Evaluating the effect of wave-particle cross diffusion in radiation belts modelling using an innovative and robust numerical scheme

EGU 2020/ ST2.7

Nourallah DAHMEN[1], Vincent MAGET[1], François ROGIER[2]
[1] ONERA/DPHY, Toulouse, France
[2]ONERA/DTIS, Toulouse, France

ONERA
THE FRENCH AEROSPACE LAB

Dynamics of electron radiation belts : Intense diffusive frame

Fokker – Planck equation for radiation belt

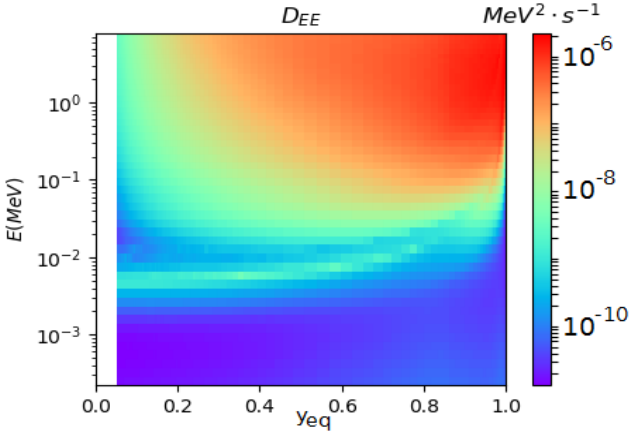
- **Multi-physics** — Field fluctuations, Coulomb interactions, collisions / frictions , wave-particle interactions, …
- **Multi-scale** — $\tau_d \gg \tau_g, \tau_b$ / $\neq$ wave characteristics…

$$\frac{\partial f}{\partial t} = \underbrace{\frac{1}{G_1}\frac{\partial}{\partial y}\Big|_{E,L}\left(G_1 D_{yy}\frac{\partial f}{\partial y}\Big|_{E,L} + G_1 D_{yE}\frac{\partial f}{\partial E}\Big|_{E,L}\right) + \frac{1}{G_2}\frac{\partial}{\partial E}\Big|_{y,L}\left(G_2 D_{EE}\frac{\partial f}{\partial E}\Big|_{y,L} + G_2 D_{yE}\frac{\partial f}{\partial y}\Big|_{y,L}\right)} + \frac{1}{G_3}\frac{\partial}{\partial L}\Big|_{\mu,J}\left(G_3 D_{LL}\frac{\partial f}{\partial L}\Big|_{\mu,J}\right)$$

Local diffusion on (y, E) in outer electron belt
Wave-particule interaction inducing D_{EE}, D_{yy} and D_{yE}

High level of inhomogeneity

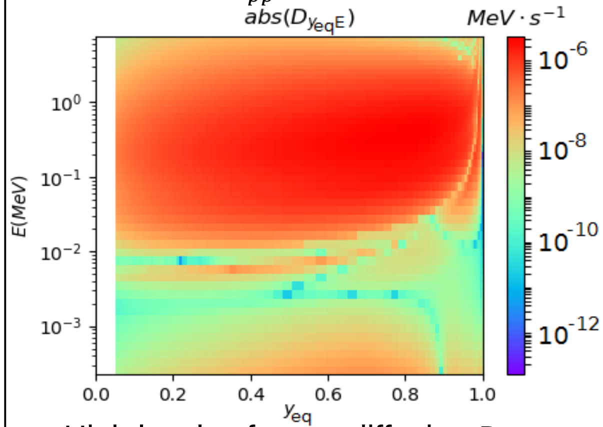
$L^* = 4.49 > L_{pp}$



- Steep spatial evolutions
- Sharp resonances

High levels of anisotropy

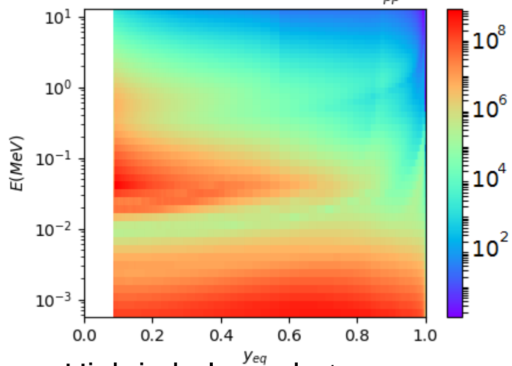
$L^* = 4.49 > L_{pp}$



- High levels of cross diffusion D_{yE} due to wave-particle interactions
- Changing sign

$\bar{\bar{D}} = \begin{pmatrix} D_{yy} & D_{yE} \\ D_{yE} & D_{EE} \end{pmatrix}$: diffusion tensor

Anisotropy ratio $= \frac{\max(eigenval(\bar{\bar{D}}))}{\min(eigenval(\bar{\bar{D}}))}$



- High imbalance between principal diffusion directions

ONERA

Harsh numerical contraints on finite difference based solvers

Typical numerical method used to solve Fokker-Planck equation

Centered finite difference (FD) approximation

+ Easy implementation

- BUT

High Inhomogeneity

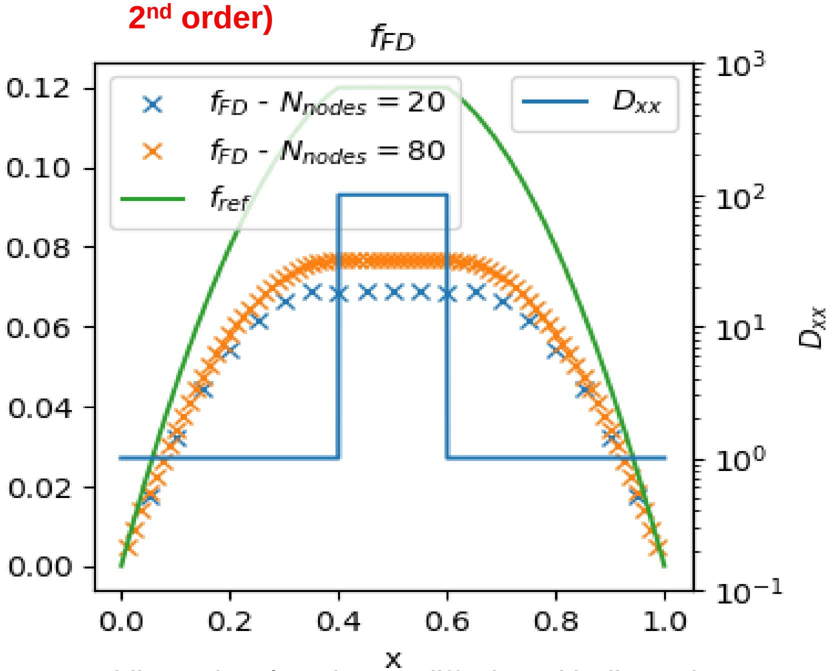
⇒ Noxious impact on FD approximation errors (loss of 2nd order)

Anisotropy + Cross diffusion

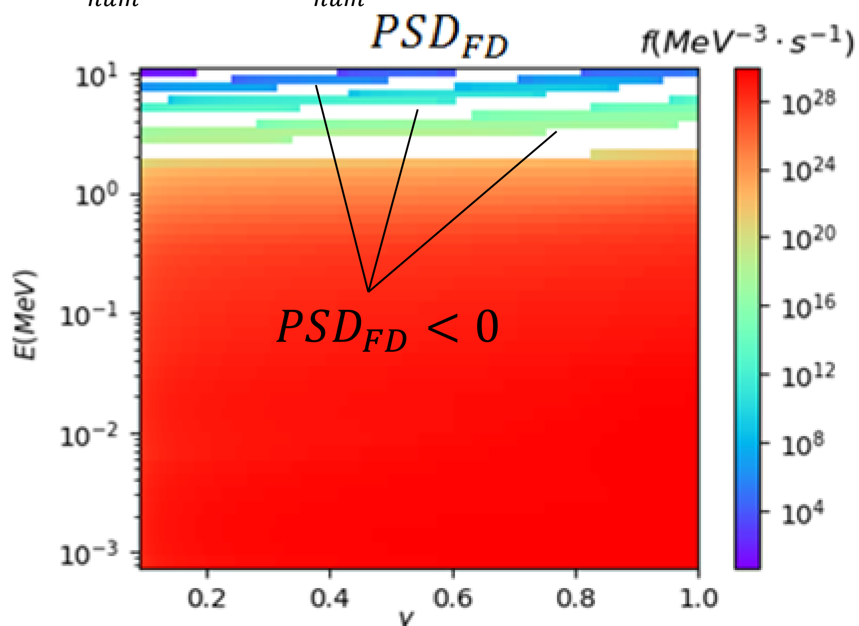
⇒ Impact on stability

⇒ Impact on physical representation

$PSD_{num} < 0 \; or \; PSD_{num} > \max$



FD test on unidimensional stationary diffusion with discontinuous coefficient



$FD\ PSD\ t_{simu} = 90\ 000s - L^* = 4.49 > L_{pp} - D_{yE} \neq 0$

Finite volume schemes : conservative and monotone alternatives

Finite volume (FV)

Conservative formulation
Mass/Energy conserved

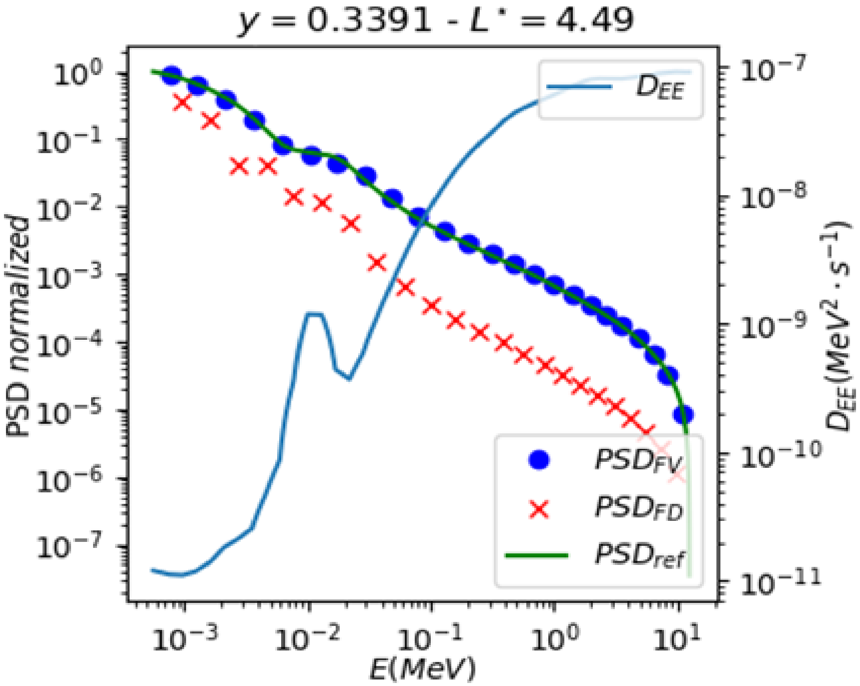
Suitable for diffusion equation

Physically relevant variants
Positivity and maximum principle preserving

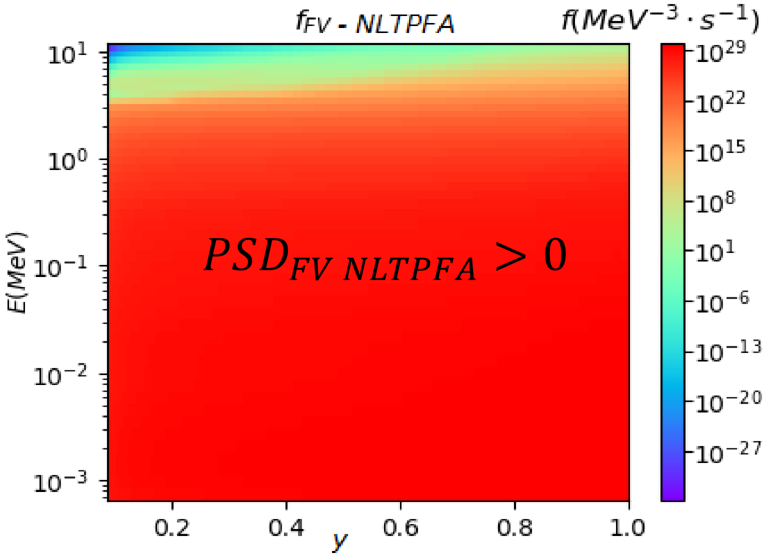
Robust scheme despite $D_{y_{eq}E} \neq 0$

Numerical flux approximation
Better handling of spatial inhomogeneity

Accurate approximations

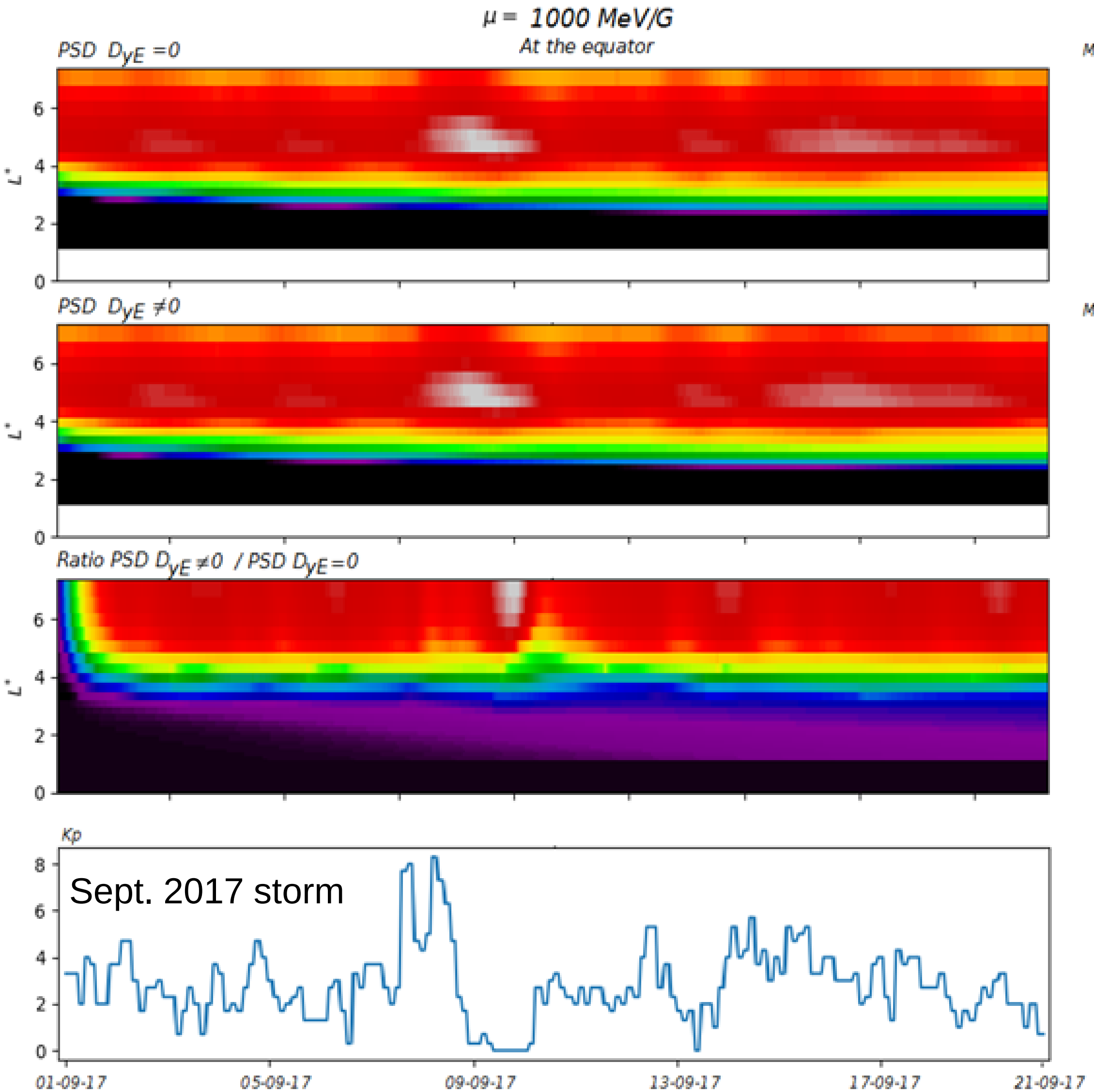


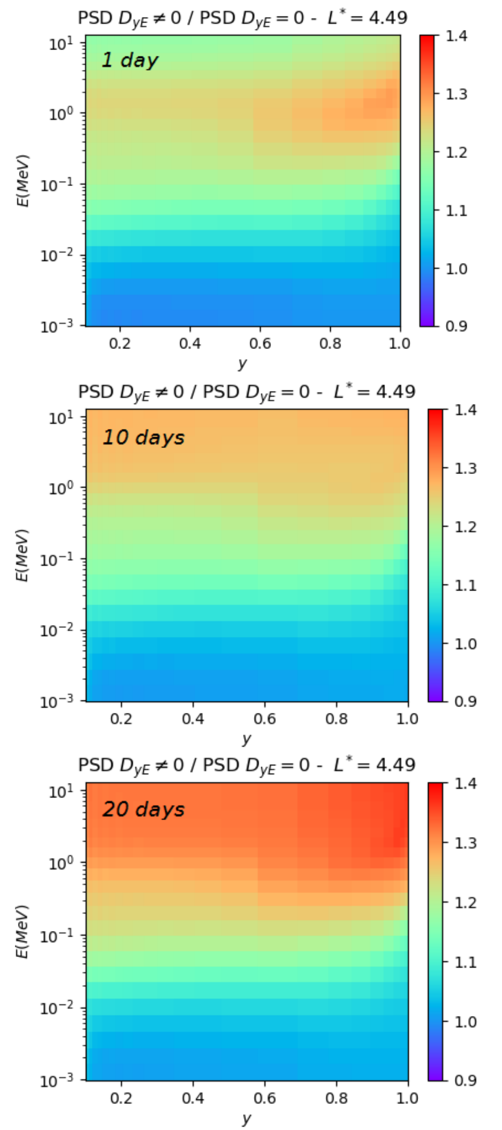
For more details: **"On the modelling of highly anisotropic diffusion for electron radiation belt dynamic codes"**
Nour Dahmen, François Rogier, Vincent Maget
Computer Physics Communications, 2020
https://doi.org/10.1016/j.cpc.2020.107342



ONERA

Real case storm simulations using the FV montone scheme (under study) - 1





Real case storm simulations using the FV montone scheme (under study)