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Target-oriented optimized survey design and quantitative comparison for 3D electrical resistivity tomography

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What is optimization

◆ Goal

The requirements for accuracy and speed are getting higher and higher

➢ Conventional solution

Increase the amount of data collected

■ New issue

Increased acquisition and calculation time

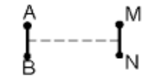
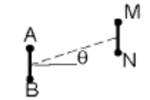
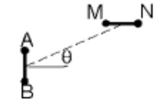
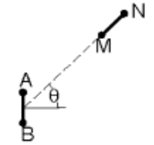
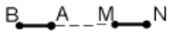
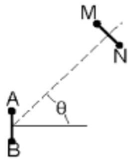
✓ Treatment

Target-oriented optimized

✓ Results

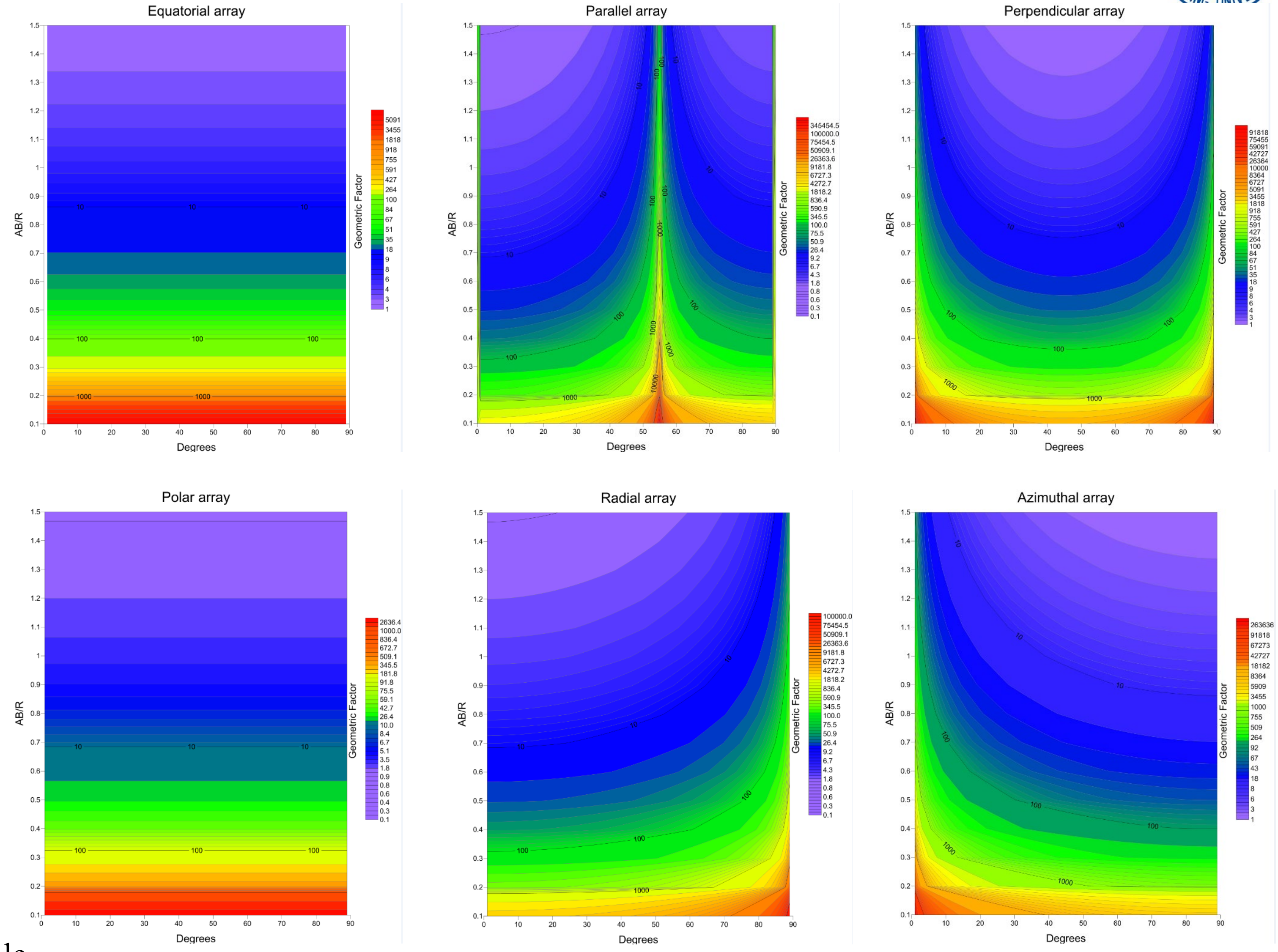
Choose higher ‘cost-effective’ configurations

Geometric factors

Layout	Nomenclature	Geometric Factor
	Equatorial array	$K_g = 2\pi r^3/bc$
	Parallel array (Broadside array)	$K_g = \dfrac{2\pi r^3}{bc(3\cos^2\theta - 1)}$
	Perpendicular array	$K_g = \dfrac{2\pi r^3}{3bc\cos\theta\ \sin\theta}$
	Radial array	$K_g=\pi r^3 / bc\cos\theta$
	Polar array (Inline array)	$K_g=\pi r^3/bc$
	Azimuthal array	$K_g=2\pi r^3 / bc\sin\theta$

Geometric factors for various dipole electrode arrays.
Note: **r** is the separation between dipole centers, **b** is the length of a measuring dipole, and **c** is the length of the source dipole.

Zhdanov,2018



Contour map of geometric factors and angles

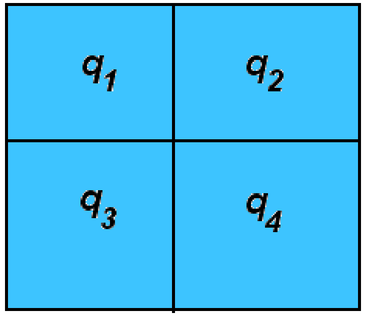
‘Compare R’ method

$$\mathbf{q}_{\text{Model}} \approx \mathbf{R}\,\mathbf{q}_{\text{Actual}}, \quad \mathbf{R} = \left(\mathbf{J}^{\mathrm{T}}\mathbf{J} + \lambda\mathbf{F}\right)^{-1}\mathbf{J}^{\mathrm{T}}\mathbf{J}$$

$$\underset{\mathbf{q}_{\text{Model}}}{\begin{pmatrix} q_{M1} \\ q_{M2} \\ q_{M3} \\ q_{M4} \end{pmatrix}} = \underset{\mathbf{R}}{\begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & R_{22} & R_{23} & R_{24} \\ R_{31} & R_{32} & R_{33} & R_{34} \\ R_{41} & R_{42} & R_{43} & R_{44} \end{pmatrix}} \underset{\mathbf{q}_{\text{Actual}}}{\begin{pmatrix} q_{A1} \\ q_{A2} \\ q_{A3} \\ q_{A4} \end{pmatrix}}$$

$$\underset{\mathbf{q}_{\text{Model}}}{\begin{pmatrix} q_{M1} \\ q_{M2} \\ q_{M3} \\ q_{M4} \end{pmatrix}} = \underset{\mathbf{R}}{\begin{pmatrix} 0.7 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.7 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.5 & 0.3 \\ 0.1 & 0.1 & 0.3 & 0.5 \end{pmatrix}} \underset{\mathbf{q}_{\text{Actual}}}{\begin{pmatrix} q_{A1} \\ q_{A2} \\ q_{A3} \\ q_{A4} \end{pmatrix}}$$

Model with 4 cells



resolution matrix: $\mathbf{R} = \left(\mathbf{G}^{\mathrm{T}}\mathbf{G} + \mathbf{C}\right)^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{G}$

For each measurement which is chosen by the target, the change in the model resolution matrix ΔR is calculated using Sherman-Morrison Rank-1 update (Wilkinson et al. 2006):

$$\Delta \mathrm{R}_{\mathrm{b}} = \frac{\mathrm{z}}{1 + (\mathrm{g}\cdot\mathrm{z})}\left(\mathrm{g}^{\mathrm{T}} - \mathrm{y}^{\mathrm{T}}\right) \qquad \text{where} \qquad \mathrm{z} = \left(\mathrm{G}_{\mathrm{b}}^{\mathrm{T}}\mathrm{G}_{\mathrm{b}} + \mathrm{C}\right)^{-1}\mathrm{g}, \ \mathrm{y} = \left(\mathrm{G}_{\mathrm{b}}^{\mathrm{T}}\mathrm{G}_{\mathrm{b}}\right)\mathrm{z}$$

Each measurement which from target selected set is ranked:

$$F_{\mathrm{CR}} = \frac{1}{m}\sum_{j=1}^{m}\frac{\Delta R_{\mathrm{b},j}}{R_{\mathrm{c},j}}$$

The average relative model resolution S: $S = \frac{1}{n}\sum_{k=1}^{n}\frac{R_{\mathrm{b},k}}{R_{\mathrm{c},k}}$

Modified Compare R method

--Target-oriented optimized

The survey design should be focused on specific target areas, which require a priori information about the subsurface properties of the data collected from a large electrode space. We select electrodes and configurations from comprehensive set firstly which meet the requirements of the target area.

After selection, the target set G_t replaces the comprehensive data set. The number of data points is far less than before and therefore the calculation time is additionally much less.

Since the data amount of G_t is much less than that of G_b , the resolution ΔR can be directly calculated with each array in G_t . So replace G_b with G_t .

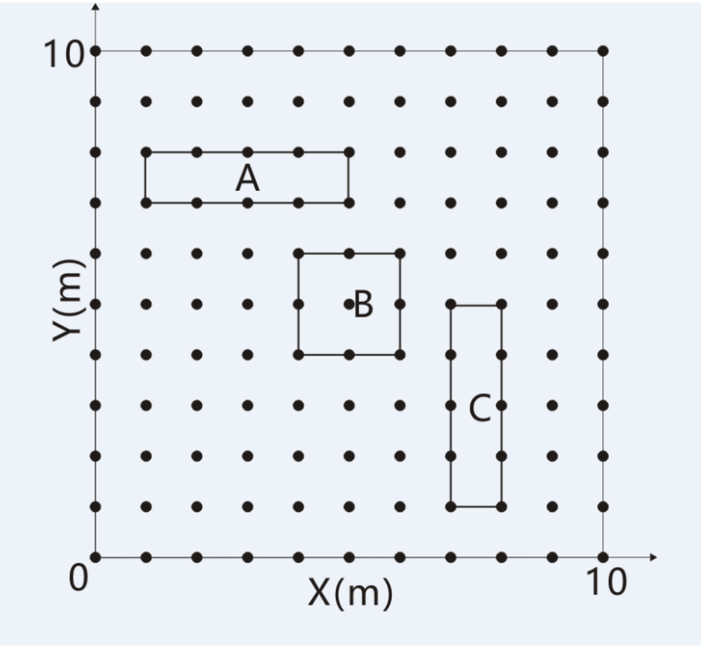
$$\Delta R_b = \frac{\mathrm{z}}{1+(\mathrm{g}\cdot\mathrm{z})}\left(\mathrm{g}^\mathrm{T} - \mathrm{y}^\mathrm{T}\right) \qquad \text{where} \qquad \mathbf{z} = \left(\mathbf{G}_\mathrm{t}^\mathrm{T}\mathbf{G}_\mathrm{t} + \mathbf{C}\right)^{-1}\mathbf{g},\ \mathbf{y} = \left(\mathbf{G}_\mathrm{t}^\mathrm{T}\mathbf{G}_\mathrm{t}\right)\mathbf{z}$$

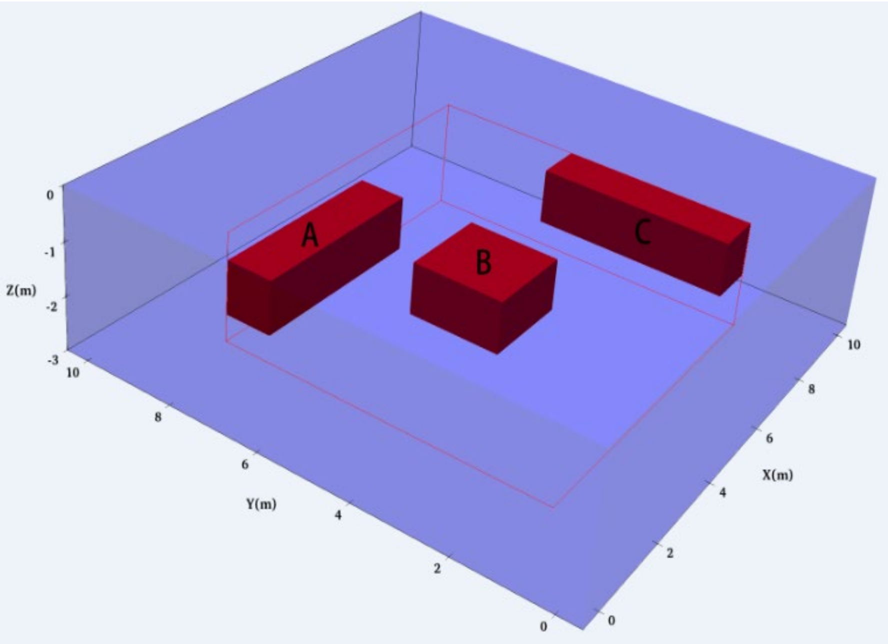
Similarly, replace R_c with R_t :

$$F_\mathrm{CR} = \frac{1}{m}\sum_{j=1}^{m}\frac{\Delta R_{\mathrm{b},j}}{R_{\mathrm{t},j}}$$

$$S = \frac{1}{n}\sum_{k=1}^{n}\frac{R_{\mathrm{b},k}}{R_{\mathrm{t},k}}$$

Synthetic test model





The comprehensive data set for the 3D ERT survey grid, which consists of the possible inline alpha and beta arrays as well as their single offset and double and triple offset line versions (Loke et al. 2014).

The comprehensive sets have 30981 arrays.
The conventional sets have 605 arrays.

Unstructured tetrahedral mesh comprising 6830 elements.

10m×10m survey grid
Electrode spacing is 1m
11 parallel lines with 11 electrodes along each line

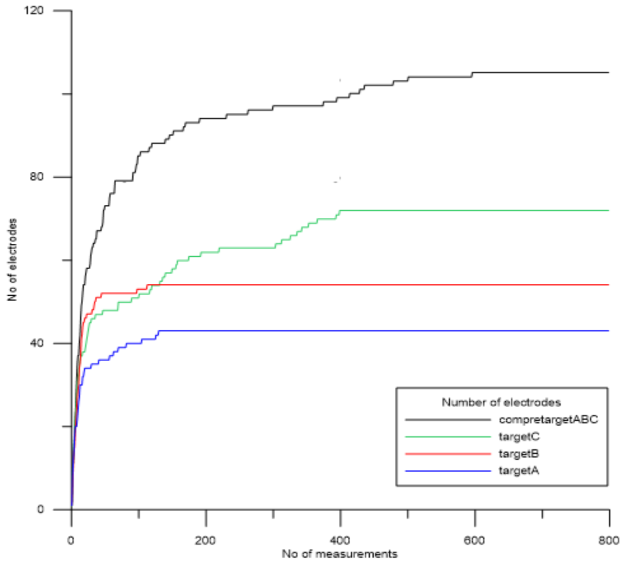
A: $4m \times 1m \times 1m$, depth range is -1m to -2m;
B: $2m \times 2m \times 1m$, depth range is -1.5m to -2.5m;
C: $1m \times 4m \times 1m$, depth range is -0.5m to -1.5m;

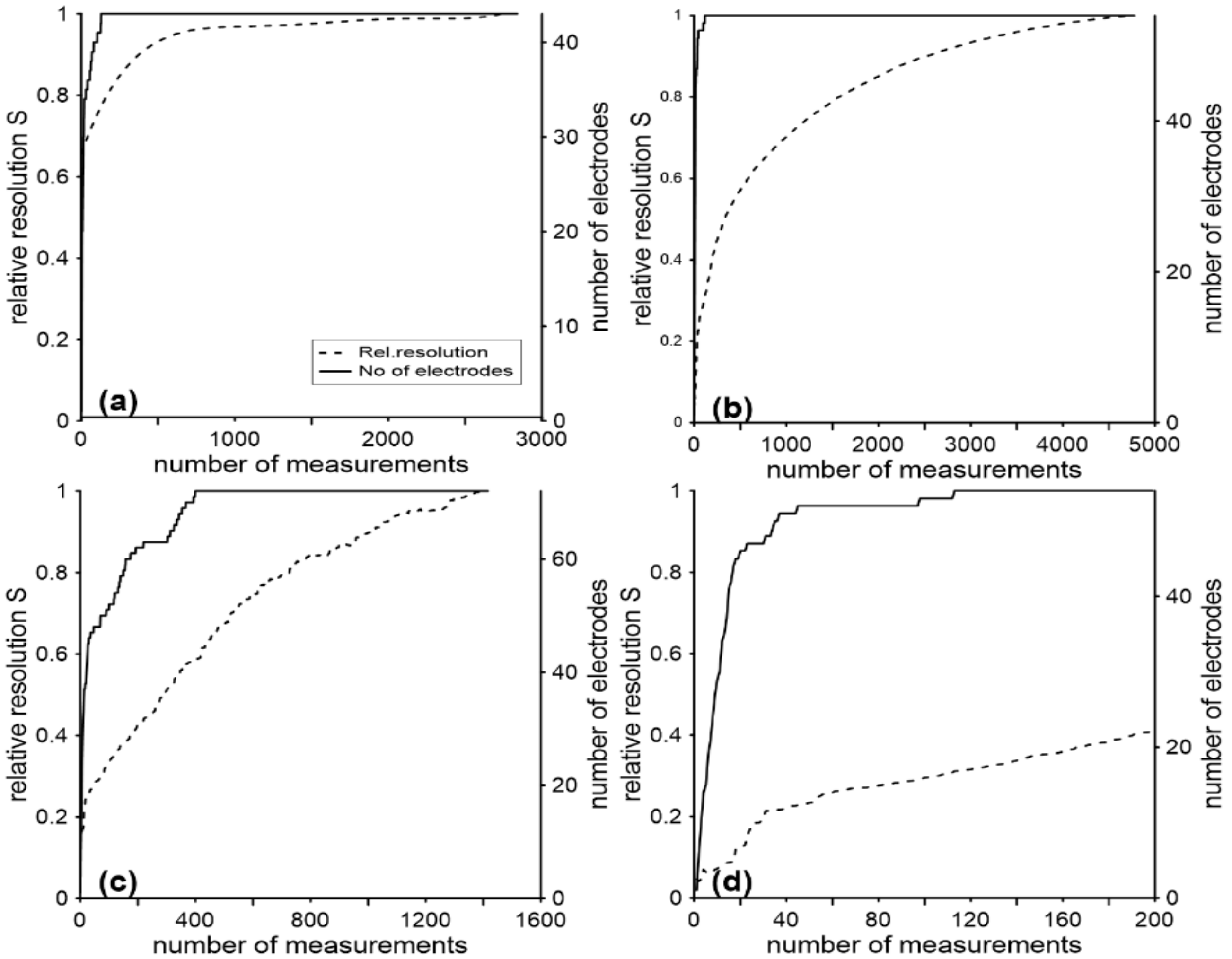
The resistivity of all three prisms is $100\Omega \cdot m$ and the background with $10\Omega \cdot m$

Target-oriented selection and the model resolution

Numbers of electrodes and measurements

Data set	Number of electrodes	Number of measurements
Comprehensive	121	30981
Target A	43	2840
Target B	54	4777
Target C	72	1416
Target all	105	9033
Conventional	121	605
Optimized	98	605





The dashed line is the relative model resolution S and the solid line is the number of electrodes, (a) target A, (b) target B, (c) target C, (d) target B (the number of measurements is only the first 200)

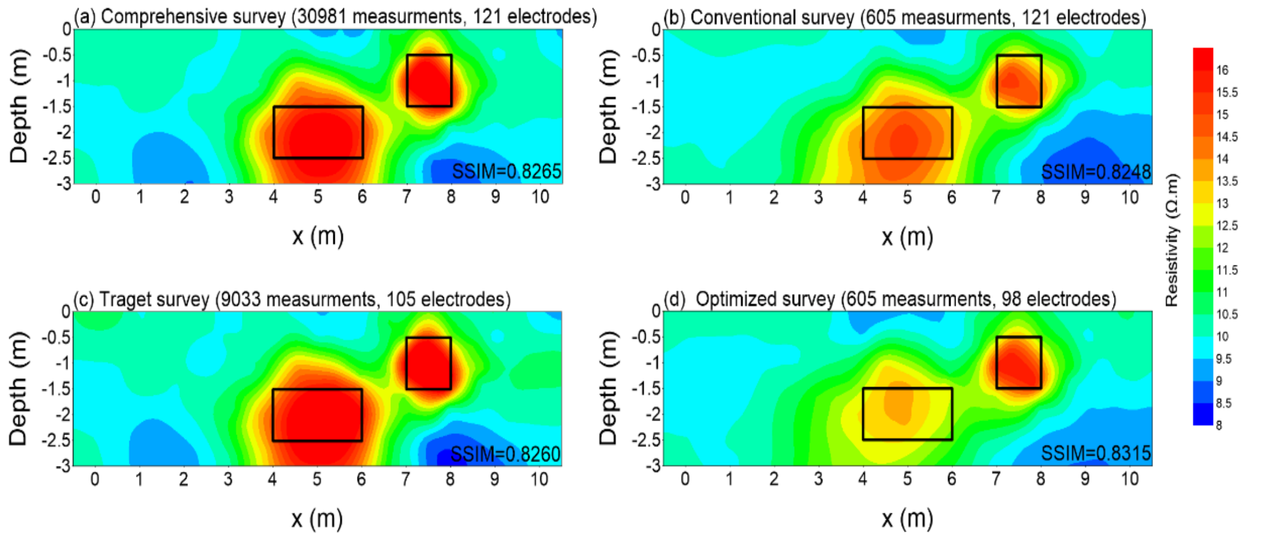
SSIM (structural similarity index)

The traditional method of evaluating RMS is not appropriate for comparing the quality of collected data by different survey designs. SSIM (structural similarity index) gives more reliable measures of image similarity better than the RMS.

It gives the value 0 when comparing the target image to a random image with the same mean and variance, and 1 if the comparison image is identical to the target. We performed SSIM calculations along the XYZ direction to get the average structural similarity values in three directions, and finally got the overall average SSIM. These values can show the structural similarity between the inversion results in all directions in 3D and also the real model.

The average SSIM of three directions

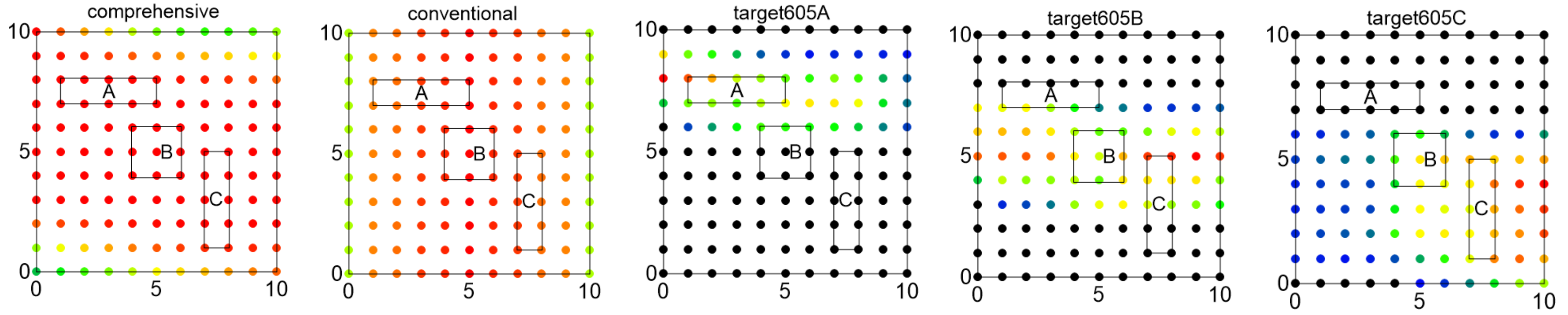
Data set	X	Y	Z	average
Comprehensive	0.8983	0.8936	0.8876	0.8932
Target all	0.8965	0.8914	0.8852	0.8910
Conventional	0.8927	0.8882	0.8836	0.8882
Optimized	0.8928	0.8883	0.8846	0.8886



Inversion for (a) comprehensive survey, (b) conventional survey, (c) target all survey (consisting A, B, C) and (d) optimized set. The actual positions of the blocks are marked by black rectangles.
Location: y=5m

Frequency of electrodes utilized

The frequency of the electrode utilized can intuitively guide us how to choose a part of the electrodes with higher utilization rate. In the subsequent optimization and selection, more reference standards are provided by this frequency.



Rainbow scale

Black is unused, blue is low frequency, red is high frequency

References

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