TGF: A New Matlab-based Software for Terrain-related Gravity Field Calculation

EGU2020-3765

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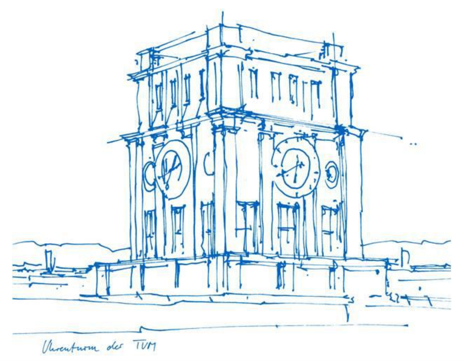

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Abstract

The TGF is a new MATLAB-based program suitable for gravity forward modelling computations in the spatial domain. It has been developed at TU Munich. Our new software is capable of calculating the gravity field generated by some topographic mass-density distribution through a combination of four different mass elements: polyhedron, prism, tesseroid and point-mass. Topographic mass density models (elevation and mass-density grids) can be used with different resolution, e.g., 1” near the computation and 15” for far-zone effects. The TGF software calculates the gravity contribution of chosen concentric zones around the calculation point. The user can manually define the radius of each zone, has the choice between ten different gravity field functionals (potential, 1st and 2nd order derivatives also known as gravity tensor).
In this contribution, the TGF software is introduced to the geodetic community and its capabilities in RTM calculation is validated. Results from numerical validation experiments of TGF are presented.

1. Forward modelling in the Space domain

- **Discretization and regularization:** as displayed in Fig. 1. Newtonian integration for i-th volume v_i with constant density $\rho_i(Q)$

$$V_i(P) = G \int_{v_i} \frac{\rho_i(Q)}{l_i(P,Q)} dv \quad (1)$$

- **Evaluation of geometric gravitational field:** closed solution of polyhedron please refer to Tsoulis (2012), of prism (Nagy et al. 2000), and numerical solution of tesseroid (Grombein et al. 2013);
- **Superposition summation:** the composite gravity effect over calculation point is then obtained:

$$V = \sum_{i=1}^{N} V_i \quad (2)$$

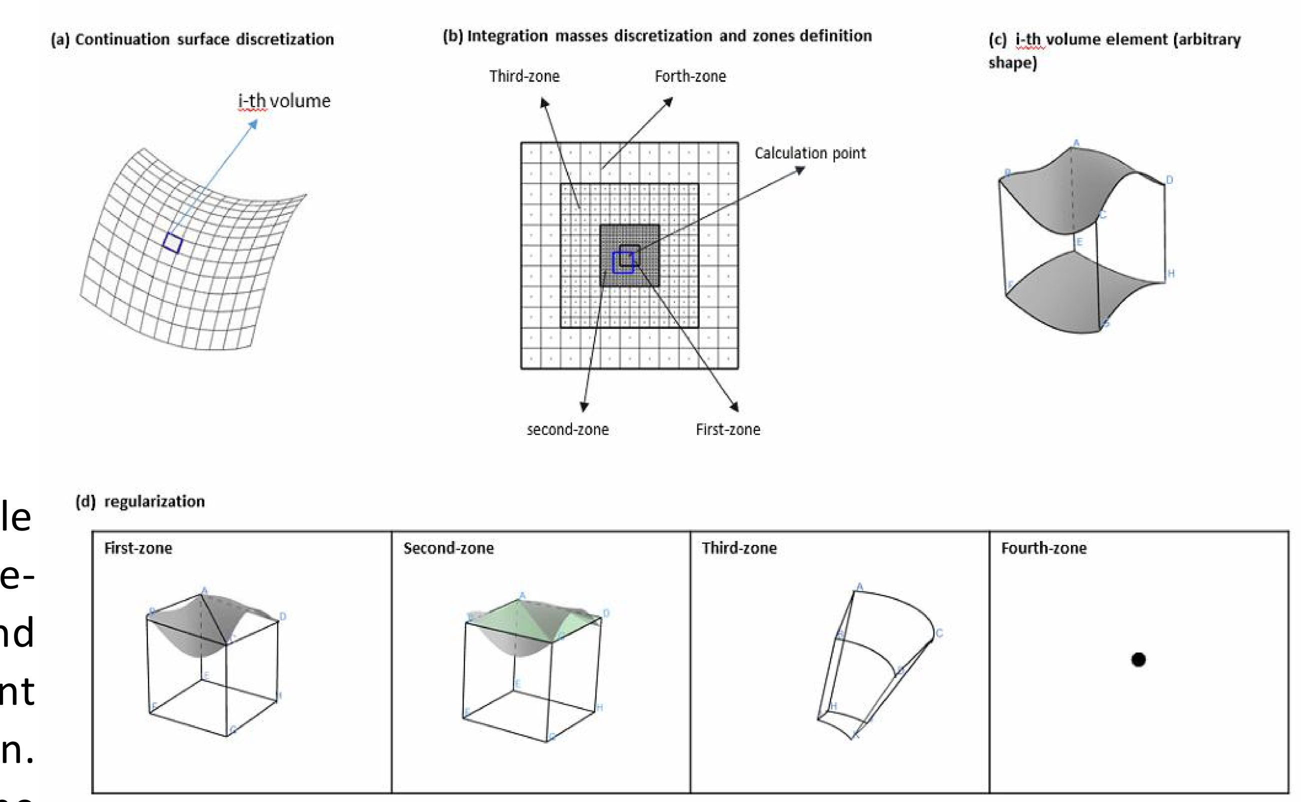

Fig 1. Discretization and regularization of the mass-distributions

Because the polyhedron using DEM grid center as corners, while prism center is coinciding with grid center, there are a square-circle leakage of half DEM resolution between prism zone and polyhedron zone shown in Fig. 2 -- representing the adjacent zone (blue square area in Fig. 1 (b)) of prism and polyhedron. These masses are evaluated based on prism assumption in the TGF software.

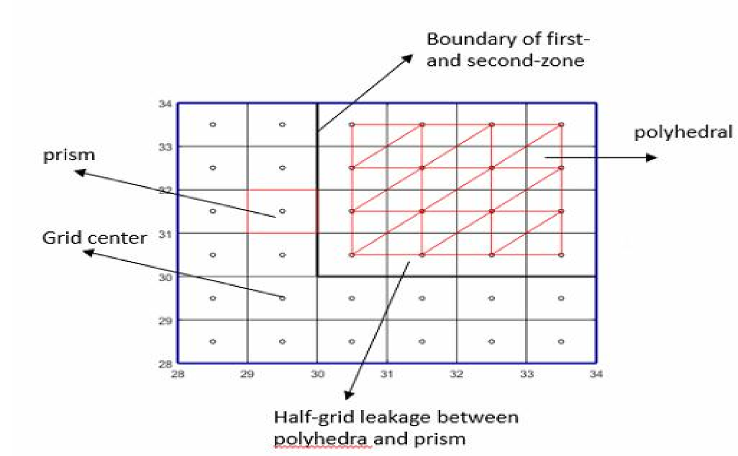

Fig. 2. Leakage problem between polyhedral-zone and prism-zone

2. TGF: structures and functions

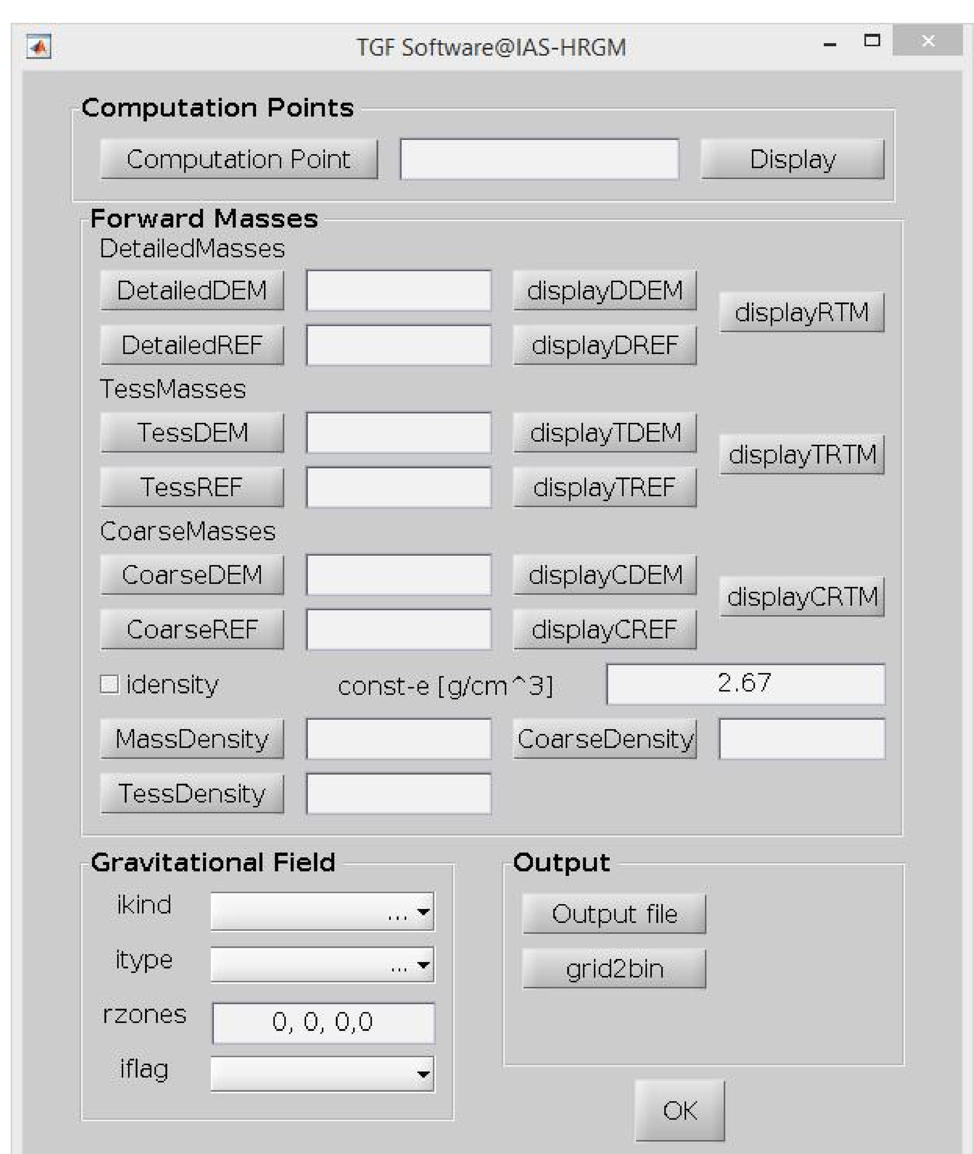

Fig 3. TGF software GUI interface

Running in GUI interface: Fig. 2 shows the working interface of TGF software. It is divided into four parts:

- **‘Computation Points’**
- ‘Forward Mass’
- ‘Gravitational Field’: parameters input (Table 1)
- ‘Output’:

$$\zeta = \frac{T}{\gamma}$$

$$\xi = -\frac{1}{r\gamma}\frac{\partial T}{\partial \varphi} \qquad \Gamma = \begin{bmatrix} T_{xx} & T_{yx} & T_{zx} \\ T_{xy} & T_{yy} & T_{zy} \\ T_{xz} & T_{yz} & T_{zz} \end{bmatrix}$$

$$\eta = -\frac{1}{r\gamma\cos\varphi}\frac{\partial T}{\partial \lambda}$$

$$\Delta g = -\frac{\partial T}{\partial r} - \frac{2}{r}T \quad (3)$$

$$\delta g = -\frac{\partial T}{\partial r}$$

Table 1. Parameter specification for TGF forward modeling

Parameter	Interpretations
idensity	Flag for mass-density: 0 – constant value; 1 – density map;
ikind	Flag for type of modelling: 1 – topographic masses; 2 – RTM masses;
itype	Specification of field functionals: $\zeta, \xi, \eta, \Delta g, \delta g, \Gamma$
rZones	Vector of four elements specifying the computation zones in [degree], rZones = [r1 r2 r3 r4]:
iflag	Flag for Earth approximation: 0 – spherical approximation; 1 – ellipsoidal approximation;

3. Efficiency in RTM gravity field calculation

- ✓ Internal testbed $\delta g_{testbed}$ with $r_1 = 0.33°, r_2 = r_1, r_3 = 1°, r_4 = 2°$;
- ✓ RTM gravity field δg_{RTM} with each set of (r_1, r_2, r_3, r_4), where $r_1 = 0{:}\,0.01{:}\,0.33°$, $r_2 = 0{:}\,0.01{:}\,0.33°$, and fixed $r_3 = 0.15°, r_4 = 2°$
- ✓ Residuals $\Delta\delta g = \delta g_{RTM} - \delta g_{testbed}$ indicate the error level that can be attributed to the vary of regularization with polyhedron and prism (Fig. 4).
- ✓ Further experiment investigates the choices of tesseroid and point-mass (Fig. 5), with $r_3 = 0.05{:}\,0.05{:}\,1°$, $r_4 = 0.2{:}\,0.1{:}\,2°$, and fixed $r_1 = 0.03°, r_2 = r_1$.

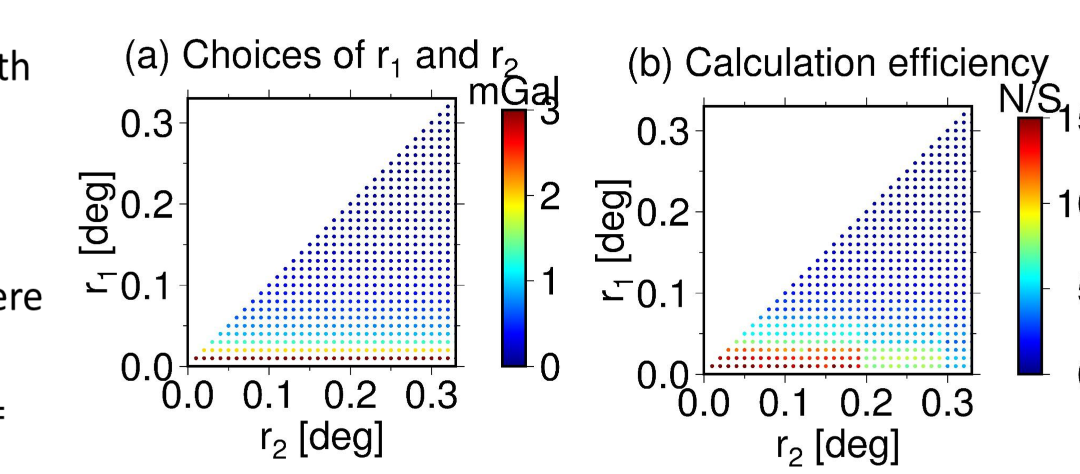

Fig 4. Forward modelling evaluation accuracy and efficiency with various radius choices of polyhedra and prism

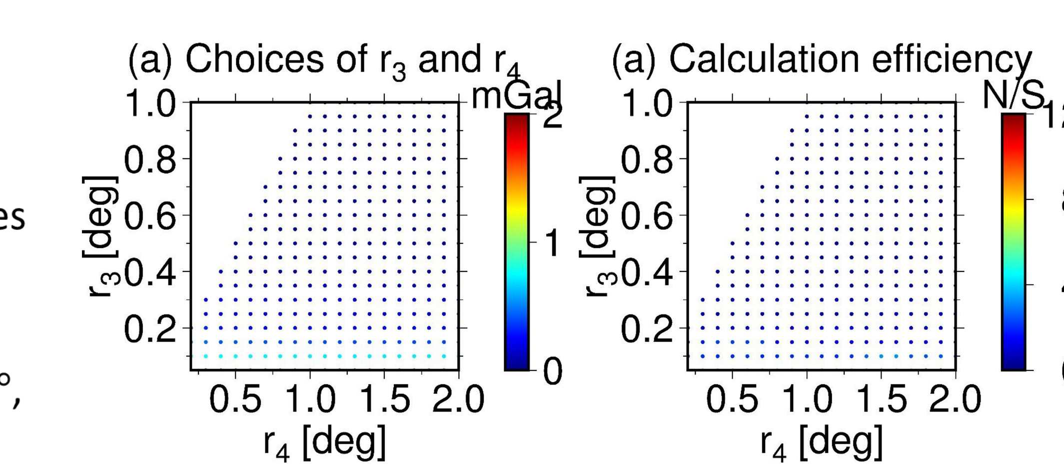

Fig 5. Forward modelling evaluation accuracy and efficiency with various radius choices of tesseroid and point-mass

$r_1 = 0.03°, r_2 = 0.03°, r_3 = 0.15°, r_4 = 0.8°$ are recommended for 1 mGal accurate RTM gravity signal retrieving using TGF software.

4. External validation for TGF in RTM gravity field calculation

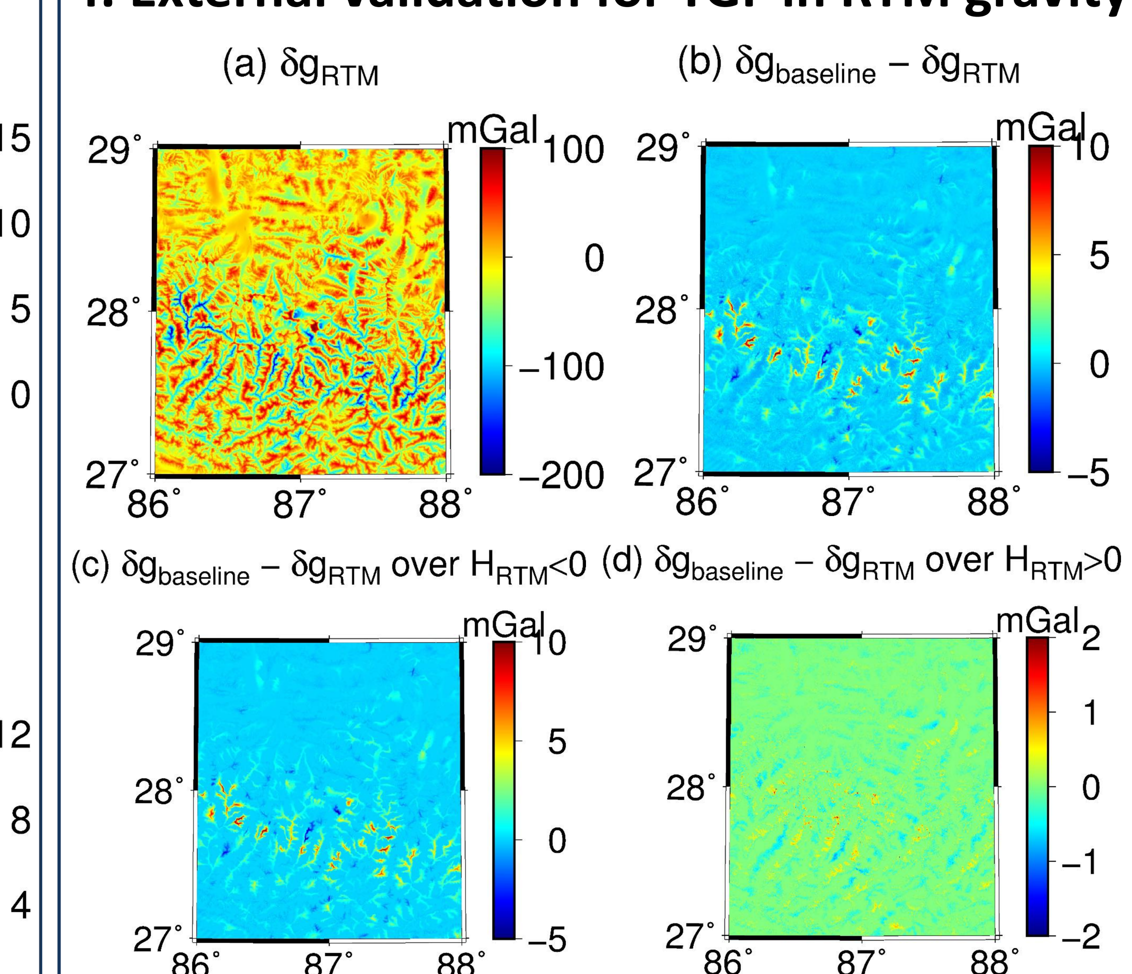

Fig 6. External validation based on baseline (Hirt et al. 2019) over Himalaya mountainous area. H_{RTM} represents RTM height.

Table 2. External validation For RTM gravity calculation over Himalayas regions (Unit: mGal).

	min	max	mean	rms
δg_{RTM}	-224.06	109.05	-11.76	39.52
$\delta g_{baseline} - \delta g_{RTM}$	-6.13	11.60	0.16	0.78
$\delta g_{baseline} - \delta g_{RTM}$ with $H_{RTM} \geq 0$	-2.28	2.36	-0.04	0.22

The RMS differences between TGF-based RTM gravity disturbances and reference values from global numerical integration and SGM were found to be smaller than 0.8 mGal over Himalaya areas (Table 2.). The extreme values generally have the synchronized distribution with negative residual height $H_{RTM} < 0$ (Fig 1.), which could be attributed to the harmonic correction approach $4\pi G\rho H_{RTM}$ (Forsberg, 1984) used in the TGF software (Hirt et al., 2019).

5. Conclusions

1) We have presented the new MATLAB-based software (TGF) for gravity forward modelling, its structure and the combination design of four methods;
2) The numerical validations demonstrated the accuracy and efficiency of TGF in the RTM forward modelling, and its potential in processing high-resolution DEMs associate with roughest topography.
3) The TGF software has been extensively tested and recently been applied in the SRTM2gravity project (Hirt et al. 2019) to convert the global 3” SRTM DEM to implied gravity effects at 28 billion computation points.

References

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