Presentation for EGU 2020

Random objective surface-wave waveform inversion

Yudi Pan, Lingli Gao, and Thomas Bohlen

Karlsruhe Institute of Technology (KIT), Germany

Related work has been accepted for publication in GEOPHYSICS:
Pan, Y. and Gao, L., accepted, Radom-objective full waveform inversion of surface waves: *Geophysics*, doi:10.1190/geo-2019-0613.1

Methodology

- **Conventional objective function in FWI**

$$\min \Phi(\mathbf{m}) = \sum_{i=1}^{N_s} \mathcal{L}[\mathbf{d}_i^{obs}, \mathbf{d}_i^{syn}(\mathbf{m})],$$

$\mathcal{L}$ is a desired measure function, *Ns* is the total number of shots.

- **Common choices of $\mathcal{L}$ in surface-wave FWI**

(Pan et al,, 2020)

L2 misfit

$$\Phi_{l_2}(\mathbf{m}) = \frac{1}{2}\sum_{i=1}^{N_s} \|\mathbf{d}_i^{syn}(\mathbf{m}) - \mathbf{d}_i^{obs}\|_2^2 \ ,$$

Envelope misfit

$$\Phi_{\mathrm{Env}}(\mathbf{m}) = \frac{1}{2}\sum_{i=1}^{N_s} \|\mathbf{e}_i^{syn}(\mathbf{m}) - \mathbf{e}_i^{obs}\|_2^2 \ , \quad \mathbf{e} = \sqrt{\mathbf{d}^2 + \mathcal{H}^2(\mathbf{d})}.$$

FK-spectra misfit

$$\Phi_{\mathrm{FK}}(\mathbf{m}) = \frac{1}{2}\sum_{i=1}^{N_s} \|\mathbf{D}_i^{syn}(\mathbf{m}) - \mathbf{D}_i^{obs}\|_2^2 \ , \ \mathbf{D} = |\mathcal{F}_{2D}(\mathbf{d})|$$

Methodology

- **Random objective waveform inversion (ROWI)**

$$\underset{\mathbf{m}}{\operatorname{argmin}} \frac{1}{2} \left\{ \begin{array}{ccc} \|\mathbf{d}_1^{syn}(\mathbf{m}) - \mathbf{d}_1^{obs}\|_2^2, & \cdots, & \|\mathbf{d}_{N_s}^{syn}(\mathbf{m}) - \mathbf{d}_{N_s}^{obs}\|_2^2 \\ \|\mathbf{e}_1^{syn}(\mathbf{m}) - \mathbf{e}_1^{obs}\|_2^2, & \cdots, & \|\mathbf{e}_{N_s}^{syn}(\mathbf{m}) - \mathbf{e}_{N_s}^{obs}\|_2^2 \\ \|\mathbf{D}_1^{syn}(\mathbf{m}) - \mathbf{D}_1^{obs}\|_2^2, & \cdots, & \|\mathbf{D}_{N_s}^{syn}(\mathbf{m}) - \mathbf{D}_{N_s}^{obs}\|_2^2 \end{array} \right\}$$

- Optimization: stochastic preconditioned steepest descent

Randomly choose one of the 3**Ns* objective functions at each iteration and optimize it with

(Shigapov, 2019)

$$\mathbf{m}_{k+1} = \mathbf{m}_k - \alpha H_a^{-1} \delta \mathbf{m}_k$$

Comp. cost: 1 iteration FWI = *Ns* iterations ROWI

Methodology

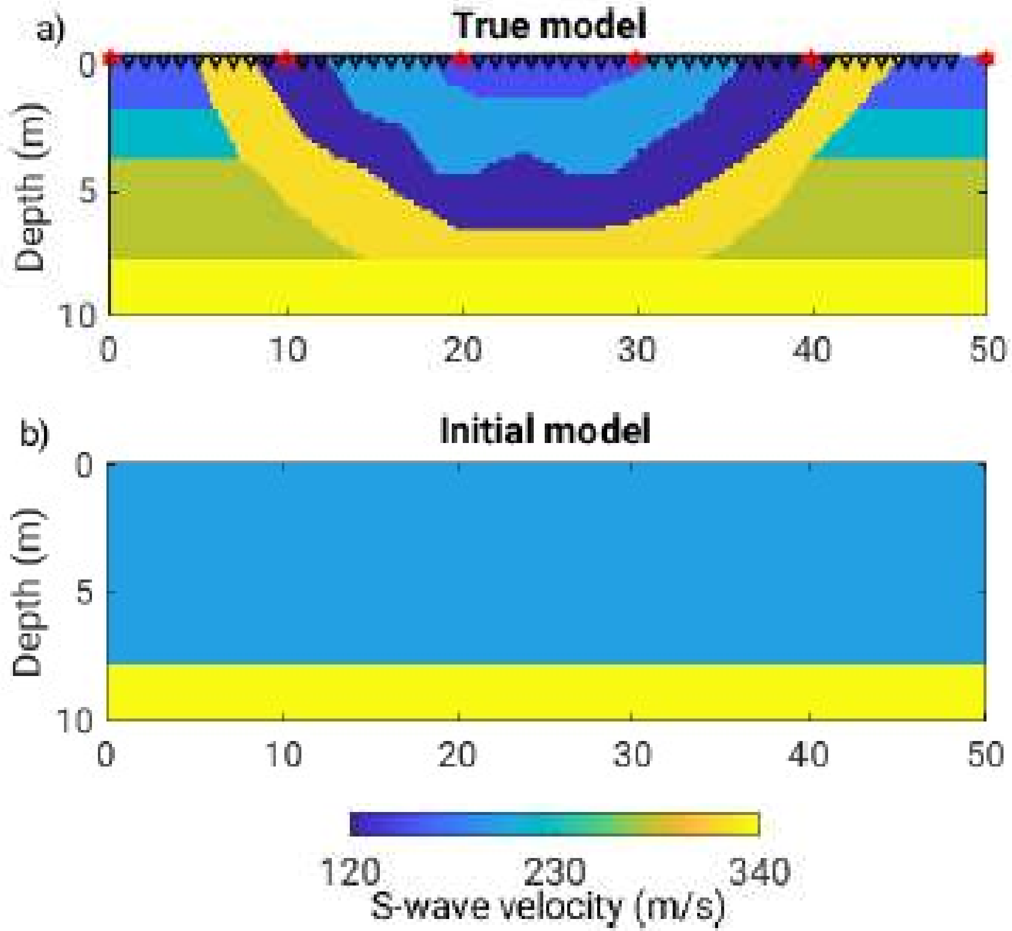
Why ROWI

$$\underset{\mathbf{m}}{\operatorname{argmin}} \frac{1}{2} \left\{ \begin{matrix} \|\mathbf{d}_1^{syn}(\mathbf{m}) - \mathbf{d}_1^{obs}\|_2^2, & \cdots, & \|\mathbf{d}_{N_s}^{syn}(\mathbf{m}) - \mathbf{d}_{N_s}^{obs}\|_2^2 \\ \|\mathbf{e}_1^{syn}(\mathbf{m}) - \mathbf{e}_1^{obs}\|_2^2, & \cdots, & \|\mathbf{e}_{N_s}^{syn}(\mathbf{m}) - \mathbf{e}_{N_s}^{obs}\|_2^2 \\ \|\mathbf{D}_1^{syn}(\mathbf{m}) - \mathbf{D}_1^{obs}\|_2^2, & \cdots, & \|\mathbf{D}_{N_s}^{syn}(\mathbf{m}) - \mathbf{D}_{N_s}^{obs}\|_2^2 \end{matrix} \right\}$$

- Gain the advantages of all measure functions;
- Using one shot per iteration: avoids redundant information;
- Envelope misfit and the stochastic nature: robust solution paths.

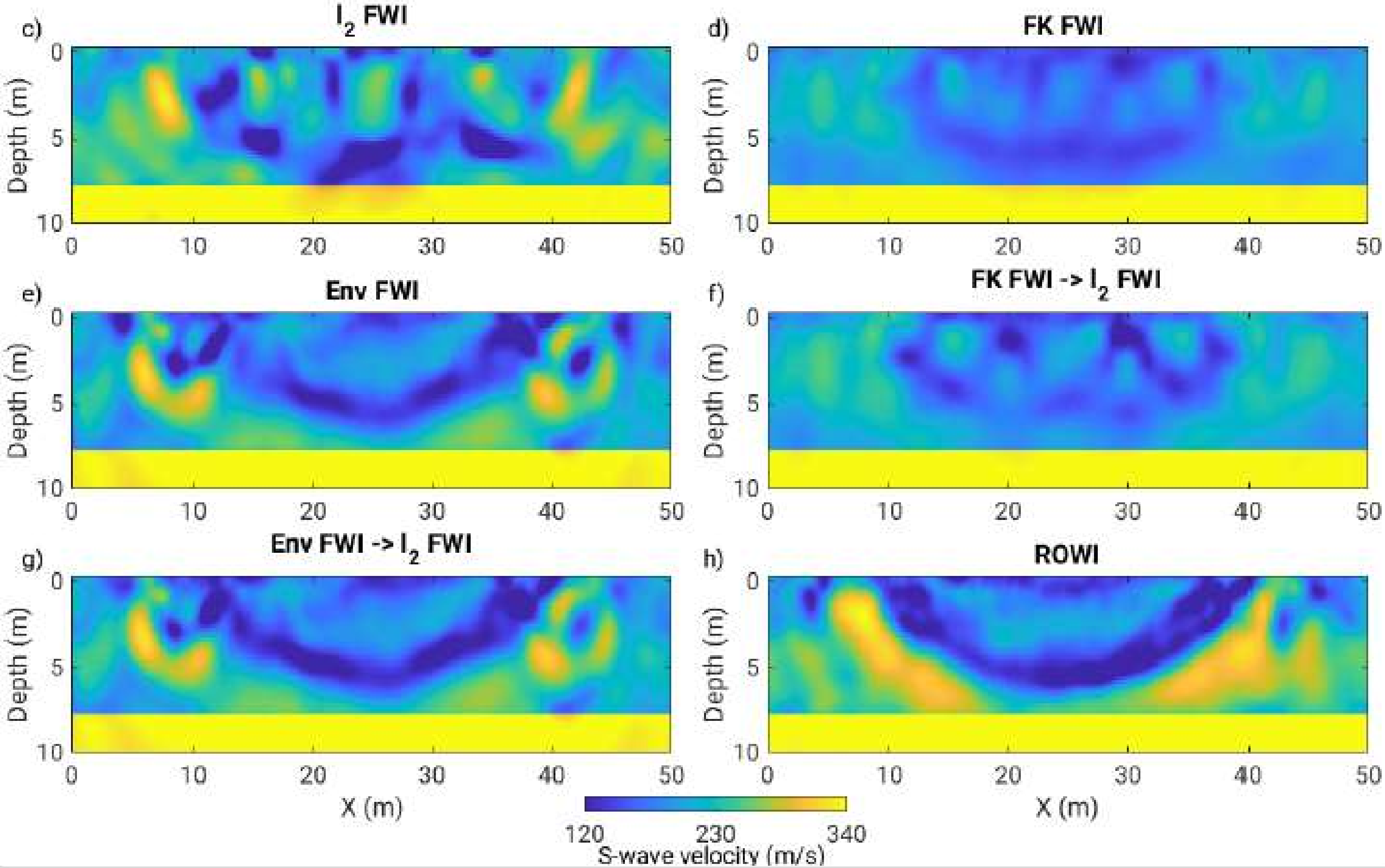
Synthetic example

- 6 vertical-force sources
- First-order Gaussian wavelet of 25 Hz
- 48 receivers
- Homogeneous *Vp, Qs, Qp,* and density models
- Rayleigh wave (P-Sv)



This figure has been published on: Pan, Y. and Gao, L., accepted, Radom-objective full waveform inversion of surface waves: *Geophysics*, doi:10.1190/geo-2019-0613.1

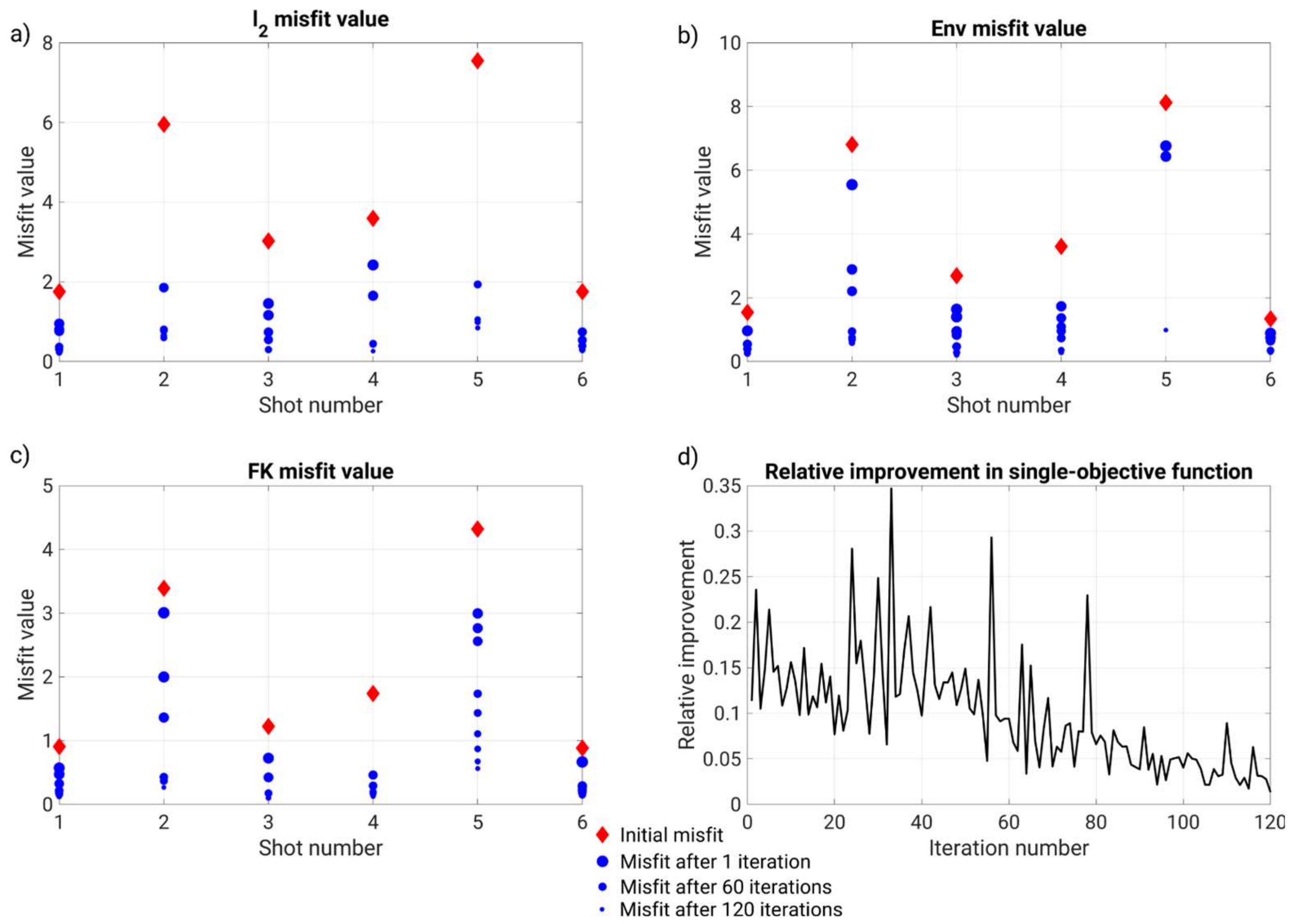
Synthetic example



This figure has been published on: Pan, Y. and Gao, L., accepted, Radom-objective full waveform inversion of surface waves: *Geophysics*, doi:10.1190/geo-2019-0613.1

Synthetic example

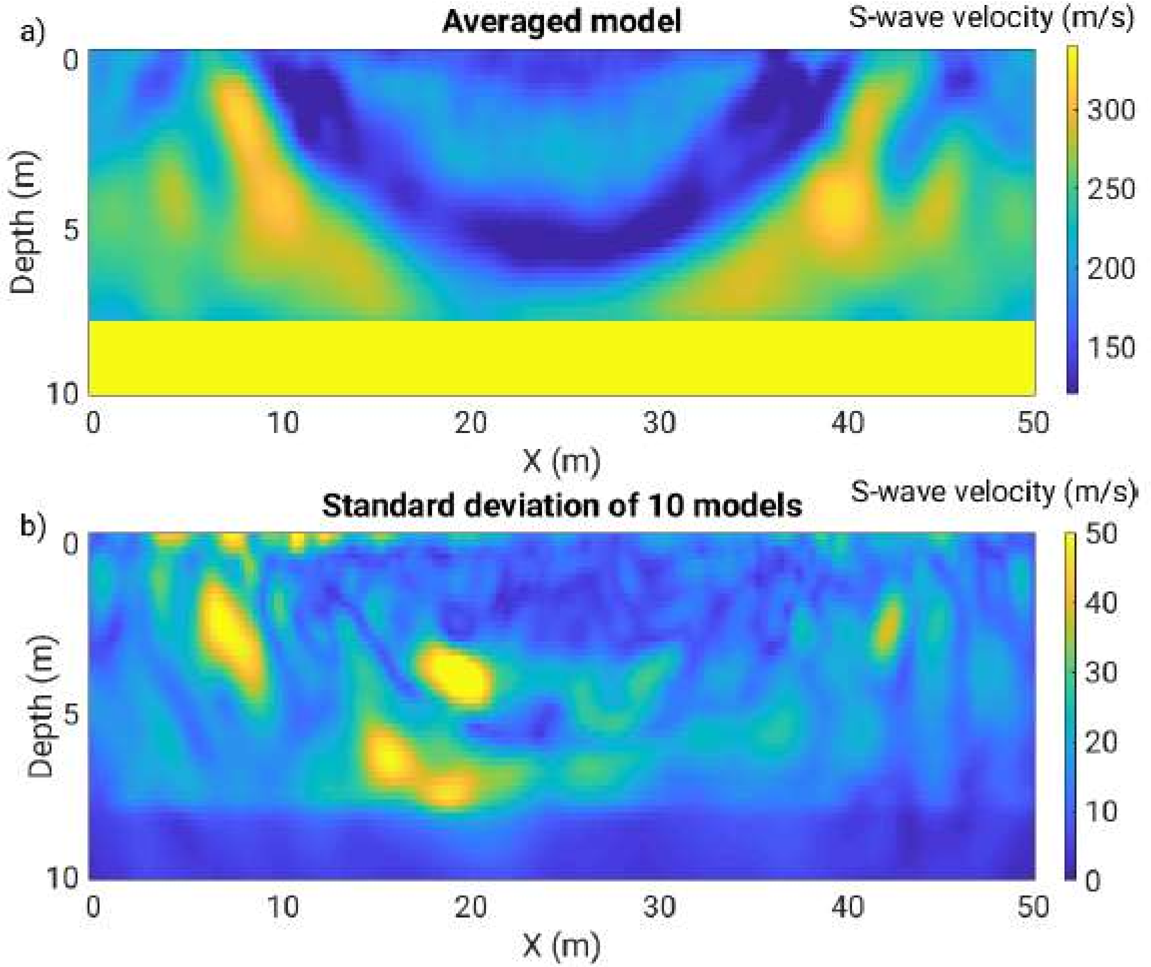
- Data misfit of ROWI



This figure has been published on: Pan, Y. and Gao, L., accepted, Radom-objective full waveform inversion of surface waves: *Geophysics*, doi:10.1190/geo-2019-0613.1

Synthetic example

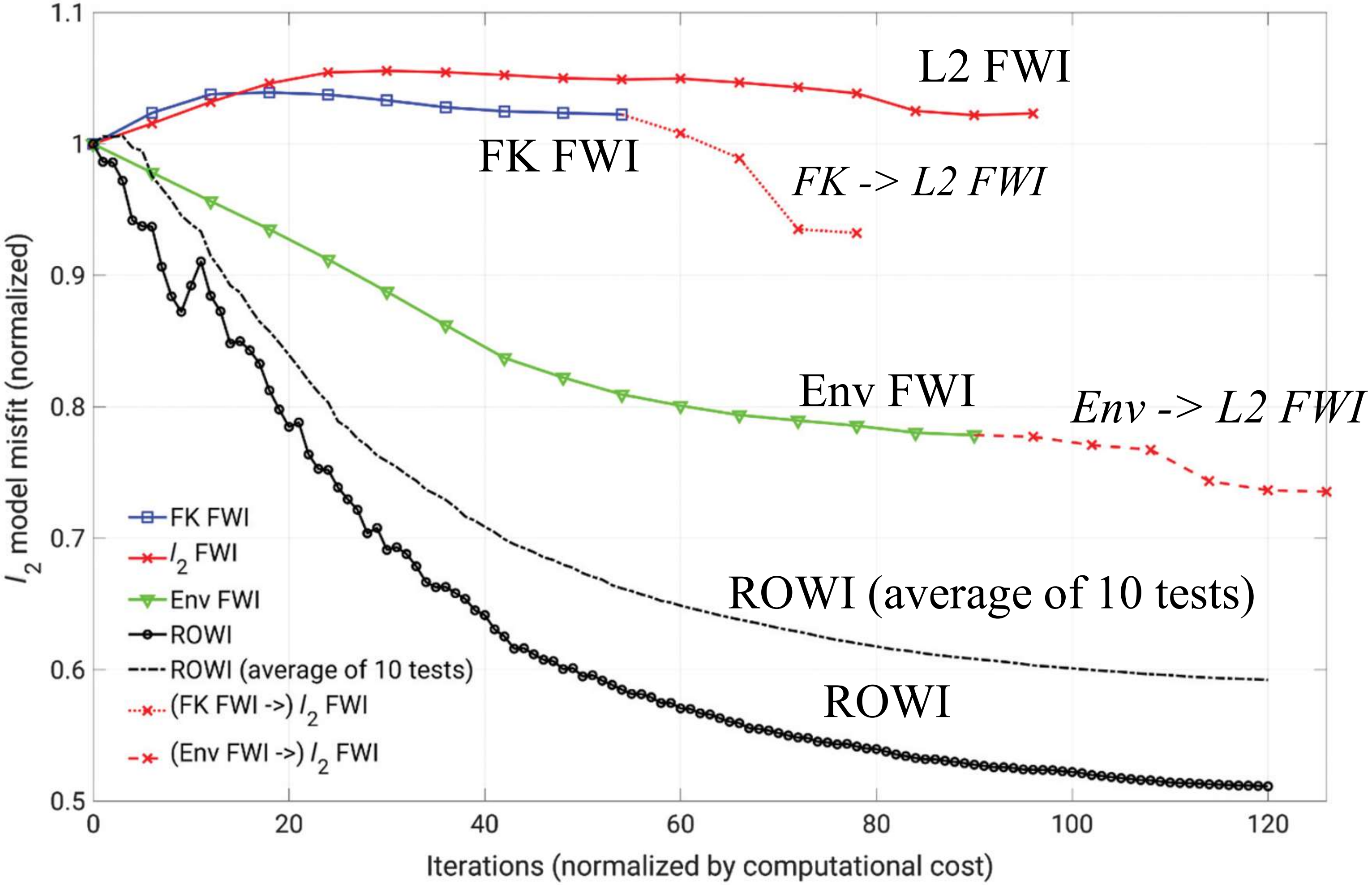
- Averaged result of 10 ROWIs with different random sequences



This figure has been published on: Pan, Y. and Gao, L., accepted, Radom-objective full waveform inversion of surface waves: *Geophysics*, doi:10.1190/geo-2019-0613.1

Synthetic example

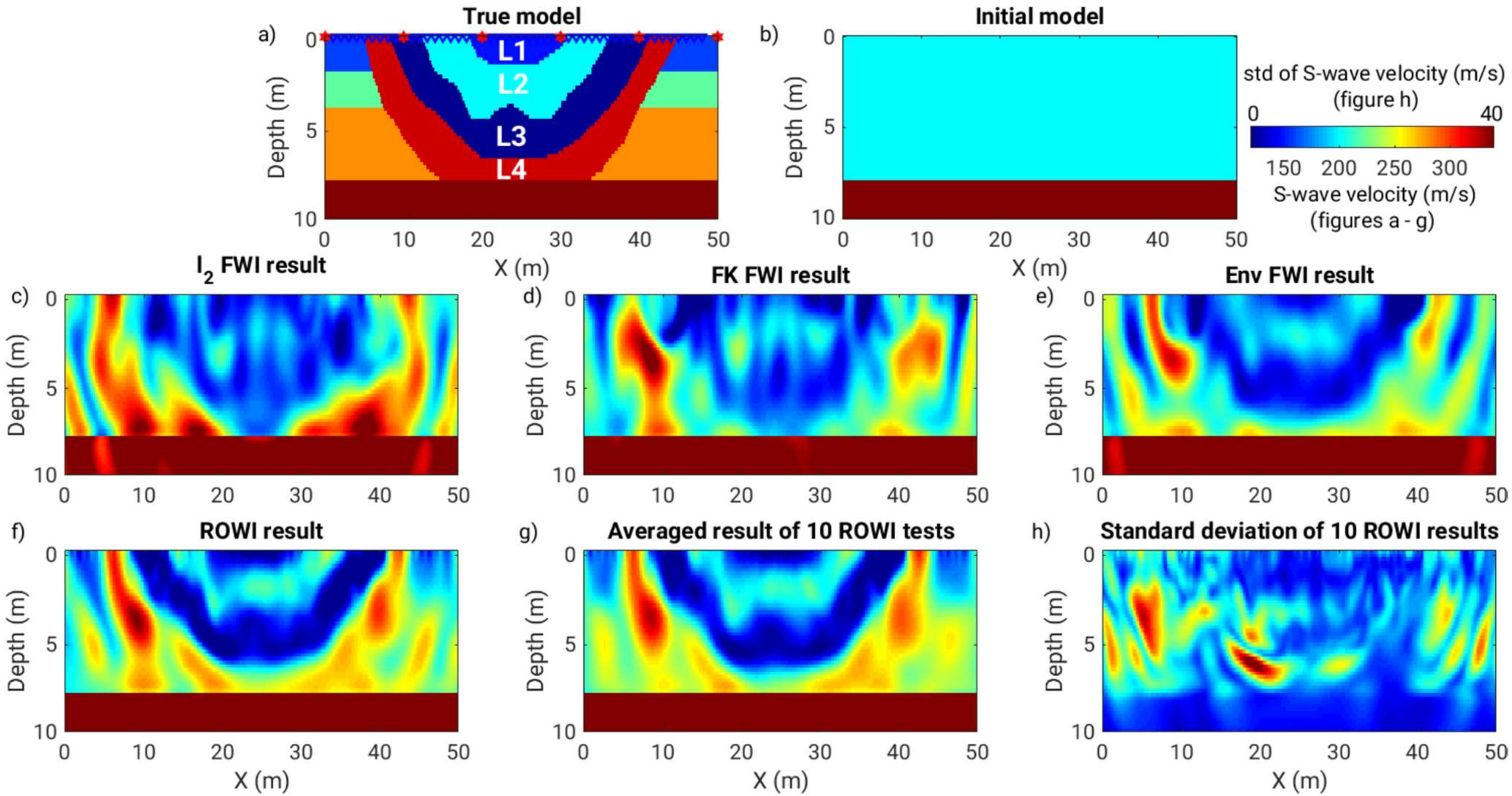
- Model misfit



 This figure has been published on: Pan, Y. and Gao, L., accepted, Radom-objective full waveform inversion of surface waves: *Geophysics*, doi:10.1190/geo-2019-0613.1

Synthetic example

- Result on Love wave (SH wave)



(Pan et al,, EAGE 2020)

Field example

Location: Rheinstetten, Karlsruhe, Germany

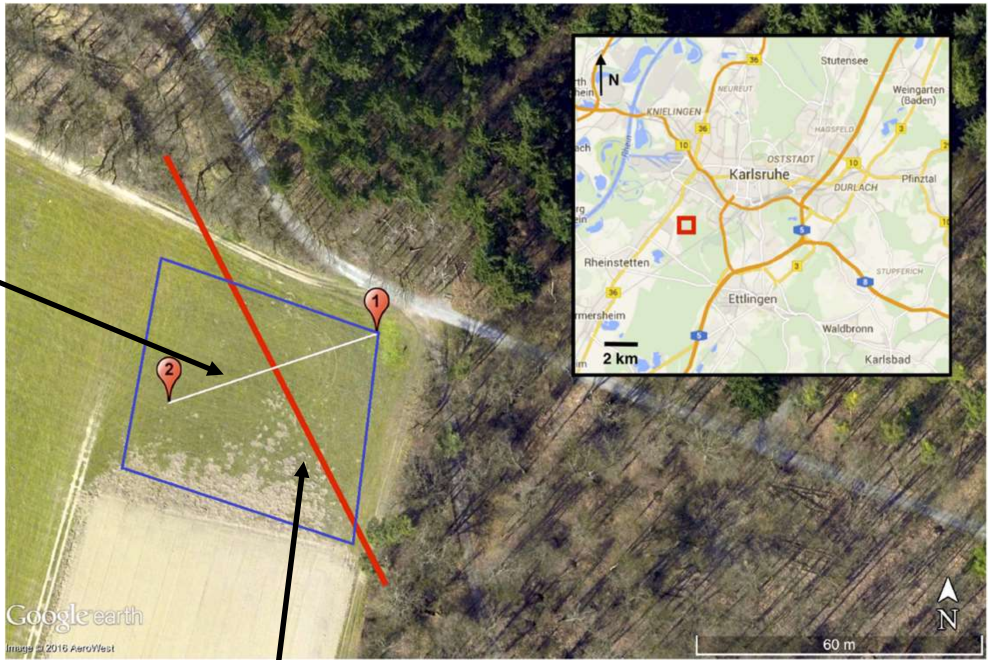
Vertical hammer source

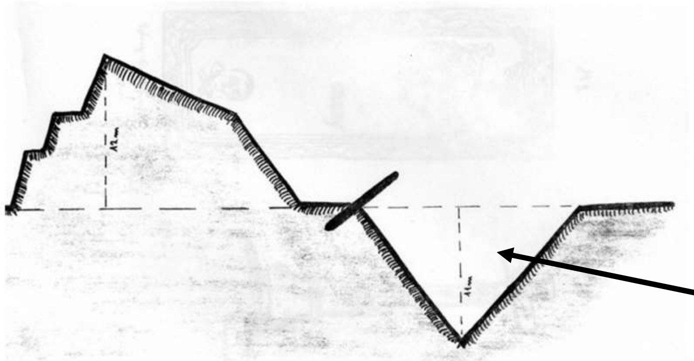
Vertical receivers (4.5 Hz)

48 traces with 1 m spacing

Source interval: 4 m

Number of shots: 12





This 'V'-shaped trench is refilled and is invisible from ground (red line)

Field example

Karlsruhe Institute of Technology

- ROWI + multiscale technique

- 6 stages with a corner freq. of [10, 20, 30, 40, 50, 60] Hz
- 60 iterations at each stage (~ 5 iterations of conventional FWI)
- 4 times of ROWI with different random sequences
- Multiparameter viscoelastic inversion (Gao et al., 2020)

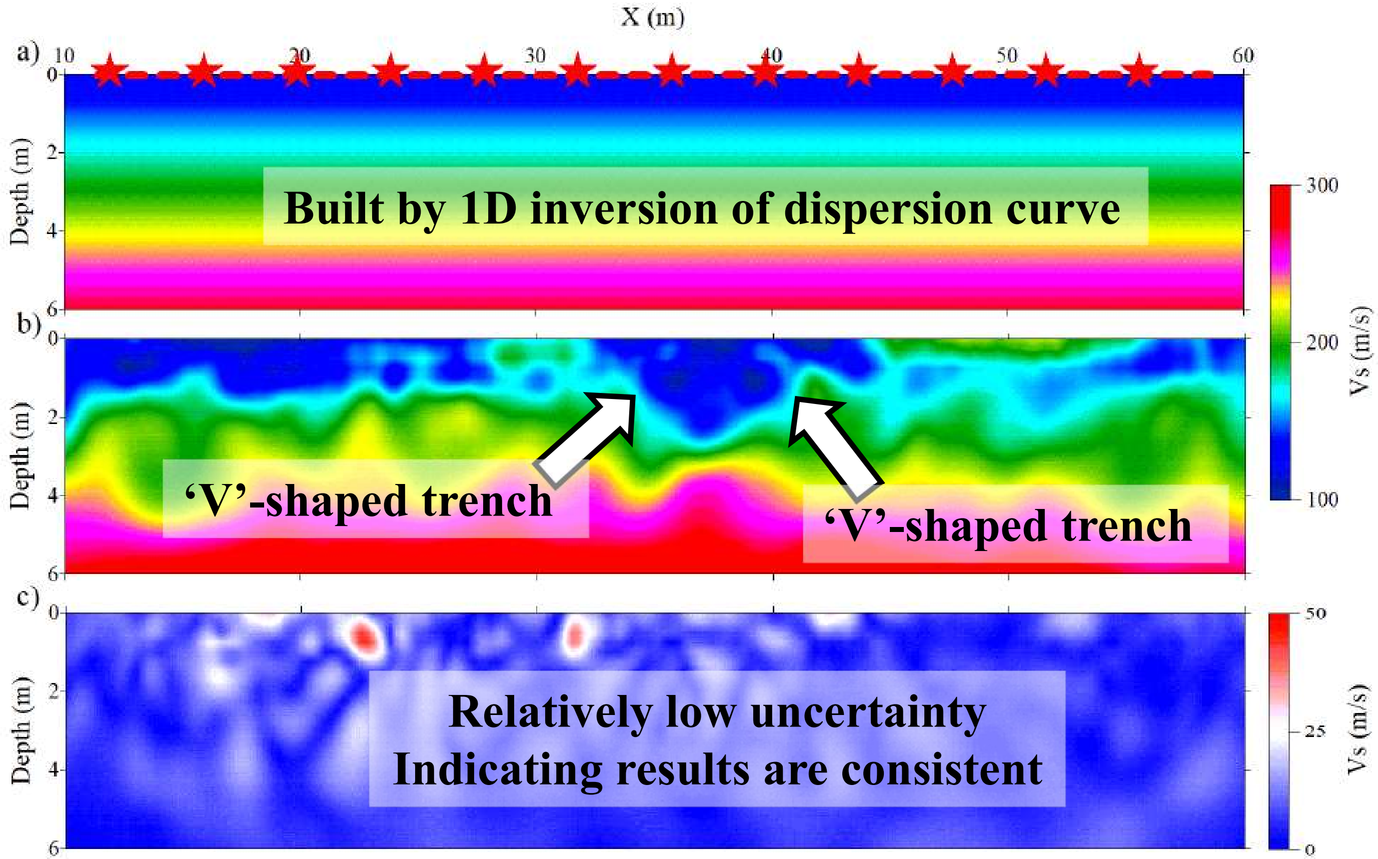
Field example

- Good initial model

Initial model

Inversion result
(average of 4 ROWIs)

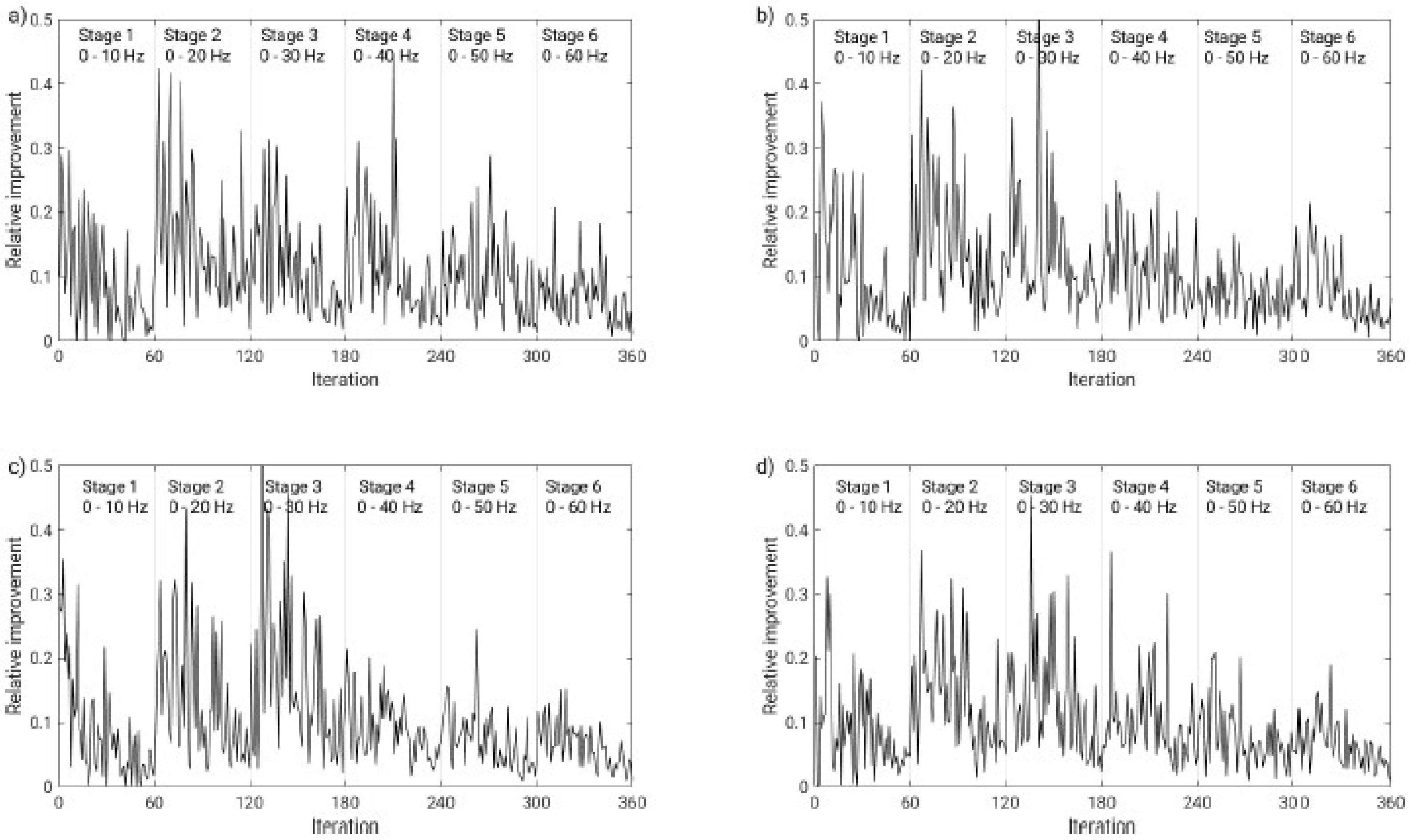
Uncertainty
(std of 4 ROWIs)



This figure has been published on: Pan, Y. and Gao, L., accepted, Radom-objective full waveform inversion of surface waves: *Geophysics*, doi:10.1190/geo-2019-0613.1

Field example

- Data misfit (relative improvement in the misfit of 4 ROWIs)

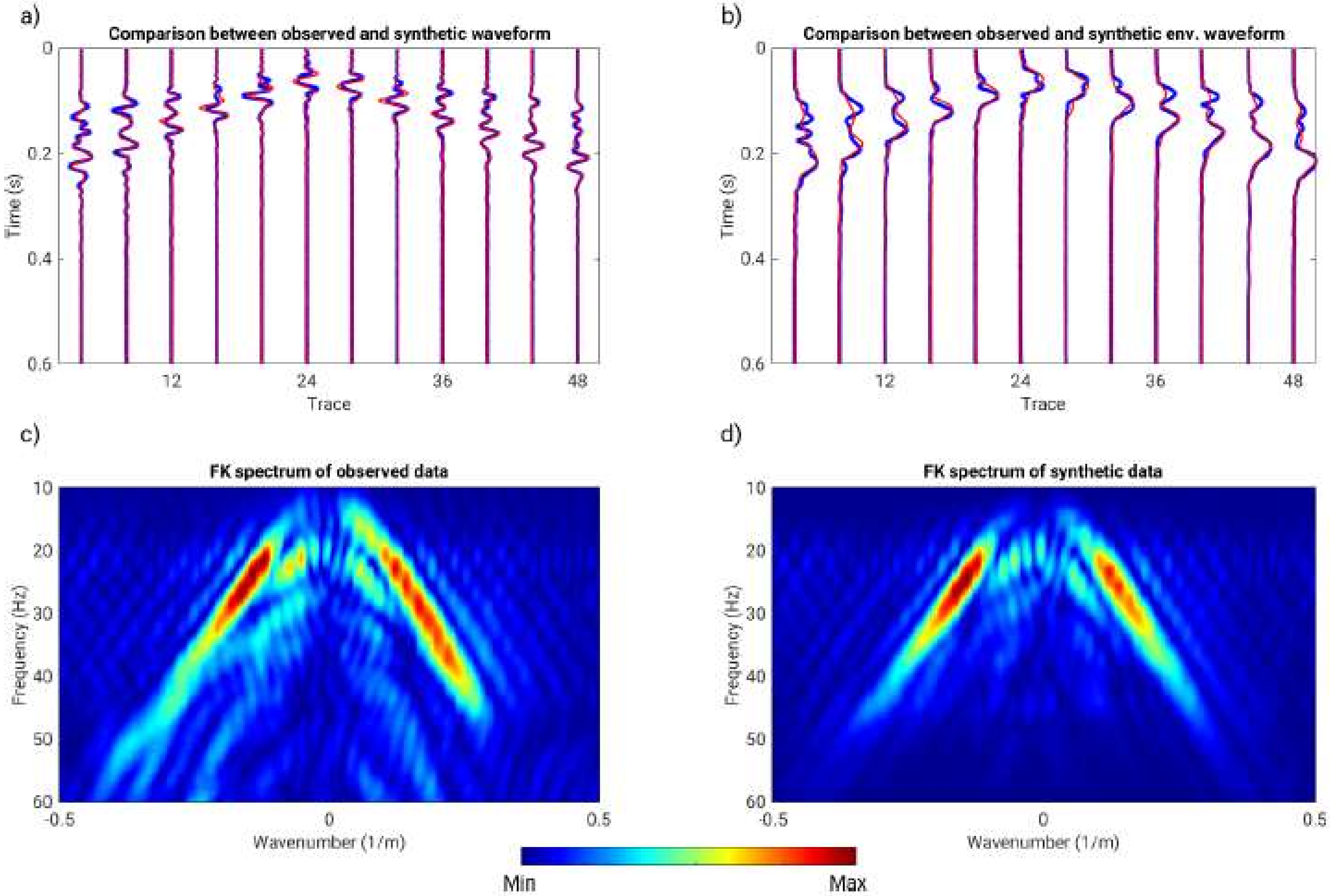


This figure has been published on: Pan, Y. and Gao, L., accepted, Radom-objective full waveform inversion of surface waves: *Geophysics*, doi:10.1190/geo-2019-0613.1

Field example

- Data fitting

central shot

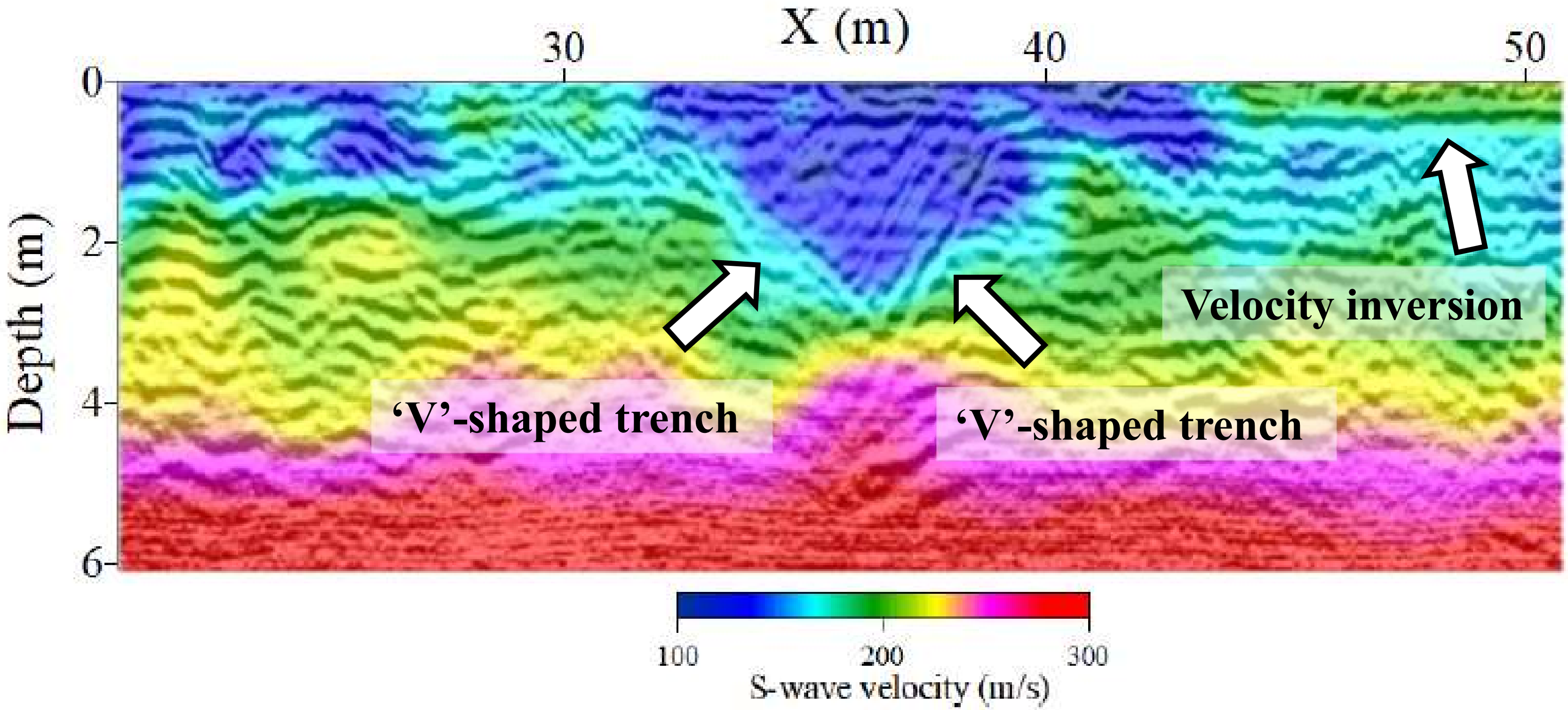


This figure has been published on: Pan, Y. and Gao, L., accepted, Radom-objective full waveform inversion of surface waves: *Geophysics*, doi:10.1190/geo-2019-0613.1

Field example

- Model evaluation

Comparison with a migrated GPR profile



This figure has been published on: Pan, Y. and Gao, L., accepted, Radom-objective full waveform inversion of surface waves: *Geophysics*, doi:10.1190/geo-2019-0613.1

Field example

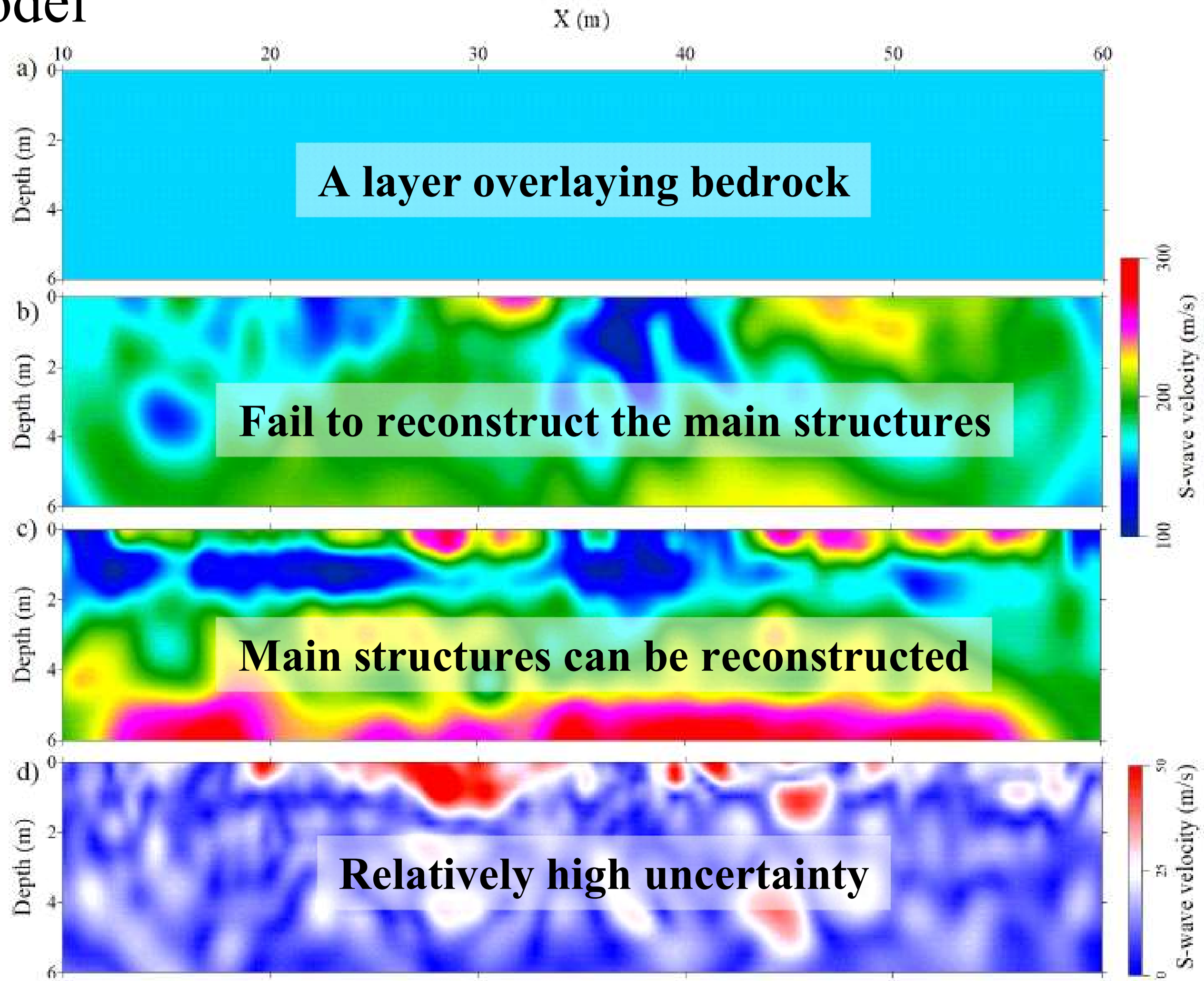
- Poor initial model

Initial model

FWI
(conventional *l2*)

ROWI
(average of 4 results)

Uncertainty
(std of 4 results)



This figure has been published on: Pan, Y. and Gao, L., accepted, Radom-objective full waveform inversion of surface waves: *Geophysics*, doi:10.1190/geo-2019-0613.1

Conclusions

- We proposed ROWI method under a multi-objective framework.
- Synthetic and field examples proved that ROWI outperforms conventional single-objective FWI.
- Standard deviation of the final models (weak Pareto solutions) provided useful information for uncertainty evaluation.

Thank you!

References

1. Gao, L., Pan, Y., and Bohlen, T., 2020, 2D multi-parameter viscoelastic shallow-seismic full waveform inversion: reconstruction tests and first field-data application: *Geophysical Journal International,* doi:10.1093/gji/ggaa198
2. Pan, Y., Gao, L., and Shigapov, R., 2020, Multi-objective full waveform inversion of shallow-seismic wavefields: *Geophysical Journal Internatio*nal, 220(3), 1619-1631.
3. Pan, Y. and Gao, L., accepted, Radom-objective full waveform inversion of surface waves: *Geophysics*, doi:10.1190/geo-2019-0613.1
4. Pan, Y., Gao, L., and Bohlen, T., accepted, Random-objective waveform inversion of shallow-seismic SH and Love waves, *82nd EAGE annual meeting*
5. Shigapov, R., 2019, Probabilistic waveform inversion: Quest for the law: Ph.D. thesis, Karlsruher Institut für Technologie (KIT).