Waveform Energy Focusing Tomography for Passive Sources

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Introduction

With the improvement of computing capacity, full waveform inversion (FWI), which exploits information of the full wave-field, becomes one of the most attractive geophysical inversion methods for not only exploration geophysics (Virieux & Operto 2009) but also global seismology (Fichtner *et al.* 2013). Despite its high resolution and successful practical applications, there still exist several obstacles to the successful application of FWI for passive earthquake sources, such as the high non-linearity for model convergence and demand for accurate source information.

We propose a new method called Waveform Energy Focusing Tomography (WEFT) for passive earthquake events, which back-propagates the seismic record from the receivers, not the data residuals like in conventional FWI, and tries to maximize the back-propagated wave-field energy around the source location over a short period around the origin time. The least-squares moment tensor migration approach is used to reconstruct the passive sources, and the Hessian matrix is approximated using either analytic expression or raytracing, which improve the accuracy of reconstructed source. Since waveform fitting is superseded by simpler energy maximization, the nonlinearity of WEFT is weaker than that of FWI, and even less-accurate initial velocity model can be used.

Methods

According to the reciprocity theorem, the energy focuses at the source location and the origin time if the seismic record is back-propagated in the correct model. So energy focusing can be served as the criterion of an optimized velocity model. The objective function (Jin & Plessix 2013) to be minimized with a negative sign is:

$$\boldsymbol{E} = -\frac{1}{2}\sum_{s=1}^{ns}\int_{\widetilde{t_0}}^{t_{max}}\int_{\mathbf{x}} w(\mathbf{x},t)|P_b(\mathbf{x},t)|^2 dt d\mathbf{x}$$

where $P_b(\mathbf{x},t)$ stands for the back-propagated wave-field, $\widetilde{t_0}$ and t_{max} define the time window. The weighting function $w(\mathbf{x},t)$ defines a tapered volume.

The derivative of $\boldsymbol{E}$ with respect to velocity can be expressed as:

$$\frac{\partial \boldsymbol{E}}{\partial c(\mathbf{x})} = \frac{1}{c^3(\mathbf{x})}\sum_{s=1}^{ns}\int_{t_0}^{t_{max}} w(\mathbf{x},t)\ddot{P}_f(\mathbf{x},t;\mathbf{x}_s)P_b(\mathbf{x},t;\mathbf{x}_s)\,dt$$

where $\ddot{P}_f(\mathbf{x},t;\mathbf{x}_s)$ indicates the second-order time-derivative of the forward-propagated wave-field and $P_b(\mathbf{x},t;\mathbf{x}_s)$ indicates the back-propagated wave-field. A quadratic line-search method (Nocedal & Wright 1999) is applied to find a step length to update model.

The moment tensor can be obtained approximately by time-reversal operation (Kawakatsu & Montagner, 2008):

$$\widehat{M}_{ij}(t) = \sum_r E_{ijn}(\boldsymbol{\xi},t;\boldsymbol{x},0) * u_n(\boldsymbol{x},-t)$$

where E_{ijn} is the strain Green's tensor (Zhao *et al*. 2006).

To give a better estimation of the moment tensor, we use the least squares solution:

$$M_{ij}(t) = (G^*G)^{-1}\widehat{M}_{ij}(t)$$

where inverse Hessian $(G^*G)^{-1}$ can be approximated using analytical expressions. In a homogeneous medium, the Green's functions for P-wave in the far field is (Aki & Richards, 1980):

$$G_{np,q} = \frac{\gamma_n\gamma_p\gamma_q}{4\pi\rho v^3} * \frac{1}{r}$$

where $r = |\mathbf{x} - \boldsymbol{\xi}|$ is the distance between a receiver and a source, and $\gamma_i = (x_i - \xi_i)/r$ is the direction cosine. In a heterogeneous medium, we use ray-tracing to better estimate γ_i. It should be emphasized that only the Green's functions in the inverse Hessian are approximated using analytical expressions and ray-tracing.

Numerical Examples

To avoid the large amount of computation, we show synthetic tests in 2D to illustrate the validity of WEFT. Considering that the analytic derivatives of the Green's functions in 2D is too complicated, we decide to reconstruct the source in 3D by transforming data from 2D to 3D. The finite-difference method is used to simulate the propagation of the wave-field according to the acoustic wave equation.

We show the comparison of the reconstructed source components in Fig. 1. After the application of least-squares term, we can find both the amplitudes and the polarity of the reconstructed moment tensor become quite consistent with the true values in all components. The gradient comparison between WEFT and conventional FWI with a moment tensor source which has polarity reversal is shown in Fig. 2. Although the gradients of WEFT are slightly different from those of FWI, overall they are similar.

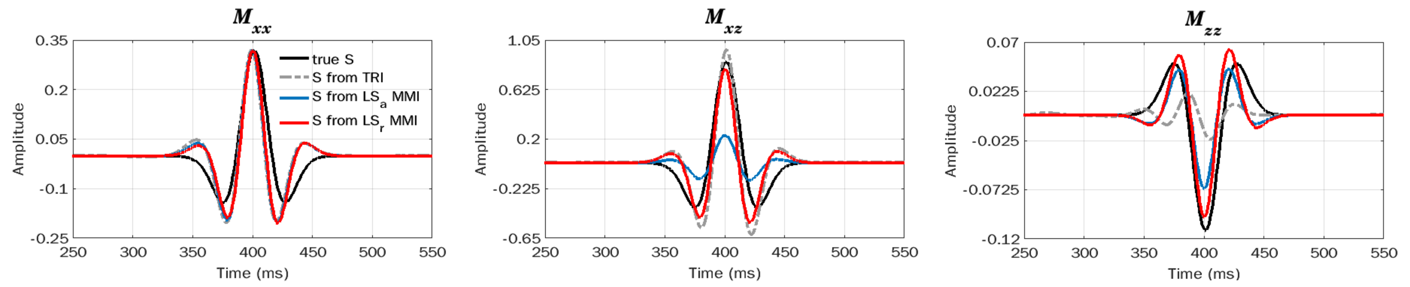

Figure 1. *The reconstructed source components of the double-couple source. The black lines are true source components. The gray dash lines indicate the reconstructed source components using time-reversal moment tensor imaging. The blue lines and red lines indicate the reconstructed source components using LS weighted moment tensor migration inversion based on the analytic method and ray tracing, respectively. The amplitudes are scaled by the actual value of M_{xx}.*

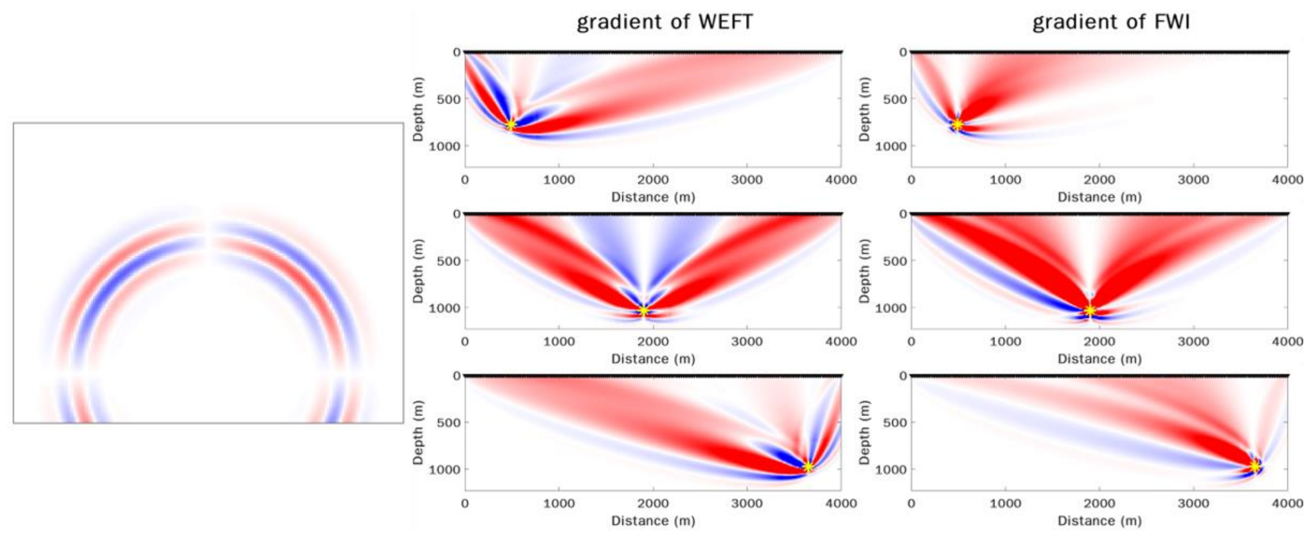

Figure 2. *The comparison of stacking gradients between WEFT (middle) and FWI (right). The yellow asterisks at the bottom denote the sources, and the black triangles at the top denote the 201 receivers with 20m spacing. The radiation pattern of the source is shown on the left.*

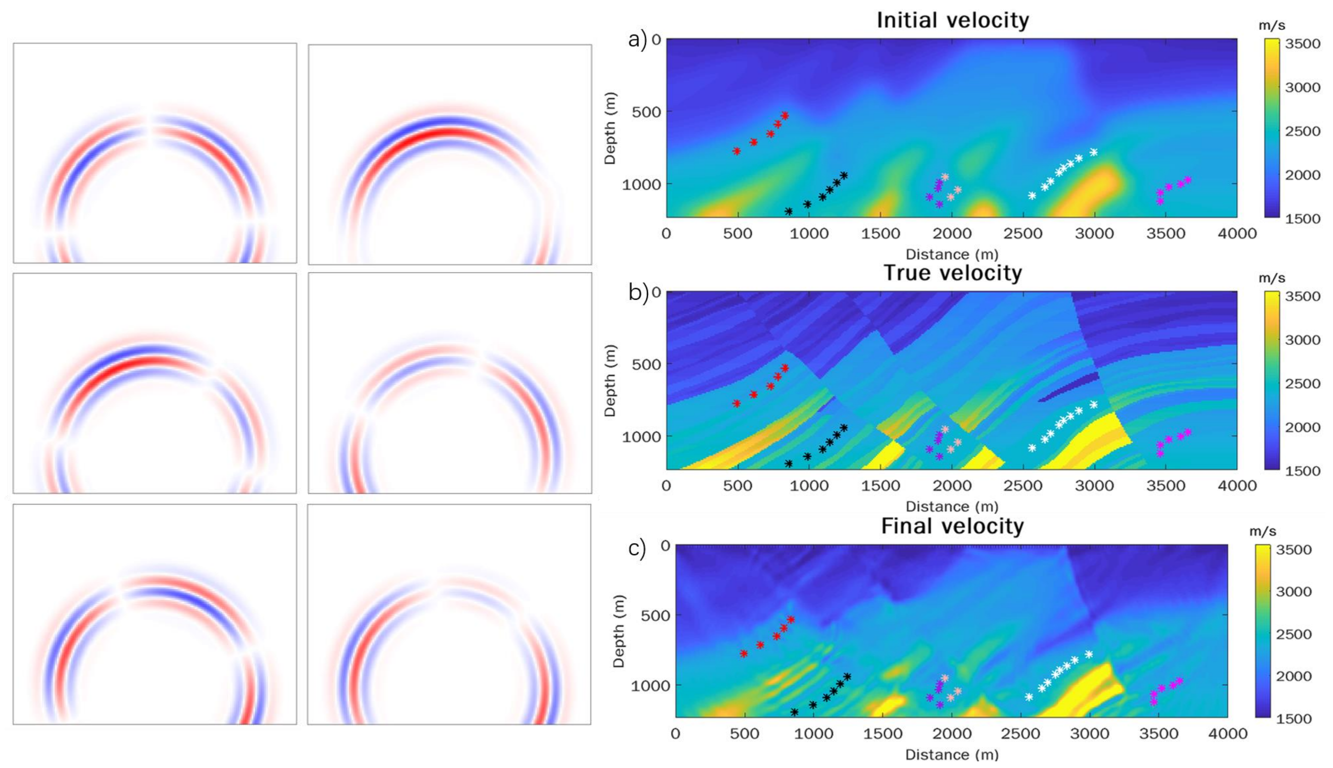

Figure 3. *Marmousi model velocity inversion using earthquakes with 6 kinds of source radiation patterns (left). The initial velocity model is shown in the first row on the right, and the true model and final inverted velocity model is shown in the second and third row, respectively. The earthquakes indicated by stars with different colors are of different moment tensors.*

In Fig. 3, we then show an example of velocity inversion using WEFT. There are six groups of earthquakes with different moment tensors (left) are used to generate the observed data. Though it is challenging for passive source waveform tomography, the inverted model compares favourably with the true model (third row on the right). The synthetic test illustrates the universality of WEFT for sources with various radiation patterns.

Conclusions

We propose a new method called waveform energy focusing tomography for passive earthquake events. Both the source time function and the moment tensor are not required before inversion. By incorporating ray-theoretical Green's function into the analytic expression of the Hessian matrix, the least-squares weighted moment tensor migration inversion is used for the source reconstruction during the WEFT, which improves the accuracy of the reconstructed source. Since WEFT concerns about energy focusing rather than waveform fitting, it has lower non-linearity and is less dependent on both the initial model and source accuracy. These advantages of WEFT make it more practical for challenging earthquake data, especially for local small magnitude earthquakes where both velocity model and earthquake source information are unknown.

References

Virieux, J. & Operto, S., 2009. An overview of full-waveform inversion in exploration geophysics, *Geophysics*, **74**, WCC1–WCC26.

Fichtner, A., Trampert, J., Cupillard, P., Saygin, E., Taymaz, T., Capdeville, Y. & Villasenor, A., 2013. Multiscale full waveform inversion, *Geophys. J. Int.*, **194**(1), 534-556.

Jin, S. & Plessix, R.-E., 2013. Stack-based full wavefield velocity tomography, in *83rd Annual International Meeting*, SEG, Expanded Abstracts, pp. 1095–1099.

Kawakatsu, H & Montagner, J-P., 2008. Time-reversal seismic-source imaging and moment-tensor inversion, *Geophys. J. Int.*, **175**, 686–688.

Nocedal, J. & Wright, S.J., 1999, Numerical optimization: Springer series in operations research and financial engineering.

Zhao, L., Chen, P. & Jordan, T.H., 2006. Strain Green tensors, reciprocity and their applications to seismic source and structure studies, *Bull. Seis. Soc. Am.*, **96**(5), 1753–1763.

Aki, K. & Richards, P., 1980. *Quantitative seismology: Theory and Methods*, W.H. Freeman, New York.