



On the use of near-neutral Backward Lyapunov Vectors to get reliable ensemble forecasts in coupled ocean-atmosphere systems

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In a nutshell: Key conclusions

The best set of Backward Lyapunov Vectors (BLVs) to build a coupled ocean-atmosphere forecasting system for long lead times are the ones associated with near-neutral or slightly negative Lyapunov exponents.

Used alone, these are also providing an appropriate ensemble spread even for the atmospheric variables, due to the swift rotation of the perturbations toward the unstable modes (First BLVs)

Their combination with the leading BLVs are key for reliable forecasts at all lead times

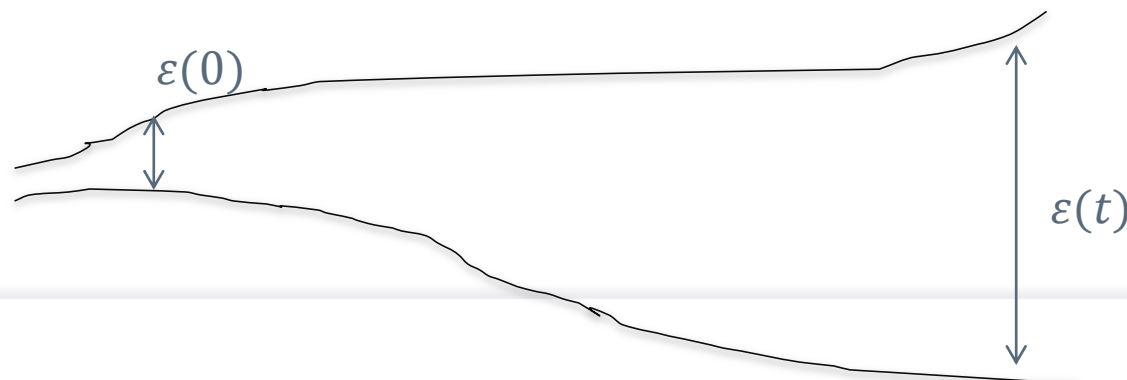
Vannitsem S. & W. Duan, 2020, On the use of near-neutral Backward Lyapunov Vectors to get reliable ensemble forecasts in coupled ocean-atmosphere systems, submitted to Climate Dynamics
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Introduction

The property of sensitivity to initial (and model) uncertainties at the origin of the degradation of the quality of forecasts of atmospheric and climate flows

Property already recognized by
Thompson (1957, Tellus) and Lorenz (1963, JAS)

From a mathematical point of view: Poincaré (1888; 1908, Science et méthode)

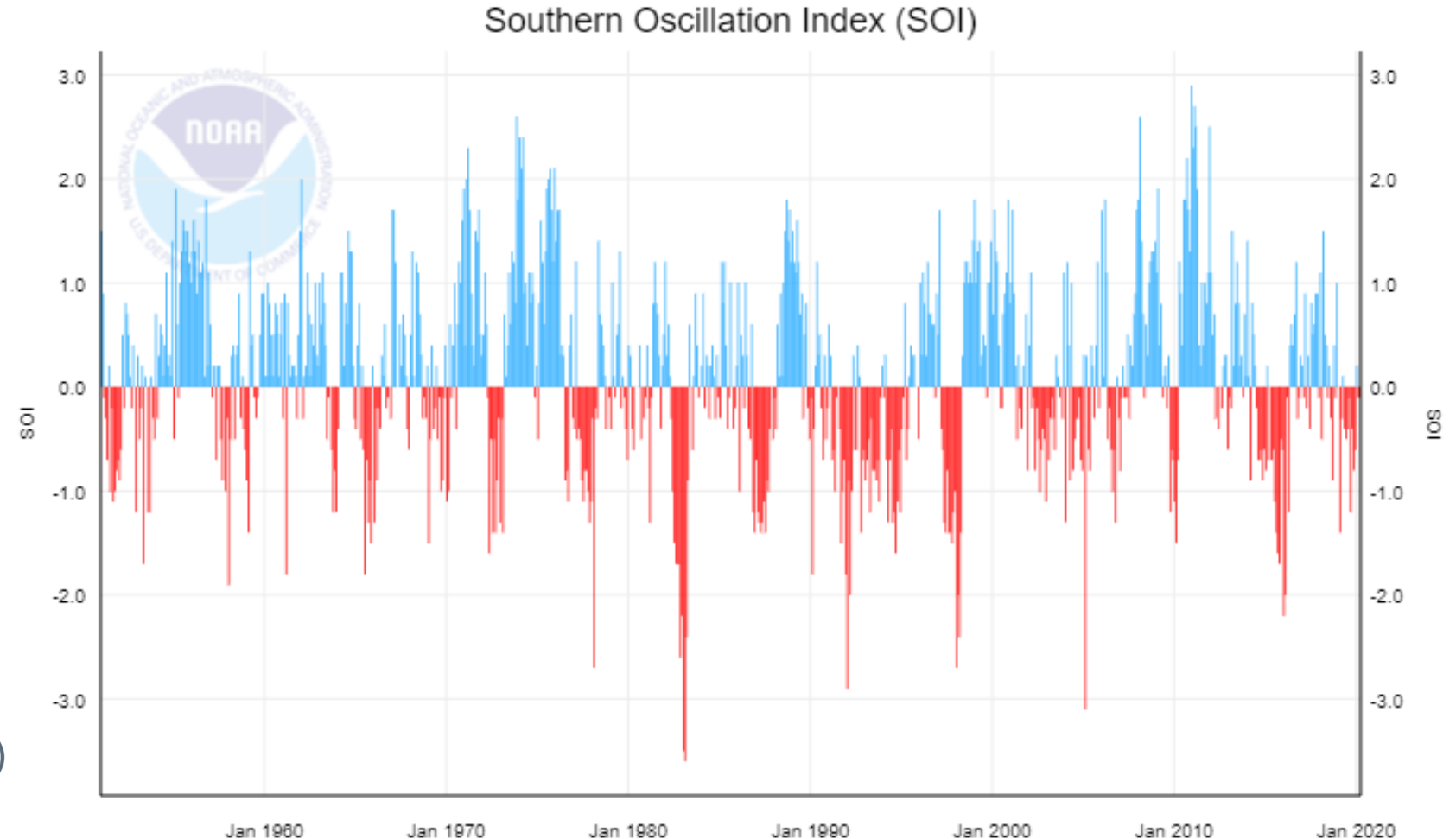


Climate variability and predictability?

One important signal:
Southern Oscillation Index

Associated with the development of
El-Nino and La-Nina in the Tropical
Regions.

El-Nino-Southern-Oscillation (ENSO)

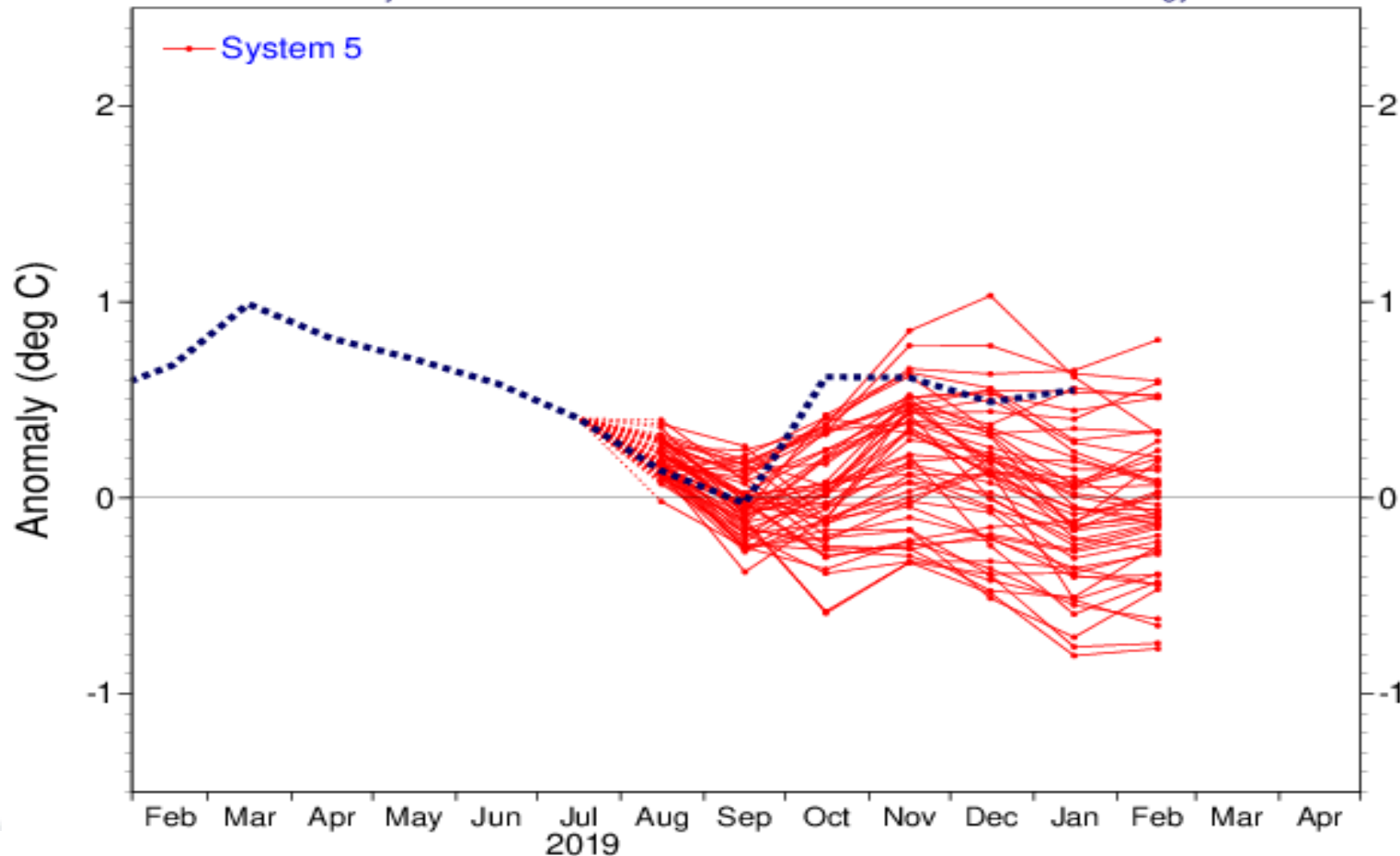


<https://www.ncdc.noaa.gov/teleconnections/enso/indicators/soi/>

Ensemble forecasts

NINO3.4 SST anomaly plume ECMWF forecast from 1 Aug 2019

Monthly mean anomalies relative to NCEP OIv2 1981-2010 climatology



From ECMWF Website
www.ecmwf.int

One example of ensemble
Forecast of Nino3.4 SST anomaly

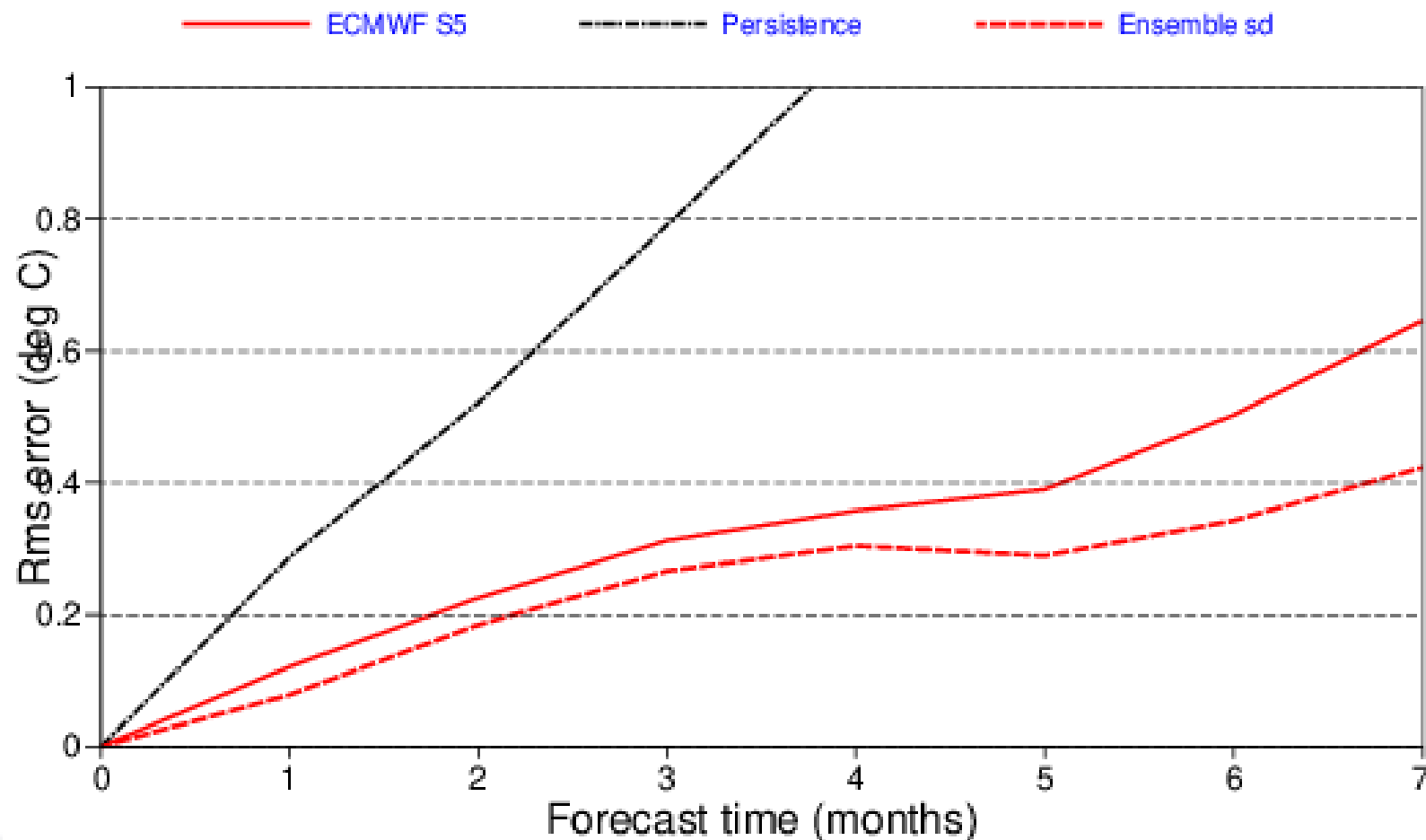
NINO3.4 SST rms errors

27 start dates from 19930201 to 20190201, bias corrected
Ensemble size is 25

From ECMWF website

Underdispersive ensembles!

Mean Square Error of the ensemble mean >> variance of the ensemble



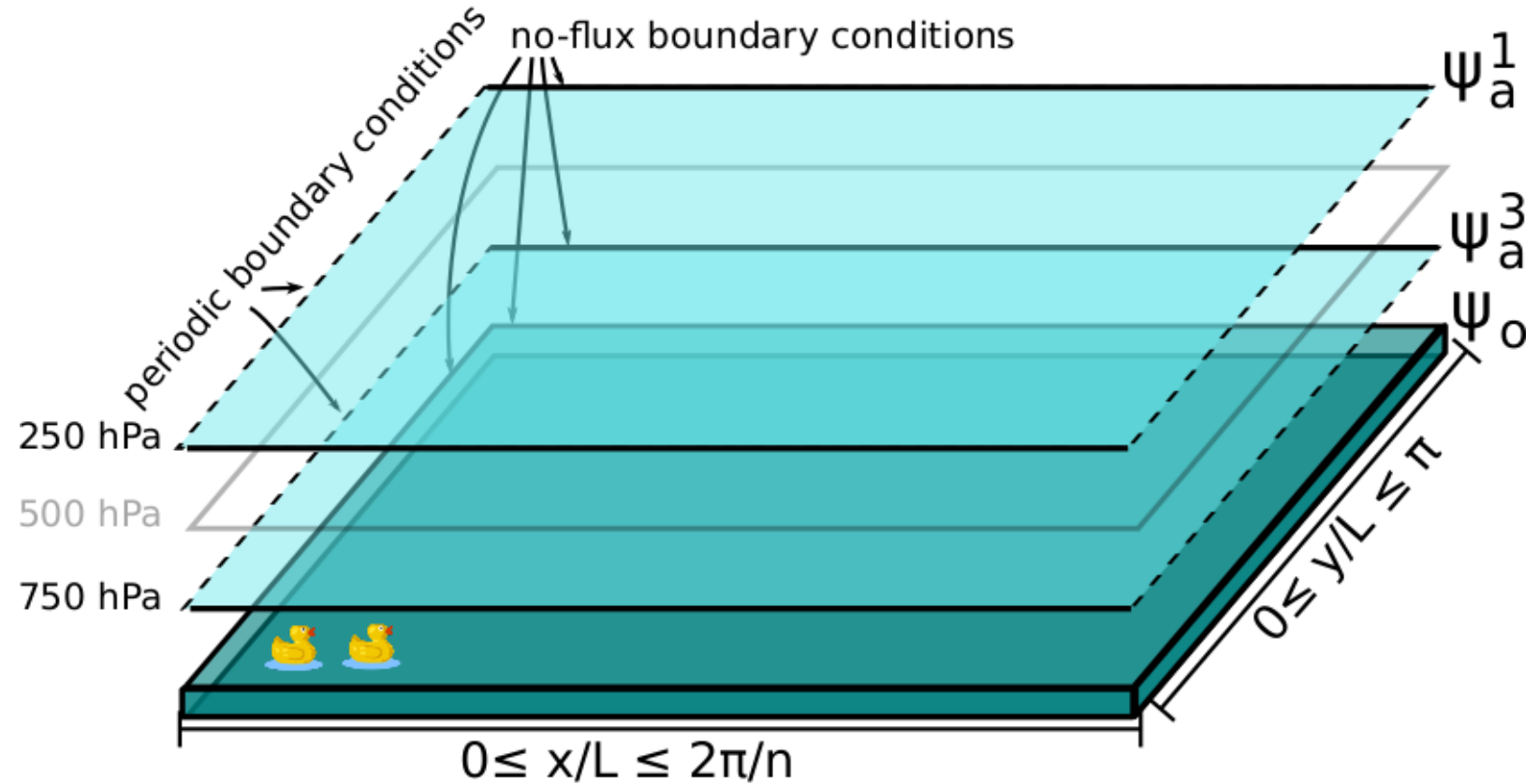
Aim of the work

Discuss the development of ensemble forecasts in coupled Ocean-Atmosphere models.

In particular: What is the best strategy for perturbing the ensembles
To get reliable forecasts for both the Ocean and the Atmosphere

The Reduced-order coupled model

- QG model for both the ocean and the atmosphere



Vannitsem et al, 2015, Physica D, 309, 71-85, 2015, (**VDDG**)

De Cruz et al 2016, Geosci. Model Develop, 9, 2793-2808, 2016. (**MAOOAM**)

Latitudinal dependence
of the radiative input

$$R_0 + C_0 \sqrt{2} \cos y$$

Surface friction strength

$$\delta = \frac{d}{f_0} = \frac{C}{\rho H f_0}$$

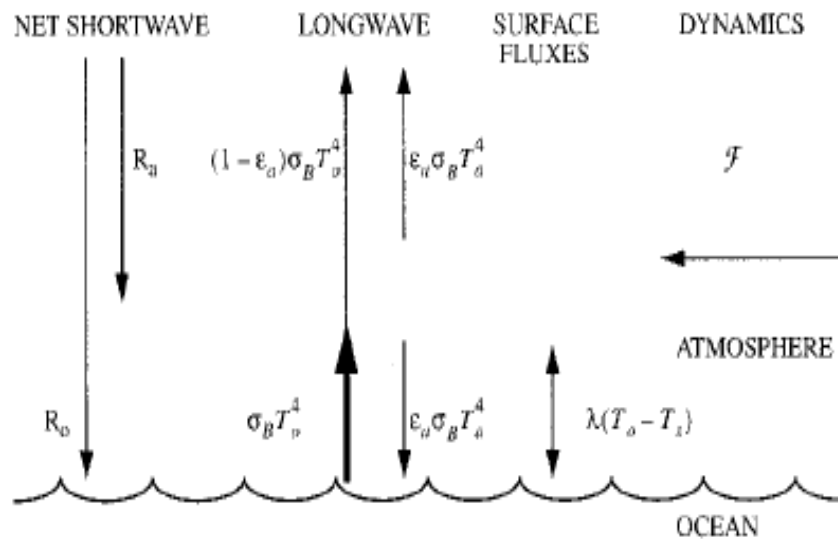
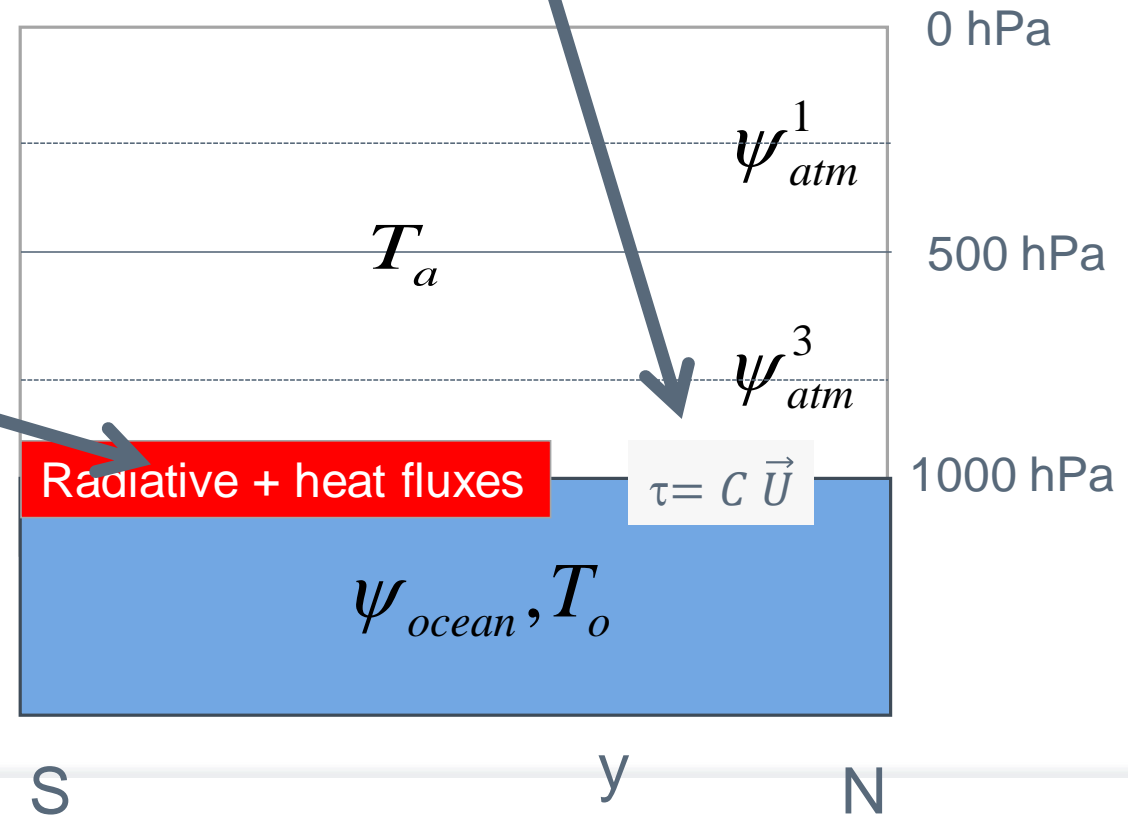


FIG. 2. Diagram of simple energy balance model on which Eqs. (1) and (2) are based. See appendix A for definition of symbols.

Barsugli & Battisti, 1998, JAS



Building a reduced order coupled ocean-atmosphere model

For the atmosphere

$$\psi = \sum_{k=1}^K \psi_k F_k$$

$$\theta = \sum_{k=1}^K \theta_k F_k$$

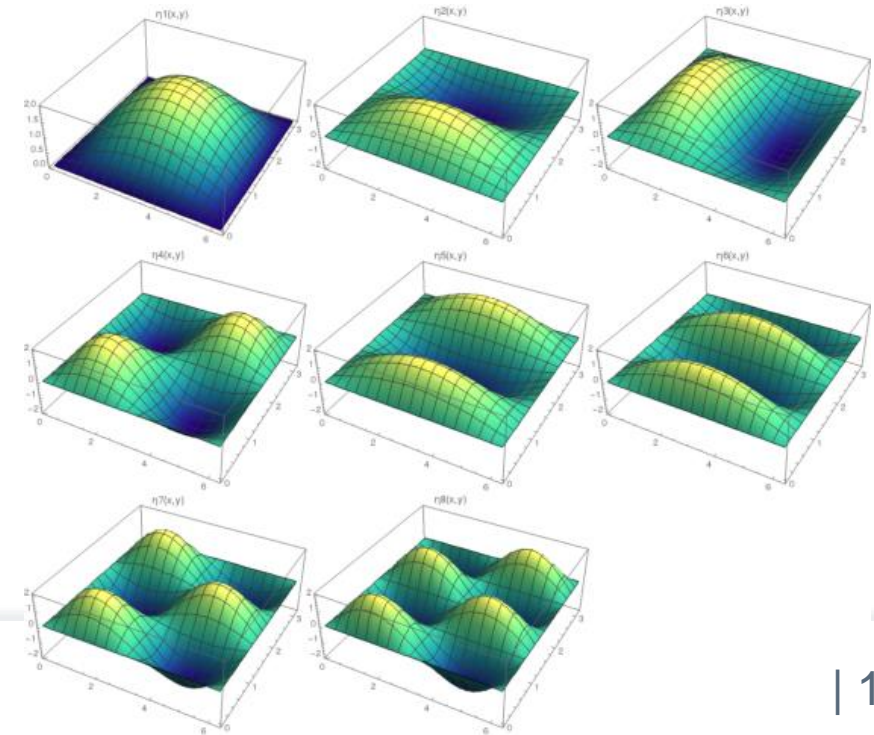
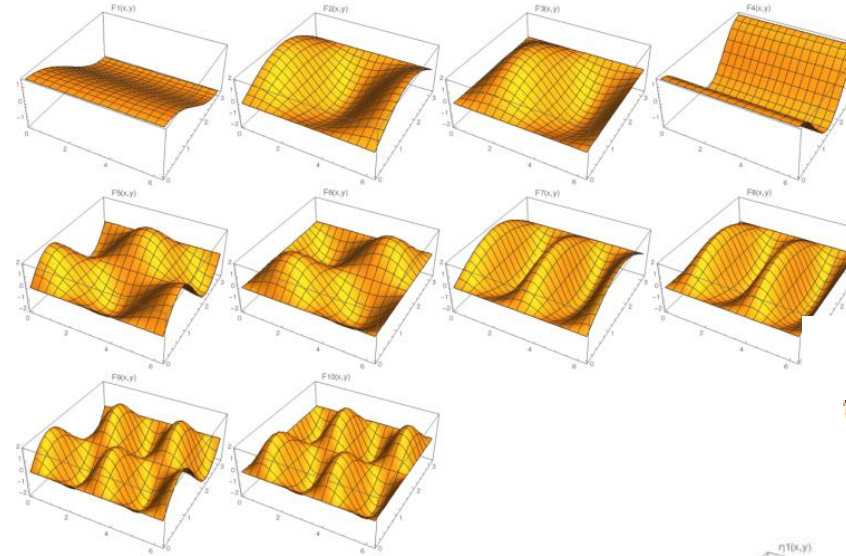
For the ocean

$$\psi_o = \sum_{i=1}^8 \psi_{o,i} \phi_i, \quad \delta T_o = \sum_{i=1}^8 T_{o,i} \phi_i$$

36-variable model

$$\dot{Z}_i = H_i + \sum_{j=1}^N L_{ij} Z_j + \sum_{j,k=1}^N B_{ijk} Z_j Z_k,$$

Vannitsem et al, Physica D, 2015



The model is available on Github

The latest version of MAOOAM: <https://github.com/Climdyn/MAOOAM/>

Arbitrary number of modes can be fixed

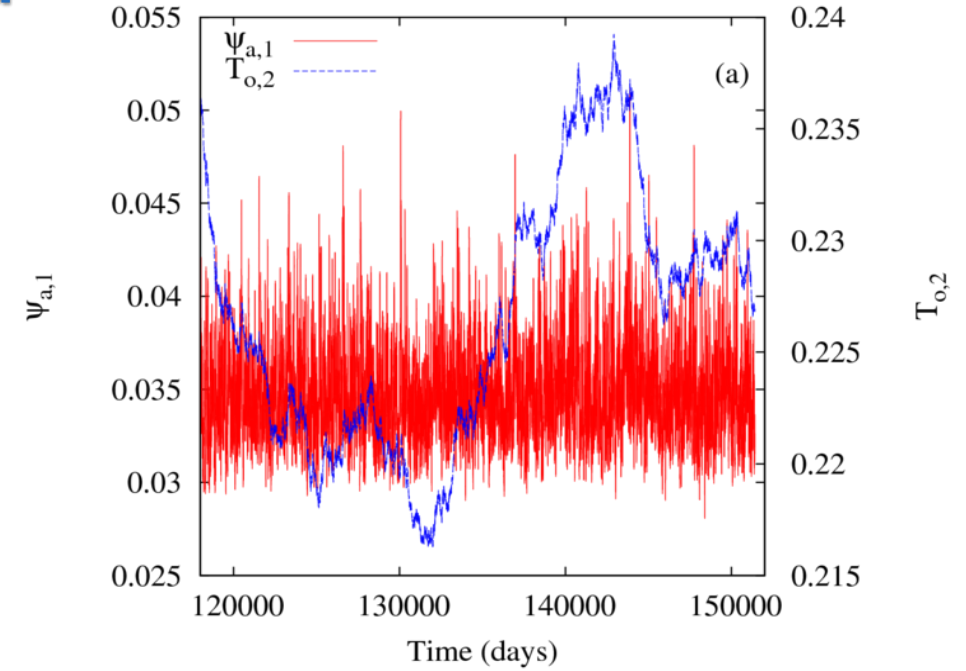
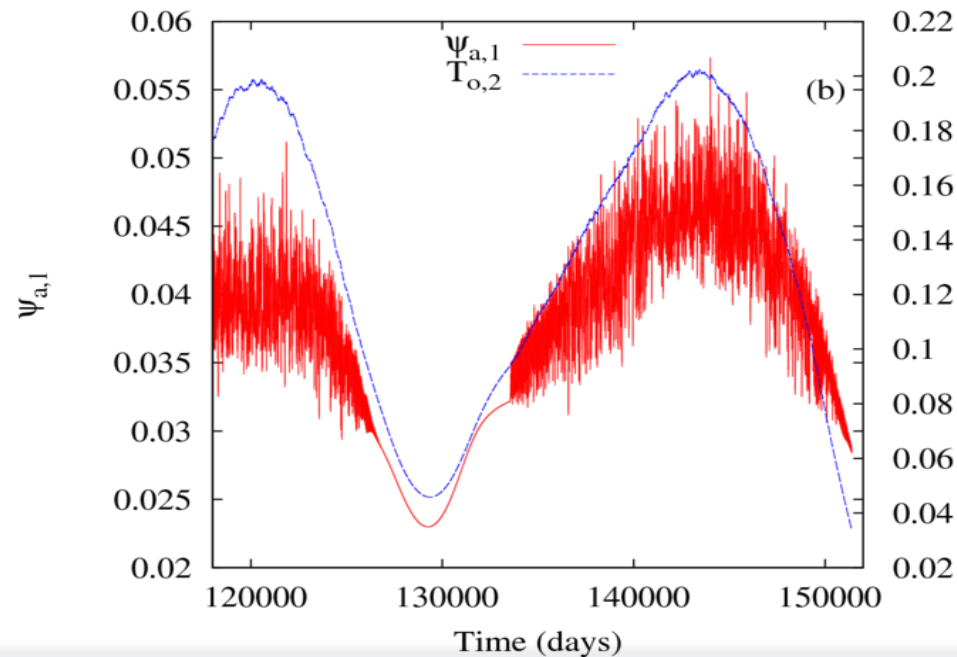
Currently new development of a version in Python (Jonathan Demaeyer)

Solutions of the model integration used

Two types of chaotic solutions investigated

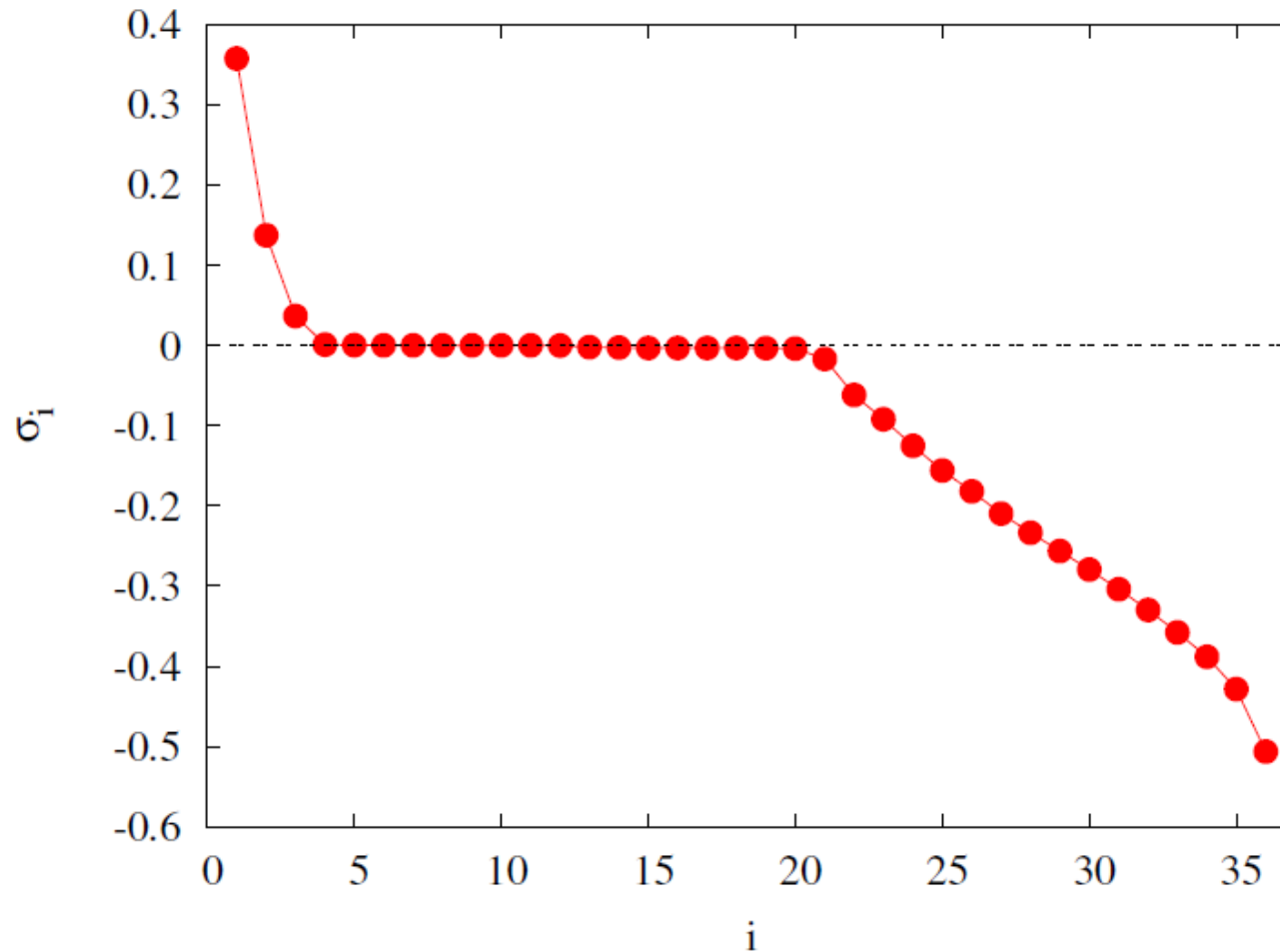
$C=0.01 \text{ kg m}^{-2} \text{ s}^{-1}$
 $H=100 \text{ m}$
 $C_o=350 \text{ W m}^{-2}$

$C=0.016 \text{ kg m}^{-2} \text{ s}^{-1}$
 $H=100 \text{ m}$
 $C_o=350 \text{ W m}^{-2}$



Lyapunov spectrum

$C=0.01 \text{ kg m}^{-2} \text{ s}^{-1}$.
 $H=100 \text{ m}$
 $C_o=350 \text{ W m}^{-2}$



To each of these
 Lyapunov exponents
 corresponds a Lyapunov
 Vector which is a local
 property
 along the trajectory

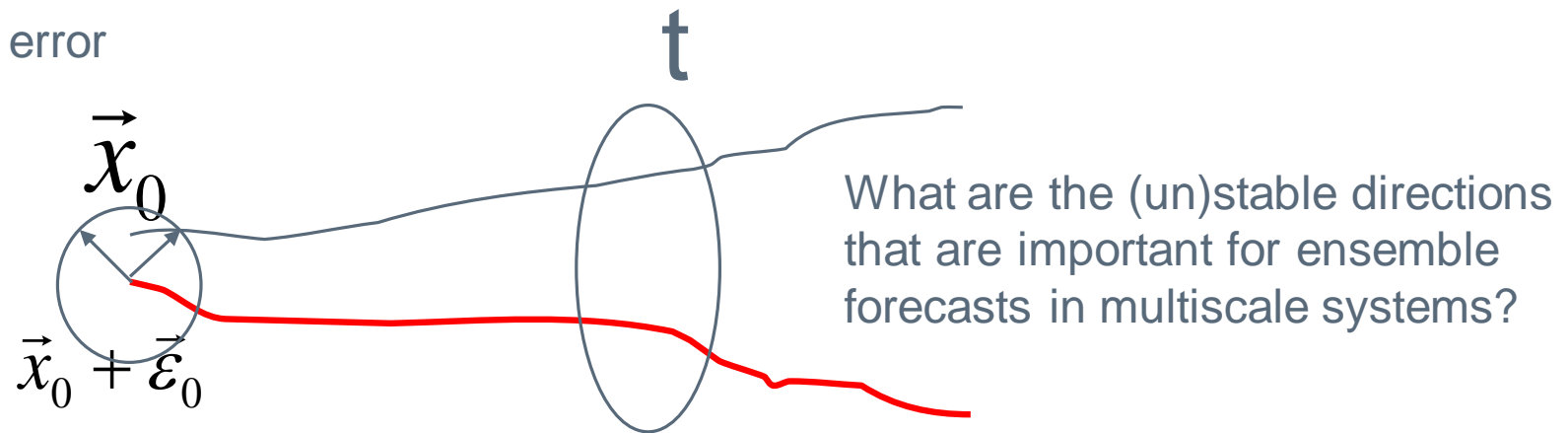
Experimental setup for ensemble forecasts

Initial error: Random

Number of ensemble members: 20

Number of realizations on the attractor of the system: 1000

No model error



Use of the Backward Lyapunov vectors to perturb the initial state

Experimental setup

There are 36 Backward Lyapunov vectors that can be considered

Experiments of ensemble forecasts with a set of Backward Lyapunov Vectors:

10 dominant ones

11 to 20

21 to 30



Random errors projected on these vectors

36 : The reference experiment since it is equivalent to the full reliable ensemble

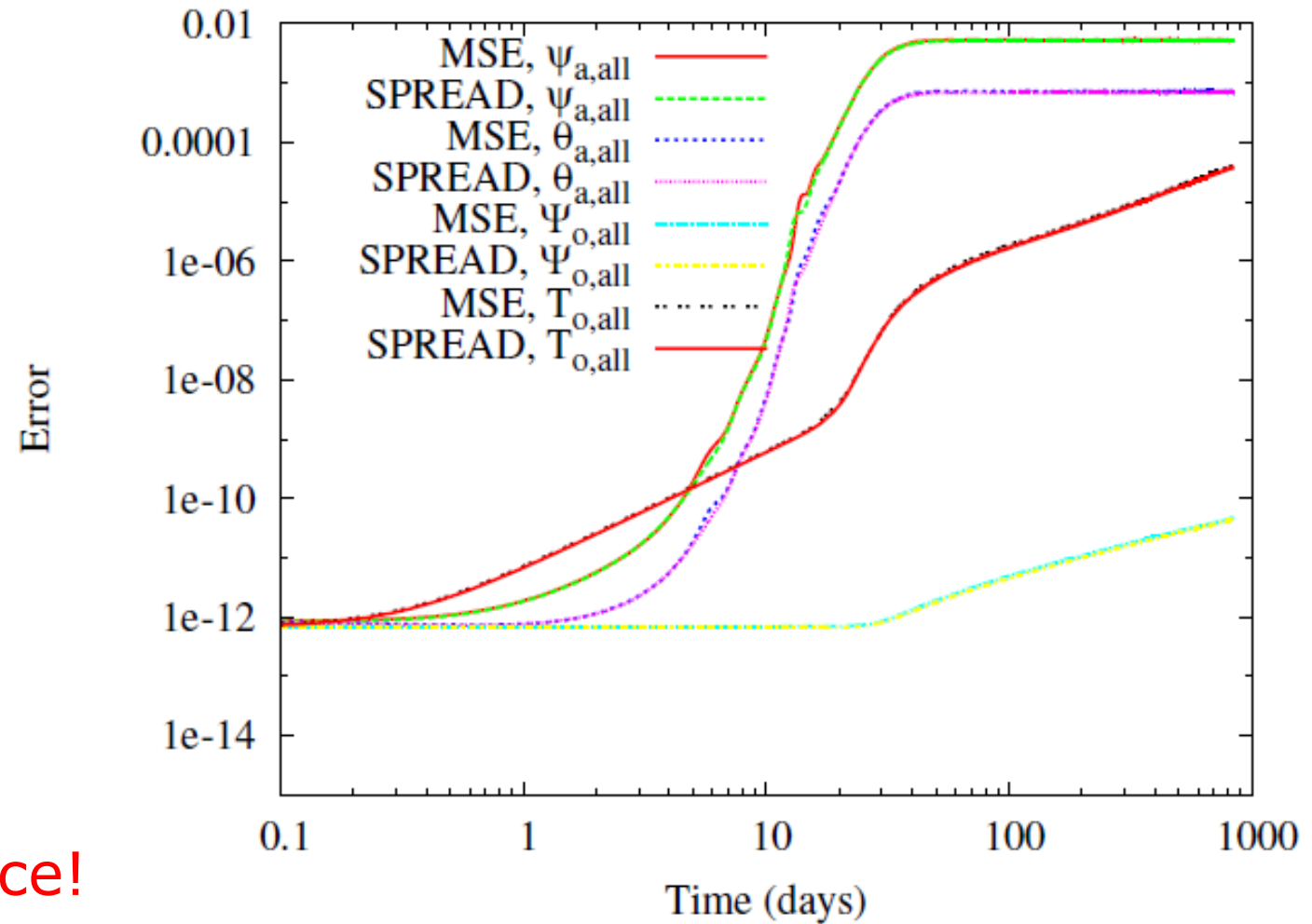
Dynamics of the error (MSE and SPREAD) for the random perturbation

Perfectly reliable ensemble!

*Mean Square Error of the ensemble mean =
variance of the ensemble*

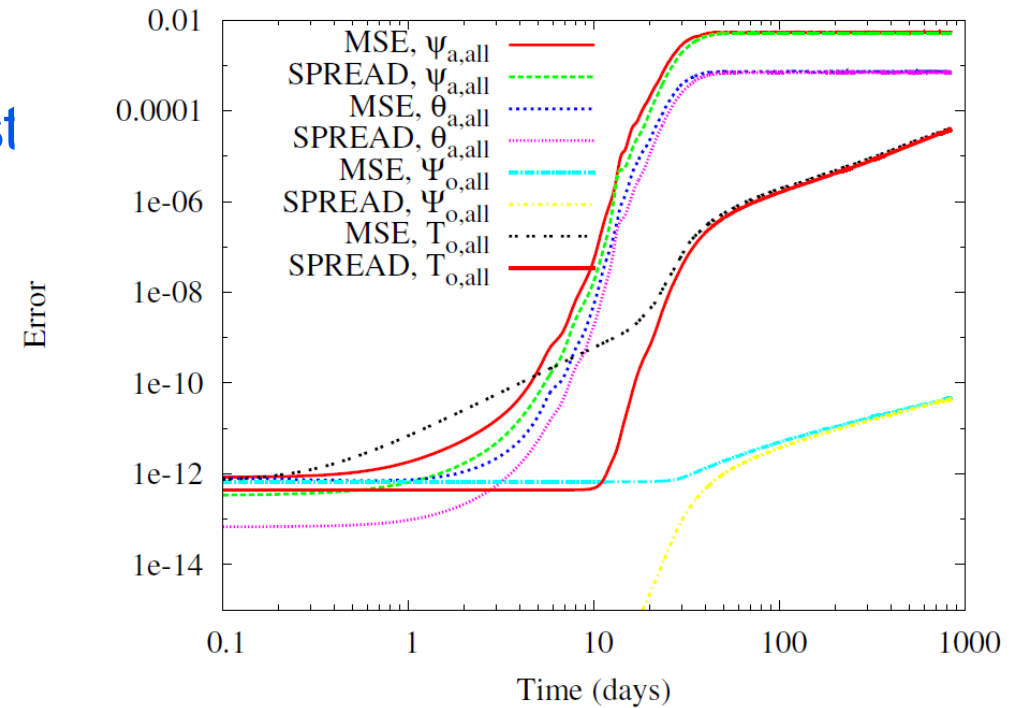
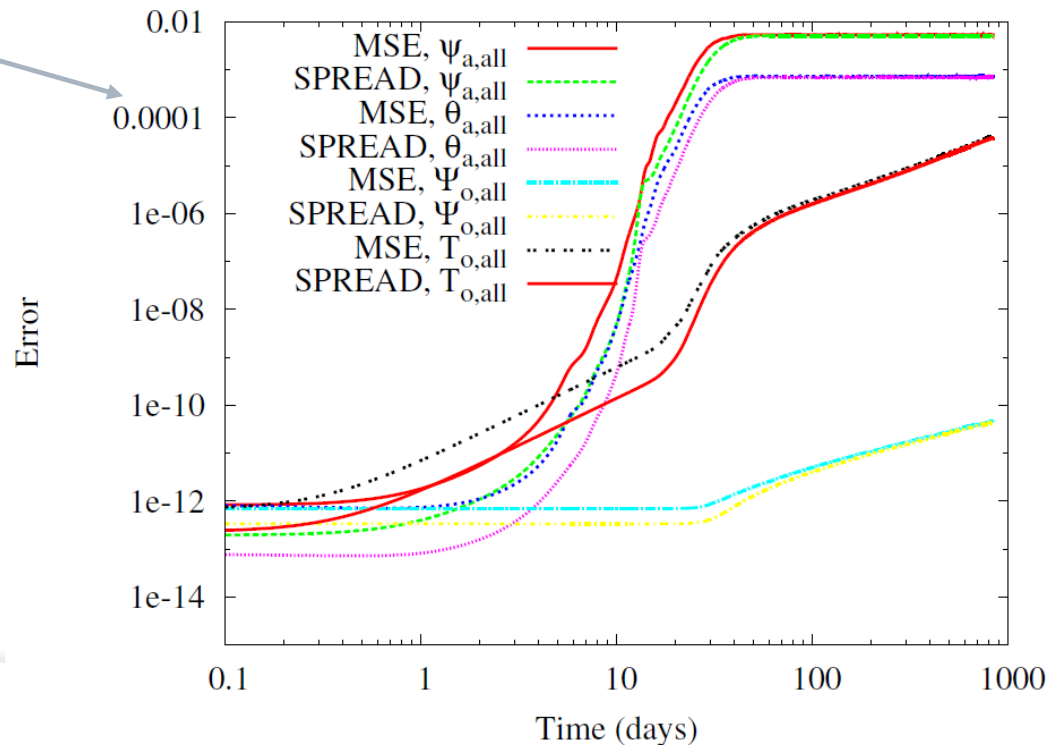
Results for the experiment
With perturbations along all
Backward Lyapunov Vectors

This experiment is the reference!



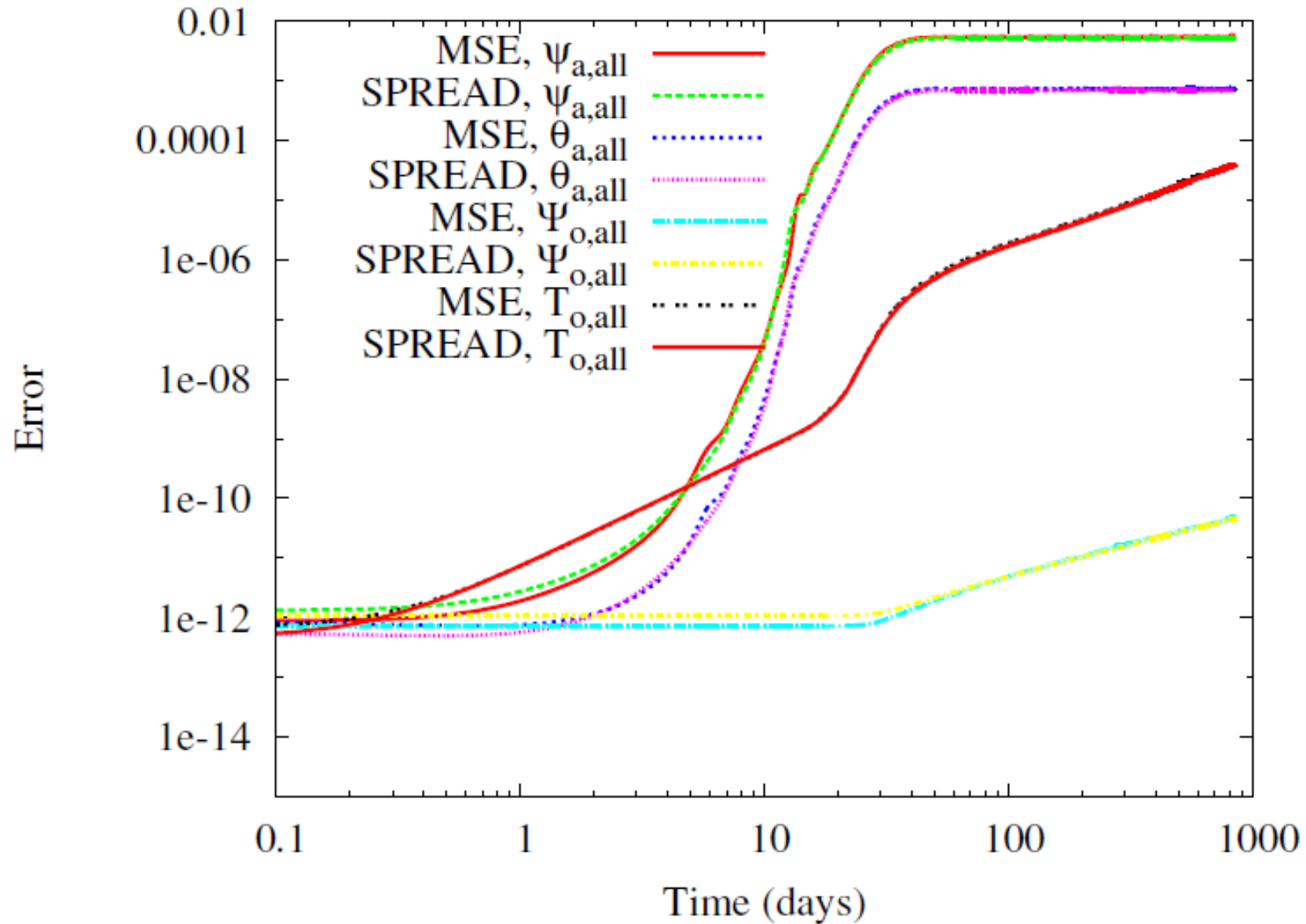
Perturbations based on the 10 first Lyapunov vectors

Perturbations based on the 11 to 20 Lyapunov vectors



Better performance if perturbations are introduced along the near-neutral Unstable modes, even for the atmosphere

A bit more tuning by changing the amplitude of the perturbations for the 11-20 LVs?

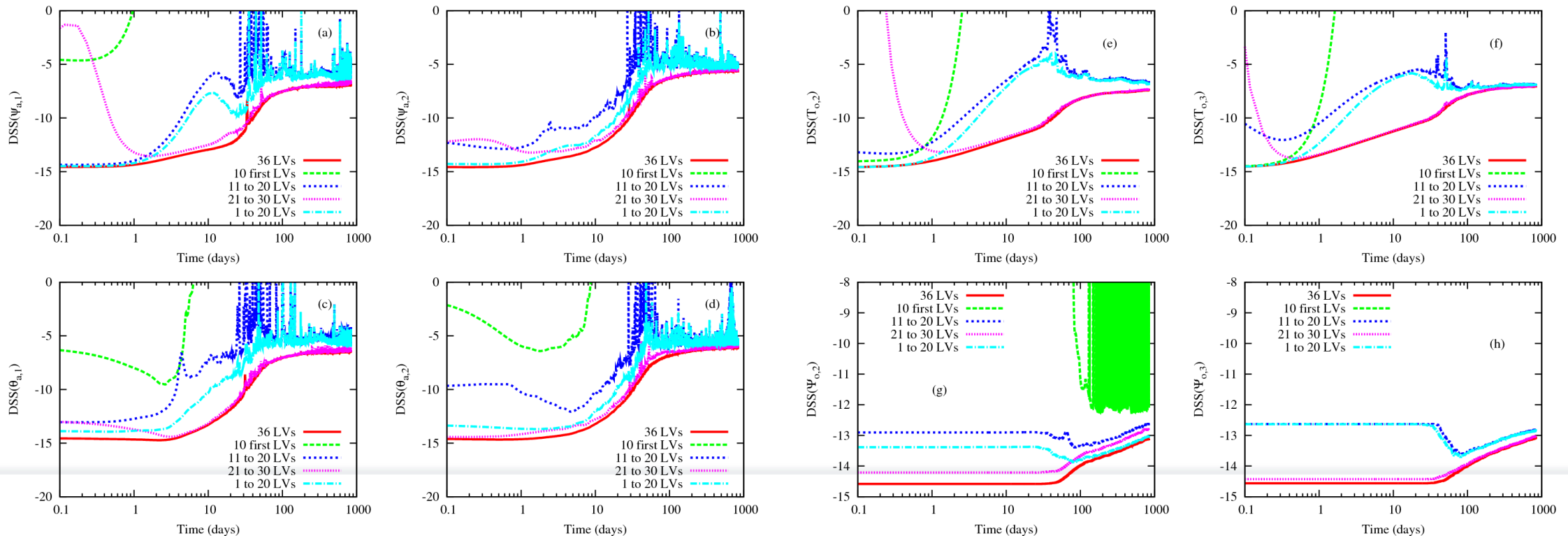


Additional considerations

For the solution with the low-frequency variability, the picture is very similar.

For long term forecasts, the use of the near-neutral and slightly negative Backward Lyapunov Vectors is key.

Use of the Dawid-Sebastiani score (Dawid and Sebastiani, 1999), the lower the better



Conclusions

The analysis of the ensemble forecasts based on the Lyapunov vectors reveals

- The best subspace in which to perturb the fields is NOT the most unstable one, because it fails to capture the variability within the ocean
- Perturbing the slow modes (near-neutral and slightly negative ones) seems to be a good approach. The atmosphere is anyway filled by the perturbations because of its fast time scales, and the rapid rotation of the perturbations along the most unstable directions.

Combining the perturbations along the unstable directions, the near neutral modes and the slightly Negative ones is a good option in this reduced-order model. This should be investigated in more Detailed coupled ocean-atmosphere models.

Reference

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- Vannitsem S. and W. Duan (2020) On the use of near-neutral Backward Lyapunov Vectors to get reliable ensemble forecasts in coupled ocean-atmosphere systems, submitted to *Climate Dynamics*, [arXiv:1911.09495](https://arxiv.org/abs/1911.09495)