Clustering and Connectivity of Fractal-Fracture Networks: Are they related?

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SUMMARY

Connectivity and flow behaviour of certain fracture networks are influenced by their fractal dimensions.

Different fractal-fracture networks having the same dimension can have distinct visual appearances in terms of clustering of fractures.

This lead to differences in connectivity and hence, flow behaviour.

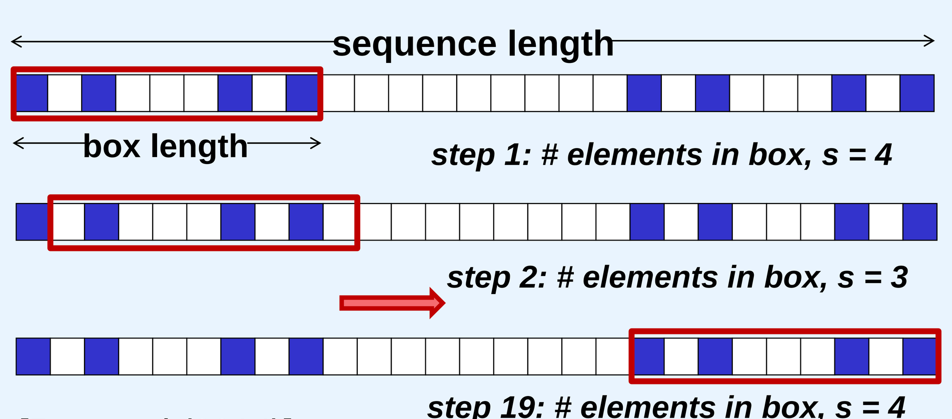
Lacunarity, a spatial clustering technique can distinguish between fracture networks belonging to a single fractal system.

Networks with similar dimensions but with distinct visual appearances are compared in terms of their lacunarity and connectivity values.

The results indicate that both the connectivity and clustering change systematically with the scale at which the networks are mapped.

In the particular case of a nested set of 7 natural maps, a good correlation is found between clustering and connectivity values.

I. What is Lacunarity?



$$L(r) = \frac{s_s^2(r)}{< s(r) >^2} + 1$$

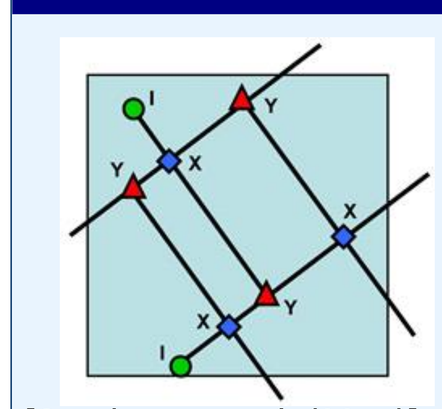
$$< L > = \frac{\sum[\log L(r) \cdot \log r]}{\sum \log r}$$

[Roy et al.(2014)]

The pattern shows a 3rd order Cantor-bar which has 8 elements distributed in an array of 27 cells. **Lacunarity**, ***L(r)*** at scale *r* = 9 is calculated by gliding a box of length 9 across the pattern and counting the number of elements, ***s*** at each step. This yields a distribution of *s (r)* from which *L (r)* is found as above

Log transformed lacunarity, log *L (r)* plotted against log r yields a 'curve'. A single value for this 'curve' is given by the second equation above

II. Connectivity of Fracture Networks

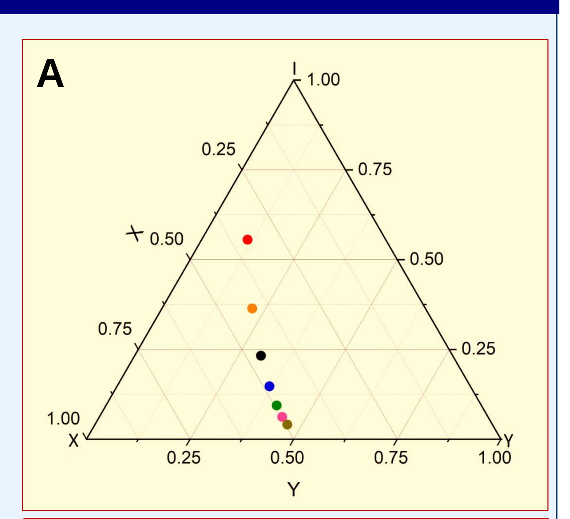


[Sanderson et al. (2015)]

$$n = \frac{4[1 - PI]}{[1 - PX]}$$

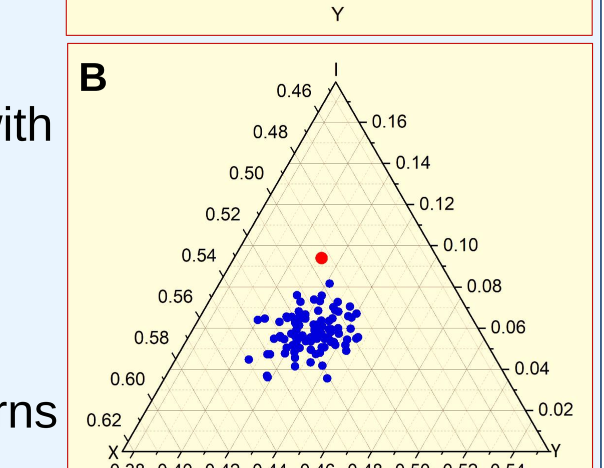
[Manzocchi (2002)]

Connectivity in terms of different types of nodes X, Y and I as shown above



Ternary diagram-A

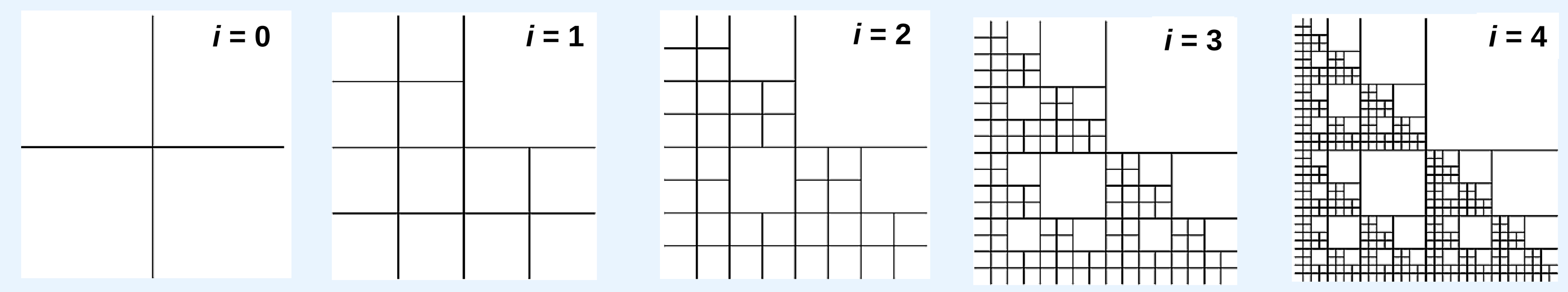
Connectivity for deterministic fractal patterns with same fractal dimension but different iterations



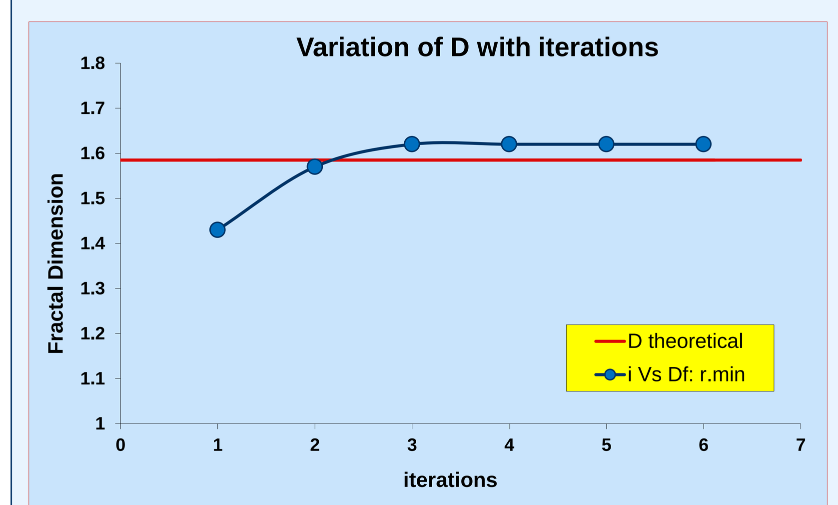
Ternary diagram-B

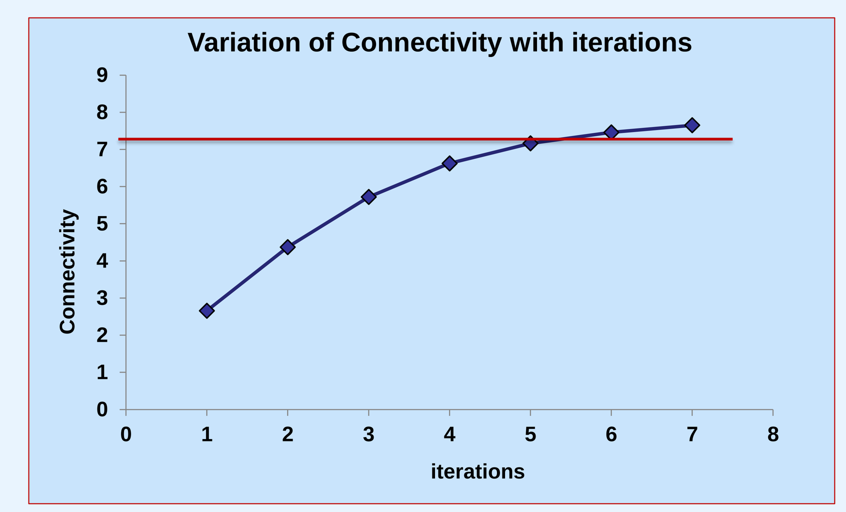
Connectivity for a deterministic fractal-fracture pattern (**red**) and random fractal-fracture patterns (**blue**) with same iteration

III. Fractal-Fracture Network: Lacunarity & Connectivity



Synthetic fractal-fracture patterns were constructed using a generalized version of the self-similar cataclasis model: *i* → iteration level

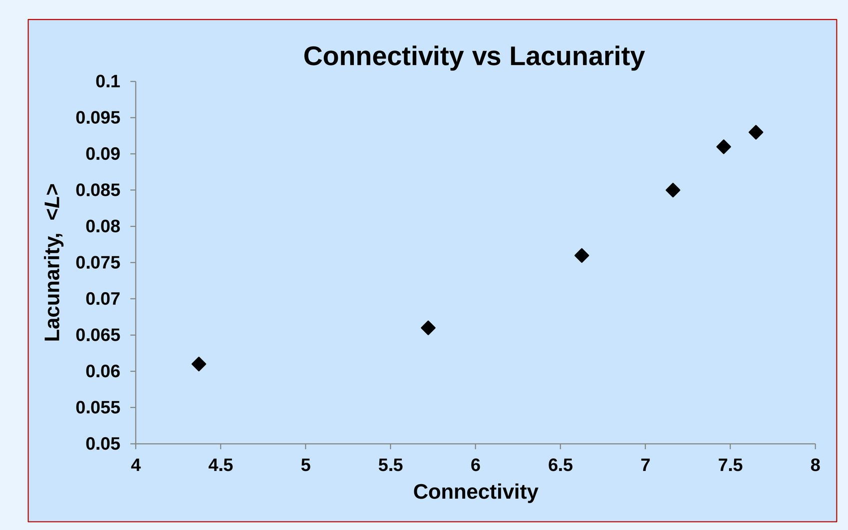




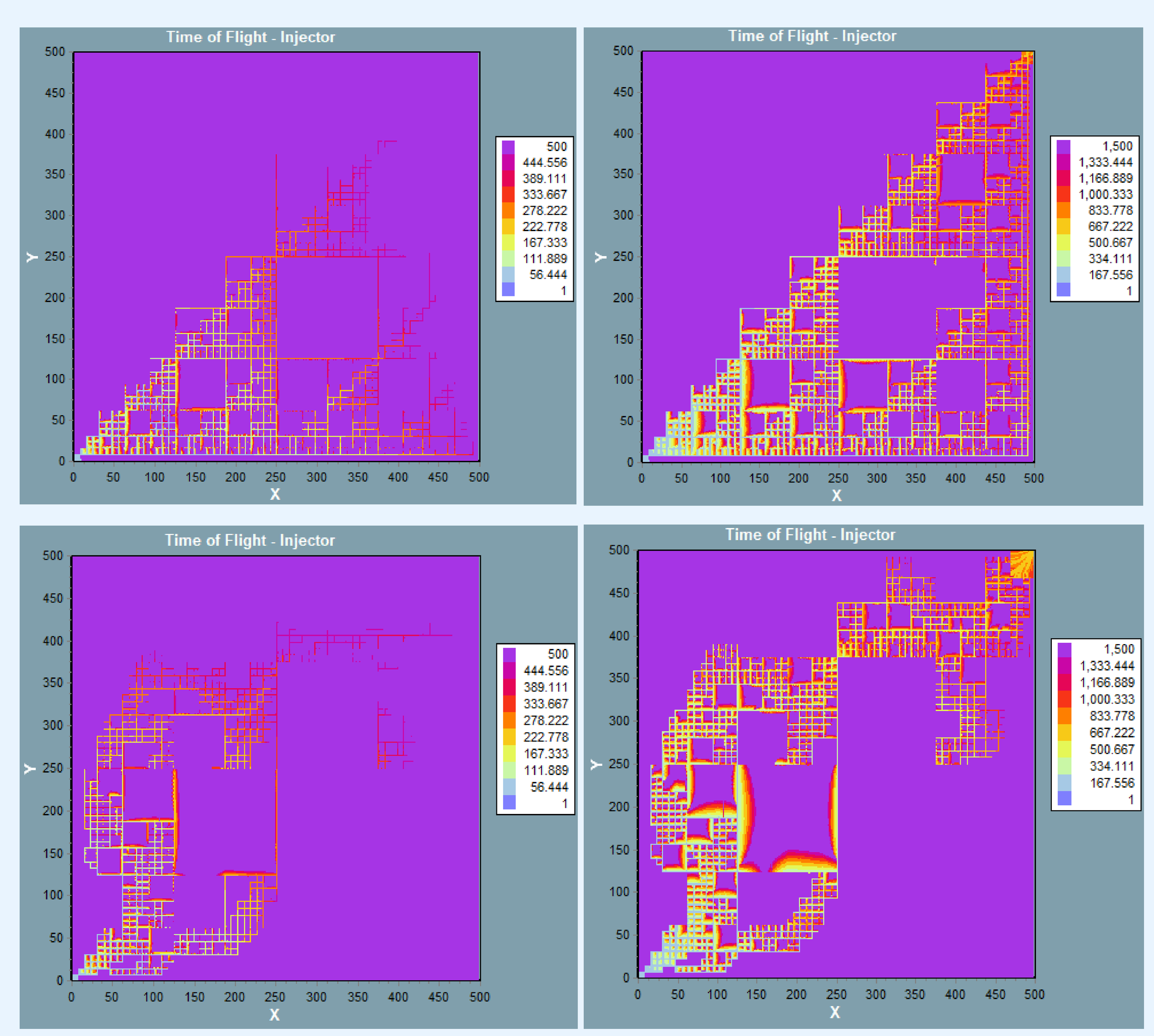
The fractal dimension stabilizes at higher iterations and so does the connectivity

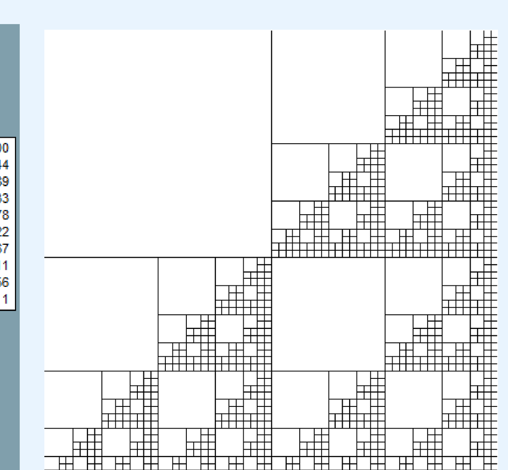
Lacunarity and connectivity show a positive correlation with each other

Does this work in case of natural fracture maps? What does iteration mean in this case?

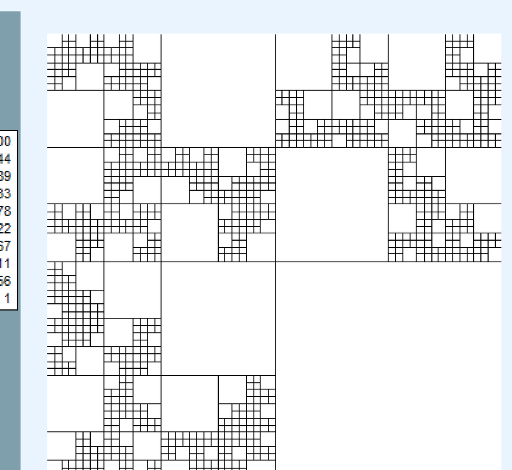


IV. Deterministic and Random Fractal Fracture Patterns





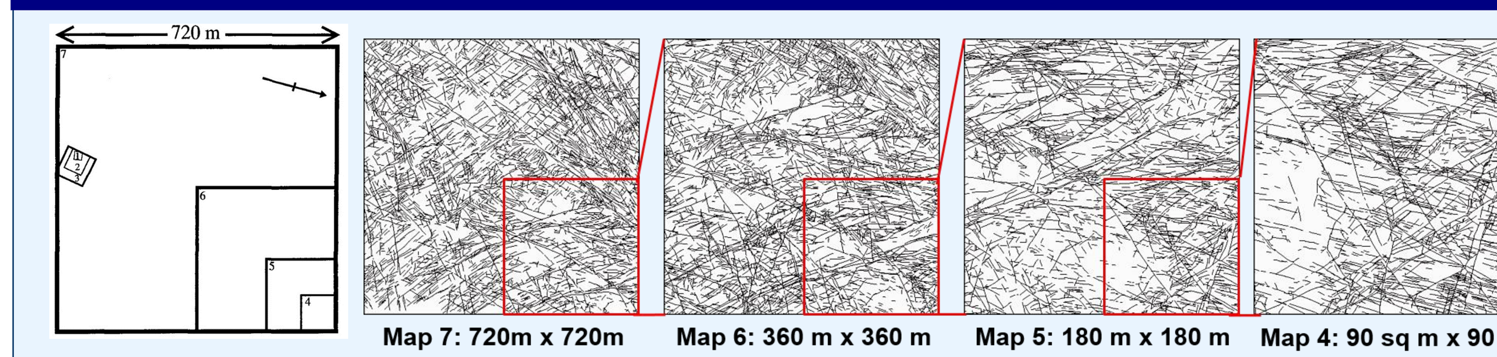
Deterministic fractal-fracture



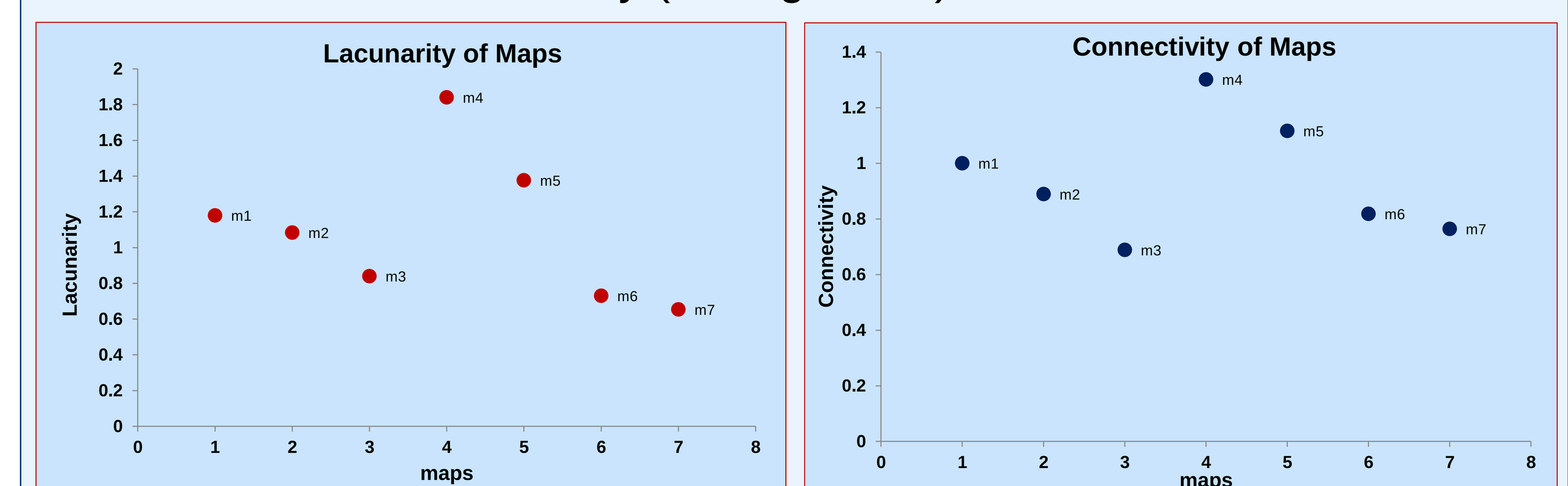
Random fractal-fracture

The time of flight (TOF) generated by flow simulation also appears to be different indicating different flow characteristics affecting the overall recovery (higher recovery for deterministic fractal-fracture network).

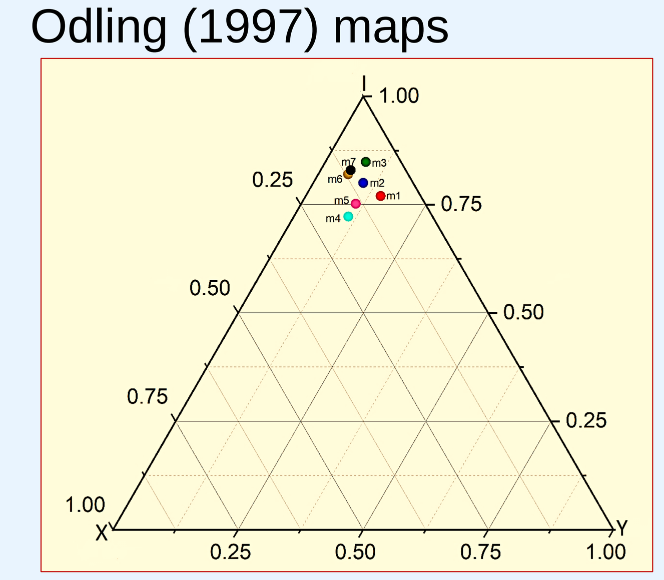
V. Natural Fracture Data

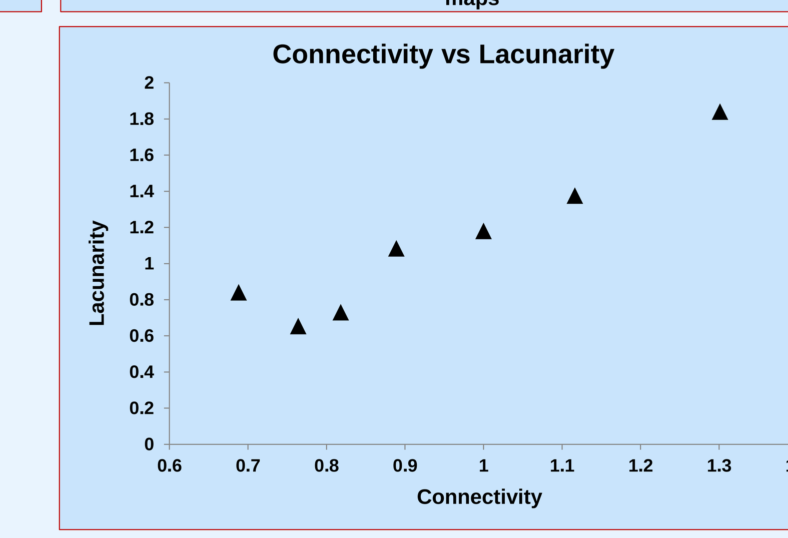


Nested Fractal-Fracture maps from Devonian sandstones exposed in the Hornelen basin, Norway (Odling, 1997).



Ternary diagram: Connectivity of Odling (1997) maps





The connectivity and Lacunarity of the 7 Odling maps are similar to each other as they belong to a single fractal system

VI. Concluding Remarks

- Maps with similar fractal dimension can have differences in terms of clustering, hence connectivity and flow properties
- As a fractal-fracture map is generated, the dimension stabilizes as the number of iterations increases. A similar behaviour is noted for the connectivity value of this pattern at different iterations
- There is a very good match between connectivity and lacunarity both in synthetic fractal-fractures as well as in a set of nested natural fracture maps belonging to the same fractal system

REFERENCES

Odling, N., (1997), Scaling and connectivity of joint systems in sandstones from western Norway, *Journal of Structural Geology*, 19(10), 1257-1271

Roy et al., (2007) Fractal characterization of fracture networks: An improved box-counting technique, *Journal of Geophysical Research*,112, B12

Manzocchi, T., (2002) The connectivity of two-dimensional networks of spatially correlated fractures, *Water Resources Research*, Vol. 38, No. 9, 1162

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