Barotropic instability of a zonal jet on the sphere:

from non-divergence through quasi-geostrophy to shallow water

THE HEBREW UNIVERSITY OF JERUSALEM

Nathan Paldor

Ofer Shamir

Chaim I. Garfinkel

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Recent development in GFD and remote sensing. Nonlinear and turbulent processes under high wind conditions

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Motivation

- The Non-Divergent (ND) and Quasi-Geostrophic (QG) equations approximate the Shallow Water (SW) equations in the limit of infinite layer thickness ($H \rightarrow \infty$)
- Are the values of H where the ND and QG models approximate the SW model similar to one another? Are these values relevant to Earth?
- We examine the linear stability of strong jets on the sphere and compare the growth rates obtained from the ND, QG and SW models.
- The limitations of the ND and QG are most applicable for strong jets which are susceptible to barotropic instability.
- SW: both the basic state and linear perturbations depend on H.
- QG: only the linear perturbations depend on H.
- ND: both the basic state and linear perturbations are independent of H.

Model jet

- We compare the growth rates of barotropic instability on a zonal jet in the ND, QG, and SW models when each model is linearised about the same mean wind profile.
- We solve for unstable modes of both strong polar and equatorial jets on the sphere for a range of layer depths typical to Earth.
- Model jet follows Hartmann (1983):

$$\bar{u}(\phi) = u_0 \text{sech}[2(\phi - \phi_0)/B] \cos \phi$$

$\bar{u}$ = Mean zonal wind

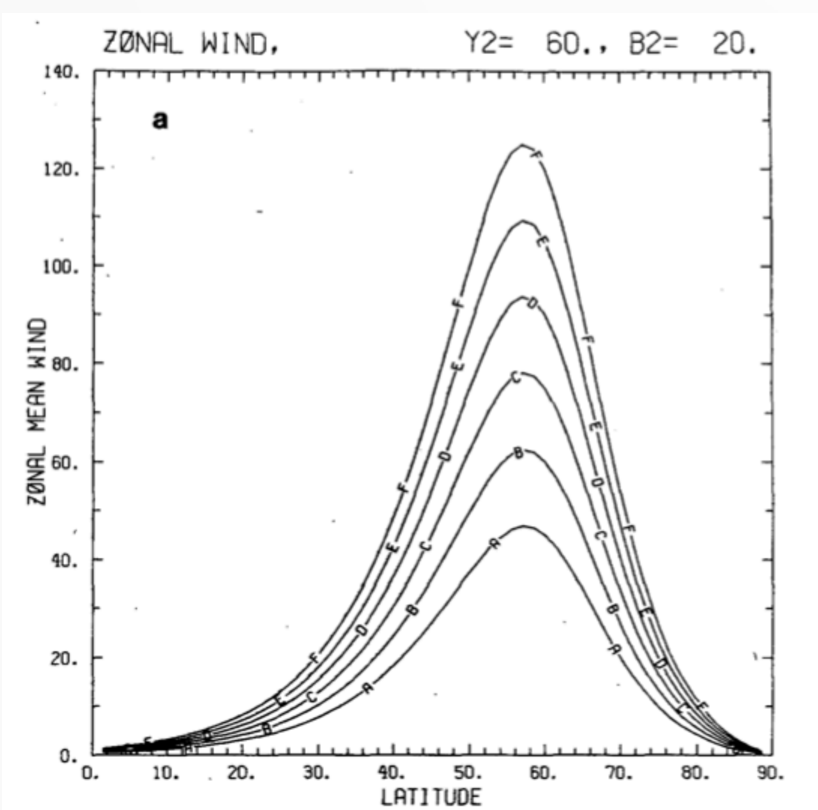
ϕ = Latitudinal coordinate

ϕ_0 = Jet's central latitude

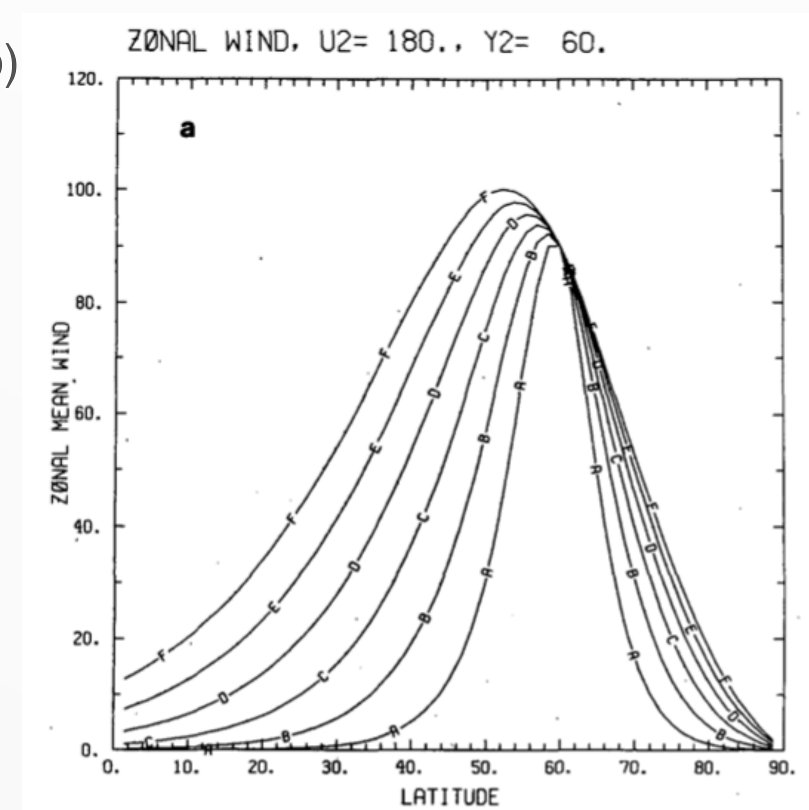
B = Jet's width

u_0 = Jet's amplitude

(a)



(b)



Figures from Hartmann (1983): Mean zonal wind for (a) $\phi_0 = 60^\circ$ and $B = 20^\circ$ and u_0 = 90, 120,150,180,210,240 m/sec and (b) u_0 = 180m/sec, $\phi_0 = 60^\circ$ and $B = 10^\circ$, 15° , 20° , 25° , 30° , 35° .

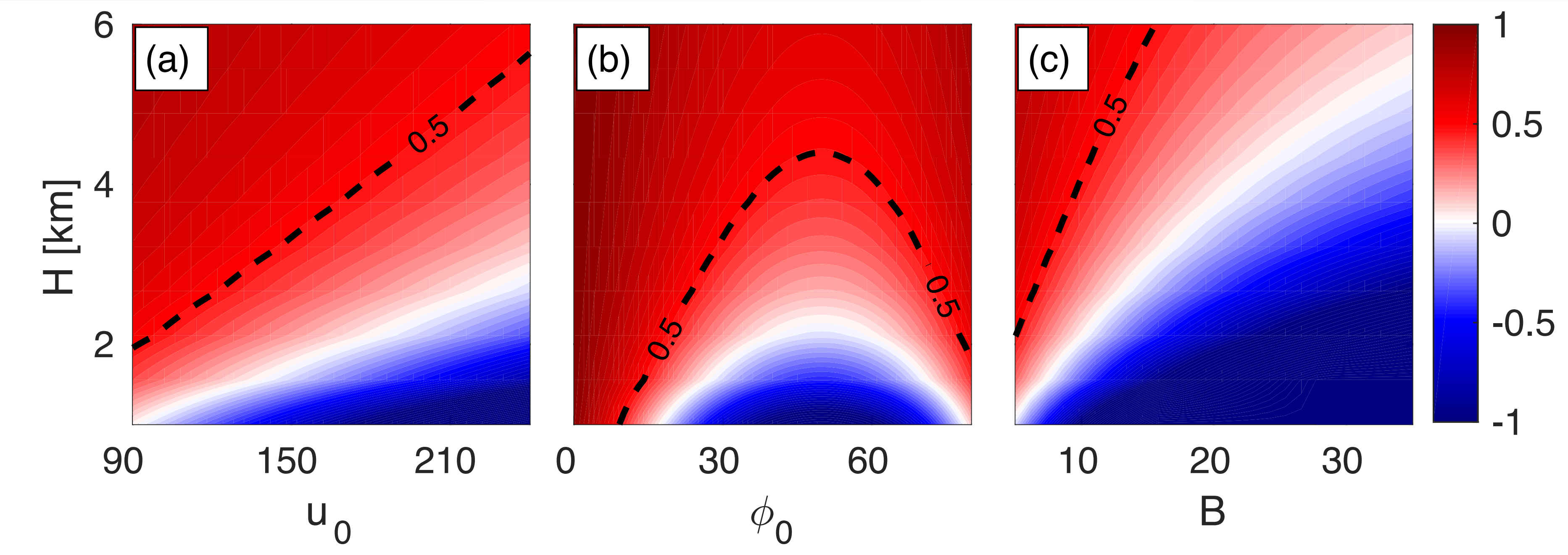
Spherical
Non-linear

Geostrophic balance

$$\bar{h}(\phi) = \bar{h}(-\frac{\pi}{2}) - \frac{a}{g}\int_{-\pi/2}^{\phi}\left[2\Omega + \frac{\bar{u}}{a\cos\phi'}\right]\sin\phi' d\phi'$$

$\bar{h}$ = Mean layer thickness
a = Earth mean radius
g = Earth gravitational acceleration
Ω = Earth angular frequency

- A complete specification requires also the mean height at a certain reference latitude
- Physically: conservation of mass => total mass = total mass at rest
- For any choice of $\bar{h}(-\pi/2)$, the mean layer thickness becomes negative at certain plausible jet-parameters

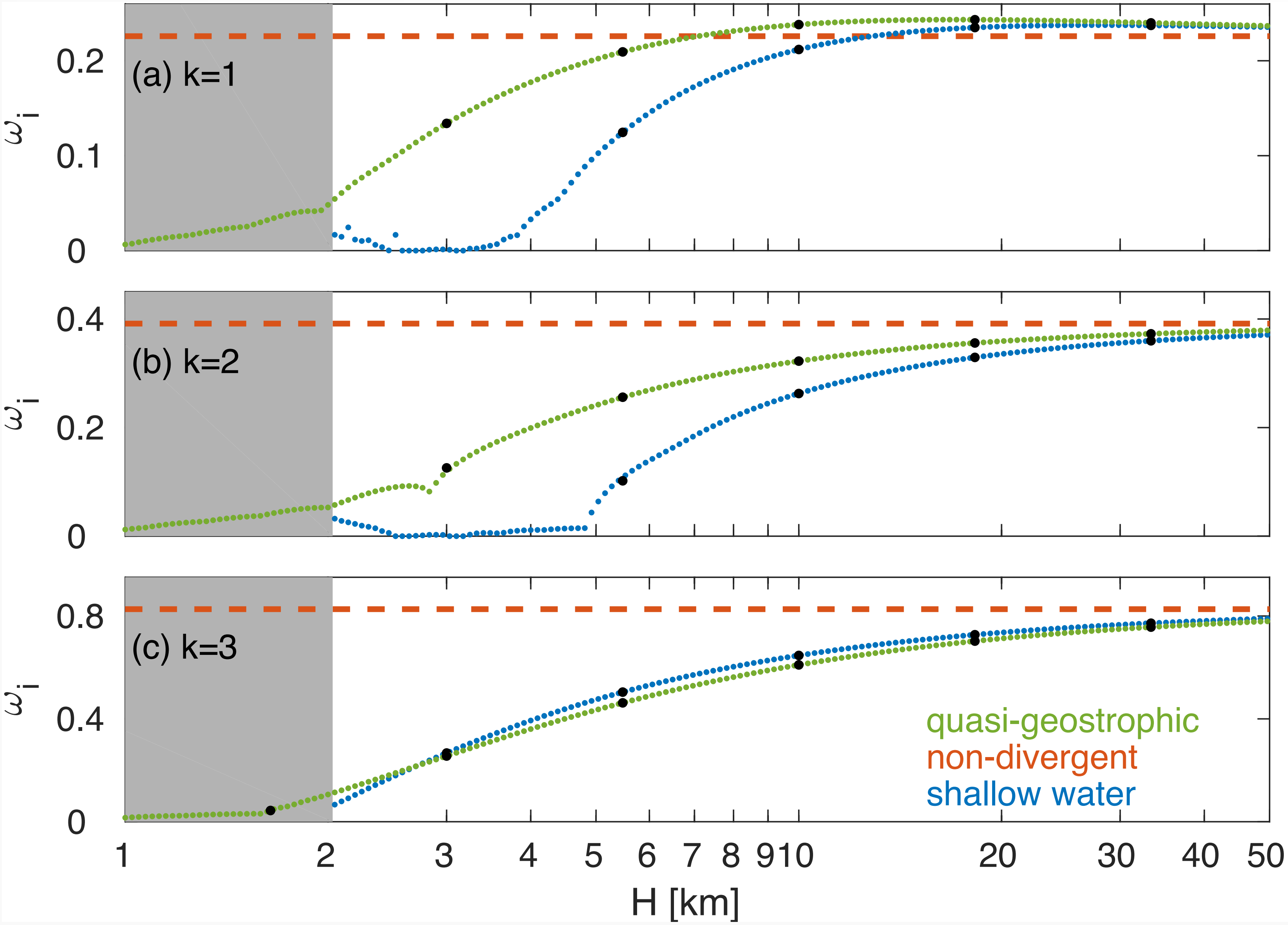


Global minimum of $\bar{h}(\phi)$ over all latitude as a function of the layer thickness at rest H and the jet-parameters u_0, ϕ_0, B

Polar jet

Growth rate of the most unstable mode

only for a layer depth that exceeds 10 km do the results of the ND model agrees to within 20% with the SW and QG models, and only at depths larger than 30 km (i.e. exceeding the thickness of the ocean or the Troposphere) the differences can be considered negligible

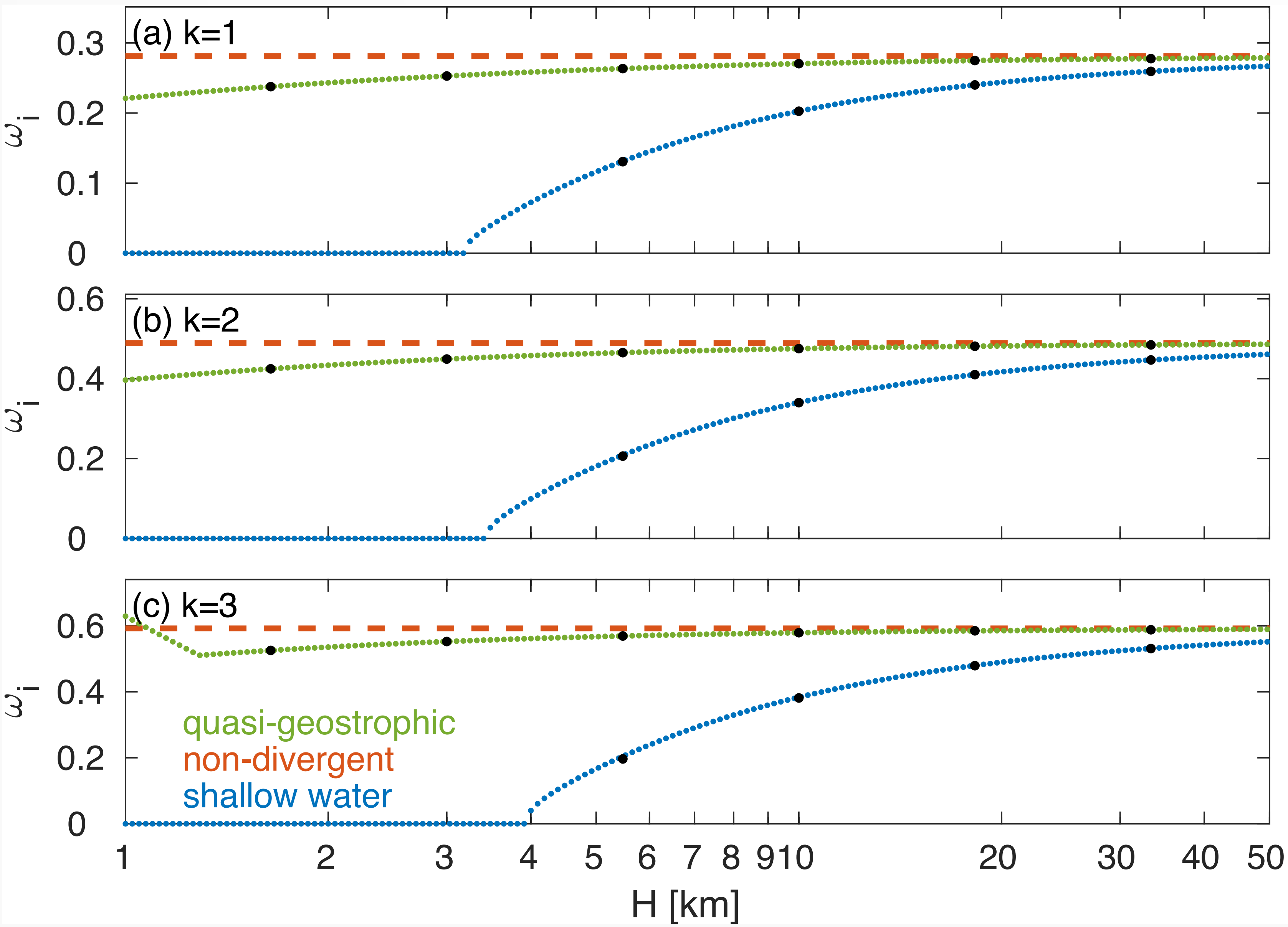


Growth rates (in units of 1/day) of the most unstable mode of the polar jet with u_0 = 180m/sec, ϕ_0 = 60° and B = 10° as a function of H for wavenumber = 1,2,3 panels(a),(b),(c), respectively. The grey shaded portion corresponds to values of H for which $\min_\phi h \leq 0$. Blue dots: SW model. Red dashed line: ND model. Green dots: QG model. Black dots: growth rates obtained using the shooting method.

Equatorial jet

Growth rate of the most unstable mode

- Like the polar jet, the differences between the models approach zero as $H \to \infty$
- Unlike the polar jet, QG model is closer to ND model as $H \to 0$
- Specifically, in the SW model the jet becomes stable for $H = 3\text{-}4$ km

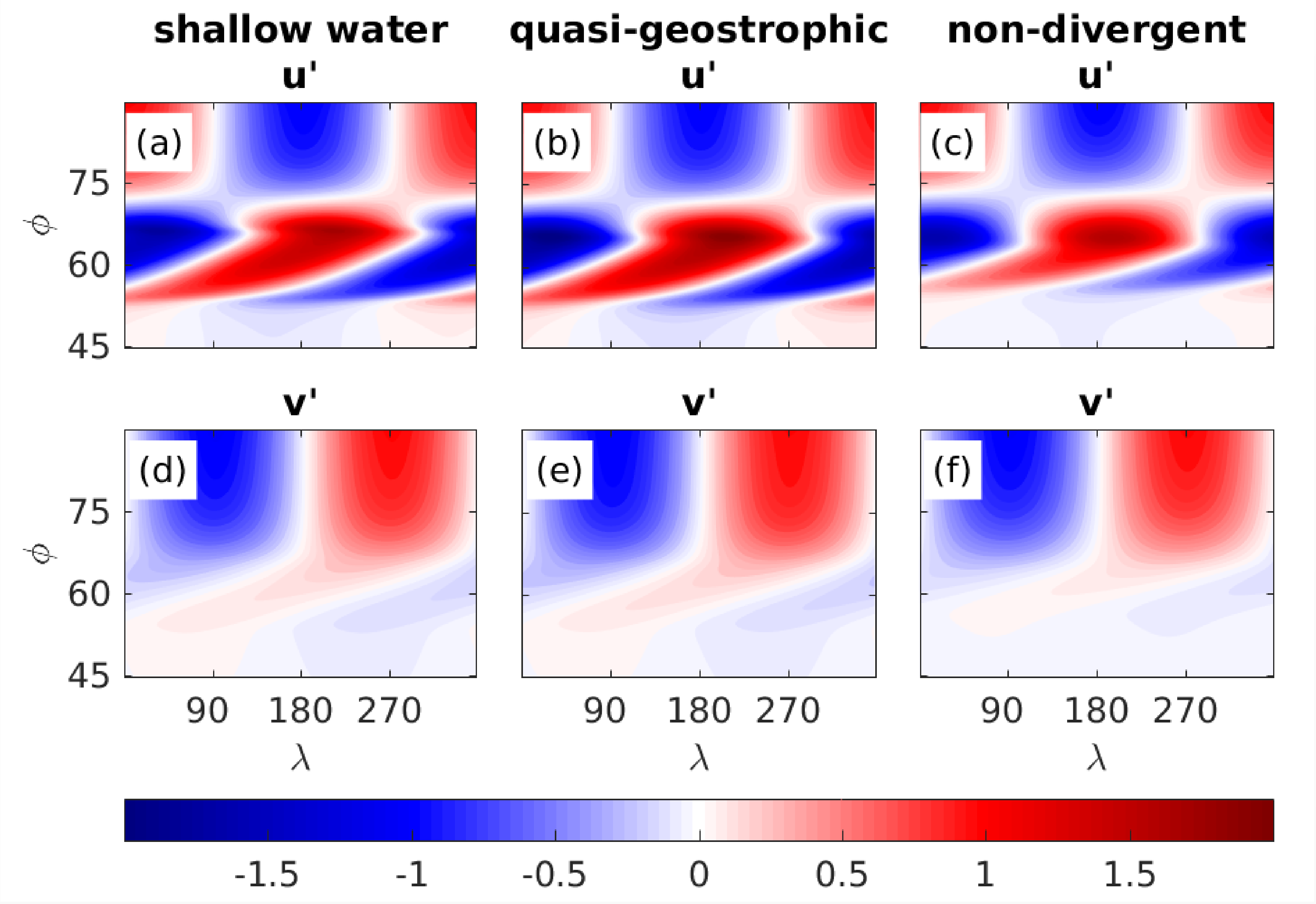


Growth rates (in units of 1/day) of the most unstable mode of the equatorial jet with u_0 = 180m/sec, $\phi_0 = 0^\circ$ and $B = 10^\circ$ as a function of H for wavenumber = 1,2,3 panels(a),(b),(c), respectively. The grey shaded portion corresponds to values of H for which $\min_\phi h \leq 0$. Blue dots: SW model. Red dashed line: ND model. Green dots: QG model. Black dots: growth rates obtained using the shooting method.

Polar jet

Structure of the most unstable mode

Though the growth rates of the most unstable mode in the SW model are substantially smaller than the QG and ND models, the structure of the most unstable mode differs only slightly in the three models.

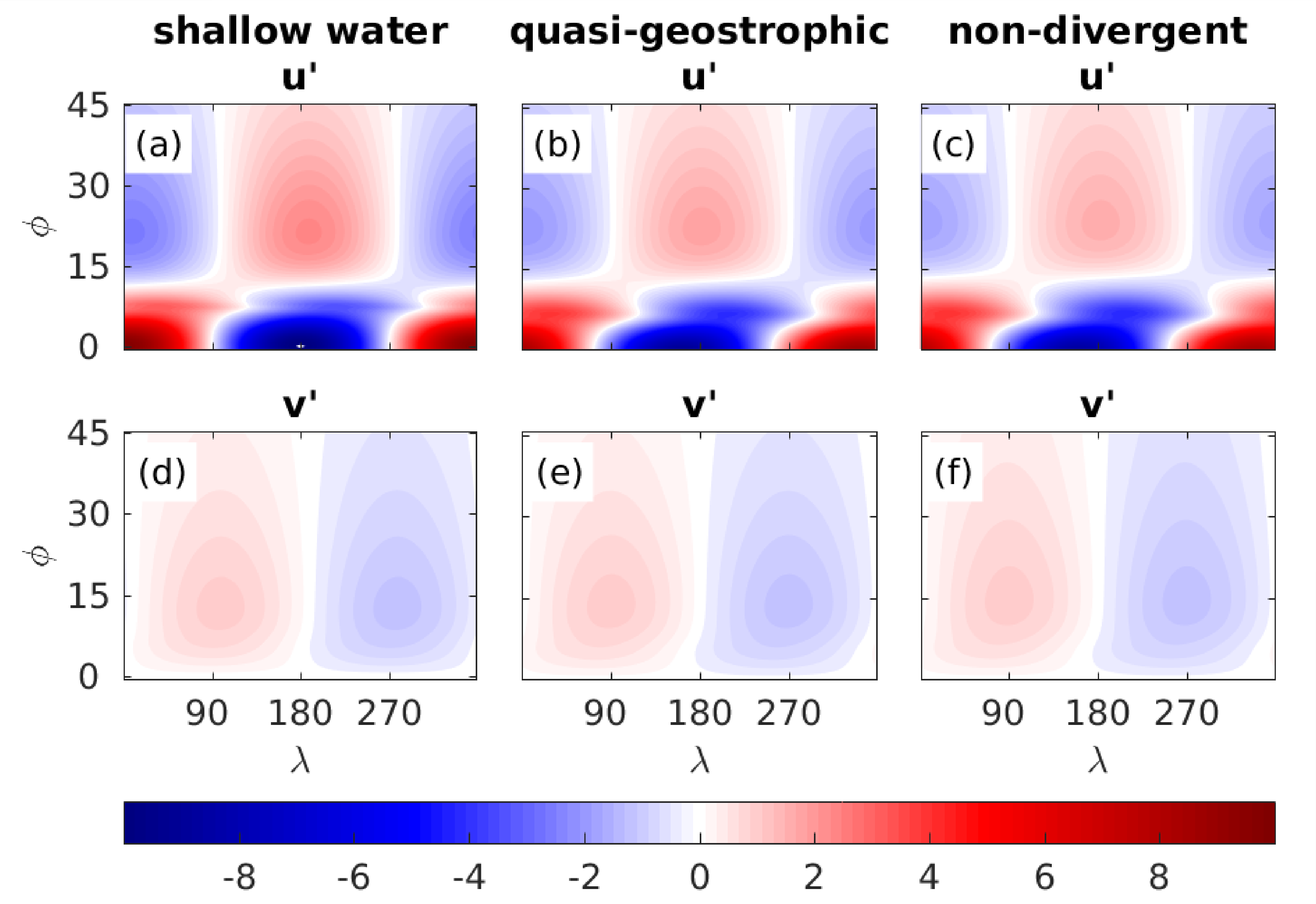


Latitude–longitude maps u' and v' of most unstable mode for a polar jet with u_0 = 180 m/sec, $\phi_0 = 60^\circ$ and $B = 10^\circ$, obtained using the SW model (a,d), the QG model (b,e) and the ND model (c,f). The solutions of the SW and QG system are shown for $H = 10$ km. In all solutions the fields' amplitudes are normalised on $\max_\phi |\hat{v}|$ in order to provide a consistent baseline for comparison.

Equatorial jet

Structure of the most unstable mode

- Like the polar jet, the structure of the most unstable mode agrees in all models
- Unlike the polar jet, the maximal zonal wind is 10 time larger than the maximal meridional wind



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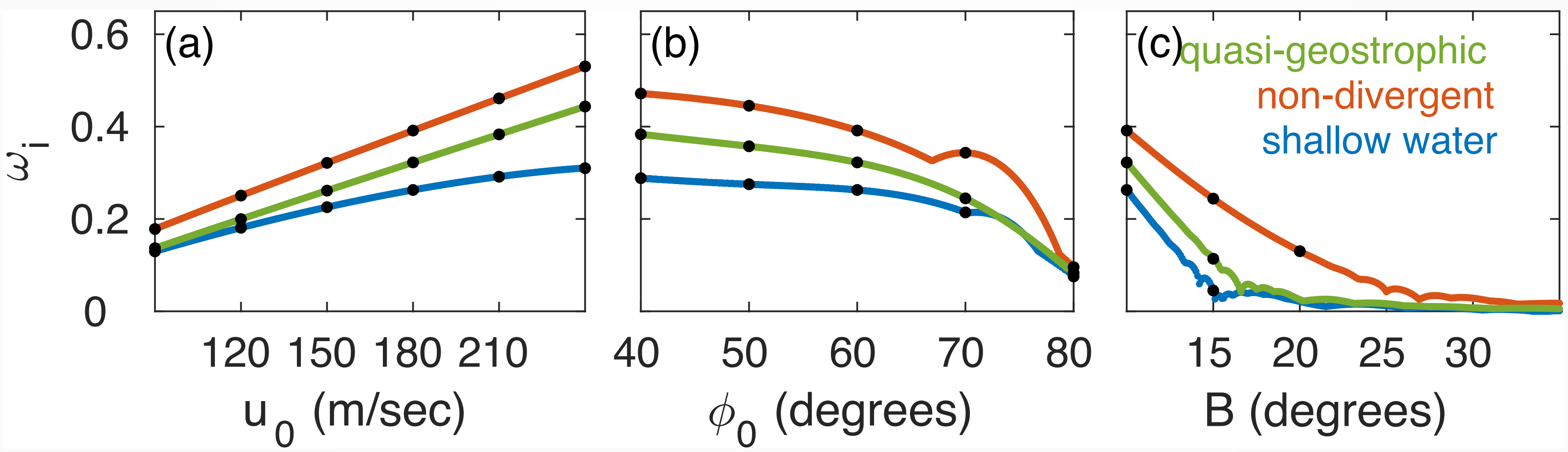
Polar jet

Parameter sensitivity

- In all cases, the growth rates are lowest in the SW model and highest in the ND model, and can be as large as 90% different (for $B = 15^{\circ}$).
- For wavenumber 1, the growth rates of the QG model become larger than the ND model as the jet strengthens.



Growth rates sensitivity to changes in the zonal wind parameters u_0 (a), ϕ_0 (b) and B (c) for wavenumber $k = 2$. (a) while holding $\phi_0 = 60^{\circ}$ and $B = 10^{\circ}$ fixed. (b) while holding $u_0 = 180$ m/sec and $B=10^{\circ}$fixed.(c) while holding u_0 =180m/sec and ϕ_0 =60°fixed. Blue: SW model with $H = 10$ km. Red: ND model. Green: QG model with $H = 10$ km. Black dots: growth rates obtained using the shooting method

Equatorial jet

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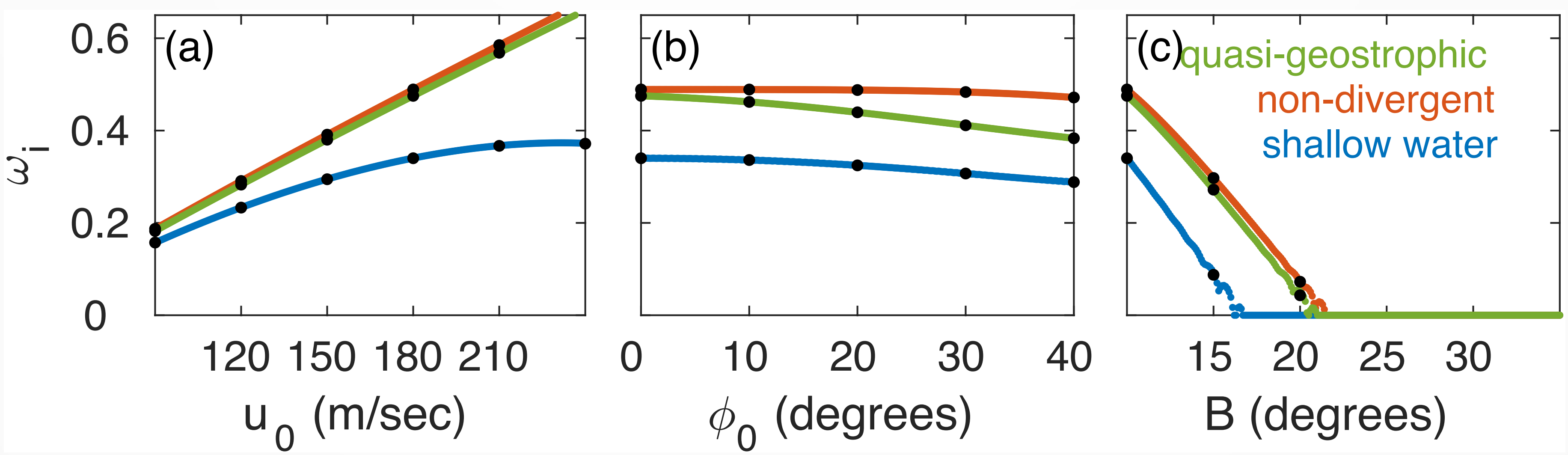

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Summary

The depth over which a layer is barotropically unstable is a crucial parameter in controlling the growth rate of small amplitude perturbations

This dependence is completely lost in the ND equation and is overly weak in the QG system

Only for depths of 30 km or more are the growth rates predicted by the ND and QG systems a good approximation to those of the SW

For depths of between 5 and 10 km, the growth rates predicted by the SW are smaller by more than 50% than those of the ND and QG

Thank you for listening!