An optimization principle for computing stationary MHD equilibria with solar wind flow

Authors: Thomas Wiegelmann[1], Thomas Neukirch[2], Dieter Nickeler[3], Iulia Chifu[1]

1. Max-Planck-Institut für Sonnensystemforschung, Göttingen, Germany
2. School of Mathematics and Statistics, University of St. Andrews, UK
3. Astronomical Institute Ondrejov, Czech Republic

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Motivation

- Global magnetic field modelling of coronal magnetic field.
- Measurements of magnetic fields are available mainly in solar photosphere.
- Current global models are either (ISSI team results, see Yeates et al. 2018)
  - Nonlinear force-free extrapolations from vector magnetograms.
  - Models including plasma effects use only the line of sight photospheric measurements as boundary condition.

- Aim: Develop a stationary MHD-code using vector magnetograms and incorporate solar wind flow.

Basic equations: stationary compressible MHD

Force balance:

$$\underbrace{\rho\left(\mathbf{v}\cdot\nabla\right)\mathbf{v}}_{\text{Plasma flow}} = \underbrace{1/\mu_0\left(\nabla\times\mathbf{B}\right)\times\mathbf{B}}_{\text{Lorentz force}} - \underbrace{\nabla\mathbf{p} - \rho\nabla\psi}_{\text{Plasma pressure and gravity forces}}$$

Solenoidal condition: $$\nabla\cdot\mathbf{B} = 0$$

Continuity equation: $$\nabla\cdot\left(\rho\mathbf{v}\right) = 0$$

Equation of state: $$p = \rho R T$$ (Here: Isothermal)

To solve the stationary MHD we define a functional of quadratic form

$$L(\mathbf{B}, \mathbf{v}, \rho) = L_{\text{force}} + L_{\text{divB}} + L_{\text{cont}} + L_{\text{angle(B,v)}}$$

$$L_{\text{force}} = \int_V \frac{\left[(\nabla \times \mathbf{B}) \times \mathbf{B} - \mu_0 \nabla(\rho R T) - \mu_0 \rho \nabla \psi - \mu_0 \rho \left(\nabla \times \mathbf{v}\right) \times \mathbf{v} - \frac{\mu_0 \rho}{2} \nabla v^2\right]^2}{B^2} \, dV$$

The plasma pressure has been replaced by assuming an isothermic equation of state and vector identities have been applied to the velocity terms

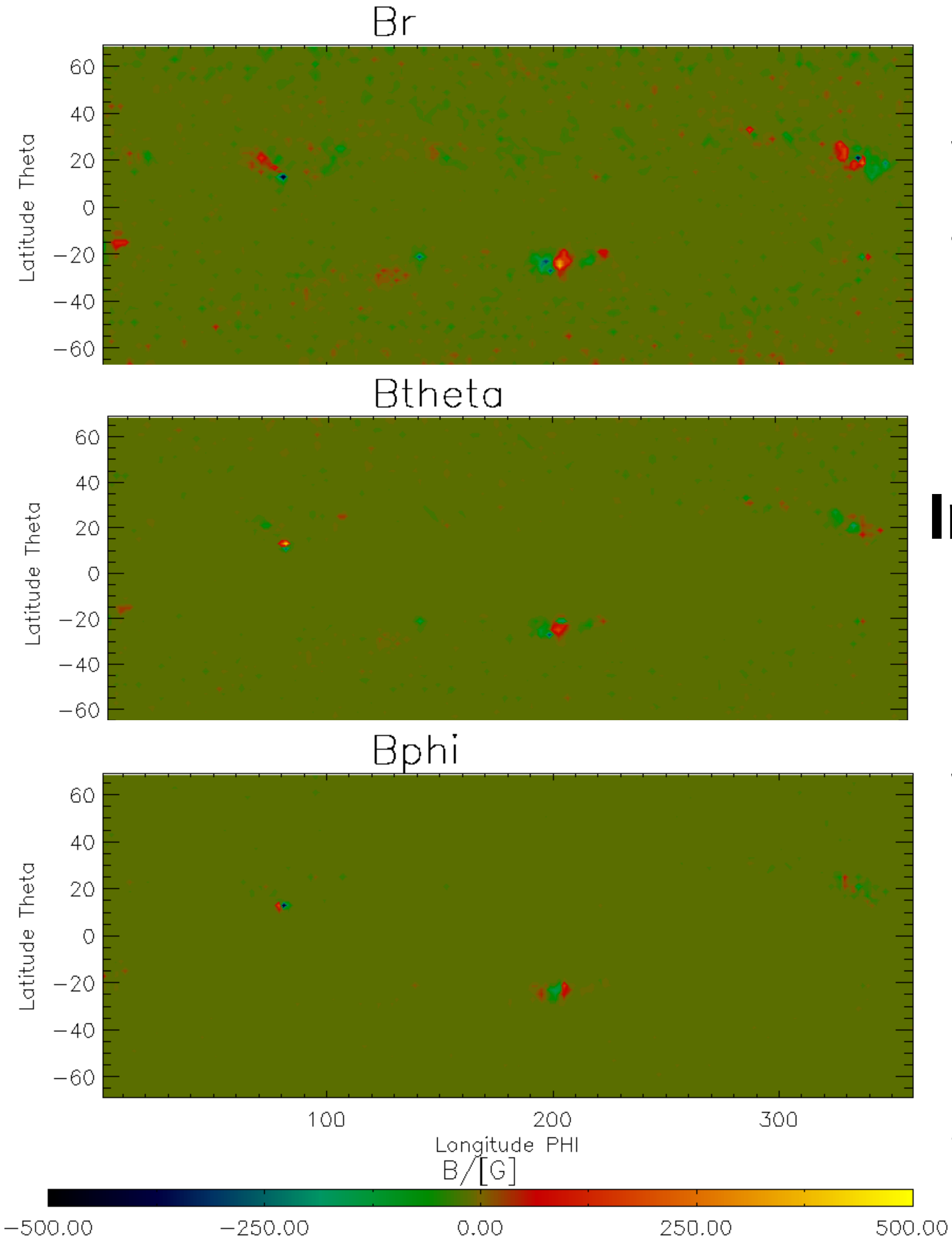
$$L_{\text{divB}} = \int_V [\nabla \cdot \mathbf{B}]^2 \, dV$$

$$L_{\text{cont}} = \int_V [\nabla \cdot (\rho \mathbf{v})]^2 \, dV$$

$$L_{\text{angle(B,v)}} = \int_V \tanh(M_A^2) \frac{[\mathbf{v} \times \mathbf{B}]^2}{v^2 \, B^2} \, dV$$

Aim of optimization: Make all terms of L small simultaneously

The functional generalizes force-free [1,2,3] and magnetostatic [4,5] optimization codes.



Application test Case

We use a synoptic vector magnetogram from SDO/HMI for Carrington rotation 2099 as boundary condition, which has been observed between 13/07/2010 and 09/08/2010. Polar regions have been cutted out.

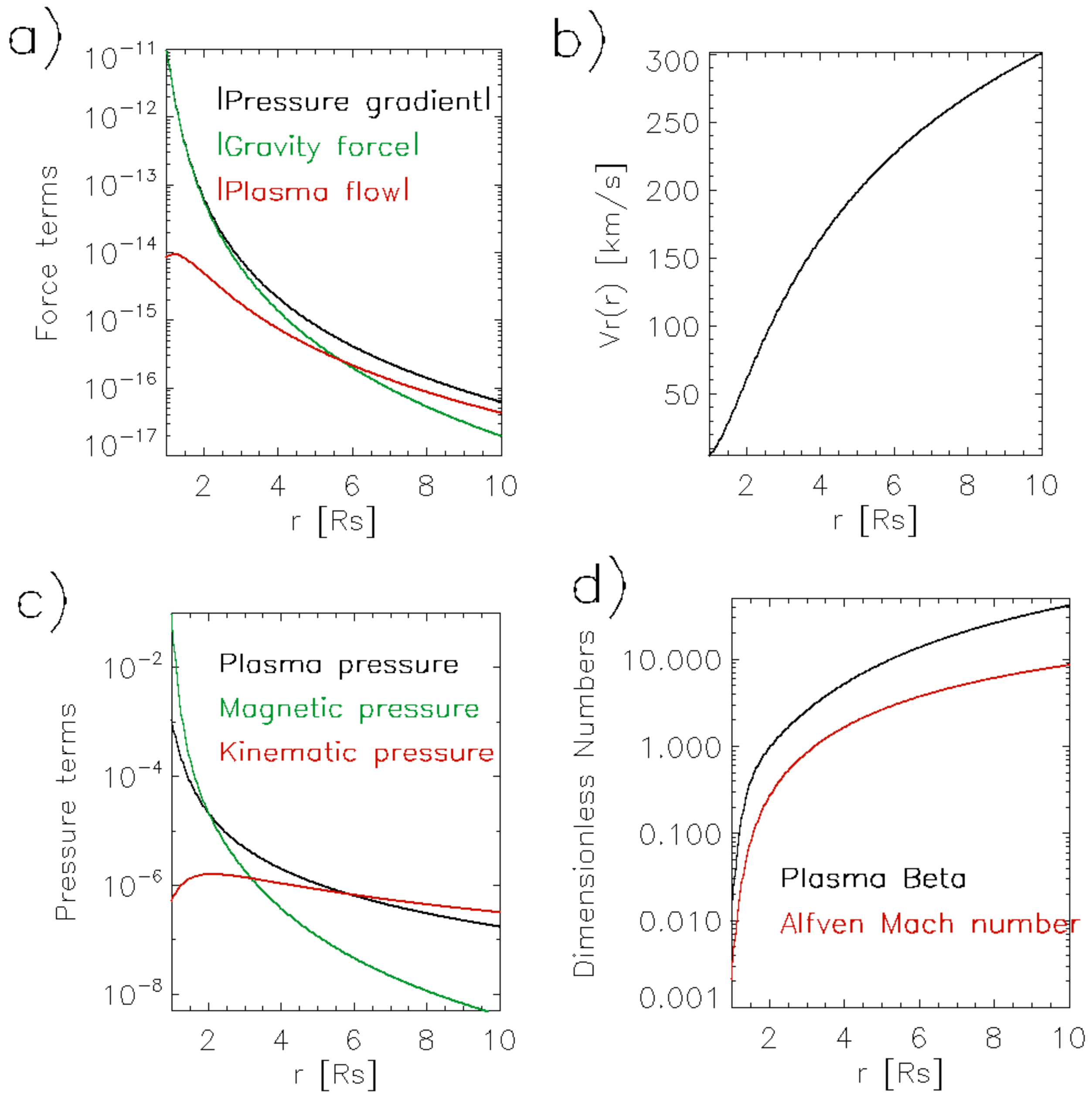
Initial force-free magnetic field

$$(\nabla \times \mathbf{B}) \times \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{B} = 0$$

With these boundary conditions we compute a 3D nonlinear force-free coronal magnetic field up to 10 solar radii with a resolution of 2 degree. For force-free fields the Lorentz force vanishes, and they also computed by optimiztion, here starting from a potential field with spherical harmonics until l=12.

Initial plasma equilibrium: Parker solar wind



$$\rho\,(\mathbf{v}\cdot\nabla)\,\mathbf{v} = -\nabla\mathbf{p} - \rho\nabla\psi$$

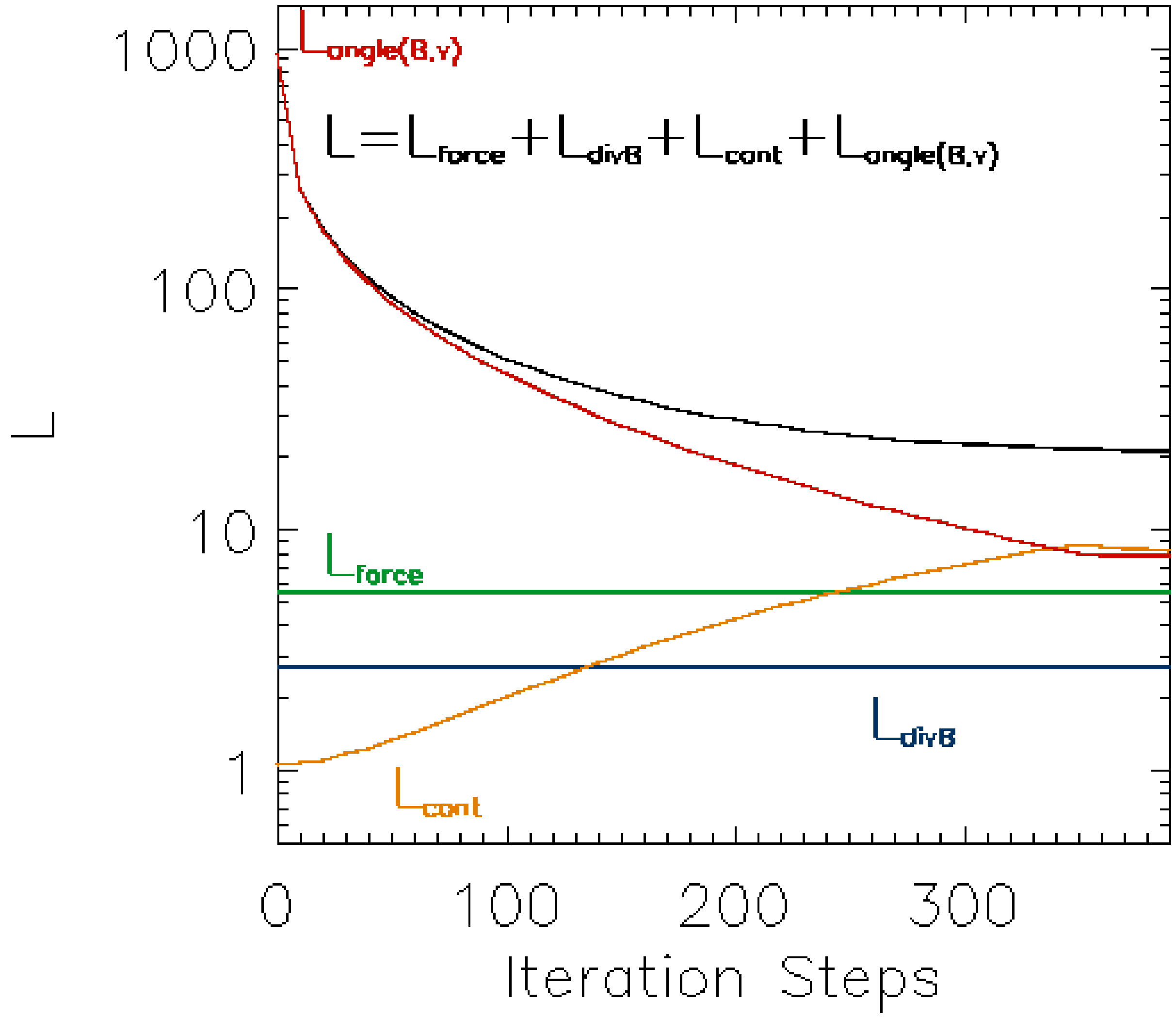
$$\nabla\cdot(\rho\mathbf{v}) = 0$$

$$p = \rho R T$$

Parker [6] found a spherically symmetric solar wind solution for these equations.

a) Shows the different force terms.
b) Solar wind velocity as function of r.
c) Plasma and kinematic pressure from Parker‘s model, magnetic pressure from force-free field model.
d) Plasma Beta and Alfven Mach number: If both quantities are small, plasma and flow forces can be neglected for computing the coronal magnetic field.

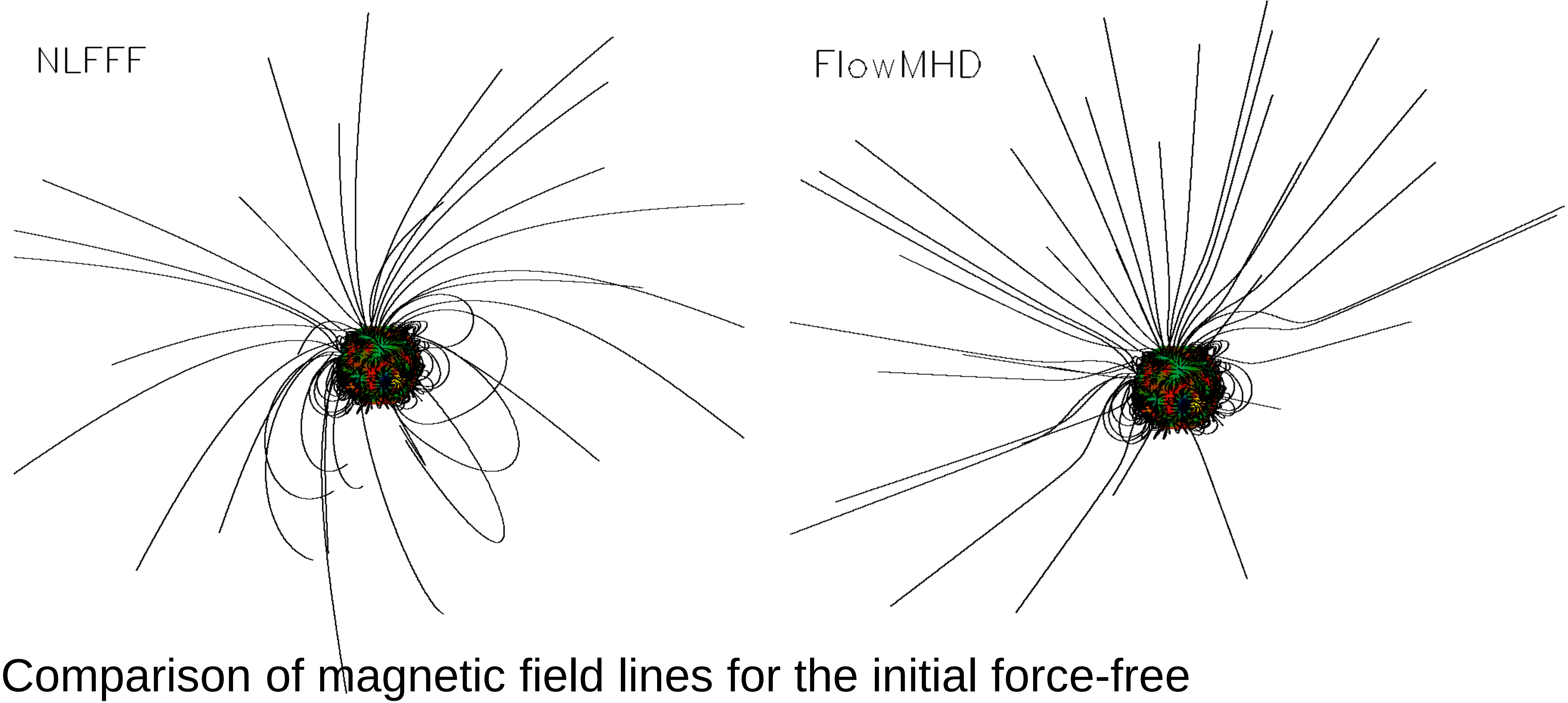
The iteration process



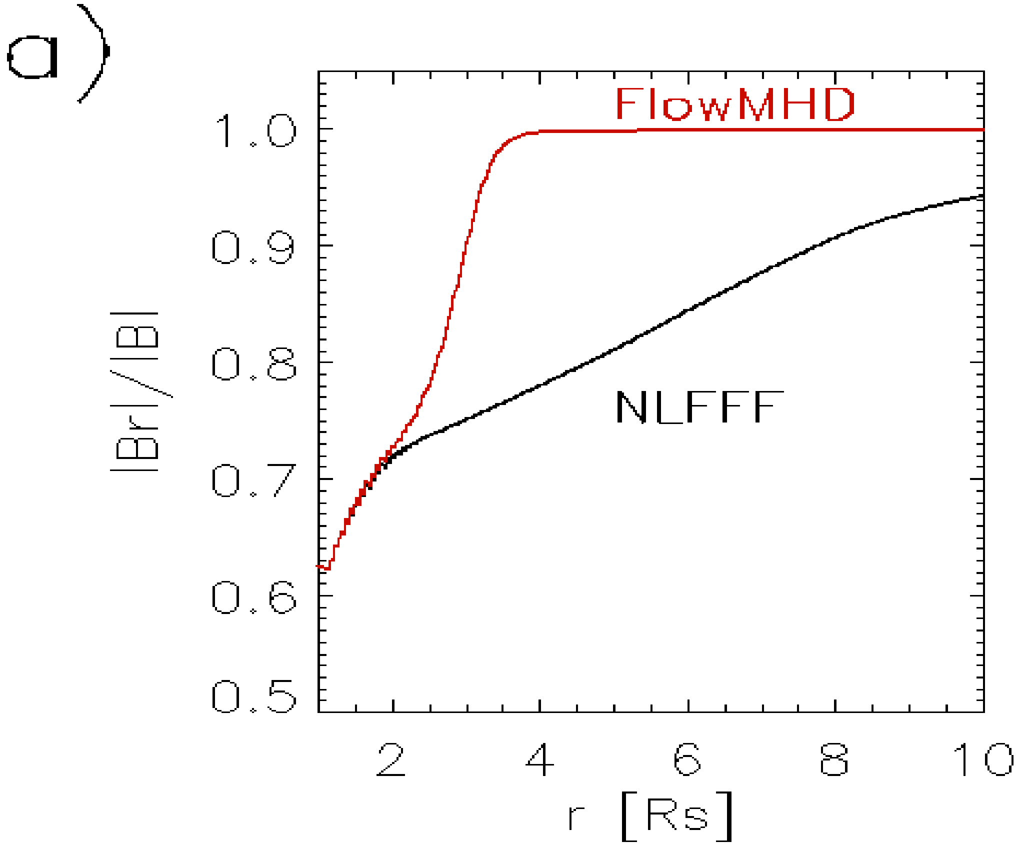
In the initial state the force-free magnetic field and plasma properties are decoupled.
During the iteration
B and V become parallel.

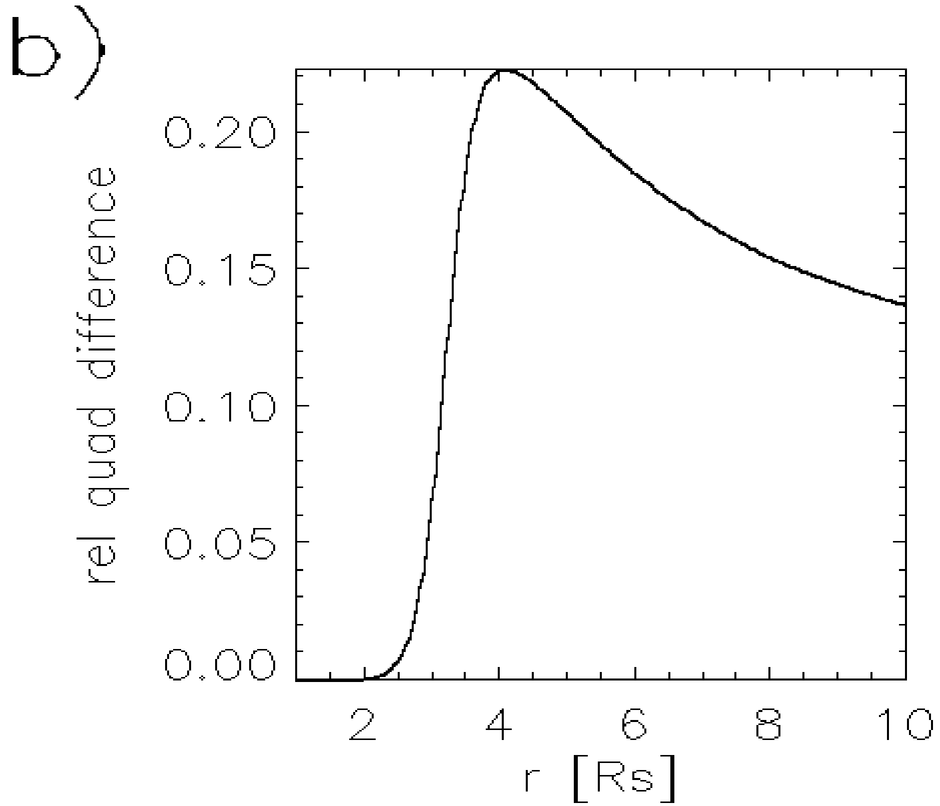
In the final state we have found a solution of the stationary compressible MHD equations with field aligned plasma flow.

The residual discretization errors are similar as for nonlinear force-free computations.



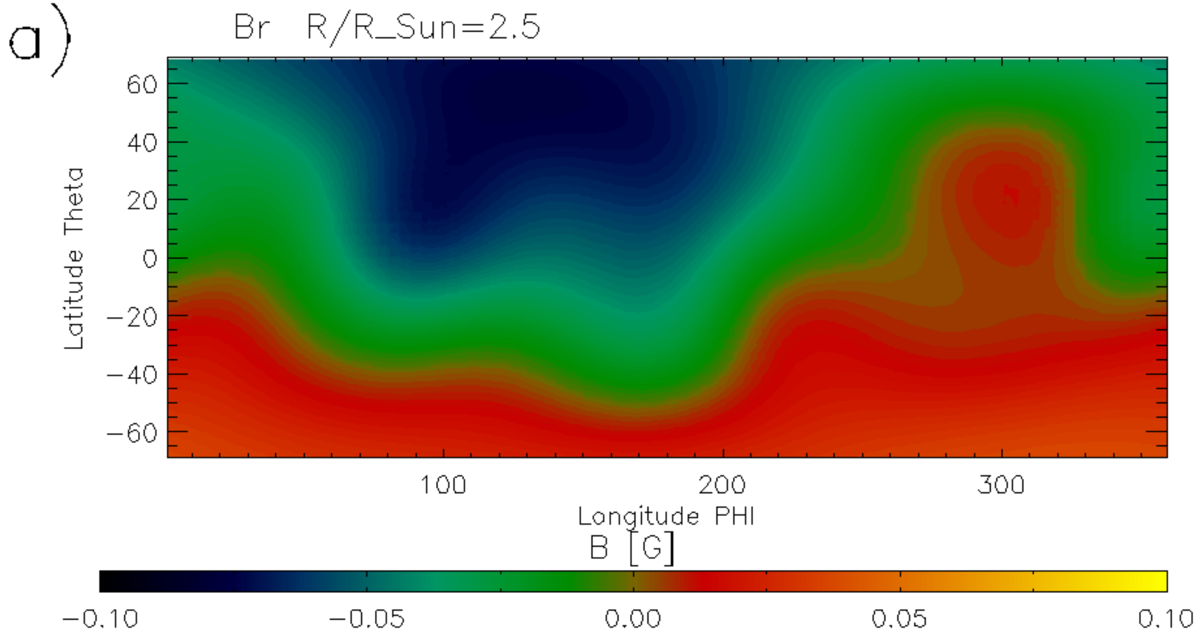
Comparison of magnetic field lines for the initial force-free field [NLFFF] and the final stationary MHD equilibrium [FlowMHD]. Solar wind in stationary MHD stretches and opens magnetic field lines. Boundary conditions are the same for both cases.

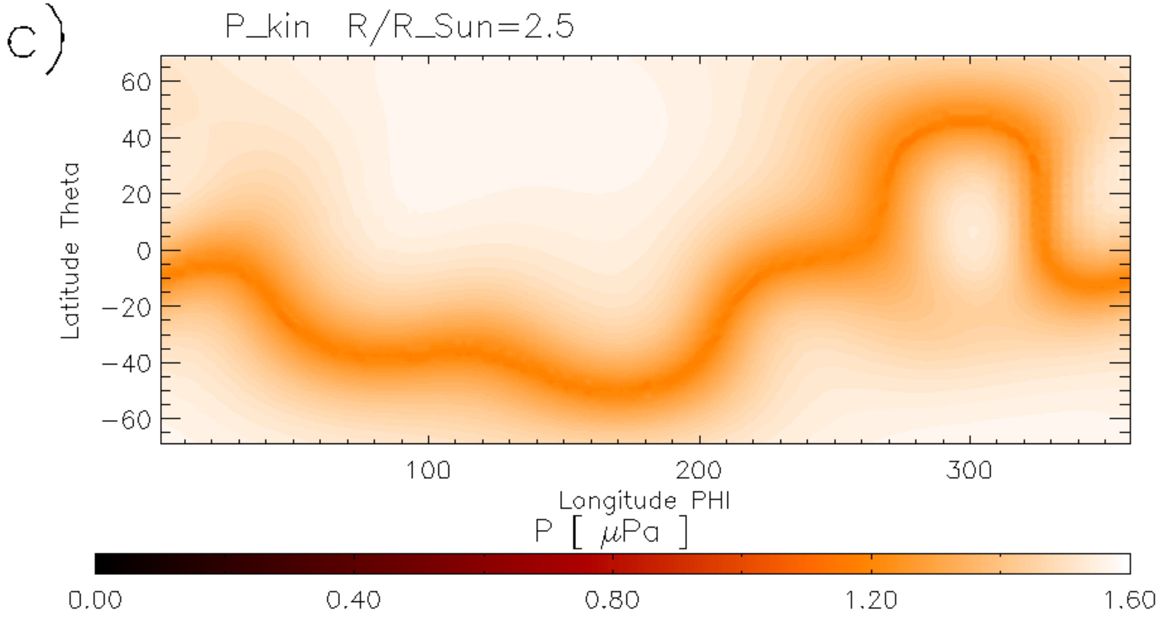


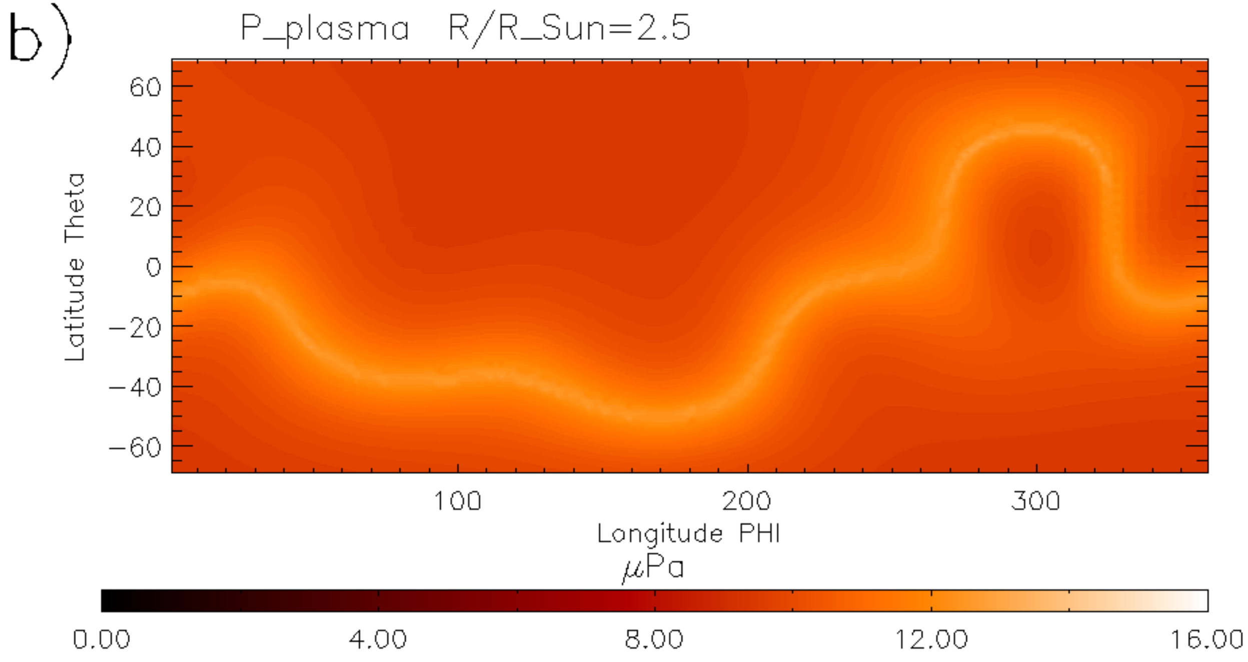


a) Compared with the initial NLFFF the stationary MHD field become more radially from about 2 solar radii on.

b) Comparison of (horizontal averaged) magnetic field vector. NLFFF and stationary MHD are almost identical below 2 solar radii, where Alfven Mach number M_A << 1.







Along the polarity inversion line of the magnetic field (panel a) the plasma pressure (panel b) becomes enhanced and the kinematic pressure (panel c) decreases.

Conclusions

- Current nonpotential global corona models (see [7]) based on vector magnetograms neglected plasma effects, while MHD-models use only line-of-sight magnetograms as boundary conditions.
- We developed a global optimization code to solve the stationary MHD-equations, which allows to incorporate the solar wind and use synoptic vector magnetograms as boundary conditions.
- The new code is the heritage of the nonlinear force-free and magneto-hydro-static magnetic reconstruction codes, based on optimization principles.
- In the lower solar corona (below about 2 Rs) the Alfven Mach number is small and the magnetic field is basically force-free.
- Above this height the solar wind plasma flow becomes important and stretches the magnetic field lines
=> Strong deviations from force-free state.

References

[1] Wheatland, M.S., Sturrock, P.A., Roumeliotis, G.: 2000, An Optimization Approach to Reconstructing Force-free Fields. Astrophys. J. 540(2), 1150.

[2] Wiegelmann, T.: 2007, Computing Nonlinear Force-Free Coronal Magnetic Fields in Spherical Geometry. Solar Phys. 240(2), 227.

[3] Tadesse, T., Wiegelmann, T., Gosain, S., MacNeice, P., Pevtsov, A.A.: 2014, First use of synoptic vector magnetograms for global nonlinear, force-free coronal magnetic field models. Astron. Astrophys. 562, A105.

[4] Wiegelmann, T., Neukirch, T., Ruan, P., Inhester, B.: 2007, Optimization approach for the computation of magnetohydrostatic coronal equilibria in spherical geometry. Astron. Astrophys. 475(2), 701.

[5] Zhu, X., Wiegelmann, T.: 2018, On the Extrapolation of Magnetohydrostatic Equilibria on the Sun. Astrophys. J. 866(2), 130.

[6] Parker, E.N.: 1958, Dynamics of the Interplanetary Gas and Magnetic Fields. Astrophys. J. 128, 664.

[7] Yeates, A.R., Amari, T., Contopoulos, I., Feng, X., Mackay, D.H., Mikic, Z., Wiegelmann, T., Hutton, J., Lowder, C.A., Morgan, H., Petrie, G., Rachmeler, L.A., Upton, L.A., Canou, A., Chopin, P., Downs, C., Druckmüller, M., Linker, J.A., Seaton, D.B., Török, T.: 2018, Global Non-Potential Magnetic Models of the Solar Corona During the March 2015 Eclipse. Space Sci. Rev. 214(5), 99.