Approximating Probabilistic Joint Inversion using Bayesian Spatial Ensemble Fusion

Gerhard Visser, Hoël Seillé, Jelena Markov

gerhard.visser@csiro.au

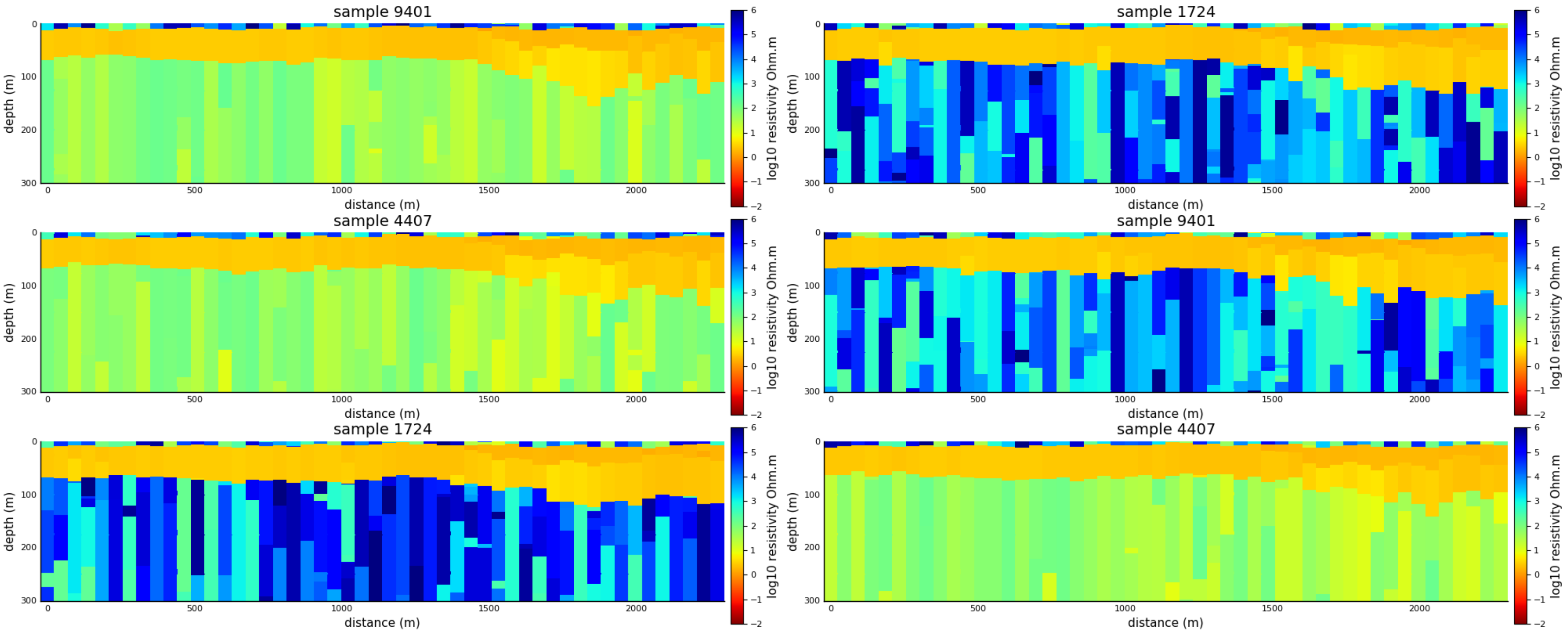
CSIRO, Deep Earth Imaging Future Science Platform

Each presentation slide is followed by a text slide describing it.

This presentation introduces Bayesian ensemble fusion as tool for approximating probabilistic joint geophysical inversion in a way we hope will lead to significant time and effort savings when scaled up.

The method will be demonstrated here using airborne electromagnetic (VTEM) and audio-magnetotelluric (AMT) data from Cloncurry in the Mount Isa province of Queensland.

Ensemble geophysical inversion



multiple plausible resistivity models derived from a single airborne EM (VTEM) flight line; uses 1D forward simulation

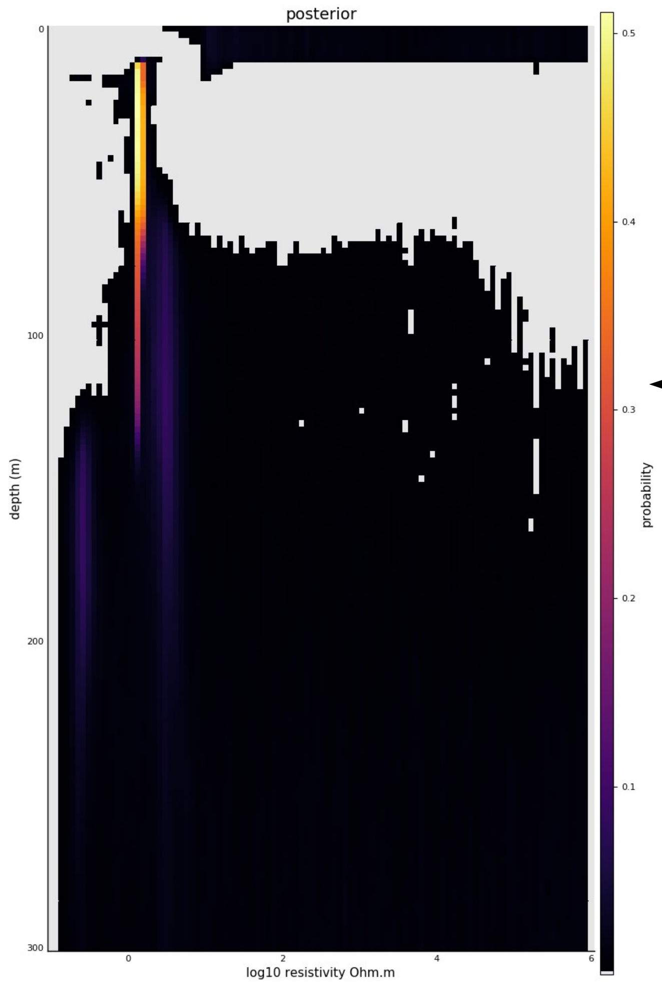
Ensemble inversion is an approach where, instead of producing a single best estimate model of the subsurface, a large set of plausible models is created.

This is a more thorough way to deal with the inherent non-uniqueness of geophysical inversion.

The image shows an example where six 2D resistivity models images were randomly sampled.

In practice, Markov chain Monte Carlo (MCMC) methods are the dominant method for creating such ensembles and our work builds on this.

Algorithmic complexity vs. Moor's law



1D resistivity ensemble from VTEM

2–3 hrs computation on single node

Moor's law

transistor count on microchips double roughly every 2 years

Algorithmic Complexity

increased resolution leads nonlinear increase in computational demands

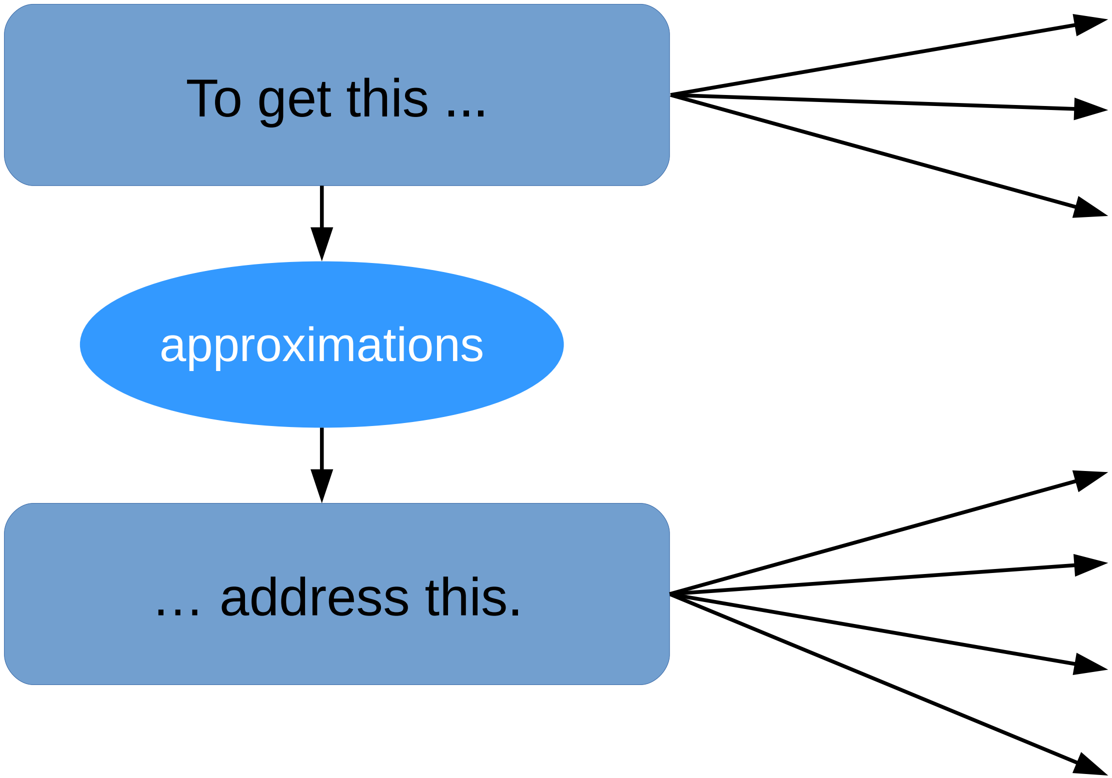
MCMC methods are extremely computationally expensive and for high dimensional model spaces can become impractical.

As a result, most existing MCMC inversion codes currently use simplified geometries. The 1D resistivity model ensemble displayed in the figure is an example.

We wish to achieve ensemble inversion on a large scale.

Due to non-linear algorithmic complexity we do not expect that waiting for more powerful hardware to be developed is an option.

Practical probabilistic workflows?



Motivations:

- explore possibilities
- test hypothesis
- quantify uncertainty

Challenges:

- computationally expensive
- simple model geometries
- hard to interpret
- hard to implement

Approximations allow us to trade accuracy for speed.

We introduce an approach that builds on three different approximations to achieve time and effort savings, both in application and development, for joint geophysical inversion.

Three approximations

1) partitioned forward modelling

2) rejection sampling

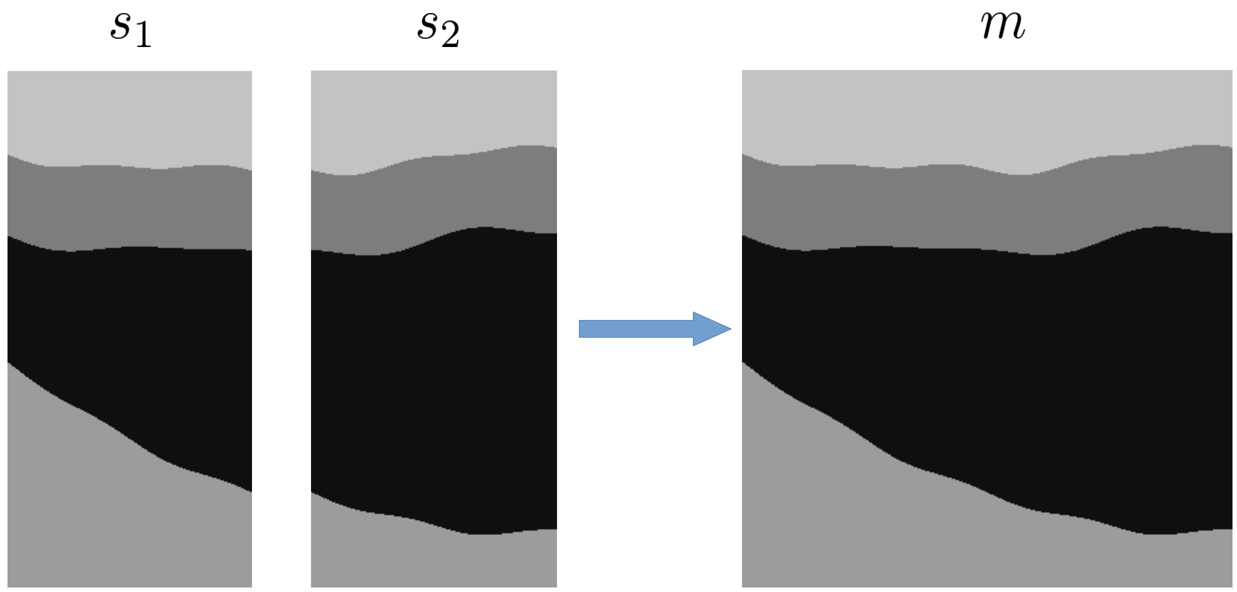
3) Bayesian ensemble fusion

These three approximations are listed and each will be described in the following slides.

Partitioned forward modelling

likelihood is approximated by simulating
forward physics on separate sub-models

$$\Pr(d|m) \approx \prod_k \Pr(b_k|s_k)$$



$$m = \text{combine}(s_1, s_2, \ldots)$$

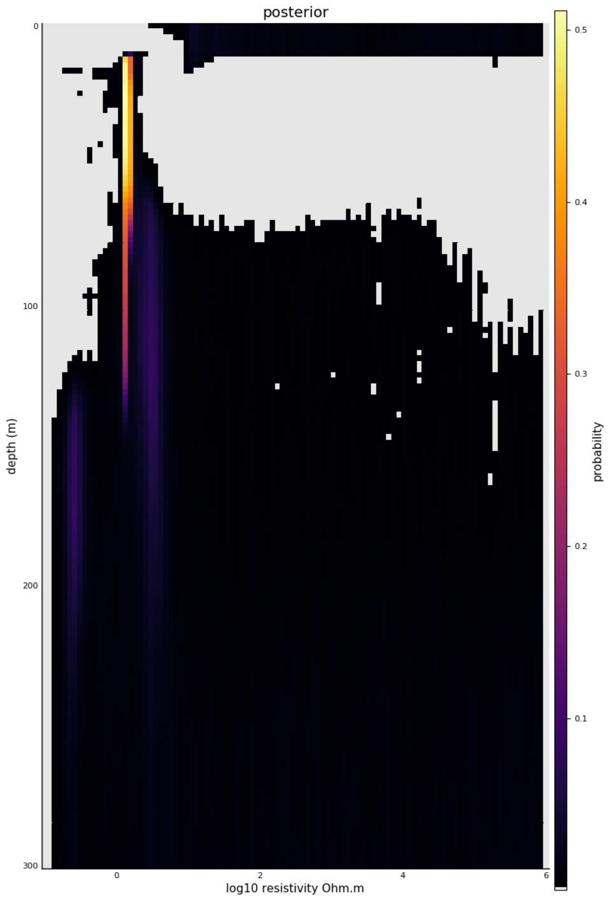
$$d = \text{combine}(b_1, b_2, \ldots)$$

By partitioning the data and performing forward physics simulations on smaller sub-models, simplified geometries can be assumed which may allow much faster forward computation.

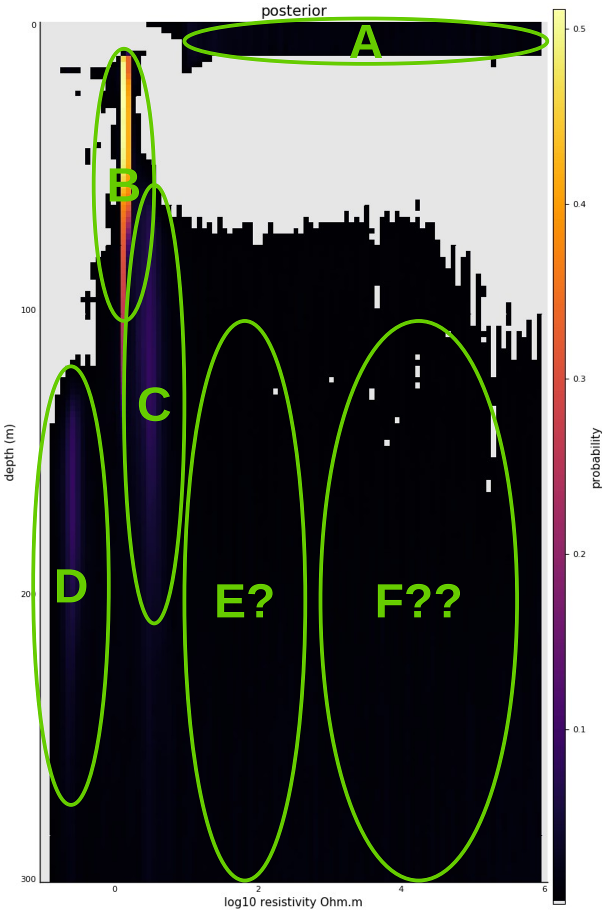
This approximation is already in common use, for example, laterally constrained inversion of AEM data computes forward physics using a series of adjacent resistivity 1D models.

Application of this approximation is not always safe and we need to consider the size and spacing of partitions carefully.

Rejection sampling using segmentation



identify layers



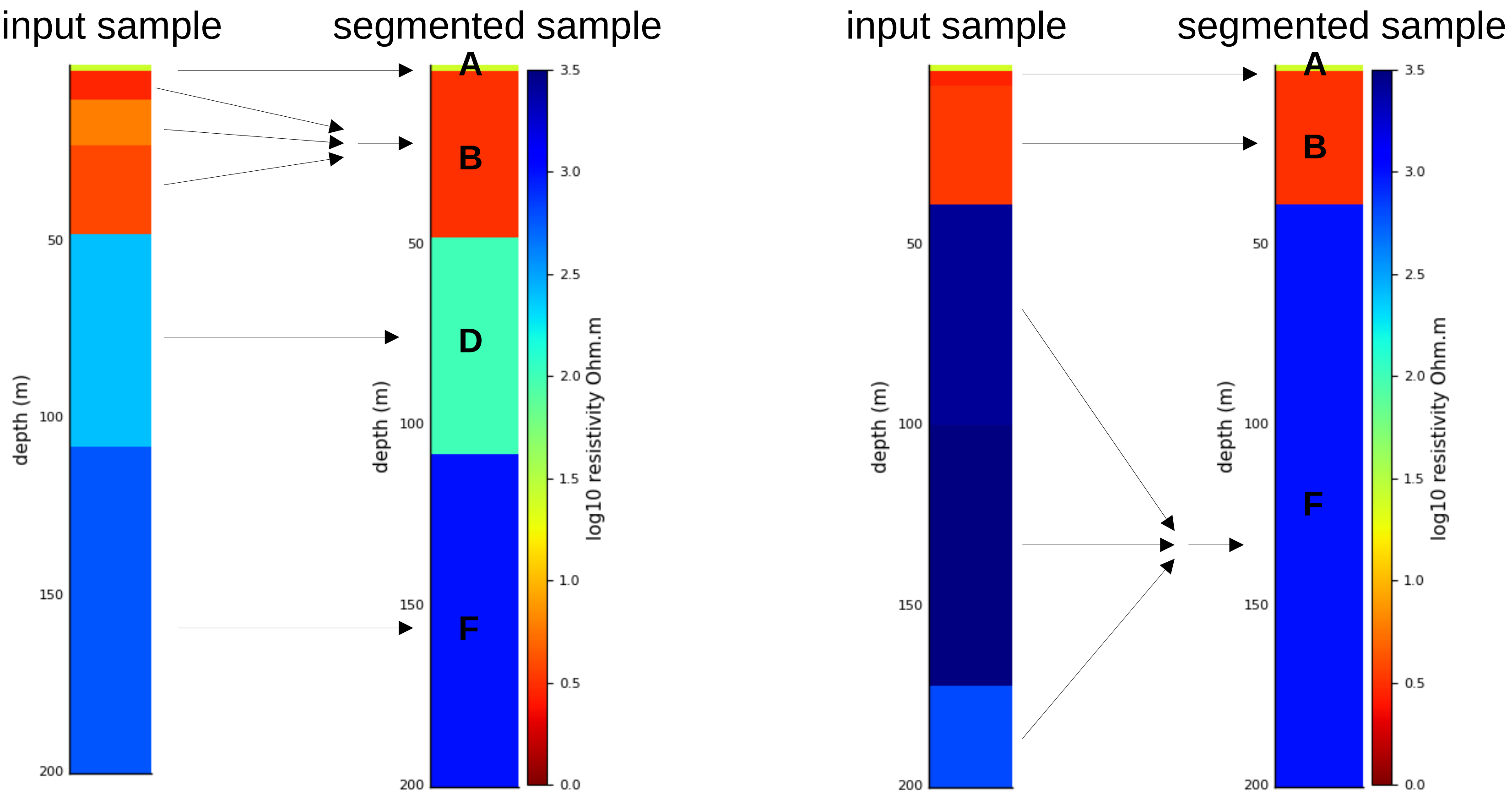
Rejection sampling can be used to apply in informative prior to an ensemble of models obtained using a less informative prior without having to repeat any forward calculations.

We use rejection sampling in conjunction with segmentation.

Here a set of 10000 1D resistivity models were sampled using transdimensional inversion from a single VTEM site. A non-informative prior was used.

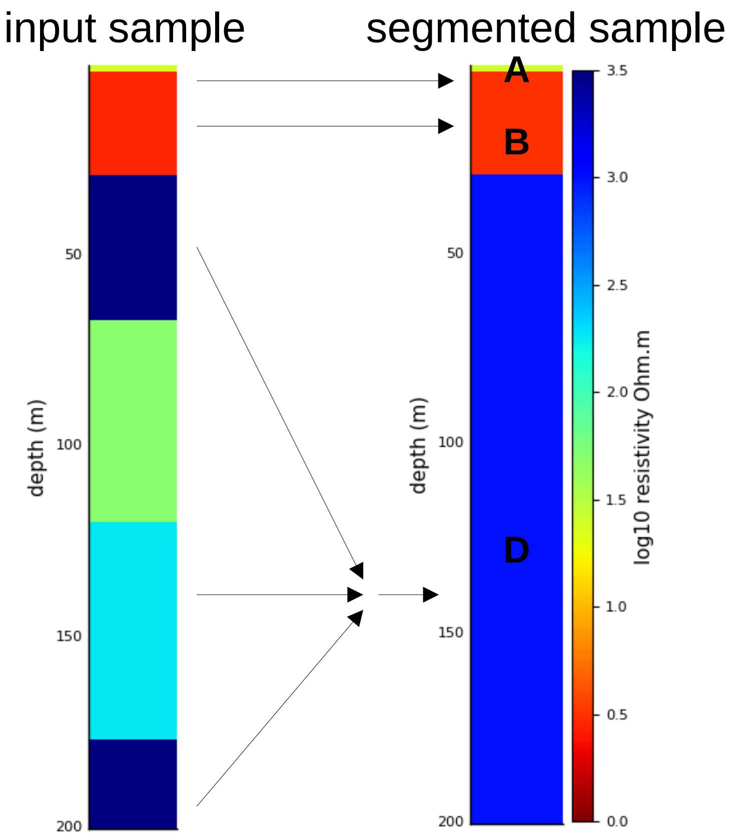
We now construct a more informative prior by define the suspected resistivity layers illustrated by the green ovals. The order in which they may occur is also restricted.

1D models can be segmented accoding to an intepretation



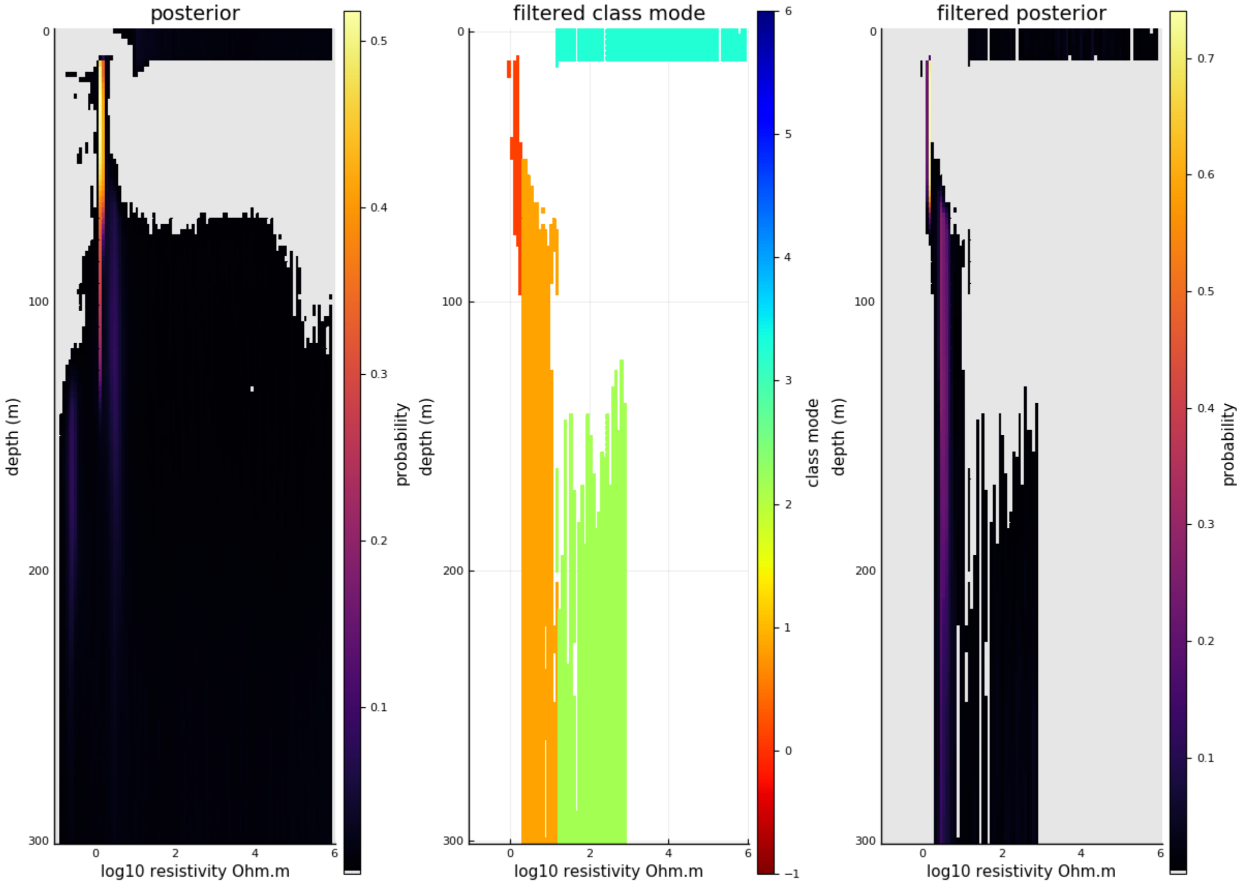
Individual 1D models can be cast into the informed parameterization using segmentation.

Samples that don’t fit the segmentation model are down-weigted or filtered out



Models that don't fit the informed prior can be down-weighted in probability or filtered out.

Possibilities are reduced



Casting an uninformed ensemble into an informed prior in this way can save time when multiple informed prior are considered. It requires no additional forward calculations

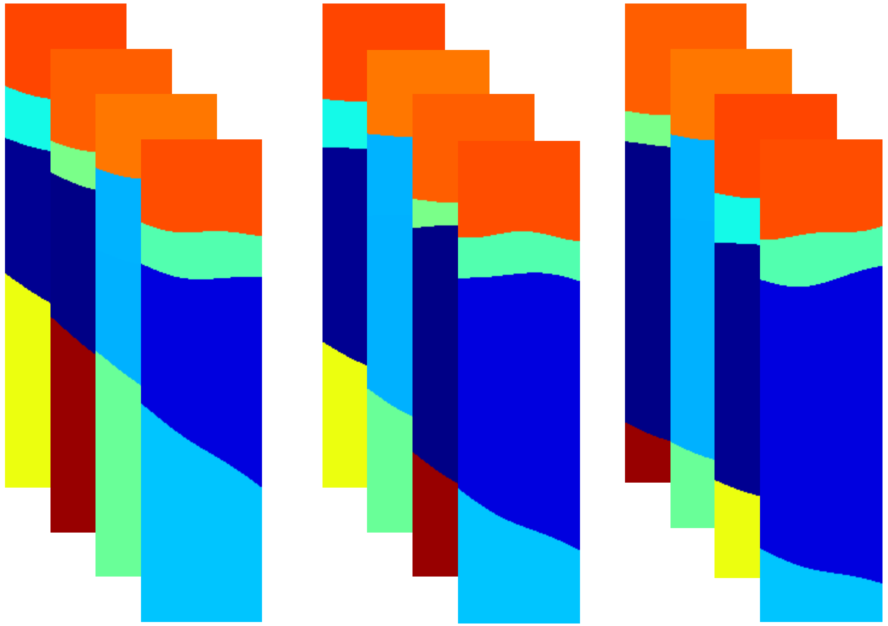
The need for using informed priors arises because there are often competing geological hypothesis and new ones may be elicited during the analysis.

The example shows an updated ensemble where the layers previously labelled 'D' and 'F' have been ruled out.

Bayesian ensemble fusion

Input:

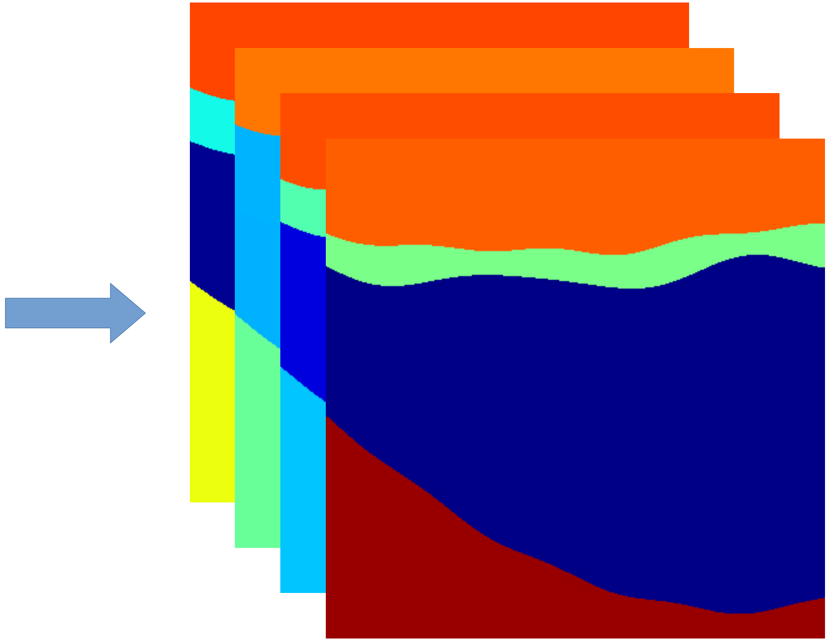
sub-model ensembles
from 3 adjacent locations



here each ensemble has 4 samples

Output:

a single combined ensemble

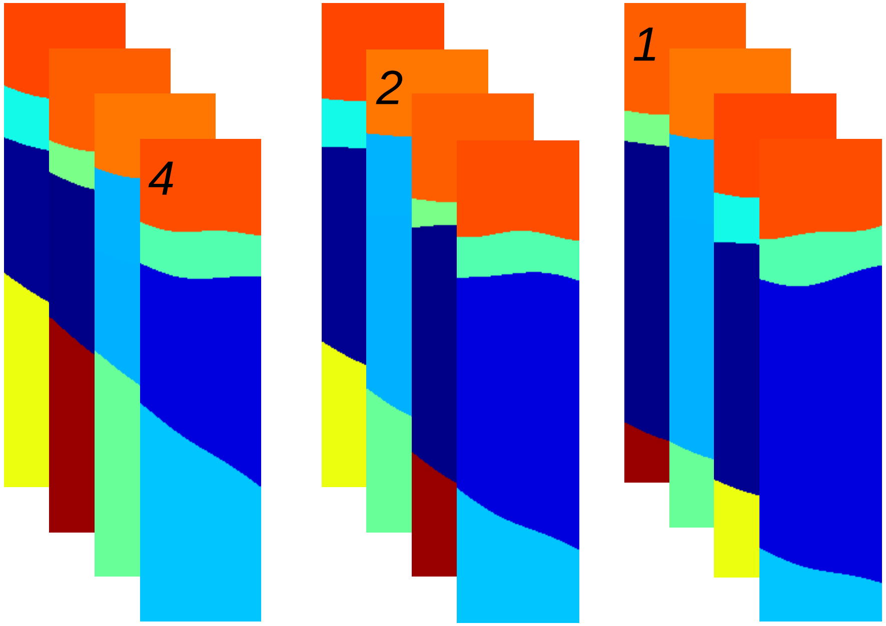


The final approximation used is Bayesian ensemble fusion which we introduce to this application.

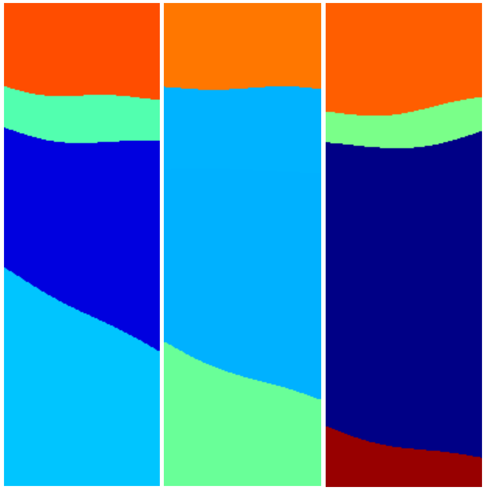
In the figure we are given three model ensemble for three adjacent locations. For simplicity each is depicted as having only 4 samples.

The goal is to process these in a way that outputs a single ensemble of larger 'fused' models.

Prior belief rules out most combinations



combination (4,2,1)

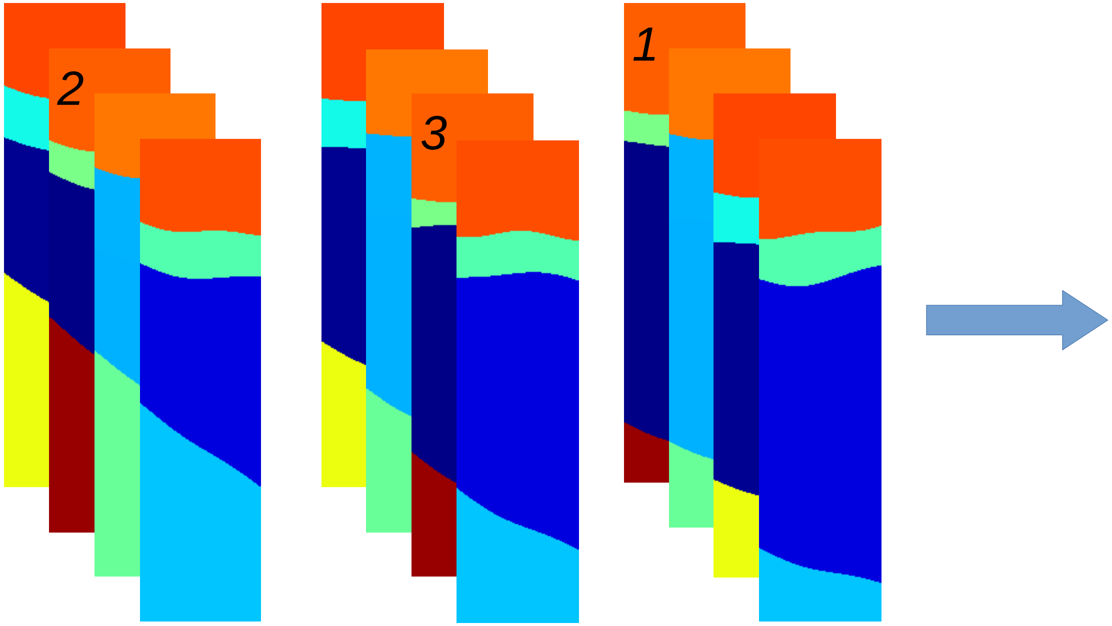


Incoherent!
low probability

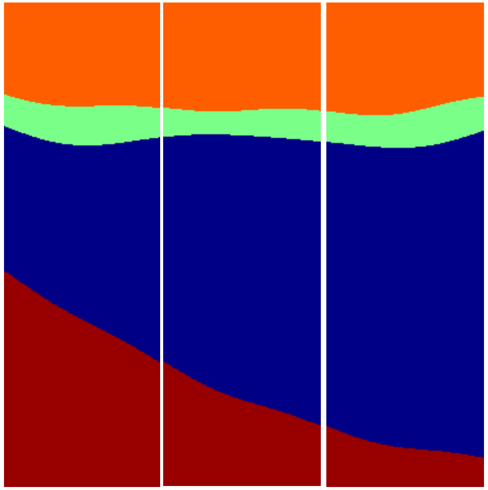
Fused models can created by simply combining samples from the three input ensembles according to their adjacency.

Most combinations created this way will lead to geologically incoherent fused models when there is significant non-uniqueness.

The fusion algorithm finds combinations consistent with prior belief



combination (2,3,1)



Coherent!
high probability

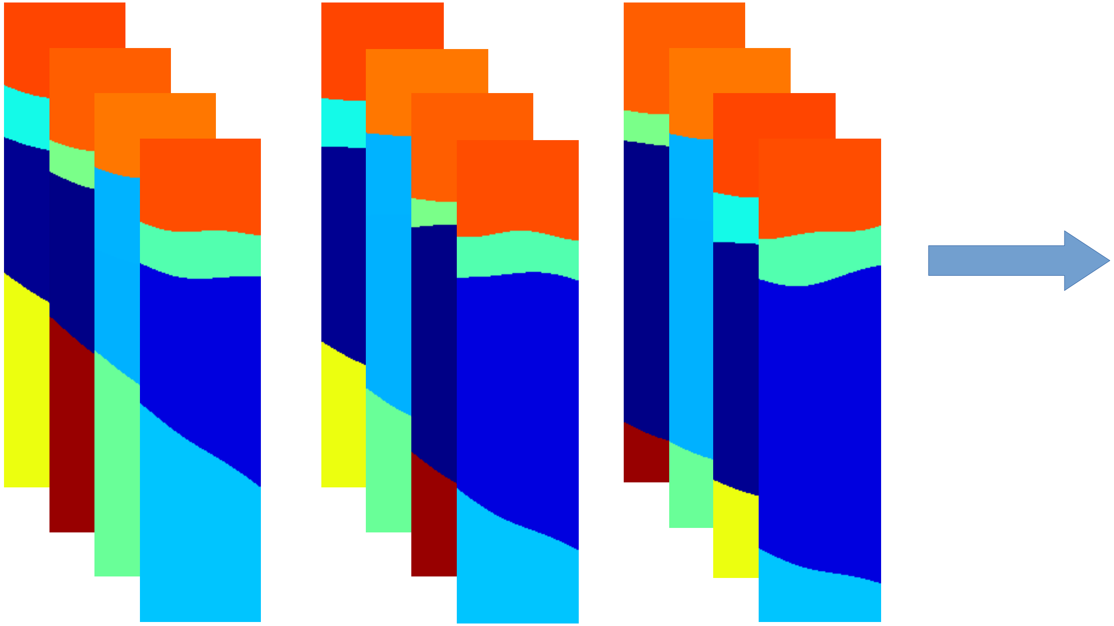
A continuity prior (i.e. regularizer) between the tree sub-models can be used to define coherence of fused models quantitatively.

An MCMC sampler is used to search the space of combinations to sample from a new objective function.

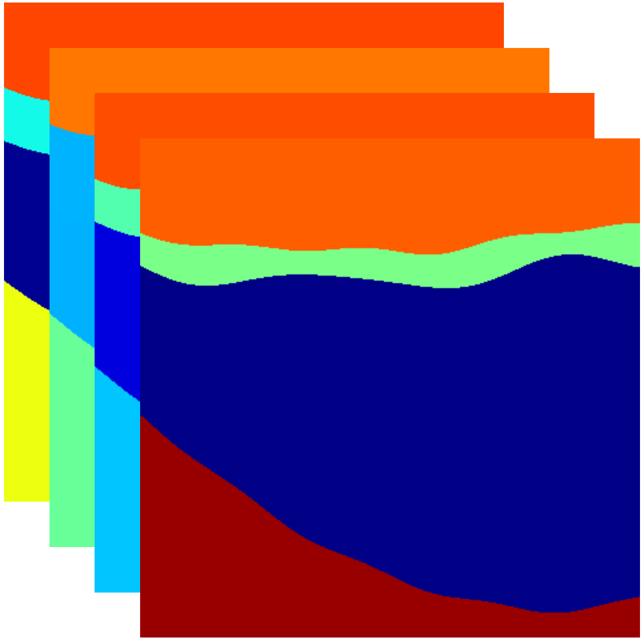
This fusion objective function is equivalent to what methods like laterally constrained inversion use. With large enough input ensembles this method of sampling converges to what an laterally constrained inversion Bayesian posterior sampler would produce.

Create an ensemble of coherent combinations

Input:
sub-model ensembles
from 3 adjacent locations



Output:
a single combined ensemble



There are several advantages to breaking up implementation into separate steps in this way,

1) the fusion program does not perform any forward calculations and can take ensembles as input from a variety of existing software

2) the effects of different geological hypotheses formulated as priors and constraints can be quickly computed without the need for repeating any expensive forward calculations

3) when new data becomes available it can be incorporated into the fusion without the need for repeating the forward calculations for existing data

4) for joint inversions, different specialists can use different ensemble inversion methods and software completely independently to produce the input ensembles

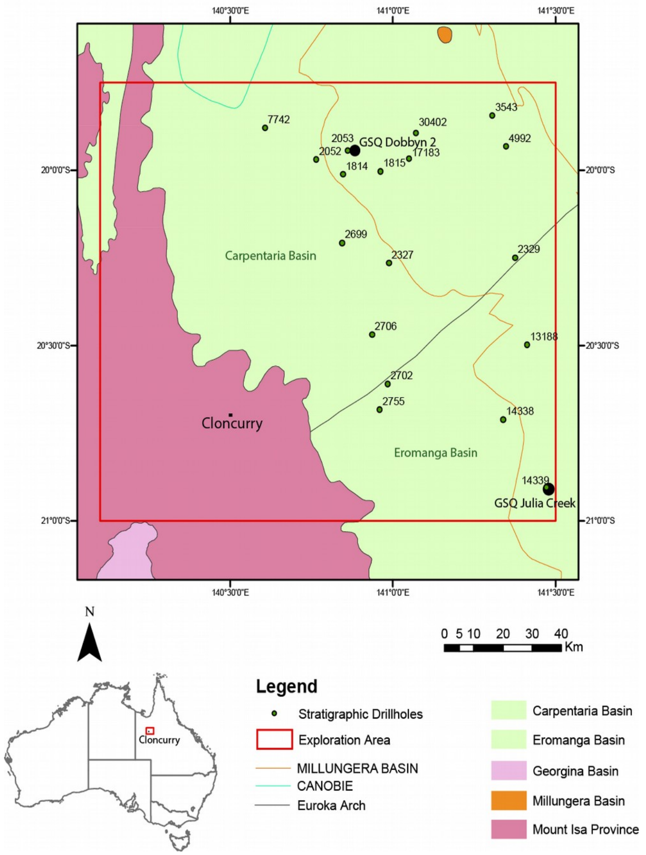
Fast probabilistic workflow

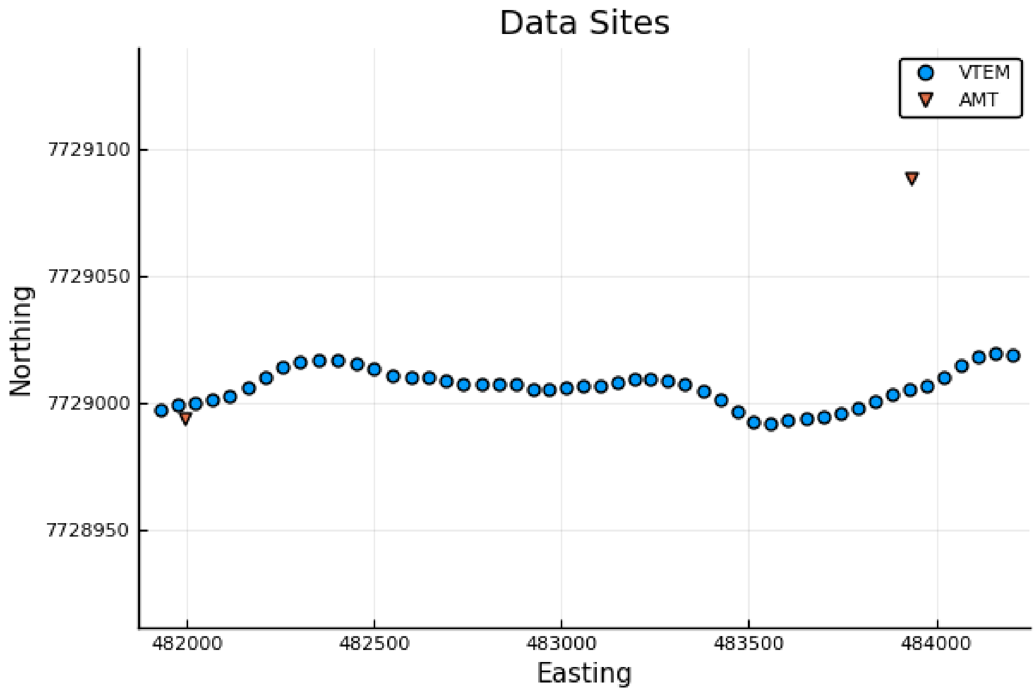
1) *partition data into subdivisions*
2) *perform uncertainty analysis separately on each subdivision*
3) *form candidate geological hypotheses*
4) *apply rejection sampling*
5) *fuse subdivision ensembles into a larger model ensemble*
6) *evaluate and return to step 3 if needed*
7) *test hypothises using less approximated methods*

This slide shows our general purpose fast probabilistic workflow.

Step 7 is not presented here and is still under development.

Airborne EM (VTEM) Line + 2 Audio-Magentotelluric Sites





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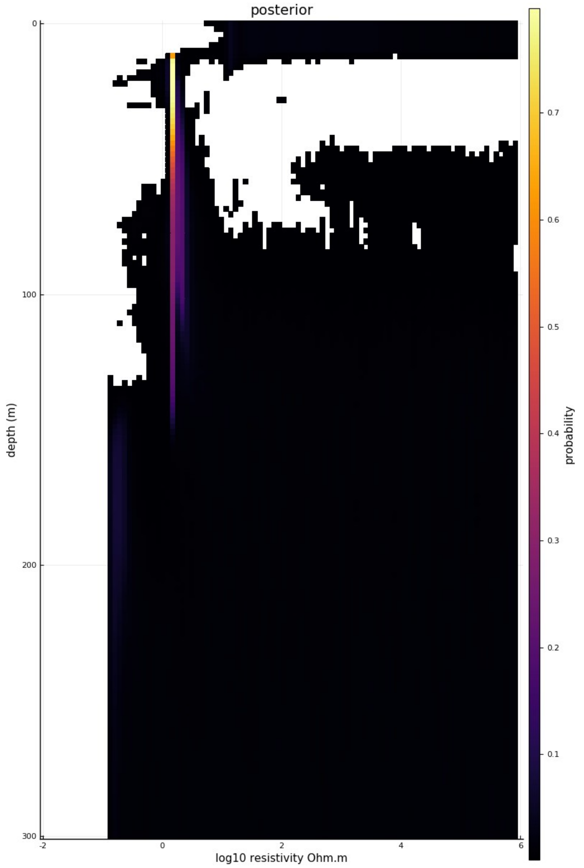
To demonstrate, we use 50 VTEM sites along a flight line and 2 AMT sites.

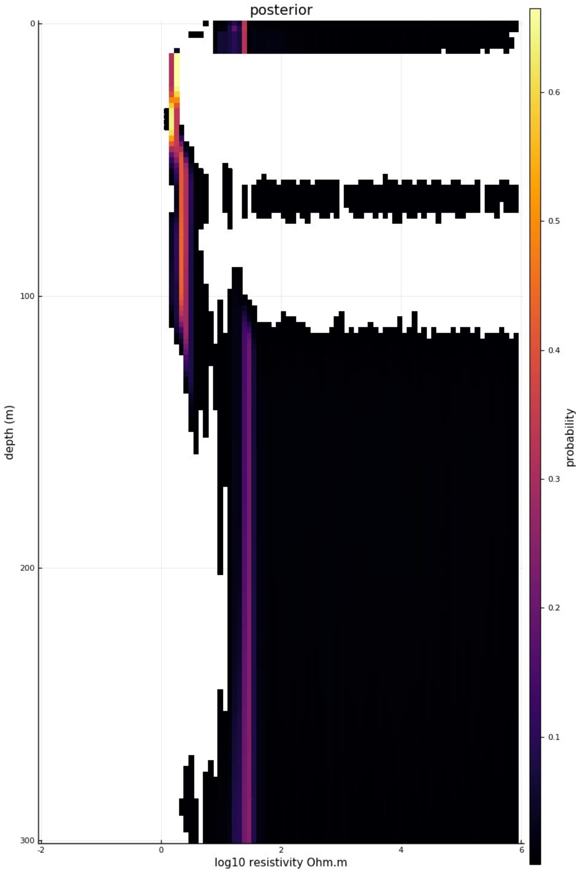
The relative locations of these are shown.

1D rj-MCMC resistivity inversion

VTEM



AMT

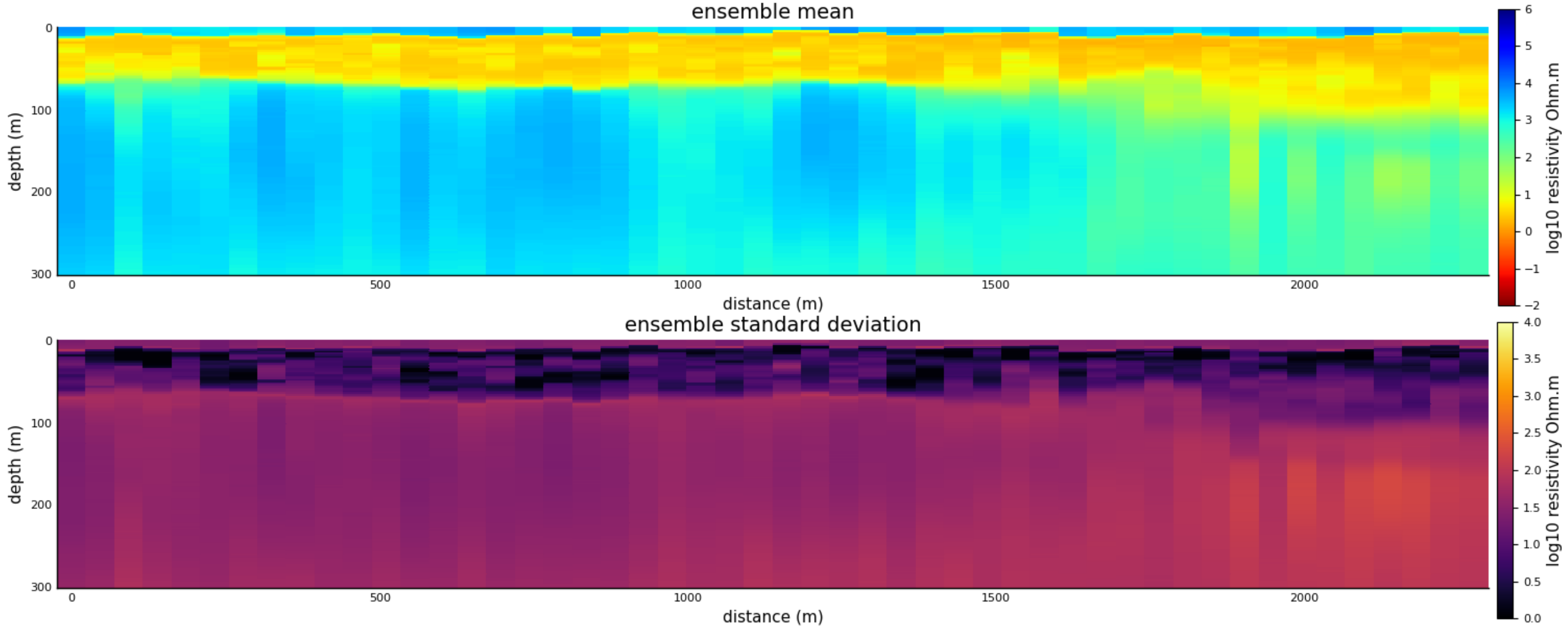


1D transdimensional inversion is first applied to each site separately using a non-informative prior.

This produces 50 VTEM and 2 AMT ensembles each containing 10000 1D resistivity models with varying numbers of layers.

The posteriors of two ensembles are summarised in the figures. The sites chosen here are about 18m apart. We can see that the AMT achieves greater resolution at depth than the VTEM. This is in part due to the physics but also because different software packages were used by two different specialists with different data processing and methods and uncertainty handling approaches.

VTEM ensemble summary statistics



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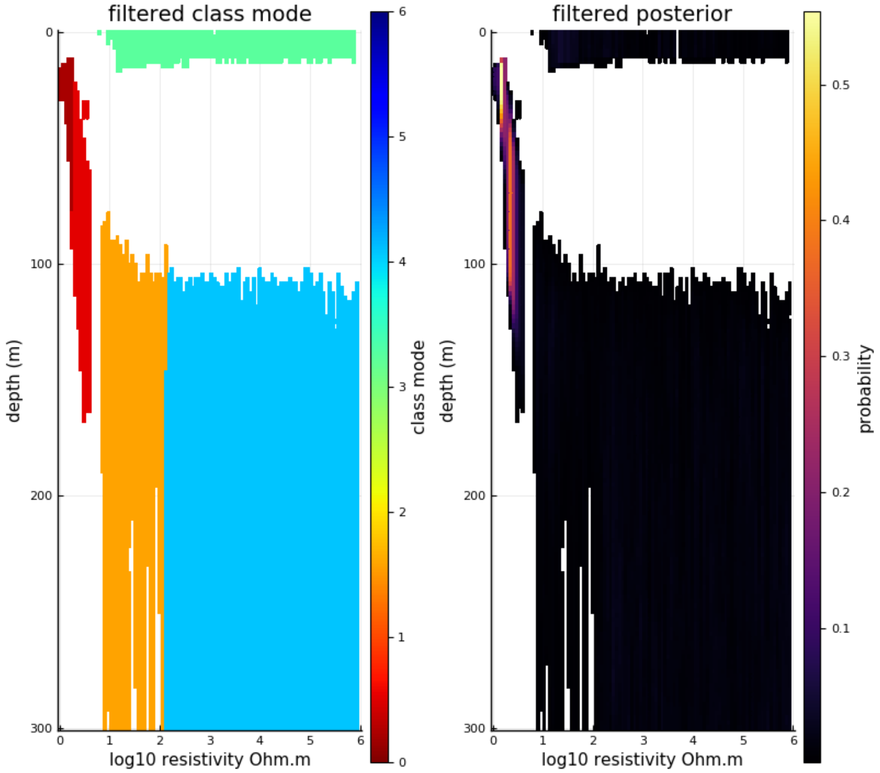
The set of 50 VTEM ensembles can be visualised by plotting summary statistics, here the mean and median.

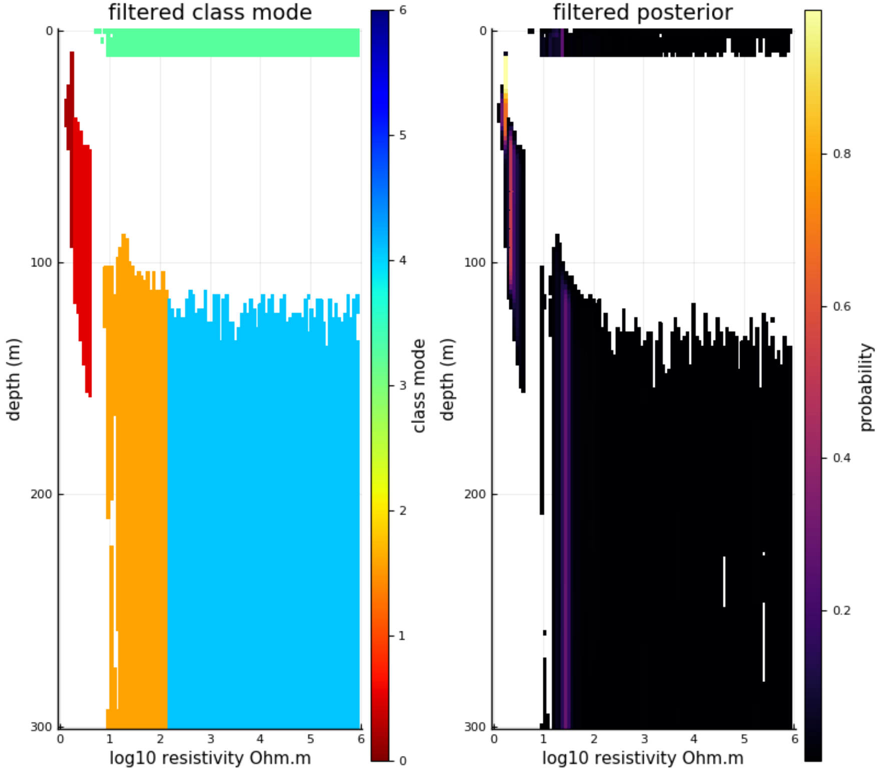
Notice that past 100m depth there is significant non-uniqueness.

Segmentation and rejection sampling

VTEM



MT

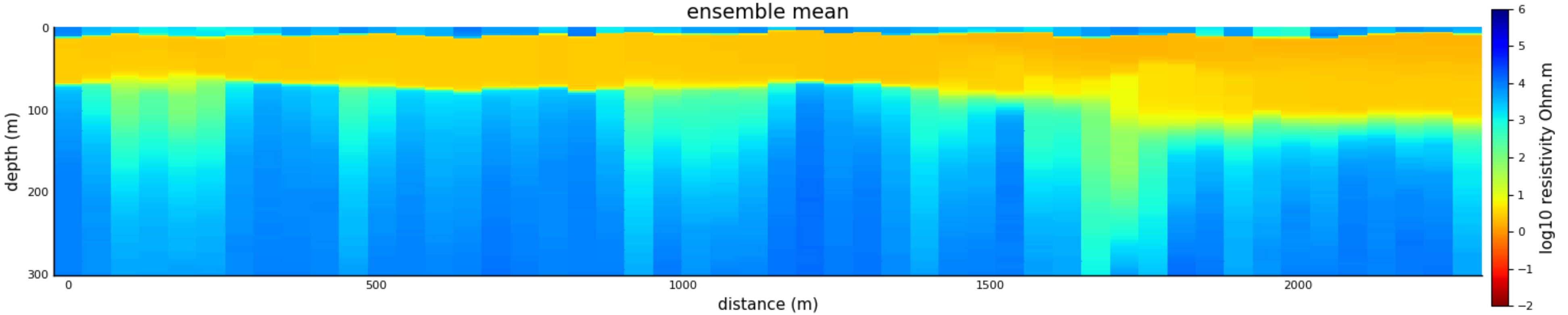


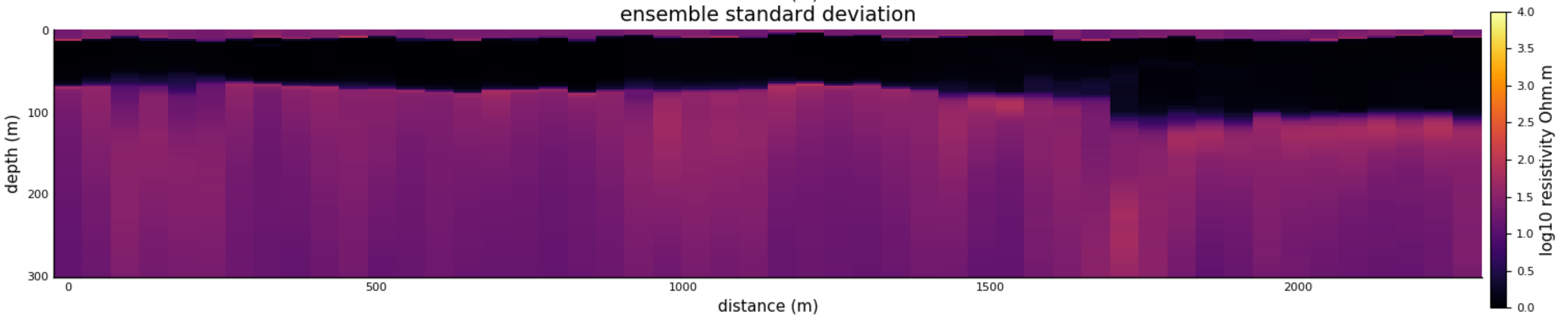
Next, segmentation is applied to impose a more informed geological hypothesis.

Here 4 resistivity layers are assumed with their properties derived (somewhat informally) from the 52 ensembles themselves and existing literature on the region which tells us to expect at least two separate sedimentary basins (red and orange) overlying the crystalline basement (blue).

The same two sites as before are shown.

VTEM ensemble statistics after applyng segmentation and rejection sampling





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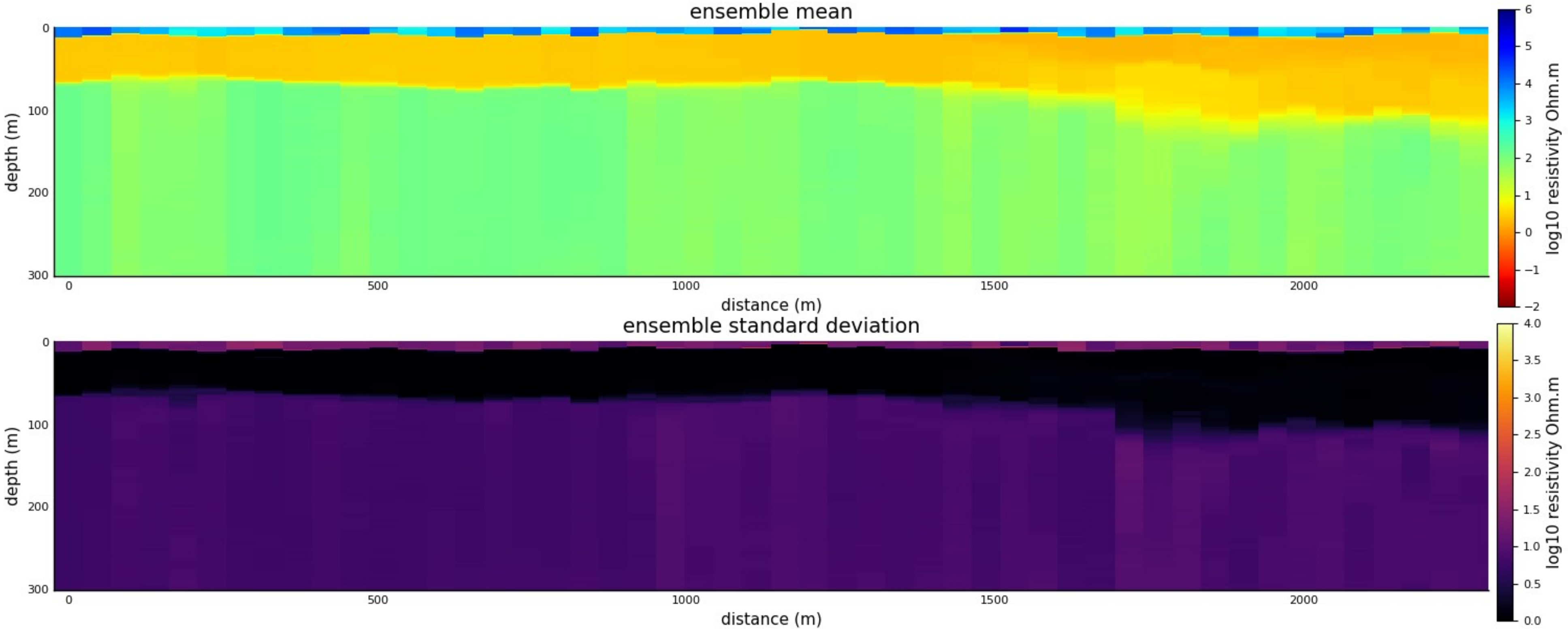
The posterior is refined and summary statistics for the 50 VTEM sites are shown.

Posterior variance is reduced as expected.

Now with lateral constaints imposed using Bayesian ensemble fusion



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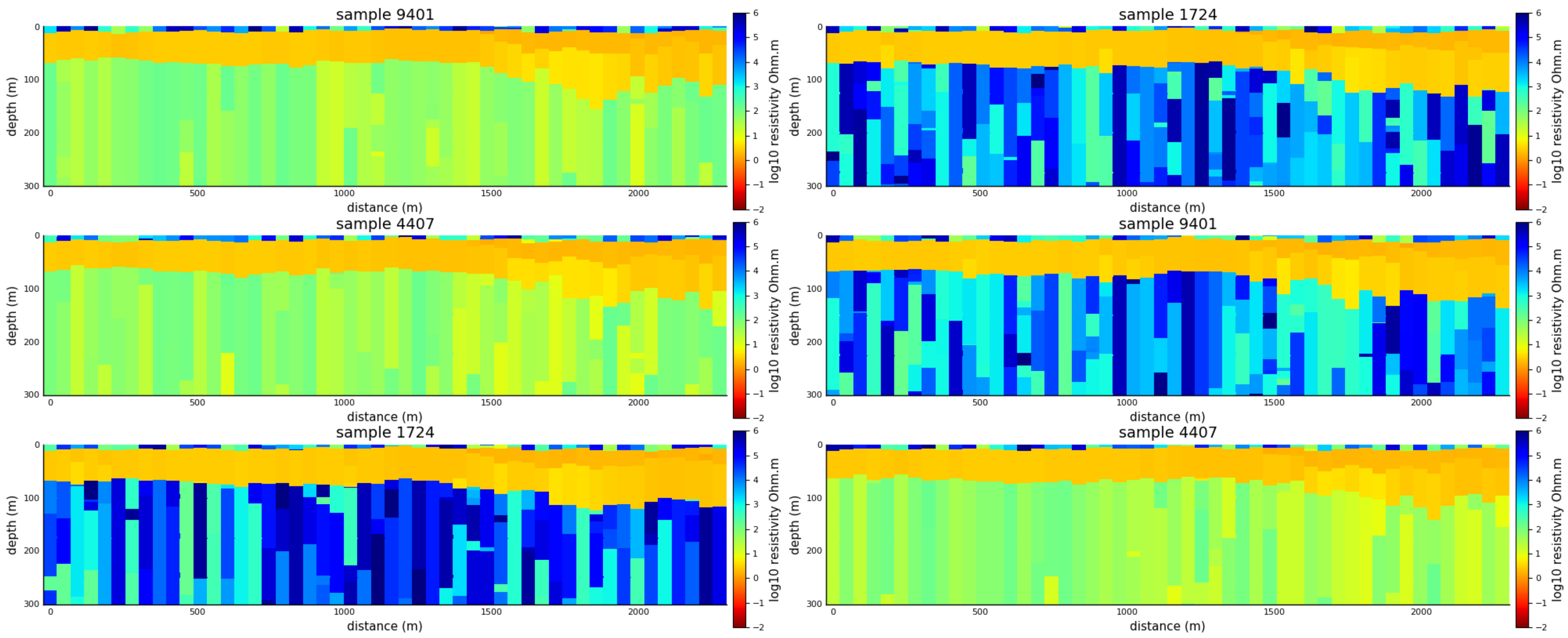
Next, Bayesian ensemble fusion is applied to impose lateral continuity between VTEM sites.

The lateral constraint is defined here in terms of adjacent layer width changes and not resistivity as it varies by different degrees within different layers.

The 2D posterior is bi-modal



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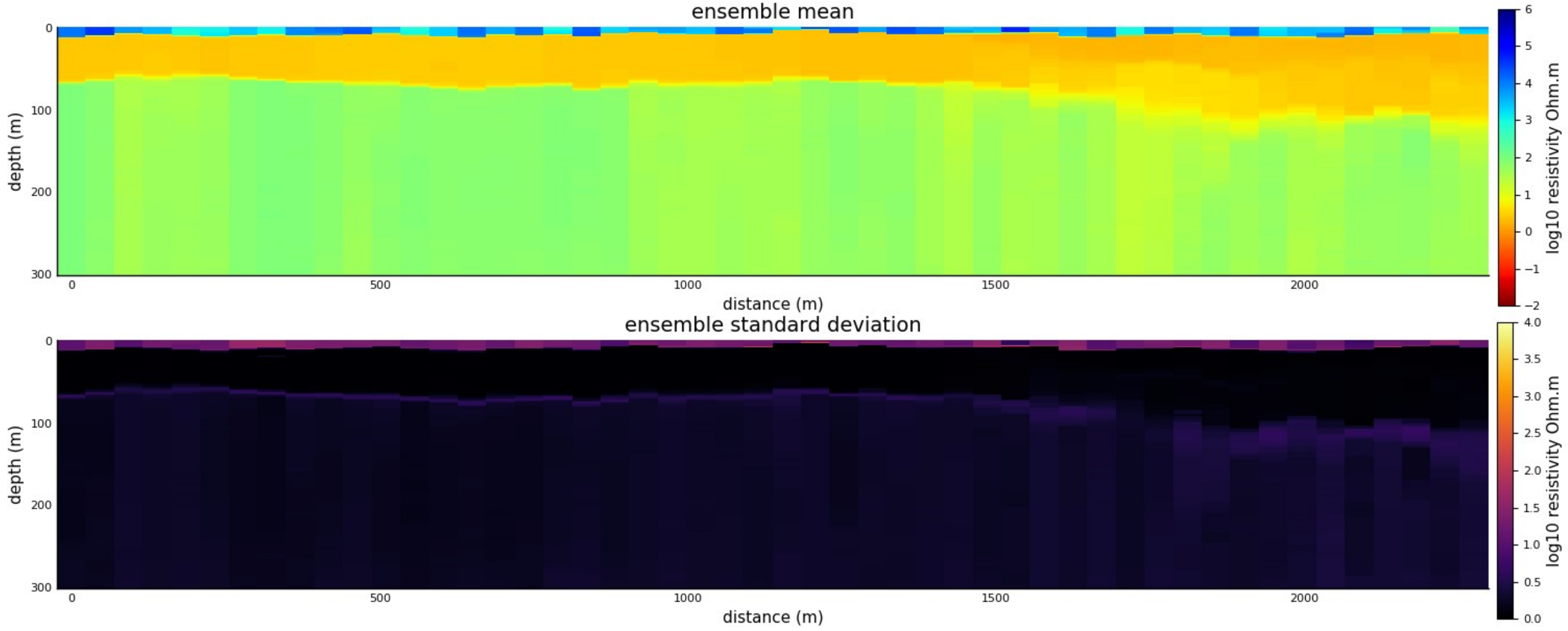
Below 100m depth, there is still significant posterior uncertainty about resistivity.

To see why we can look at the individual 2D fused models. Six randomly selected 2D models are displayed.

Notice that the posterior is bimodal because the VTEM data by itself cannot tell us if the deepest later is sedimentary of crystalline basement.

Bayesian ensemble fusion was repeated but now with the two MT sites included



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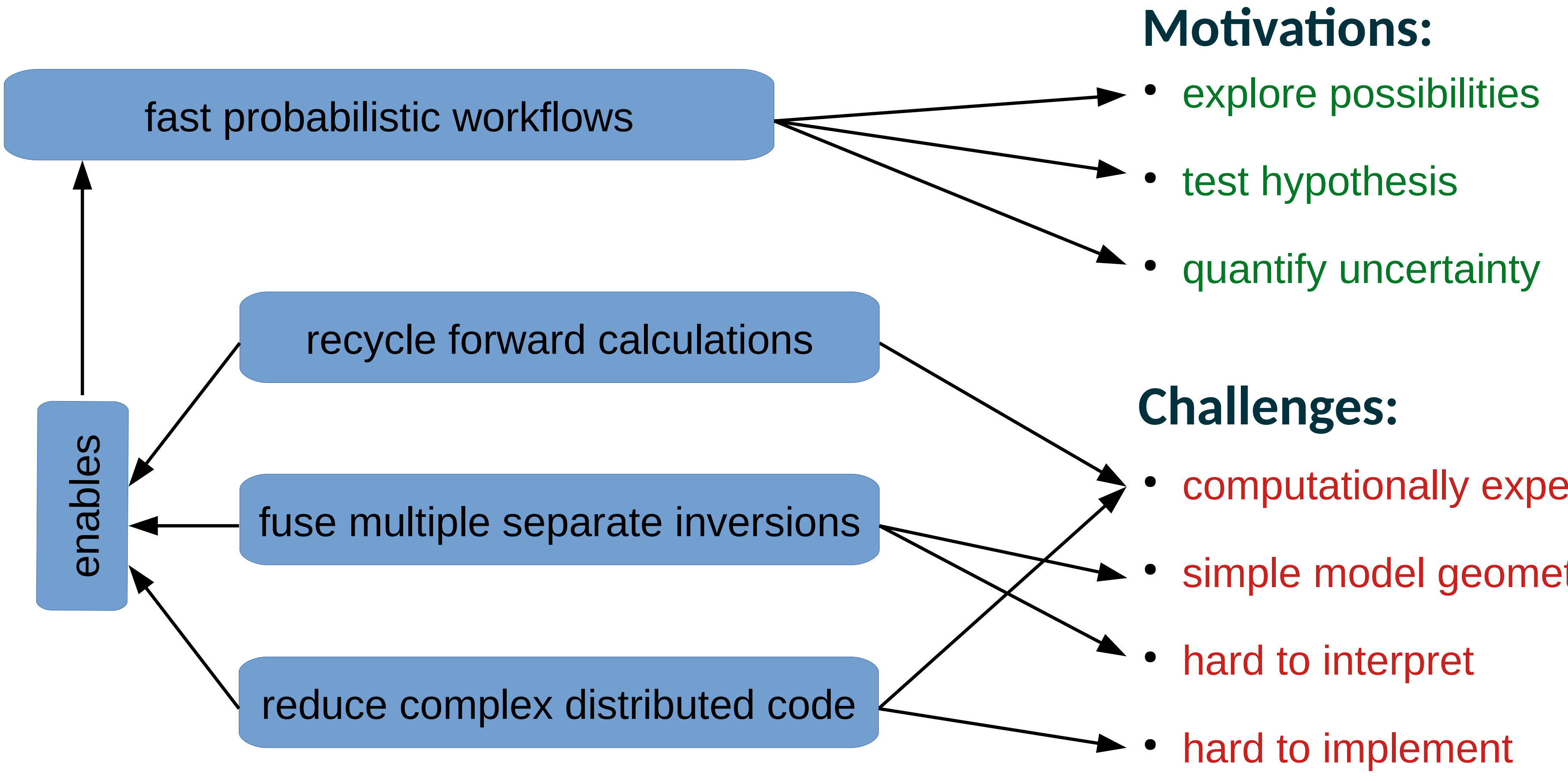
Finally, the lateral constraints on adjacent layer widths are extended to also conform to those of our 2 AMT sites.

The result is that one of the modes in the posterior now dominates. We conclude that the crystalline basement is probably not present within the shallowest 300m here, which is consistent with other sources of geological information.t.

Conclusion


fast probabilistic workflows
recycle forward calculations
fuse multiple separate inversions
reduce complex distributed code
enables
Motivations:
explore possibilities
test hypothesis
quantify uncertainty
Challenges:
computationally expensive
simple model geometries
hard to interpret
hard to implement

There a several benefits to our approach,

1) different and existing software can be used by different specialists to create the input ensembles, which reduces the need for complex coordination and simplifies coding

2) forward calculations are performed once and then stored to be recycled in many subsequent fusions

3) many inversions of the same data, or different combinations thereof, can then be performed using different priors, constraints and geological interpretations, at very little additional computational cost

4) when new data becomes available it can be incorporated into the fusion without the need for repeating the forward calculations for existing data

Because coherence between input ensemble is formulated by coupling interface depths and not petrophysical parameters, it should be possible to fuse ensembles produced by geophysical methods sensitive to different petrophysical properties.

In continuing work we are looking to incorporate ensemble estimates of interface depths derived from seismic data.

Thank You

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