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Unification of global vertical height system using precise frequency signal links

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Introduction

- The realization of International Height Reference System (IHRS) plays a key role in Earth science.
- A main component of the IHRS realization is the global vertical datum unification.
- It requires the connection of the existing local vertical height reference systems (VHS).
- The connection of VHSs is difficult when they are far apart or separated by the ocean.

Introduction

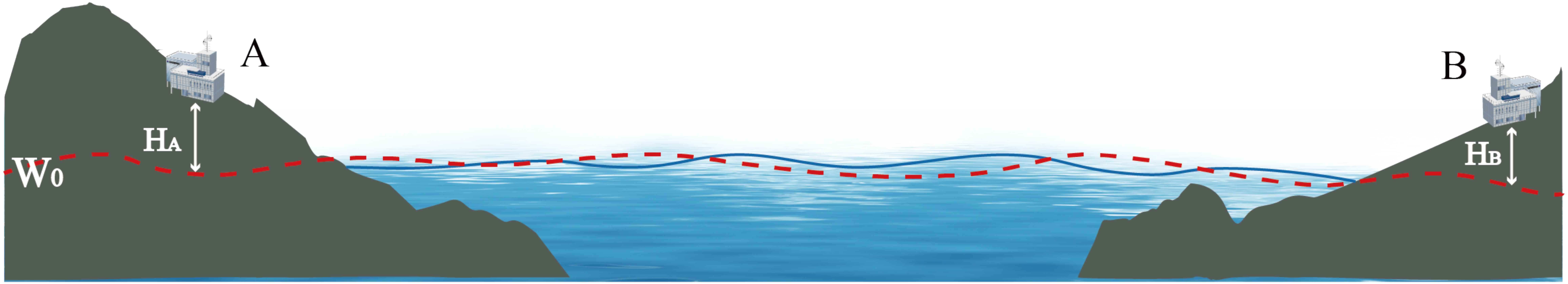

Fig. 1 Red dashed curve denotes the zero-height surface (an equipotential surface), and the solid blue curve denotes the mean sea level.

- In this report we formulate a framework for connecting two local VHSs
- Basic principle: the theory of General Relativity (GR)
- Basic method: ultra-precise frequency signal links between satellites and ground stations

Introduction

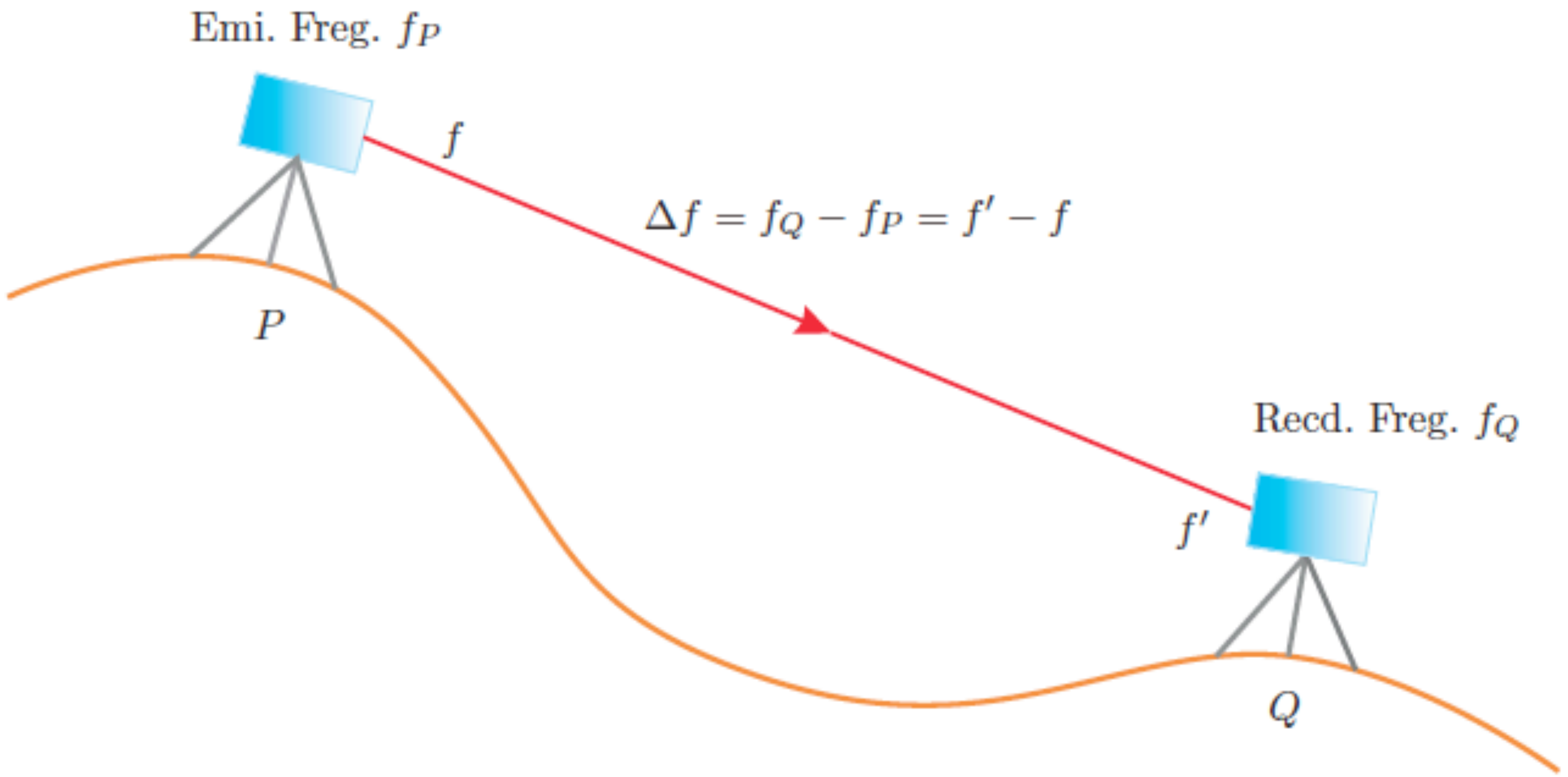

Fig. 2. A receiver at Q receives signal with frequency f emitted by an emitter at P

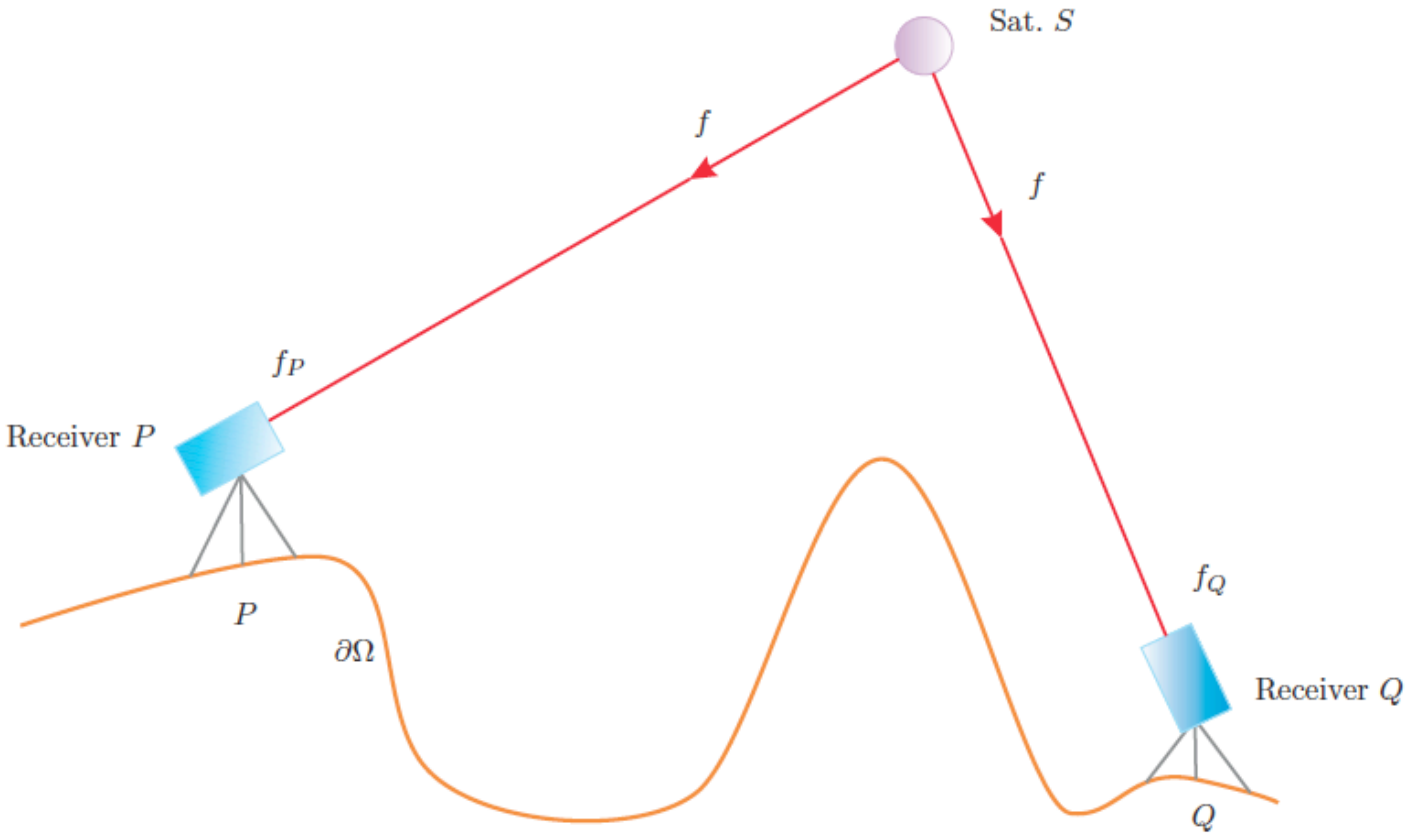

Fig. 3. P and Q receive signals from Sat. S simultaneously. Geopotential difference between P and Q is determined based on frequency shift between P and Q

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Height reference system and the SFST method

Vertical height reference system

- According to IHRS, the geopotential on the geoid is a constant value W_0, and the vertical coordinates are defined as

$$C_P = -\Delta W_P = W_0 - W_P$$

- The geopotential number C_p can be converted to a physical height H_P by the following equation

$$H_P = \frac{C_P}{\hat{g}} = \frac{W_0 - W_P}{\hat{g}}$$

- If H_P is orthometric height (OH), which s a geometric length measured along the plumb-line

$$\hat{g} = \bar{g} = \frac{1}{H_P}\int_0^{H_P} g(h)dH_P$$

Height reference system and the SFST method

Vertical height reference system

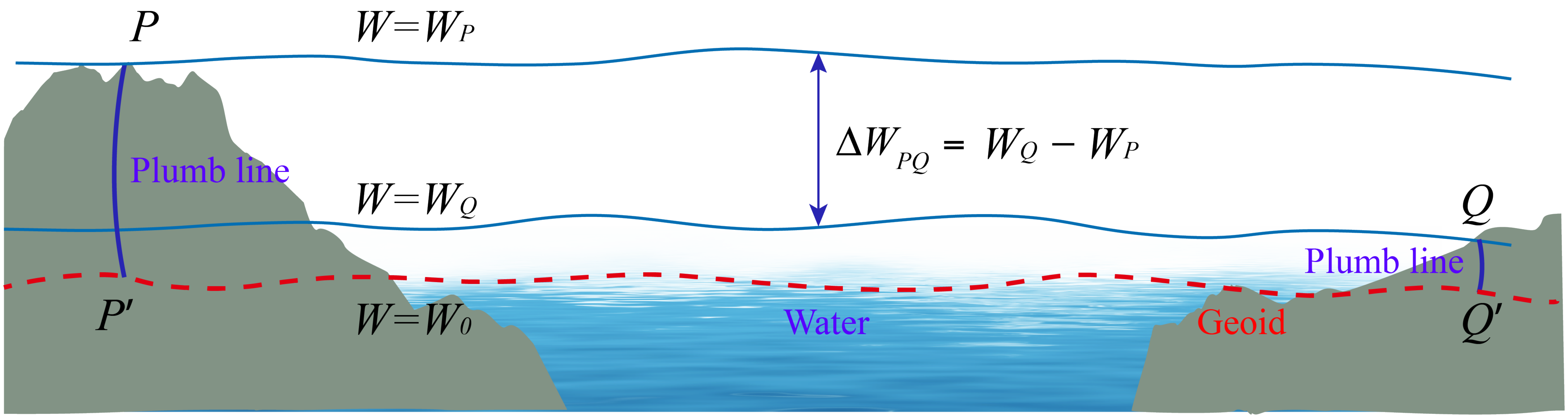
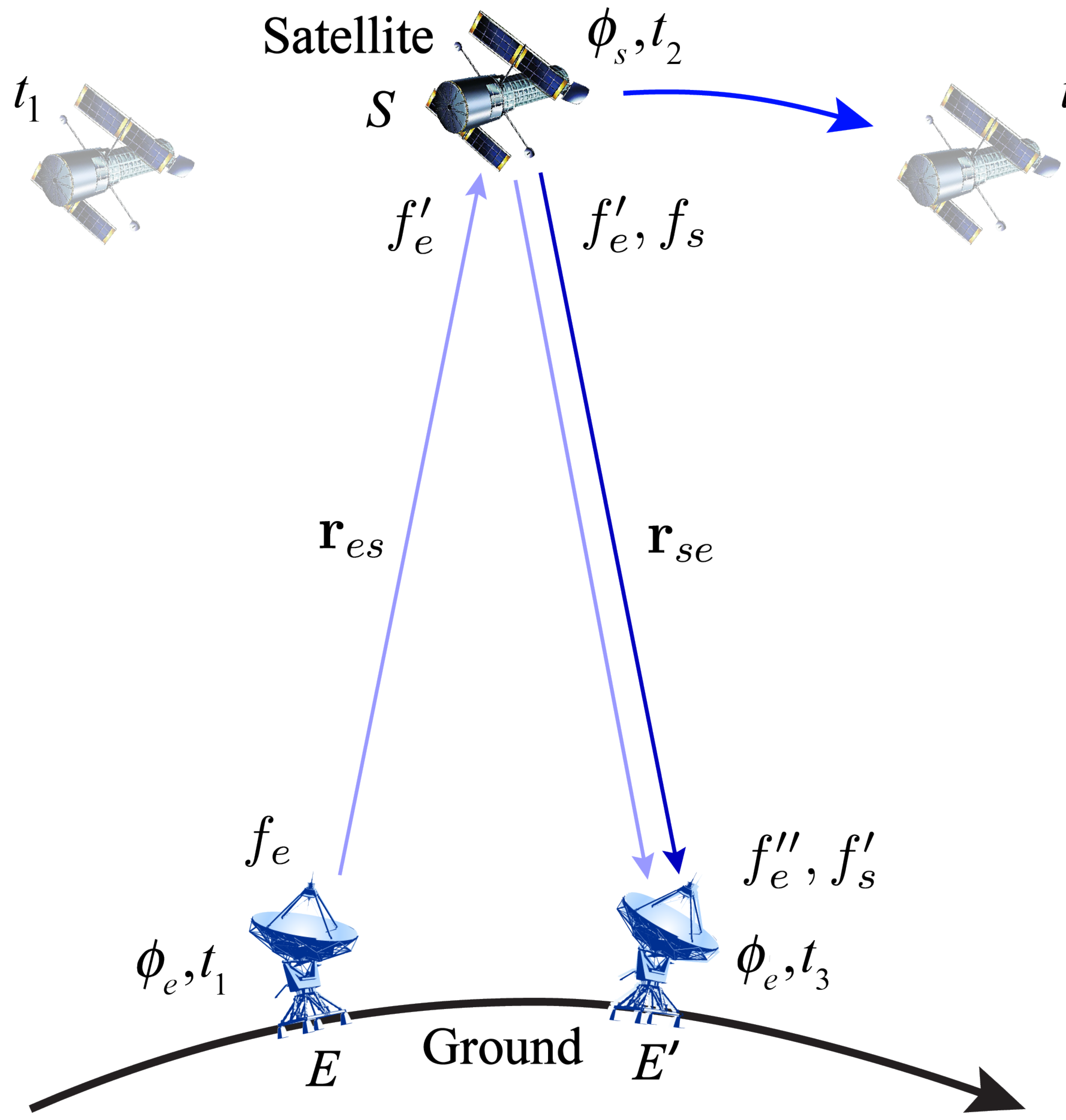


Fig. 4 Red dashed red curve denotes the global geoid, the two solid blue curves denote the $W = W_P$ and $W = W_Q$ surfaces, respectively. Bold blue curve denotes the plumb-line, along which the height integration is executed.

Height reference system and the SFST method

SFST method

- Fig 3 The **satellite frequency signal transmission (SFST)** method: ground station E emits a frequency signal f_e at time t_1. Satellite S transmits the received signal f'_e and emits a frequency signal f_s at time t_2 . The ground station receives signal f''_e and f'_s at time t_3 at position P' . ϕ is gravitational potential, $\mathbf{r}$ is position vector, $\mathbf{v}$ is velocity vector.



Height reference system and the SFST method

SFST method

- By proper combination of three frequencies, the gravitational potential (GP) difference between satellite and ground is given by

$$\frac{\Delta\phi_{es}}{c^2} \equiv \frac{\phi_s - \phi_e}{c^2} = \frac{\Delta f}{f_e} - \frac{v_s^2 - v_e^2}{2c^2} - \sum_{i=1}^{4} q^{(i)} + \Lambda f + O(c^{-5})$$

▫ $\Delta\phi_{es} = \phi_s - \phi_e$ —— GP difference; ▫ $\sum_{i=1}^{4} q^{(i)}$ —— high order amounts;

▫ Λf —— the sum of all correction terms (such as ionospheric and tropospheric effects)

- Theoretical precision can reach a relative precision of 10^{-19}. (Shen ZY et al. 2017)

Height reference system and the SFST method

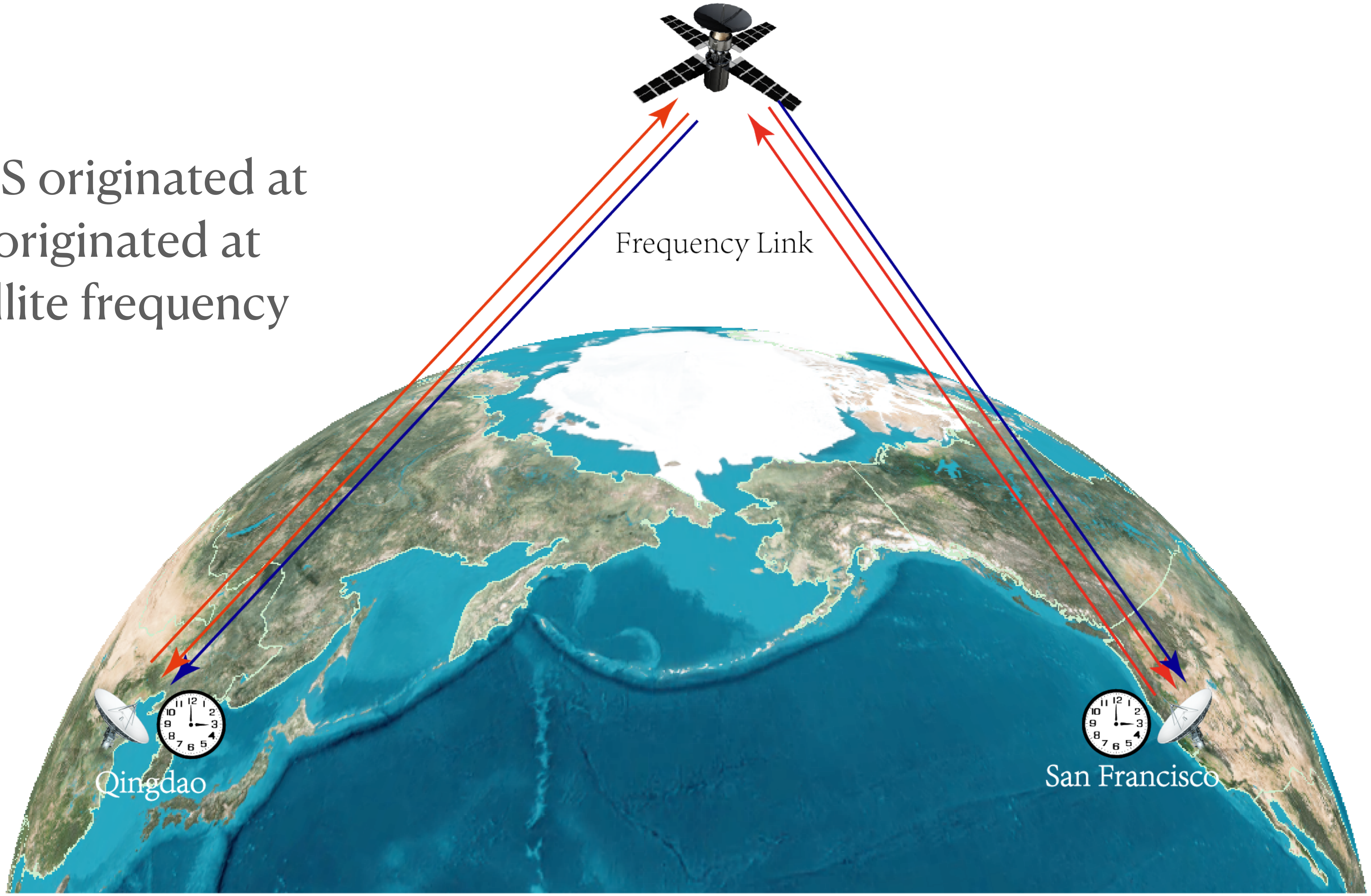
Determination of height difference between two ground height datum stations

- A SFST link can determine the gravitational potential difference between a satellite and a ground site.
- The satellite can serve as a ”bridge” to connect two ground sites, and the gravitational potential difference between these two sites can be obtained.
- Given the gravitational potential difference between two ground sites, the OH difference between them can be determined.
- Consequently the VHSs of the two ground sites are connected.

Height reference system and the SFST method

Determination of height difference between two ground height datum stations

Fig. 4 Connection of China HS originated at Qingdao datum and USA HS originated at San Francisco datum via satellite frequency signal transmission.



Height reference system and the SFST method

Determination of height difference between two ground height datum stations

- The geopotential difference is determined by $\Delta W_{PQ} = (\phi_Q - \phi_P) + (Z_Q - Z_P)$
- Suppose the height of point P is given, then the height of point Q can be obtained by

$$H_P = \frac{W_0 - W_P}{\bar{g}_P} \qquad H_Q = \frac{W_0 - W_Q}{\bar{g}_Q} = \frac{W_0 - W_P - \Delta W_{PQ}}{\bar{g}_Q}$$

- $\bar{g}_i$ is usually replaced by the following formula $\bar{g}_i = g_i + 4.24 \times 10^{-5} H_i$
- A practical formula for determining H_Q can be expressed as

$$H_Q = \frac{H_P \cdot (g_P + 4.24 \times 10^{-5} H_P) - \Delta W_{PQ}}{g_Q + 4.24 \times 10^{-5} H_Q}$$

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Simulation experiments

Experiment setup

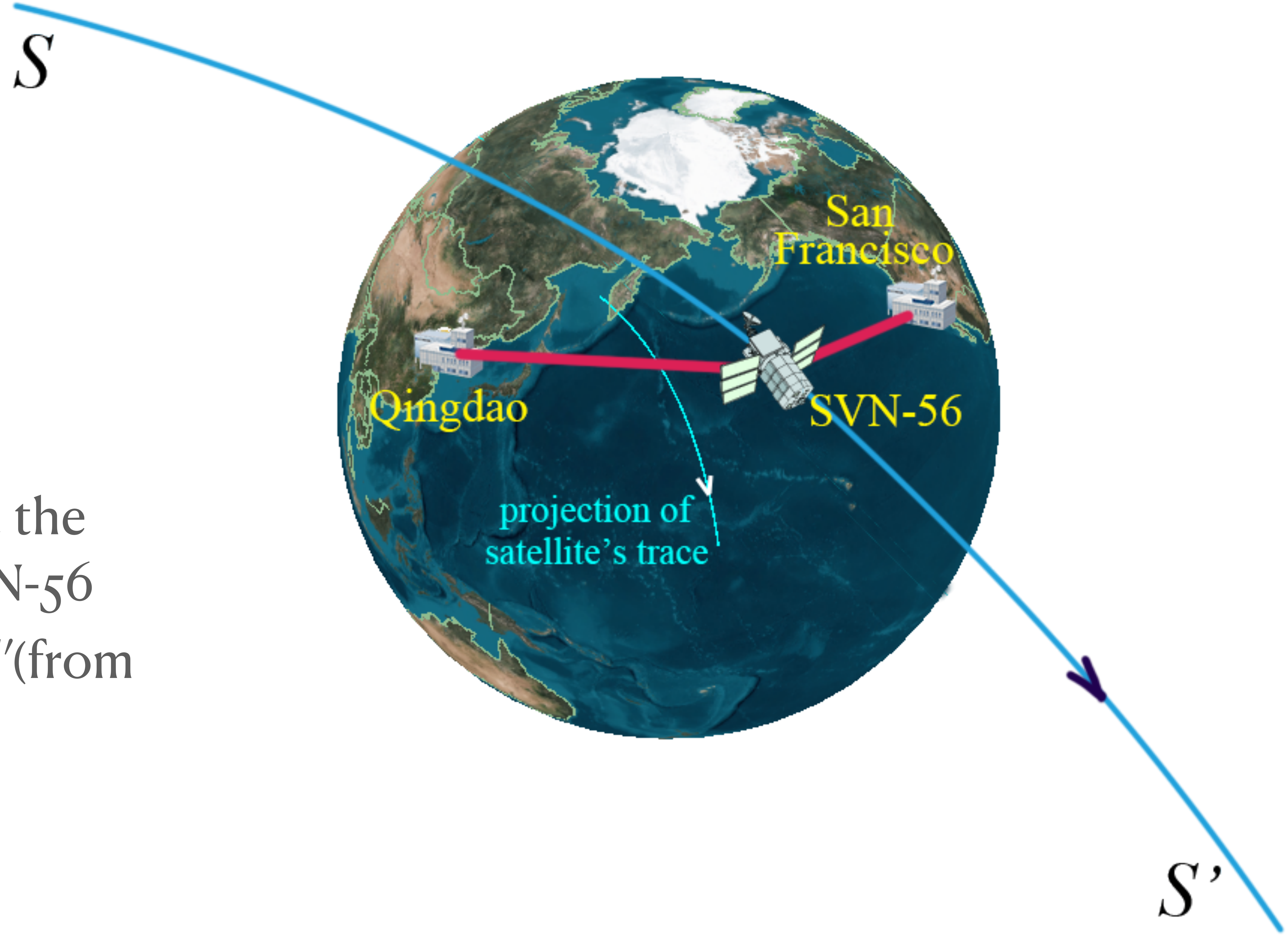

Fig. 5 Experiments are conducted at the time duration when the satellite SVN-56 moves from position S to position S'(from 7:00 am to 8:30 am, March 30, 2019)

Simulation experiments

Experiment setup

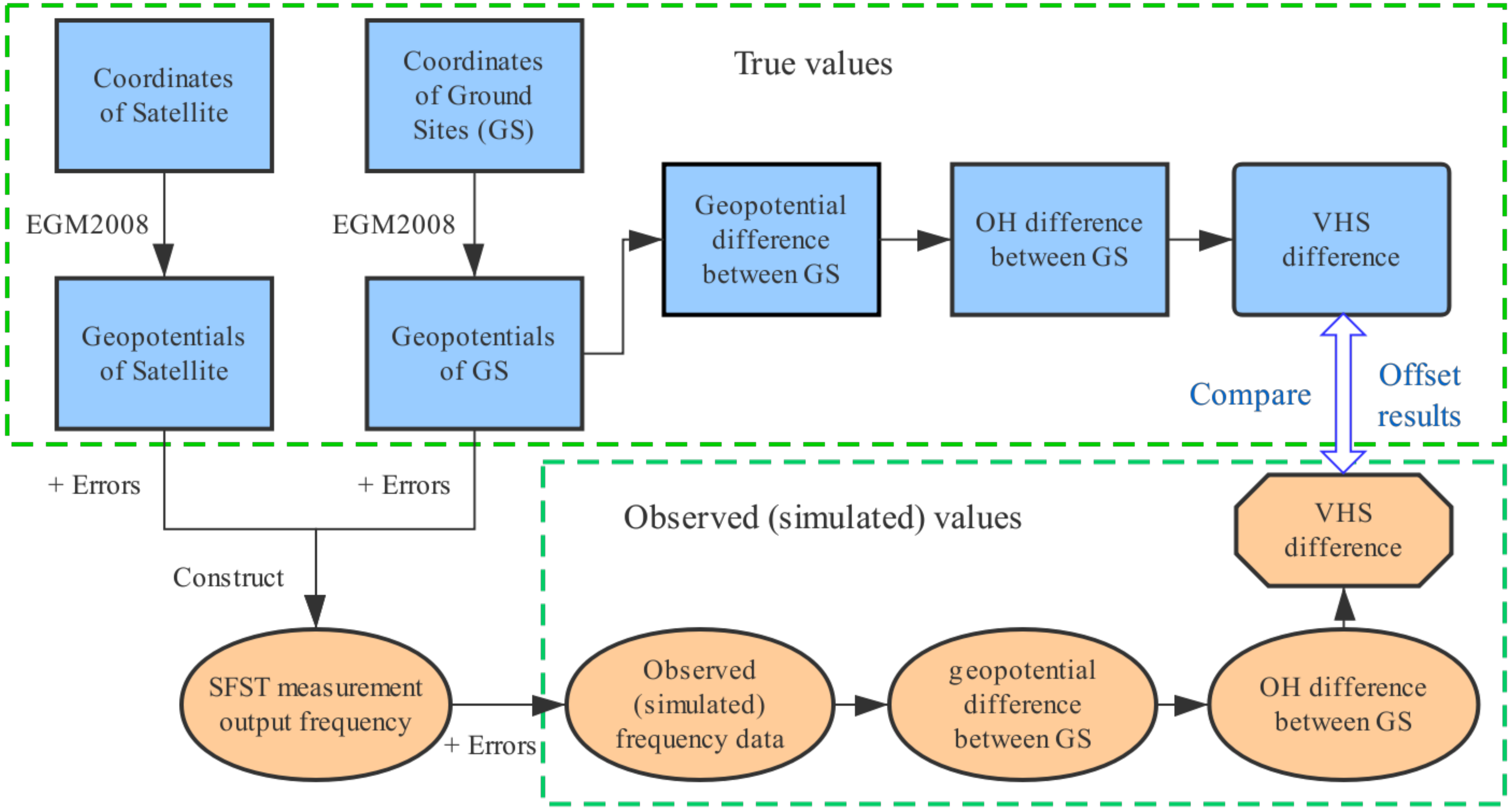

Fig. 4 The scheme of the simulation experiment

Simulation experiments

Input values

Table 1 The input datas used in simulation experiments.

Entities	Values of Parameter
Satellite	SVN-56 (GPS Navigation Sat.)
Qingdao DS	(36.06974° N, 120.32172° E, 77.472 m)
San Francisco DS	(37.76985° N, 122.46616° W, 75.878 m)
Gravity field model	EGM2008
Ionospheric model	International Reference Ionosphere
Tropospheric model	Earth Global Reference Atmospheric Model
Tide correction	ETERNA
Observation duration	from 7:00 am to 8:30 am, March 30, 2019
Mearsurement interval	5 s
Height systems diff.	1.000 m (China HS is higher than US HS)

Simulation experiments

Input values

Table 2 Error magnitudes of different error sources in determining the gravitational potential difference between a satellite and a ground station.

Influence factor	(Residual) Error magnitude in $\Delta f / f_e$
ionospheric correction residual	$\delta f_{ion} \sim 5.5 \times 10^{-19}$
tropospheric correction residual	$\delta f_{tro} \sim 1.9 \times 10^{-19}$
tidal correction residual	$\delta f_{tide} \leq 10^{-18}$
position & velocity	$\delta f_{vepo} \sim 3.4 \times 10^{-19}$ (10 mm and 0.1mm/s)
asynchronism	$\delta f_{delay} \sim 10^{-19}$ (below 1 ms)
clock error	$\delta f_{osc} \sim 4.8 \times 10^{-17}$

Simulation experiments

Experiment results

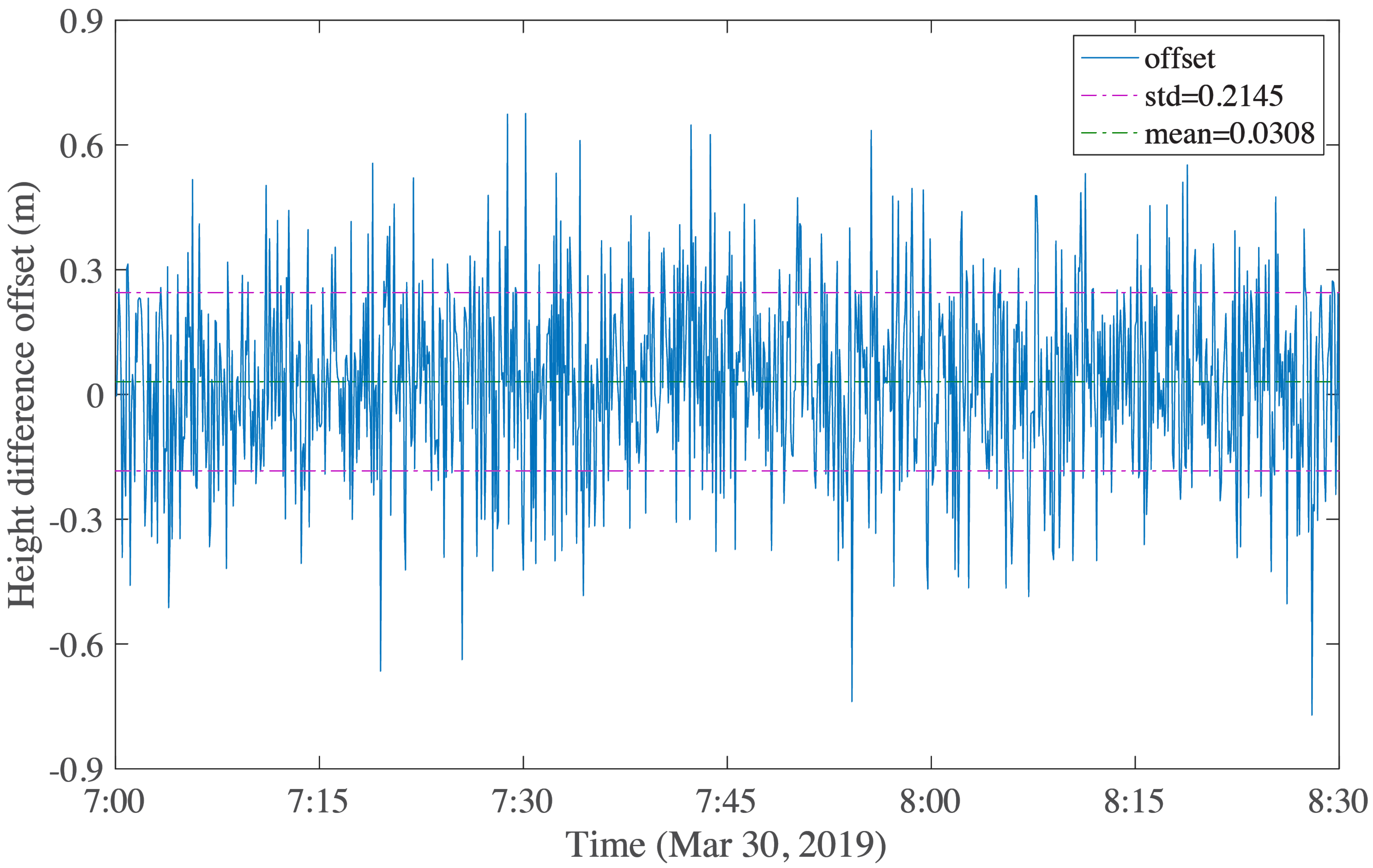

Fig. 6 The offset between true values and estimated values of Height datum difference determined by SVN-56 satellite.

Simulation experiments

Experiment results

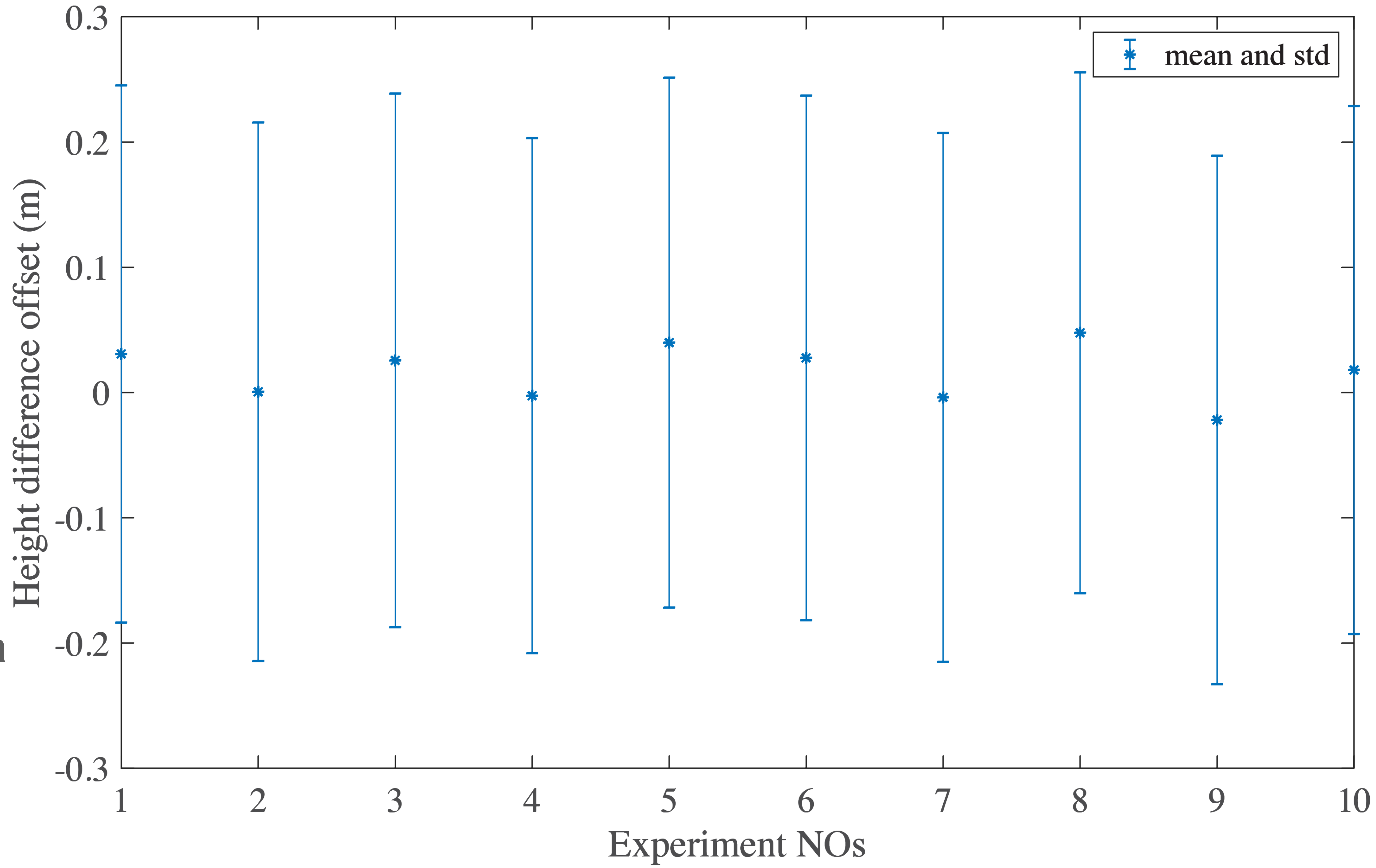

Fig. 7 The mean offset values and STD values of Height datum difference in 10 different simulation experiments.

Simulation experiments

Experiment results

Table 3 The results of 10 simulation experiments.

Experiment No.	Height diff. between China's VHS and US' VHS (m)	Offset to true value (1 m)	STD (m)
1	1.0308	0.0308	0.2145
2	1.0061	0.0061	0.2151
3	1.0257	0.0257	0.2130
4	0.9975	-0.0025	0.2056
5	1.0399	0.0399	0.2116
6	1.0277	0.0277	0.2094
7	0.9961	-0.0039	0.2112
8	1.0477	0.0477	0.2079
9	0.9781	-0.0219	0.2110
10	1.0180	0.0180	0.2108
Average	1.0168	0.0168	0.2110

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Conclusions

- SFST method is very promising for unifying different local VHSs.
- Simulation experiments show that the deviation between "observed" result and the true value is around 2 cm, with an accuracy level (STD) of 2 decimeters in 1.5 h.
- The precision of the VHS connection mainly depend on the stability of atomic clocks. In this report we assume their stabilities reach the level of 4.8×10^{-17} in one second.
- Considered the high cost of atomic clocks, the best practice is to adopt the SFST method as a supplement of conventional methods in global VHS unification

Thank you for your attention