictionary for the IPMP algorithms.

The term LIPMP algorithm summarizes the

- Learning Regularized Functional Matching Pursuit (LRFMP) algorithm and the
- Learning Regularized Orthogonal Functional Matching Pursuit (LROFMP) algorithm.

Approach

An LIPMP algorithm follows the same routine as the respective IPMP algorithm (click here). However, $\mathcal{D}$ is the infinite set of spherical harmonics up to a certain degree, all possible Slepian functions as well as all possible Abel–Poisson low and band pass filters. Then we additionally compute a finite "dictionary of candidates" (α_{N+1}, d_{N+1}) in each iteration which consists of one candidate from each type of trial function. The candidates from the Slepian functions as well as the Abel–Poisson low and band pass filters are obtained by solving constrained non-linear optimization problems.

- The chosen trial functions constitute a "learnt" dictionary which can be used in future runs of the IPMP algorithms. A maximal spherical harmonic degree is also learnt.
- The LIPMP algorithms are standalone approximation algorithms for inverse problems as well. In particular, they are advancements of the IPMP algorithms as they supersede the need to choose the dictionary.

Idea
Structure
Theoretical aspects

← Back to overview

About the learning algorithms

Idea

Structure

Theoretical aspects

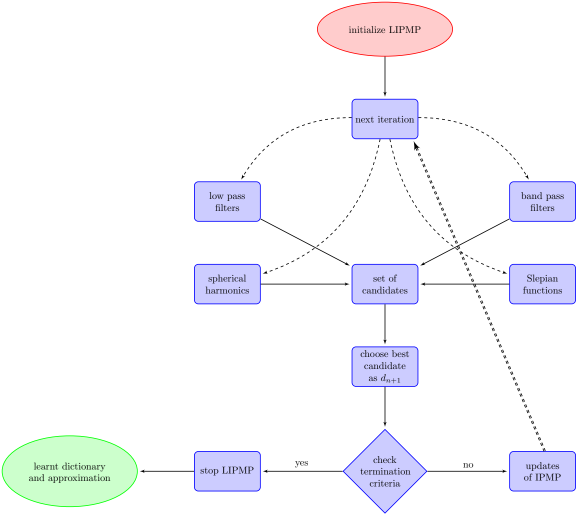
← Back to overview

For an overview of the algorithm's structure click here.

The algorithm starts in the red and terminates in the green circle.

The different types of arrows are only used for an improved visibility.

✗ Close figure



About the learning algorithms

With respect to the convergence of the algorithms:

- Due to their very similar structure, the LRFMP algorithm inherits the results of the RFMP algorithm. In particular, this means, the approximation converges towards the solution of the regularized normal equation.
- Unfortunately, for the ROFMP algorithm, there exist not as many results as for the RFMP algorithm. Those that exist have technical assumptions that are in question for the LROFMP algorithm.

With respect to the learnt dictionaries:

- Optimal dictionaries must be infinite by construction. Hence, it is unlikely that a learnt dictionary will converge towards an optimal one. However, the LIPMP algorithms already work with an optimal dictionary.
- In the LRFMP algorithm, the sequence of learnt dictionaries

$$\mathcal{D}_0^*\left(f_0, \mathcal{T}\neg, \lambda, y\right) := \begin{cases} \{f_0\}, & f_0 \not\equiv 0, \\ \emptyset, & \text{else}, \end{cases}$$

$$\mathcal{D}_{N+1}^*\left(f_0, \mathcal{T}\neg, \lambda, y\right) := \mathcal{D}_N^*\left(f_0, \mathcal{T}\neg, \lambda, y\right) \cup \{d_{N+1}\}, \quad N \in \mathbb{N},$$

  is a sequence of well-working dictionary. That means, in the limit $N \to \infty$, it will be able to represent the solution of the regularized normal equation.
- Note that a learnt dictionary depends particularly on the operator, the regularization parameter and the data (i. e. the inverse problem at hand).

Experiments

Settings
Comparison
Standalone
← Back to overview

We

- consider EGM2008, GRACE and synthetic data, respectively.
- use a regularly distributed grid of 12684 grid points (if not stated otherwise).
- assume that the data are given at 500 km satellite height (if not stated otherwise).
- include 5 % Gaussian noise.
- choose the tested regularization parameter with a minimal relative approximation error.
- terminate the algorithms if the relative data error falls below the noise level. Furthermore, we terminate the algorithms after 1000 iterations at the latest.
- compare the learnt dictionary with a manually chosen dictionary as well as consider the LIPMP algorithms as standalone approximation algorithms.
- use a manually chosen dictionary of 95152 trial functions (click here).
- include a starting dictionary of 13903 trial functions (click here).
- apply the learnt dictionary only iteratively (i. e. in the N-th iteration of an IPMP algorithm only the first N learnt dictionary elements can be chosen).
- use the same regularization parameter for learning and applying the learnt dictionary.
- set some technical values.

We show a selection from our latest results.

✗ Close figure

$$[N^{\mathrm{m}}]_{\mathrm{SH}} = \{\, Y_{n,j} \mid n = 0, \ldots, 25;\ j = -n, \ldots, n\}$$

$$[S^{\mathrm{m}}]_{\mathrm{SL}} = \Big\{ g^{(k,5)}\left(\left(c, A(\alpha,\beta,\gamma)\varepsilon^3\right), \cdot\right) \Big| \; c \in \left\{\frac{\pi}{4}, \frac{\pi}{2}\right\},\ \alpha \in \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\},\ \beta \in \left\{0, \frac{\pi}{2}, \pi\right\}, \; \gamma \in \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\},\ k = 1, \ldots, 36 \Big\}$$

$$[B_K^{\mathrm{m}}]_{\mathrm{APK}} = \left\{ \frac{K(x,\cdot)}{\|K(x,\cdot)\|_{\mathrm{L}^2(\mathbb{S}^2)}} \;\middle|\; |x| \in Z,\ \frac{x}{|x|} \in X^{\mathrm{m}} \right\}$$

$$[B_W^{\mathrm{m}}]_{\mathrm{APW}} = \left\{ \frac{W(x,\cdot)}{\|W(x,\cdot)\|_{\mathrm{L}^2(\mathbb{S}^2)}} \;\middle|\; |x| \in Z,\ \frac{x}{|x|} \in X^{\mathrm{m}} \right\}$$

X^{m} contains 4551 regularly distributed grid points on $\mathbb{S}^2$

$$Z = \{0.75,\ 0.80,\ 0.85,\ 0.89,\ 0.91,\ 0.93,\ 0.94,\ 0.95,\ 0.96,\ 0.97\}$$

$$\Rightarrow \mathcal{D}^{\mathrm{m}} = [N^{\mathrm{m}}]_{\mathrm{SH}} \cup [S^{\mathrm{m}}]_{\mathrm{SL}} \cup [B_K^{\mathrm{m}}]_{\mathrm{APK}} \cup [B_W^{\mathrm{m}}]_{\mathrm{APW}}$$

✗ Close figure

$$[N^{\mathrm{s}}]_{\mathrm{SH}} = \{Y_{n,j} \mid n = 0, \ldots, 100;\ j = -n, \ldots, n\}$$

$$[S^{\mathrm{s}}]_{\mathrm{SL}} = \Big\{ g^{(k,5)}\left(\left(c, A(\alpha,\beta,\gamma)\varepsilon^3\right), \cdot\right) \Big| \; c \in \left\{\frac{\pi}{4}, \frac{\pi}{2}\right\},\ \alpha \in \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\},\ \beta \in \left\{0, \frac{\pi}{2}, \pi\right\},\ \gamma \in \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\},\ k = 1, \ldots, 36 \Big\}$$

$$[B_K^{\mathrm{s}}]_{\mathrm{APK}} = \left\{ \frac{K(x,\cdot)}{\|K(x,\cdot)\|_{\mathrm{L}^2(\mathbb{S}^2)}} \;\middle|\; |x| = 0.94,\ \frac{x}{|x|} \in X^{\mathrm{s}} \right\}$$

$$[B_W^{\mathrm{s}}]_{\mathrm{APW}} = \left\{ \frac{W(x,\cdot)}{\|W(x,\cdot)\|_{\mathrm{L}^2(\mathbb{S}^2)}} \;\middle|\; |x| = 0.94,\ \frac{x}{|x|} \in X^{\mathrm{s}} \right\}$$

X^{m} contains 123 regularly distributed grid points on $\mathbb{S}^2$

$$\Rightarrow \mathcal{D}^{\mathrm{s}} = [N^{\mathrm{s}}]_{\mathrm{SH}} \cup [S^{\mathrm{s}}]_{\mathrm{SL}} \cup [B_K^{\mathrm{s}}]_{\mathrm{APK}} \cup [B_W^{\mathrm{s}}]_{\mathrm{APW}}$$

Experiments

EGM2008	GRACE May 2008	GRACE 2009

EGM2008 data is used.

The regularization parameter is chosen as $10^{-9}\|y\|_{\mathbb{R}^\ell}$ in all experiments.

The final relative noise level is slightly below the noise level.

See also a comparison of the absolute approximation errors obtained in the RFMP (click here) and the ROFMP (click here) algorithm, respectively.

algorithm	RFMP	RFMP	ROFMP	ROFMP
size of dictionary	95152	$\leq$ 637	95152	$\leq$ 550
iterations	957	662	766	577
final relative RMSE	0.000466	0.000471	0.000463	0.000467
CPU-runtime in h	514.03	507.22	561.76	665.76

Settings
Comparison
Standalone
← Back to overview

✗ Close figure

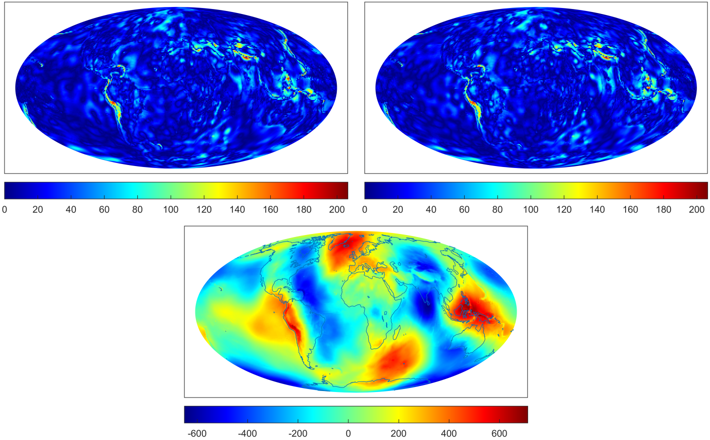


Figure: Absolute approximation errors obtained by the RFMP algorithm with the manually chosen (upper left) and the learnt dictionary (upper right). In the lower row, the solution is presented. The colour scale is adapted in the upper row plots for a better comparison. All values in m^2/s^2.

✗ Close figure

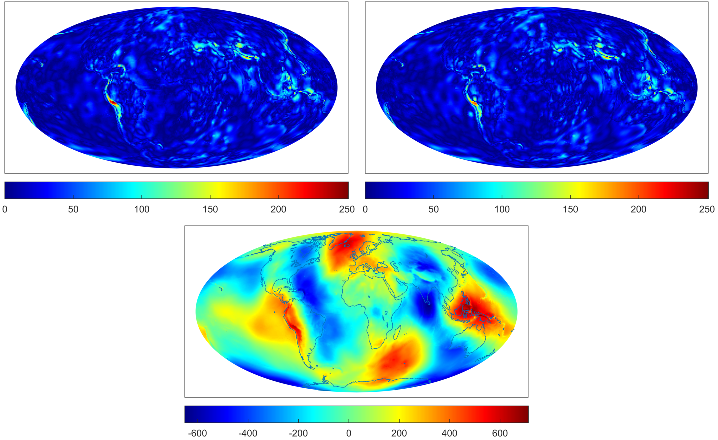


Figure: Absolute approximation errors obtained by the ROFMP algorithm with the manually chosen (upper left) and the learnt dictionary (upper right). In the lower row, the solution is presented. The colour scale is adapted in the upper row plots for a better comparison. All values in m^2/s^2.

Experiments

EGM2008	**GRACE May 2008**	GRACE 2009

GRACE data from May 2008 is used.

The regularization parameter is chosen as $10^{-4}\|y\|_{\mathbb{R}^\ell}$ in all experiments.

The final relative noise level is slightly below the noise level.

See also a comparison of the absolute approximation errors obtained in the RFMP (click here) and the ROFMP (click here) algorithm, respectively.

algorithm	RFMP	RFMP	ROFMP	ROFMP
size of dictionary	95152	$\leq$ 384	95152	$\leq$ 303
iterations	393	483	274	306
final relative RMSE	0.000340	0.000335	0.000328	0.000330
CPU-runtime in h	522.09	341.16	528.67	372.31

✗ Close figure

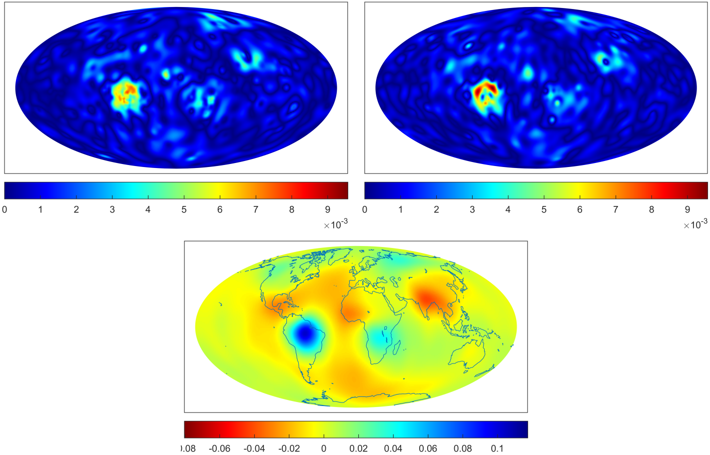


Figure: Absolute approximation errors obtained by the RFMP algorithm with the manually chosen (upper left) and the learnt dictionary (upper right). In the lower row, the solution is presented. The colour scale is adapted in the upper row plots for a better comparison. All values in m^2/s^2.

✗ Close figure

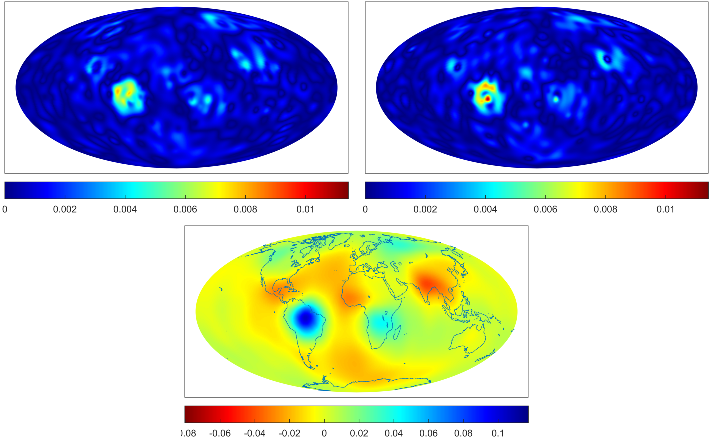


Figure: Absolute approximation errors obtained by the ROFMP algorithm with the manually chosen (upper left) and the learnt dictionary (upper right). In the lower row, the solution is presented. The colour scale is adapted in the upper row plots for a better comparison. All values in m^2/s^2.

Experiments

EGM2008	GRACE May 2008	**GRACE 2009**

From the LRFMP algorithm, we learn a dictionary for each month in 2008. We use their union to approximate GRACE data from May 2009.
The regularization parameter is chosen as $10^{-4}\|y\|_{\mathbb{R}^\ell}$ in all experiments.
The final relative noise level is slightly below the noise level.

See also a comparison of the absolute approximation errors obtained (click here).

algorithm	RFMP	RFMP
size of dictionary	95152	$\leq$ 6701
iterations	399	620
final relative RMSE	0.000335	0.000346

✗ Close figure

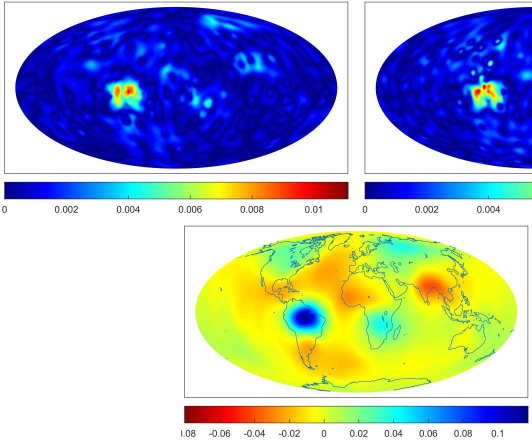


Figure: Absolute approximation error obtained by the RFMP algorithm using the manually chosen dictionary (upper left) and using the learnt GRACE dictionary (upper right). In the lower row, the solution is presented. The scale is adapted in the upper row plots to improve the comparability. All values in m^2/s^2.

Experiments

Approximation	Regular grid	Irregular grid	Synthetic data

The EGM2008 data is used without any satellite height.
The regularization parameter is chosen as $10^{-9}\|y\|_{\mathbb{R}^\ell}$ in all experiments.
The algorithm terminates after 1000 iterations.

algorithm	LRFMP	LROFMP
final relative data error	0.075293	0.075210
final relative RMSE	0.000249	0.000253
absolute approximation error	click here	click here

Settings
Comparison
Standalone

← Back to overview

✗ Close figure

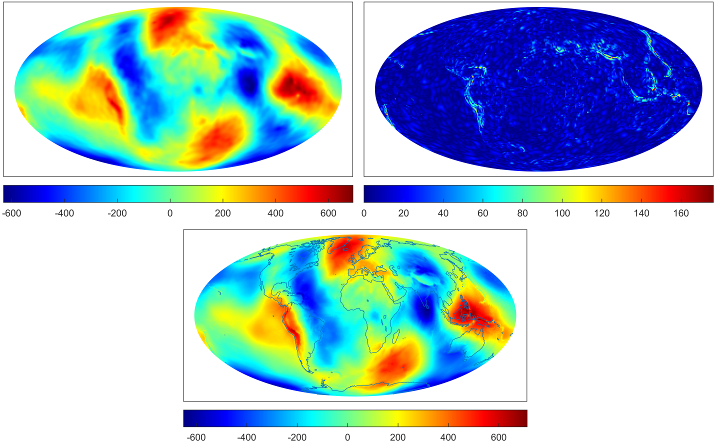


Figure: Approximation (upper left) and absolute approximation error (upper right) obtained by the LRFMP algorithm. In the lower row, the solution is presented. All values in m^2/s^2.

✗ Close figure

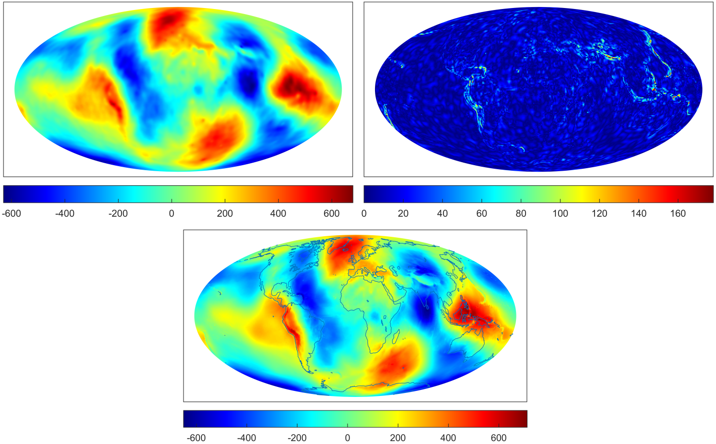


Figure: Approximation (upper left) and absolute approximation error (upper right) obtained by the LROFMP algorithm. In the lower row, the solution is presented. All values in m^2/s^2.

Experiments

Approximation	**Regular grid**	Irregular grid	Synthetic data

The EGM2008 and the GRACE (May 2008) data are considered here. The regularization parameter is chosen as $10^{-9}\|y\|_{\mathbb{R}^\ell}$ in all experiments with the EGM2008 data and $10^{-4}\|y\|_{\mathbb{R}^\ell}$ in all experiments with the GRACE data.
The final relative data error is equal to or slightly less than the noise level.

See also a comparison of the absolute approximation errors obtained in the LRFMP (click here) and the LROFMP (click here) algorithm, respectively.

algorithm	LRFMP	LRFMP	LROFMP	LROFMP
data	EGM2008	GRACE	EGM2008	GRACE
iterations	637	384	550	303
final relative RMSE	0.000471	0.000338	0.000465	0.000318

✗ Close figure

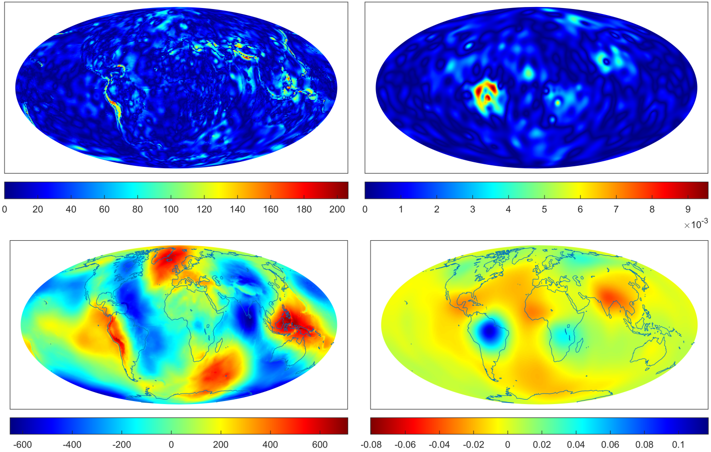


Figure: Absolute approximation error obtained by the LRFMP algorithm (upper row) for EGM2008 (left) and GRACE (May 2008, right) data. In the lower row, the solutions are presented. All values in m^2/s^2.

✗ Close figure

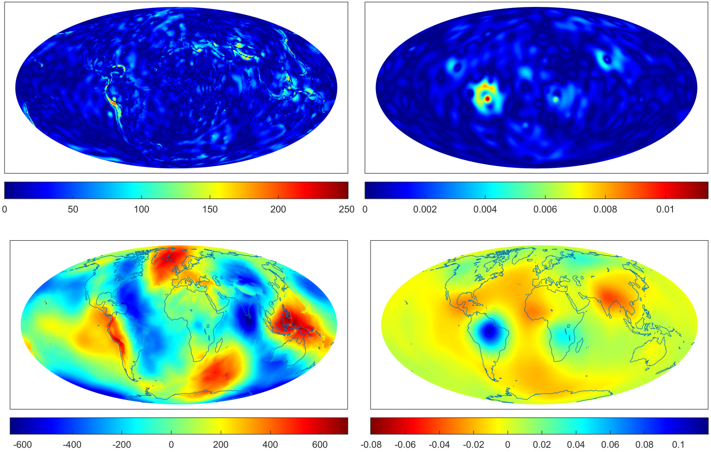


Figure: Absolute approximation error obtained by the LROFMP algorithm (upper row) for EGM2008 (left) and GRACE (May 2008, right) data. In the lower row, the solutions are presented. All values in m^2/s^2.

Experiments

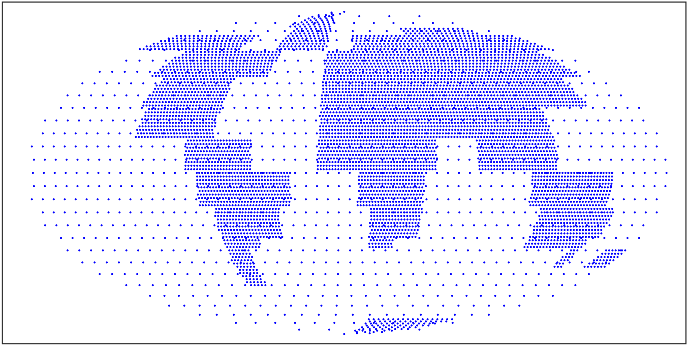
Approximation	Regular grid	**Irregular grid**	Synthetic data

An irregular data grid of 6968 grid points is used (click here) for EGM2008 data.
The regularization parameter is chosen as $5 \cdot 10^{-9}\|y\|_{\mathbb{R}^\ell}$ in all experiments.
The final relative data error is slightly less or equal to the noise level.

algorithm	LRFMP	LROFMP
iterations	975	983
final relative RMSE	0.000472	0.000521
absolute approximation error	click here	click here

Settings
Comparison
Standalone
← Back to overview

✗ Close figure



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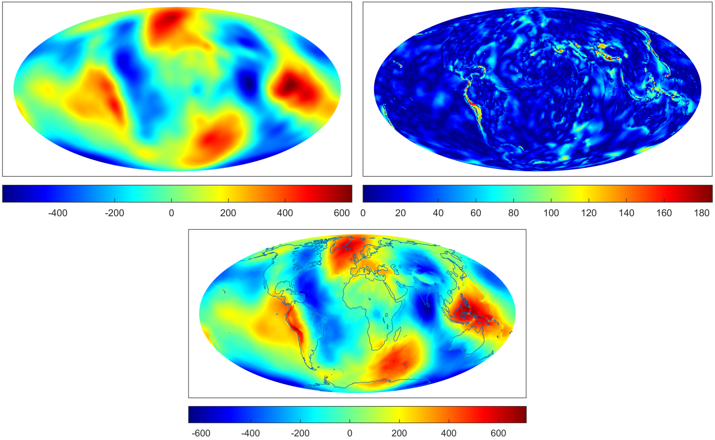

Figure: Approximation (upper left) and absolute approximation error (upper right) obtained by the LRFMP algorithm. In the lower row, the solution is presented. All values in $\mathrm{m}^2/\mathrm{s}^2$.

✗ Close figure

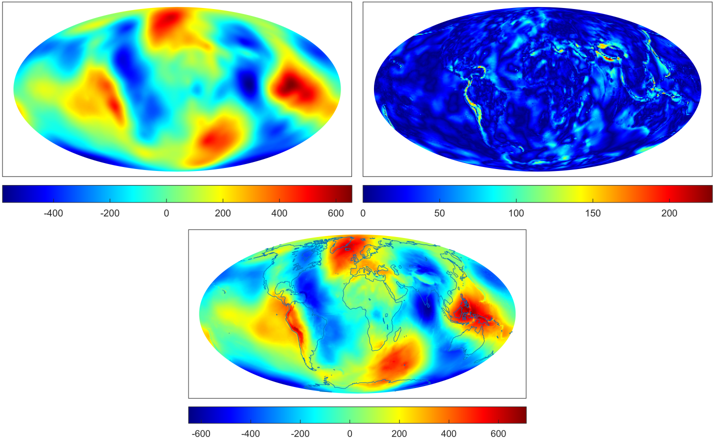

Figure: Approximation (upper left) and absolute approximation error (upper right) obtained by the LROFMP algorithm. In the lower row, the solution is presented. All values in m^2/s^2.

Experiments

Approximation	Regular grid	Irregular grid	**Synthetic data**

The synthetic data consists of 3 spherical harmonics and 3 Abel–Poisson low pass filters. Thus, we also consider only these types of trial functions in the LROFMP algorithm.
The regularization parameter is chosen as $10^{-8}\|y\|_{\mathbb{R}^\ell}$.
The final relative data error is slightly below the noise level.
Only the spherical harmonics in the solution are chosen.

algorithm	LROFMP
iterations	24
final relative RMSE	0.000076
chosen filters	click here

Settings
Comparison
Standalone
← Back to overview

✗ Close figure

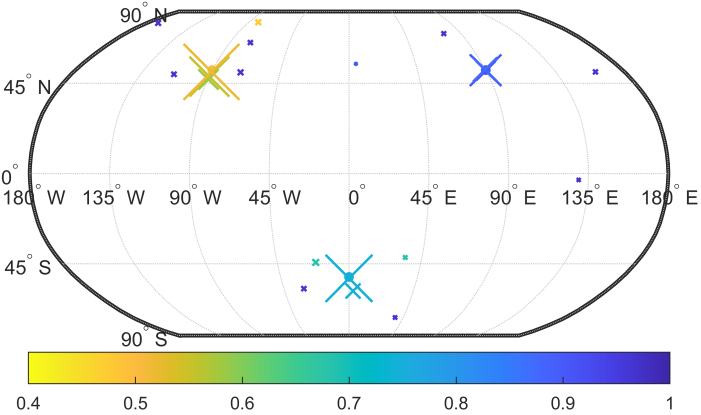

Figure: Chosen Abel–Poisson low pass filters for perturbed data for synthetic data. The dots stand for the centres $x/|x|$ of the solutions. The crosses symbolize the centres $x/|x|$ of the chosen Abel–Poisson low pass filters. For both of them, the colour represents the scale $|x|$. The size of the crosses is scaled by the absolute value of the related chosen coefficients α (and a fixed multiple for improved visibility).

Conclusions and Outlook

We

- ✓ can learn a dictionary for the IPMP algorithms.
- ✓ developed advanced approximation algorithms for inverse problems simultaneously.
- ✓ showed the applicability for both tasks in numerical tests.
- ✓ considered some theoretical aspects.
- ✓ experienced: less storage demand, mostly less runtime, sparser dictionaries, similarly good approximations.

All in all, if the types of trial functions, the available storage or the runtime are critical, we advocate to use an LIPMP, in particular the LRFMP, algorithm with spherical harmonics as well as Abel–Poisson low and band pass filters. Otherwise, the IPMP algorithms provide good approximations as well.

We want to

- ✗ use more data.
- ✗ consider further geoscientific problems: for instance from seismology.
- ✗ determine more suitable values of the regularization parameter.
- ✗ gain further theoretical insights.

← Back to overview

For more details...

V. Michel and N. Schneider (2020), A first approach to learning a best basis for gravitational field modelling. GEM – International Journal on Geomathematics, https://doi.org/10.1007/s13137-020-0143-5.

N. Schneider (2020), Learning Dictionaries for Inverse Problems on the Sphere, submitted PhD-Thesis, Geomathematics Group Siegen, University of Siegen.

Further literature on the IPMP algorithms are listed on the website of the Geomathematics Group Siegen

https://www.uni-siegen.de/fb6/geomathe/publications/index.html?lang=de.

See, in particular, the works of Fischer, Leweke (former Orzlowski), Michel, Telschow and Kontak.

Thank you for your interest in my work. If you have any questions, please do not hesitate to ask them, for instance, in the

session's chat on Tuesday, May 5, 14:00-15:45

or reach out via the contact details given at

https://www.uni-siegen.de/fb6/geomathe/staff/schneider.html?lang=de&lang=de.

← Back to overview