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# Gravity inversion with depth normalization

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# Introduction

- The inverse problem of gravity exploration (e.g. 2D problem) for mesh model can be formulated as a solution of an underdetermined system of linear algebraic equations:

$$A\sigma = V_z, \tag{1}$$

$A$ – gravity effects' matrix, $\sigma$ – vector of density, $V_z$ – observed gravity field

$$A = \begin{vmatrix} Vz_{11}(x_1) & Vz_{12}(x_1) & \dots & Vz_{MN}(x_1) \\ Vz_{11}(x_2) & Vz_{12}(x_2) & \dots & Vz_{MN}(x_2) \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ Vz_{11}(x_L) & Vz_{12}(x_L) & \dots & Vz_{MN}(x_L) \end{vmatrix} \qquad \sigma = \begin{vmatrix} \sigma_{11} \\ \sigma_{12} \\ \dots \\ \sigma_{1N} \\ \sigma_{21} \\ \dots \\ \sigma_{M1} \\ \dots \\ \sigma_{MN} \end{vmatrix} \qquad V_z = \begin{vmatrix} V_z(x_1) \\ V_z(x_2) \\ \dots \\ \dots \\ \dots \\ V_z(x_L) \end{vmatrix}$$

# Introduction

- A standard approach to solving this problem is to minimize the loss function:

$$L = \| A\sigma - V_z \|^2 + \frac{1}{C} \| \sigma^2 - \sigma_a^2 \|^2 \rightarrow min \qquad (2)$$

$\sigma_a$ – vector of prior density, $C$ – inverse of regularization strength

- Traditionally, the minimum of the loss is found using optimization methods, in particular the gradient descent method:

$$\sigma_{n+1} = \sigma_n - \alpha \nabla_{\sigma} L \qquad (3)$$

$\alpha$ - constant or changes during iterations (but constant for each cell, i.e. constant within iteration)

# Problem

- Underdetermined system has an infinite number of solutions;

- Loss function L is most sensitive to near-surface cells, therefore, the traditional approach allows to select models with a contrast near-surface layer. Such models can be used, for example, for source-based continuation. However, they are not suitable for geological interpretation:

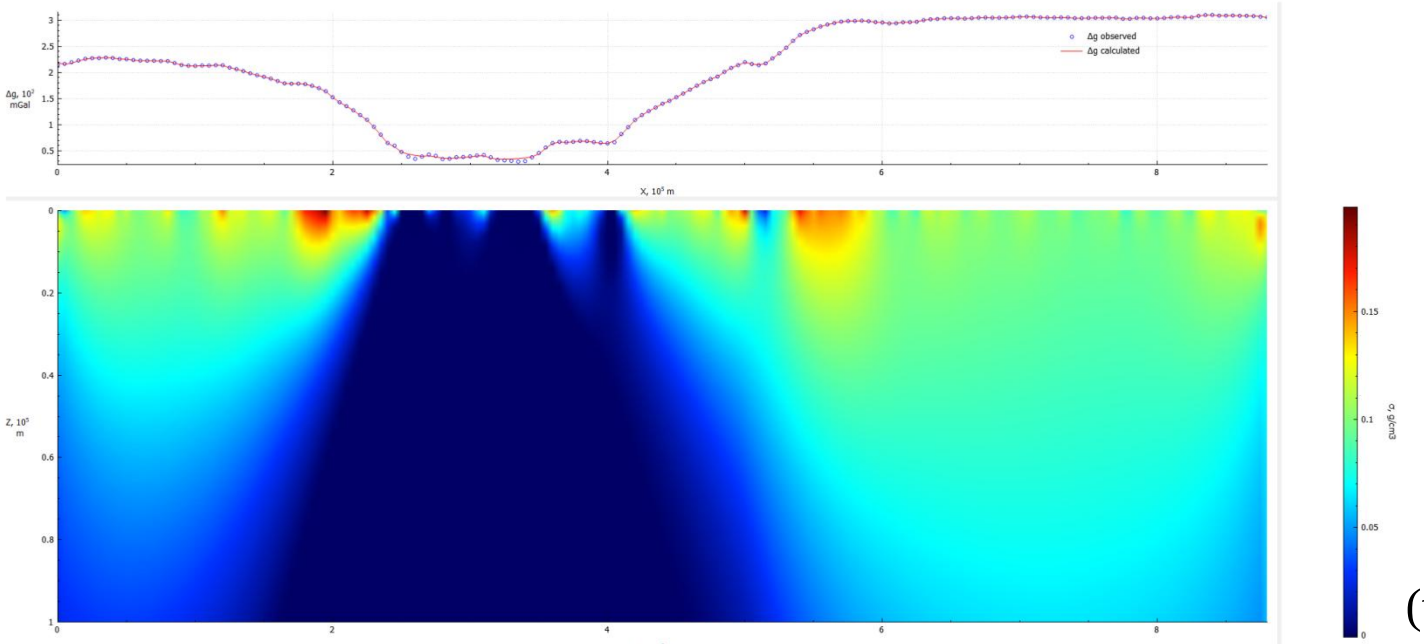



Fig. 1 Example of inversion result (traditional approach)

# Proposed solution

- Since sensitivity of loss function decreases with depth, a variable parameter $\alpha$ value can be used to solve this problem.

- For example, $\alpha$ can be depth dependent function**.** Particularly, $\alpha$ can be $\sim z^n$, the power n is a hyperparameter and is selected by the interpreter

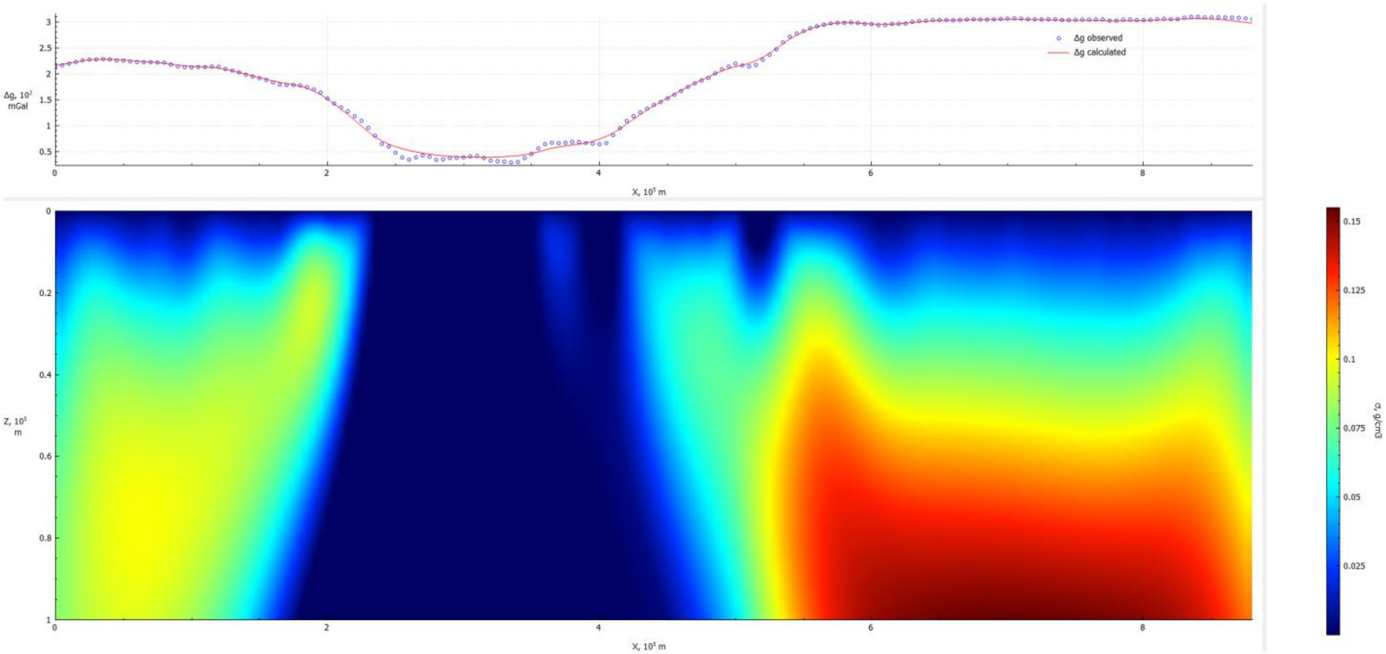


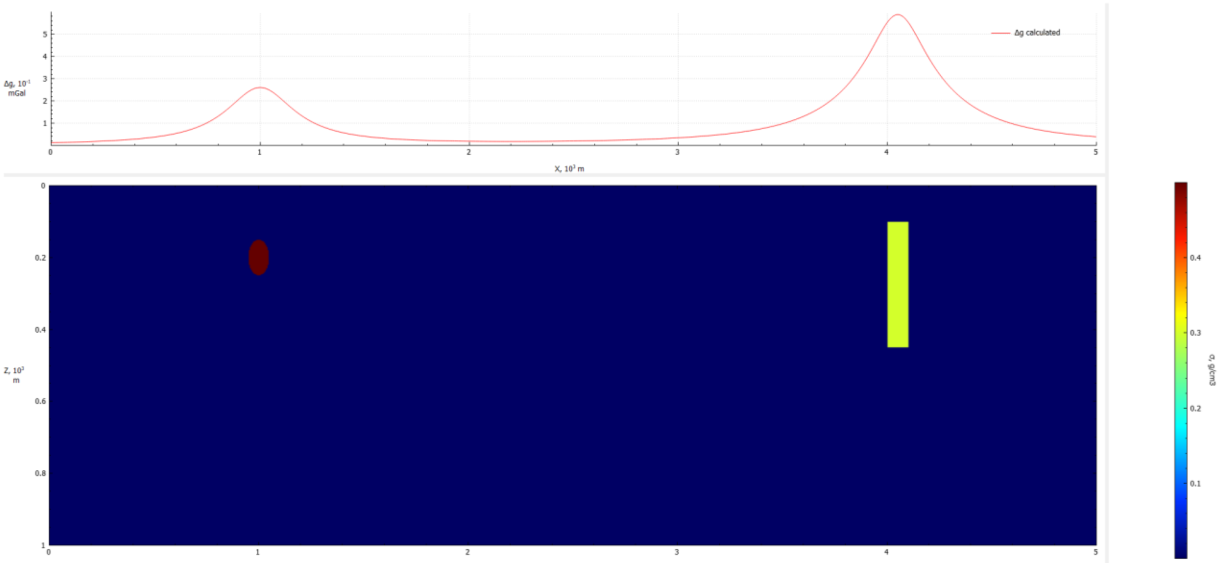

Fig. 2 Example of inversion result (proposed approach)

# Test model #1

- The first test model is 2D and consists of two anomalous objects: an infinite horizontal cylinder and a vertical rectangular prism.
- Length of profile is 5 km, maximum depth is 1 km;
- Theoretical gravity of this model was used as observed field;
- No prior data used;

Table 1. Test model #1 parameters

| Parameter | Cylinder | Prism |
|---|---|---|
| Excess density ($g/cm^3$) | 0.5 | 0.3 |
| Z top (m) | 150 | 100 |
| Z bottom (m) | 250 | 450 |
| X min (m) | 950 | 4000 |
| X max (m) | 1050 | 4100 |
| $\Delta V_z$ max (mGal) | 0.26 | 0.59 |




- For 2D gravity modeling we used GravInv2D software (*www.gravinv.ru*)

Fig. 3 Test model # 1 with calculated gravity

# Result #1

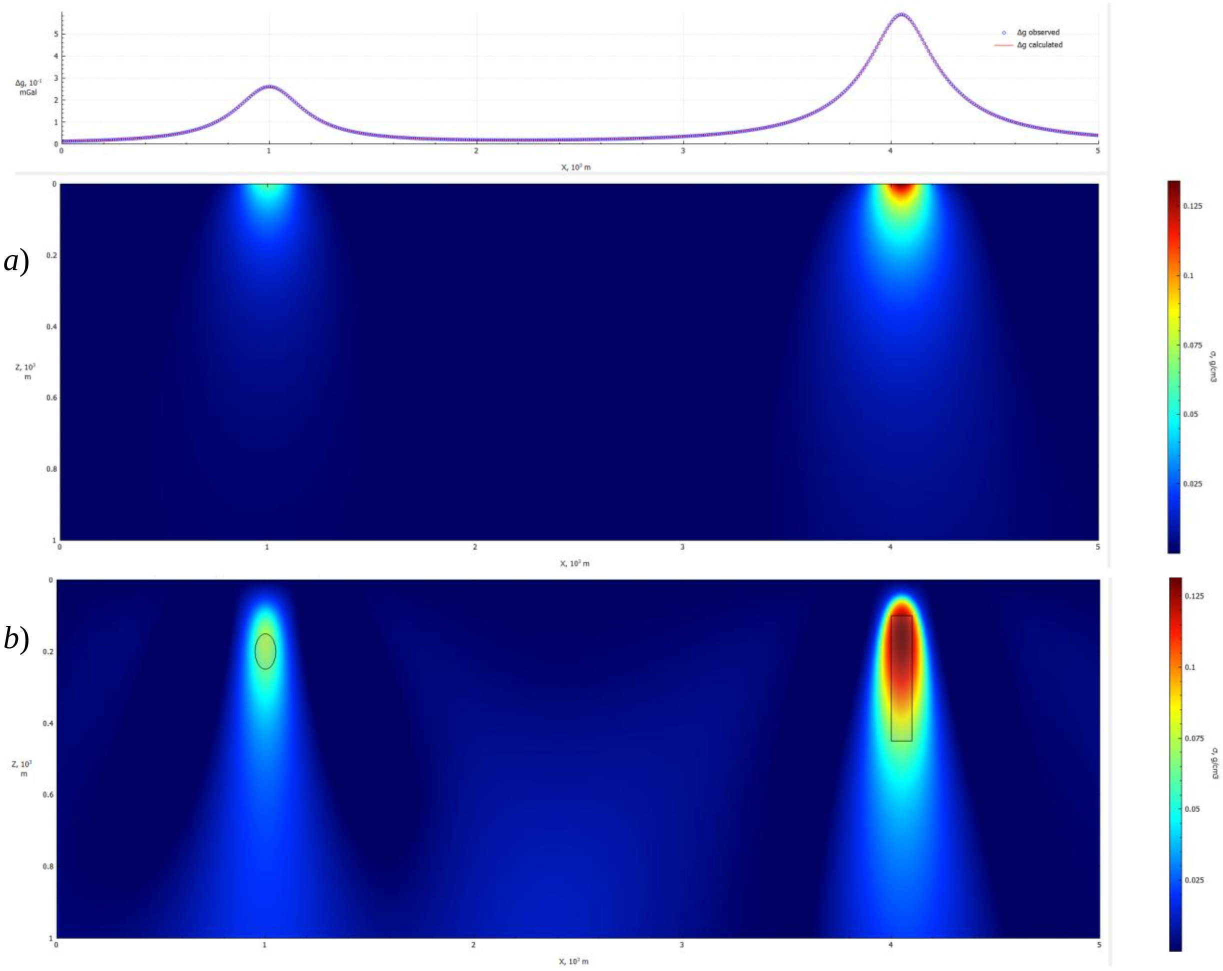



Fig. 4 Inversion results: *a*) α = const; *b*) α ~ $z^{1.5}$.
Blue dots – gravity field of test model;
Red line – gravity field of selected model.

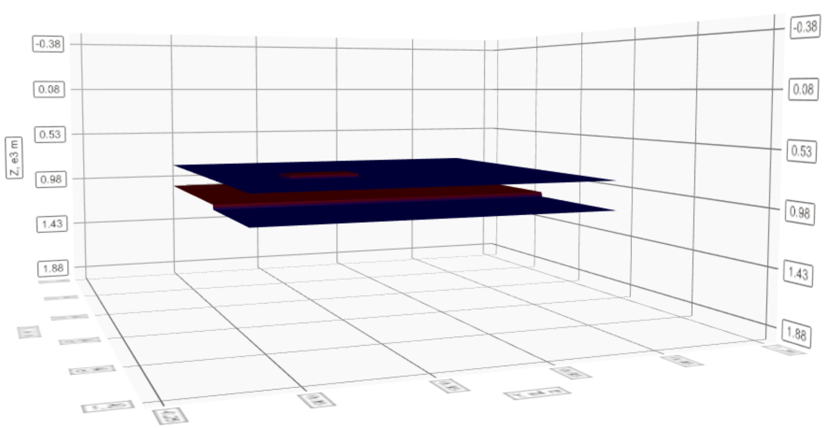
- RMS between observed and calculated field < 1% in both cases;
- Contrast near-surface layer at constant α;
- α, increasing with depth allows to fit the model, that better shows the position of anomalous objects;
- Both results allow to define horizontal position of objects;
- Both results represent density underestimation.

# Test model #2

- The second model is 3D and consists of three layers (two boundaries). First boundary has inlier, second boundary has fault.
- Size of area is 10 km x 10 km, maximum depth is 1.5 km;
- No prior data used;
- For 3D gravity modeling we used GravInv3D software (*www.gravinv.ru*)

Table 2. Test model #2 parameters

| Parameter | Inlier | Fault |
|---|---|---|
| Excess density ($g/cm^3$) | 0.3 | 0.2 |
| Z top (m) | 770 | 1000 |
| Z bottom (m) | 800 | 1040 |
| X min (m) | 5500 | 6000 |
| X max (m) | 6500 | 6000 |
| Y min (m) | 2000 | - ∞ |
| Y max (m) | 4000 | + ∞ |
| $\Delta V_z$ max (mGal) | 0.10 | 0.29 |



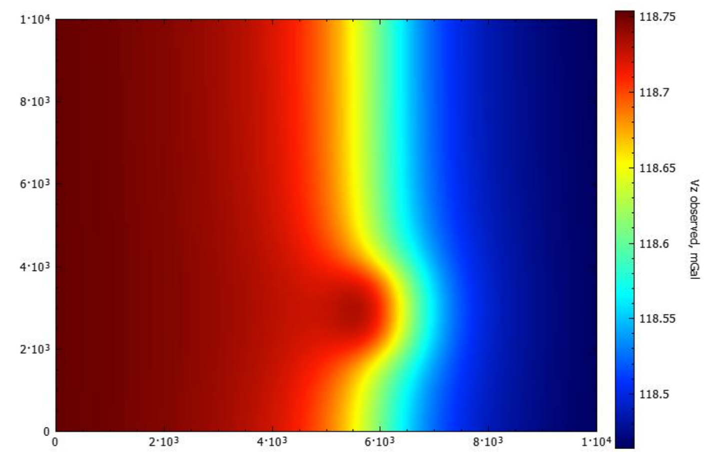




Fig. 5 Test model #2 with calculated gravity

# Result #2

a)

b)

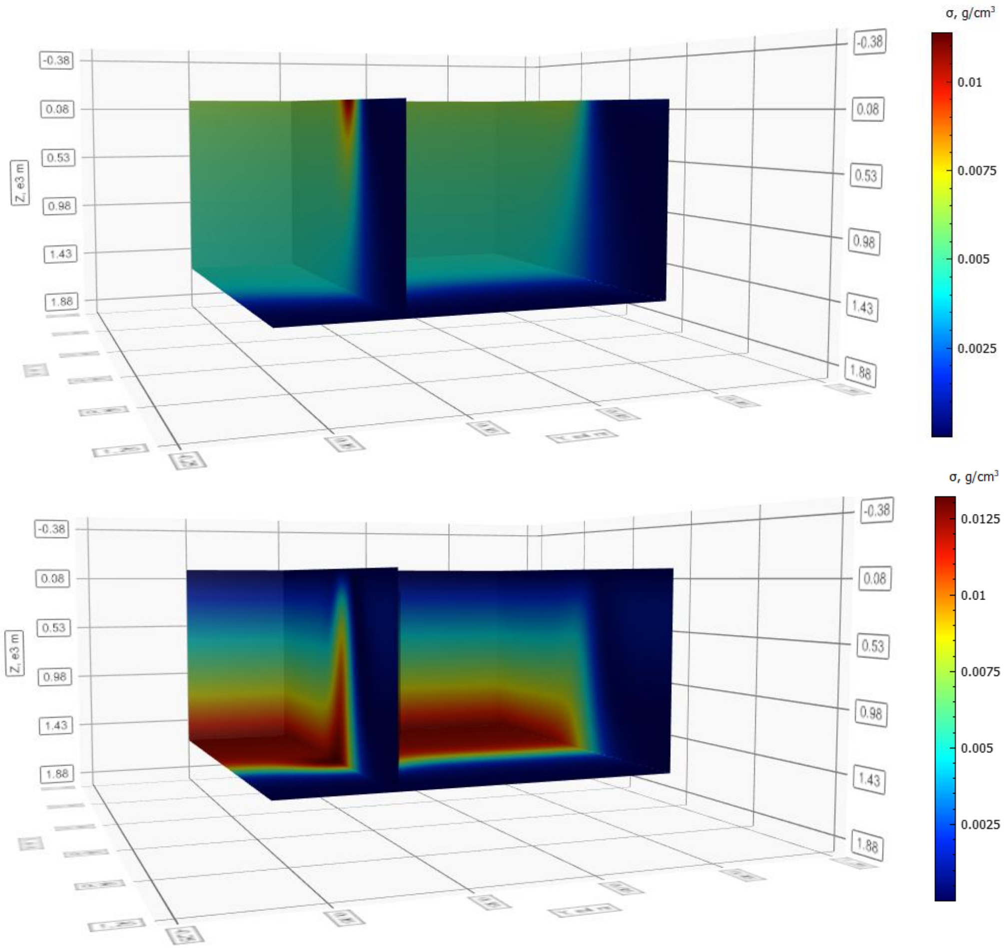



Fig. 6 Inversion results: *a*) $\alpha$ = const; *b*) $\alpha \sim z^{1.5}$

- RMS between observed and calculated field < 1% in both cases;
- Contrast near-surface layer above inlier and positive excess densities at all depths above fault ($\alpha$ = const);
- Extremum of density near mass center of inlier and maximum of density gradient near fault ($\alpha \sim z^{1.5}$);
- Both results allow to define horizontal position of objects;
- Both results represent density underestimation.

# Result #2

- Maximum of top density anomalous area corresponds to mass center of the inlier;

- Area of density gradient maximum (bottom anomalous area) corresponds to the top of fault;

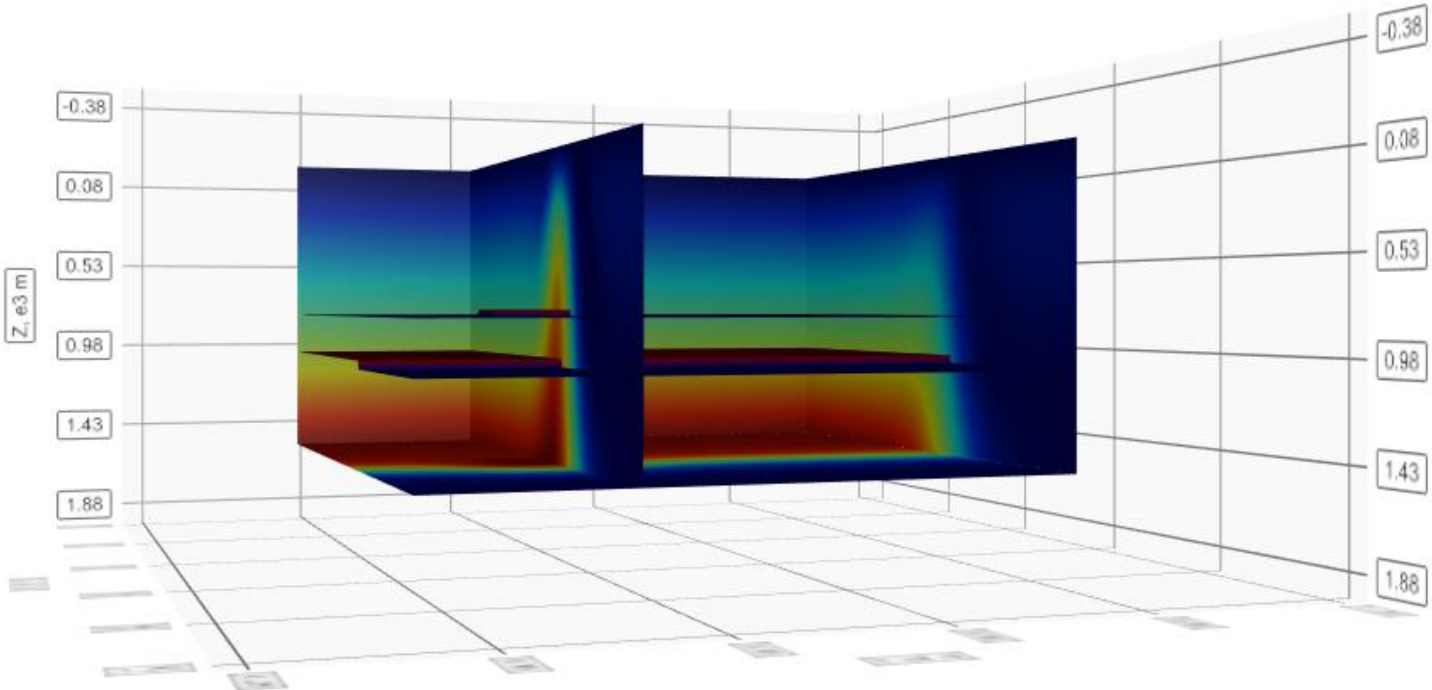



Fig. 7 Inversion result ($\alpha \sim z^{1.5}$) with test model

# Conclusions

- Using of depth dependent descent step allows to include deep cells in model selection process;
- Tests represent that models, selected with depth dependent $\alpha$, have density extremum at points, close to mass center, if anomalous object is isolated. Also the approximate boundaries of the object can be highlighted;
- In the case of semi-infinite objects, the proposed approach allows to estimate the depth of the upper edge by maximum of density gradient;
- To achieve the best estimate, we propose to select several models with different values of $\alpha$ power from the interval [1; 2];
- Proposed approach allows to estimate approximate position of anomalous objects even without prior data;
- Prior data must be used to determine the anomalous density and more precise boundaries.

Thank you for your attention!