



Utrecht University

Department of Physics



Extreme sensitivity and climate tipping points

Anna von der Heydt^{1,2} and Peter Ashwin³

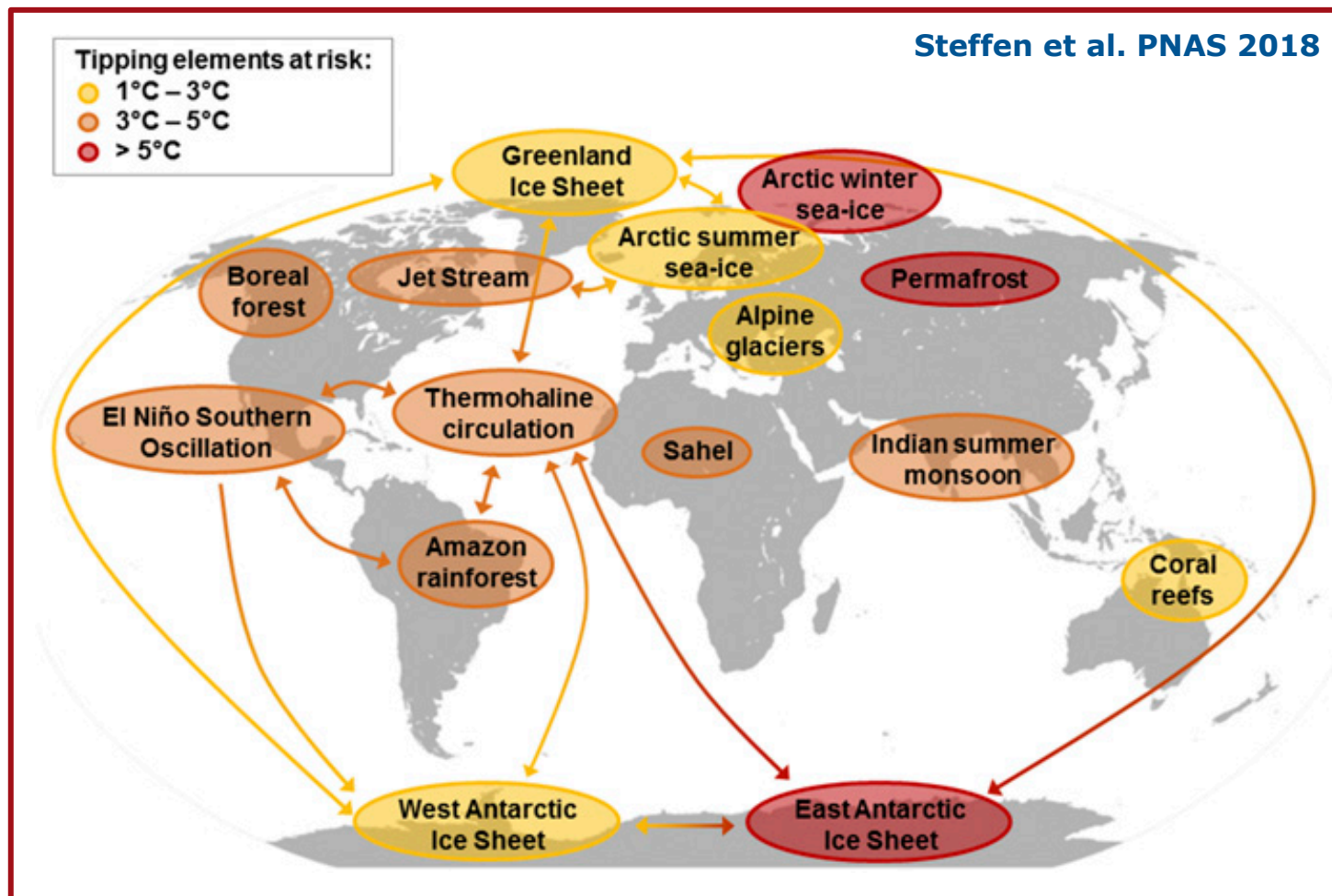
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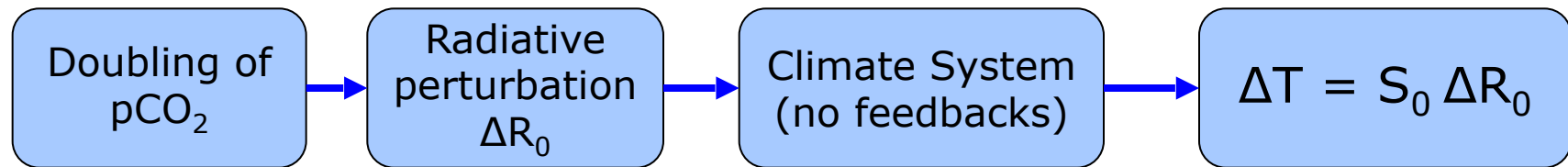
Tipping elements in the climate system



Need a comprehensive framework to quantify the Climate Response!

What is climate sensitivity?

- Equilibrium change in global mean surface temperature after a doubling of the atmospheric CO₂ concentration.

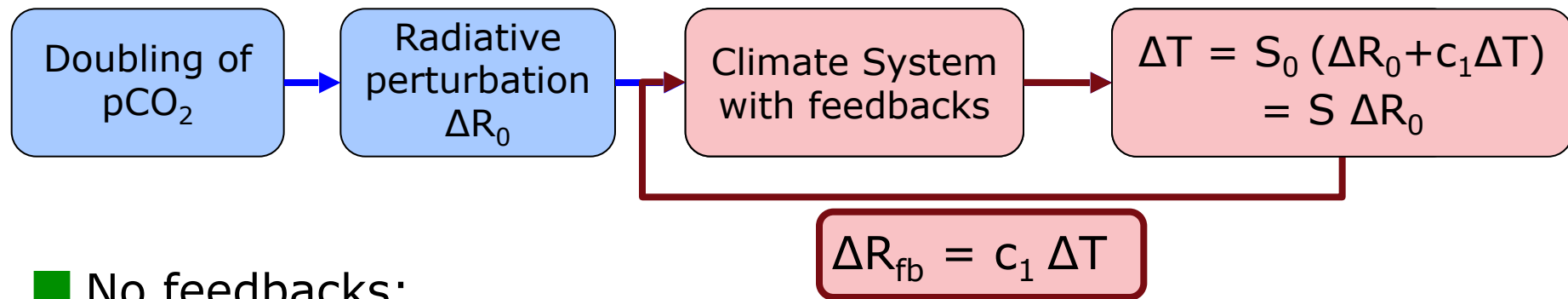


- No feedbacks:

Planck response $S_0 = 0.3 \text{ K}/(\text{W}/\text{m}^2)$

What is climate sensitivity?

- Equilibrium change in global mean surface temperature after a doubling of the atmospheric CO₂ concentration.



- No feedbacks:

Planck response $S_0 = 0.3 \text{ K}/(\text{W}/\text{m}^2)$

- With feedbacks:

$$S = \Delta T / \Delta R_0$$

Quantifying climate sensitivity: problems

■ Timescales and equilibrium

- ★ Slow and fast feedback processes.
- ★ Timescale separation.

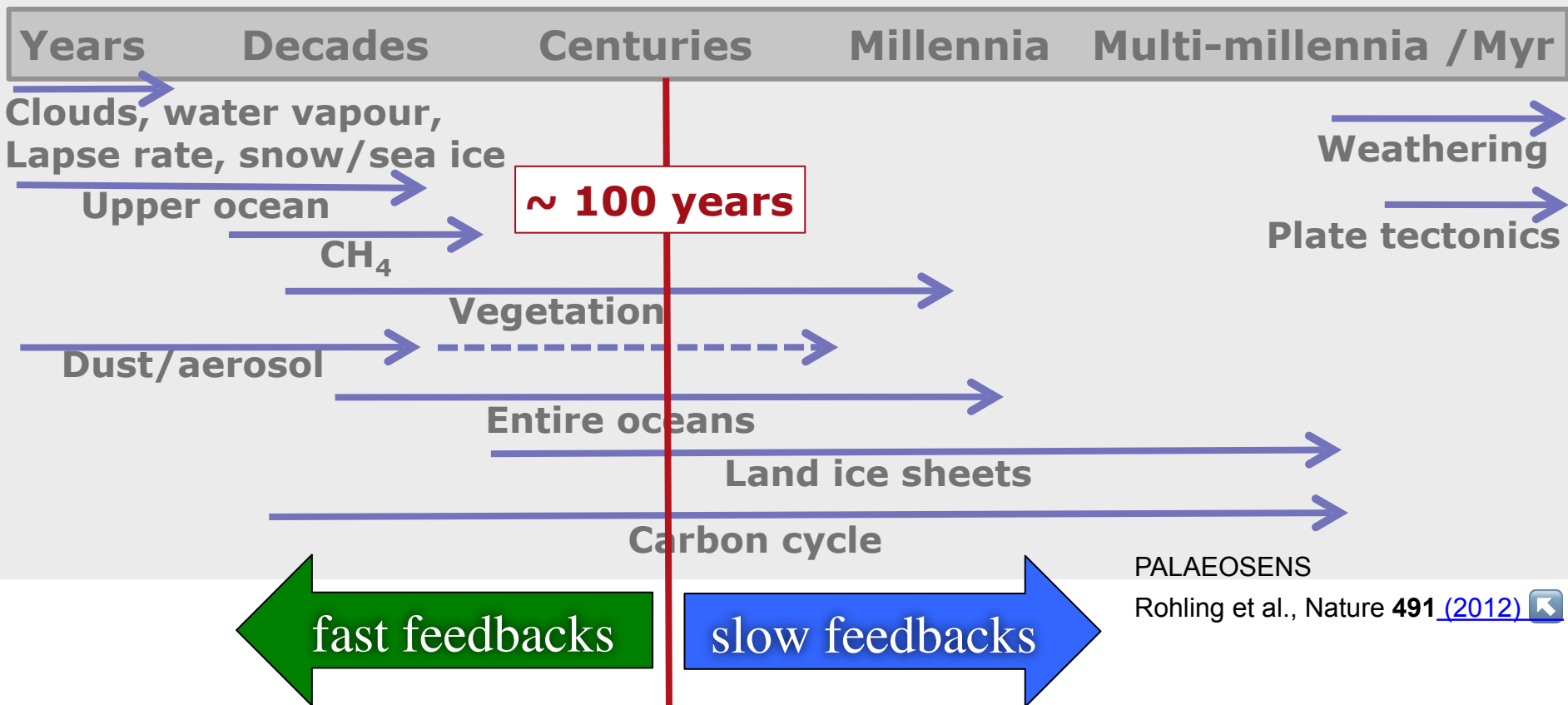
■ Dependence on the background climate

- ★ (Fast) feedback processes are not “constant”.

■ Tipping points in the climate system

- ★ New ‘flavours’ of climate sensitivity.
- ★ Extremes in climate sensitivity vs probability of tipping.

Time scales & equilibrium



Earth system sensitivity

'Correct' for slow feedbacks, e.g.

'Equilibrium' sensitivity S:

$$S^p = S_{[\text{CO}_2]} = \frac{\Delta T}{\Delta R_{[\text{CO}_2]}}$$

$$S_{[\text{CO}_2, LI]} = \frac{\Delta T}{\Delta R_{[\text{CO}_2]} + \Delta R_{[LI]}}$$

$$S_{\text{forcing, slow}} = \frac{\Delta T}{\Delta R_{\text{forcing}} + \Delta R_{\text{slow}}}$$

Distributions of climate sensitivity - origin of uncertainty?

■ Uncertainty from observations, model, unaccounted processes

- ★ Big uncertainties in quantification for radiative forcing.
- ★ Palaeoclimate: Big uncertainty in climate reconstruction.

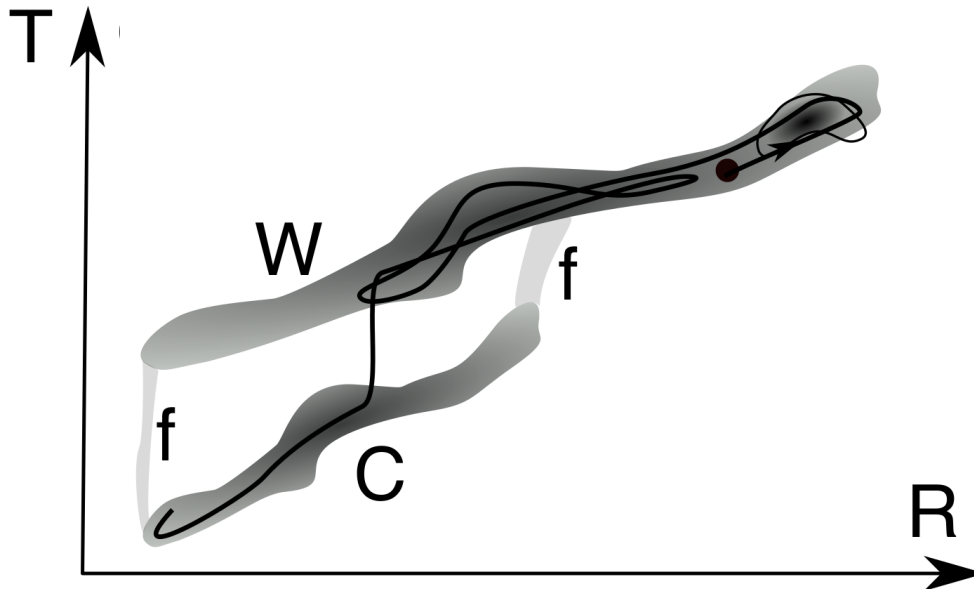
■ Climate dynamics:


- ★ feedback processes change with background climate!
- ★ Very high climate sensitivity:
 - nonlinearities in the climate system - evidence for tipping?

Palaeoclimate sensitivity S: trajectory on a 'climate attractor'

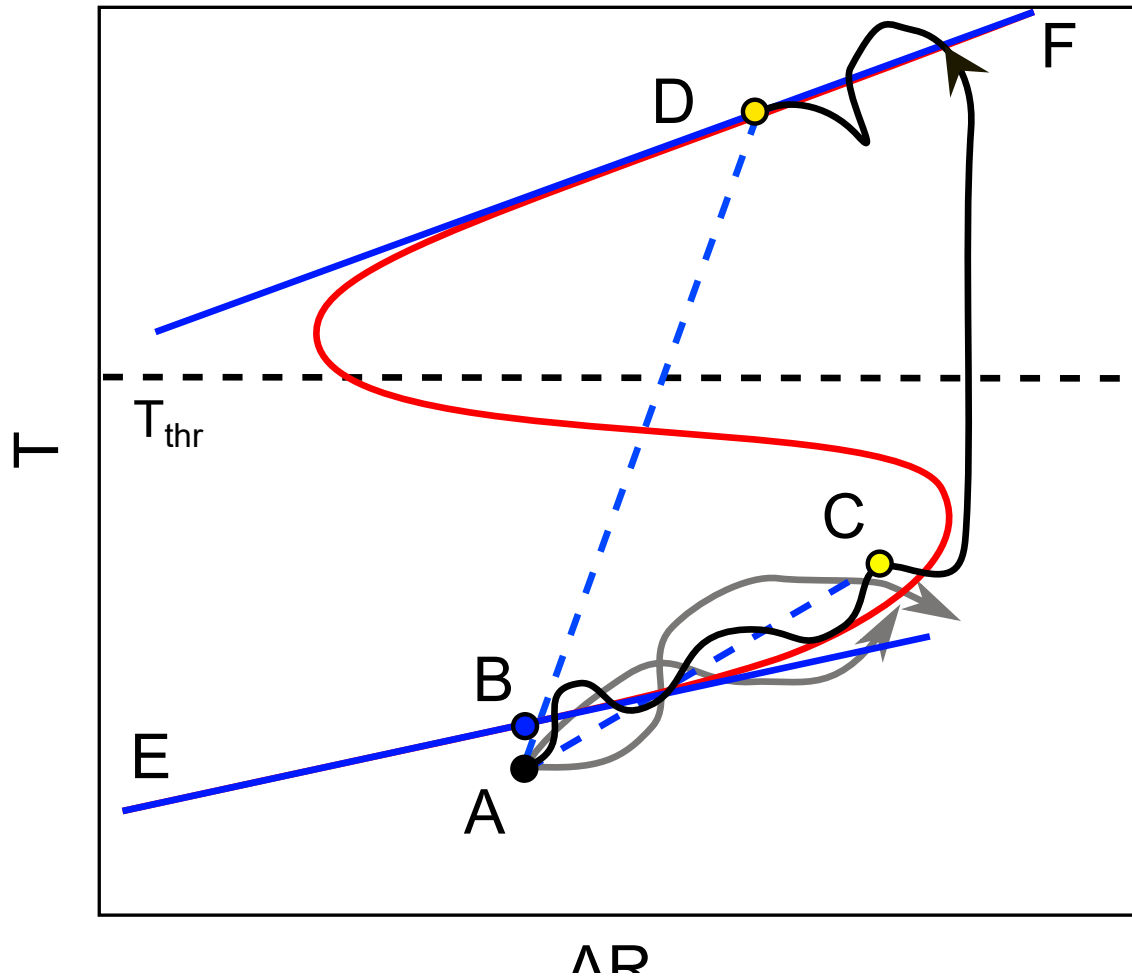
$$c_T \frac{dT}{dt} = R_{\text{forcing}} + R_{\text{slow}} + R_{\text{fast}} - R_{\text{OLW}}$$

$$S_{\text{forcing,slow}} = \frac{\Delta T}{\Delta R_{\text{forcing}} + \Delta R_{\text{slow}}} \approx \frac{dT}{d(R_{\text{forcing}} + R_{\text{slow}})}$$



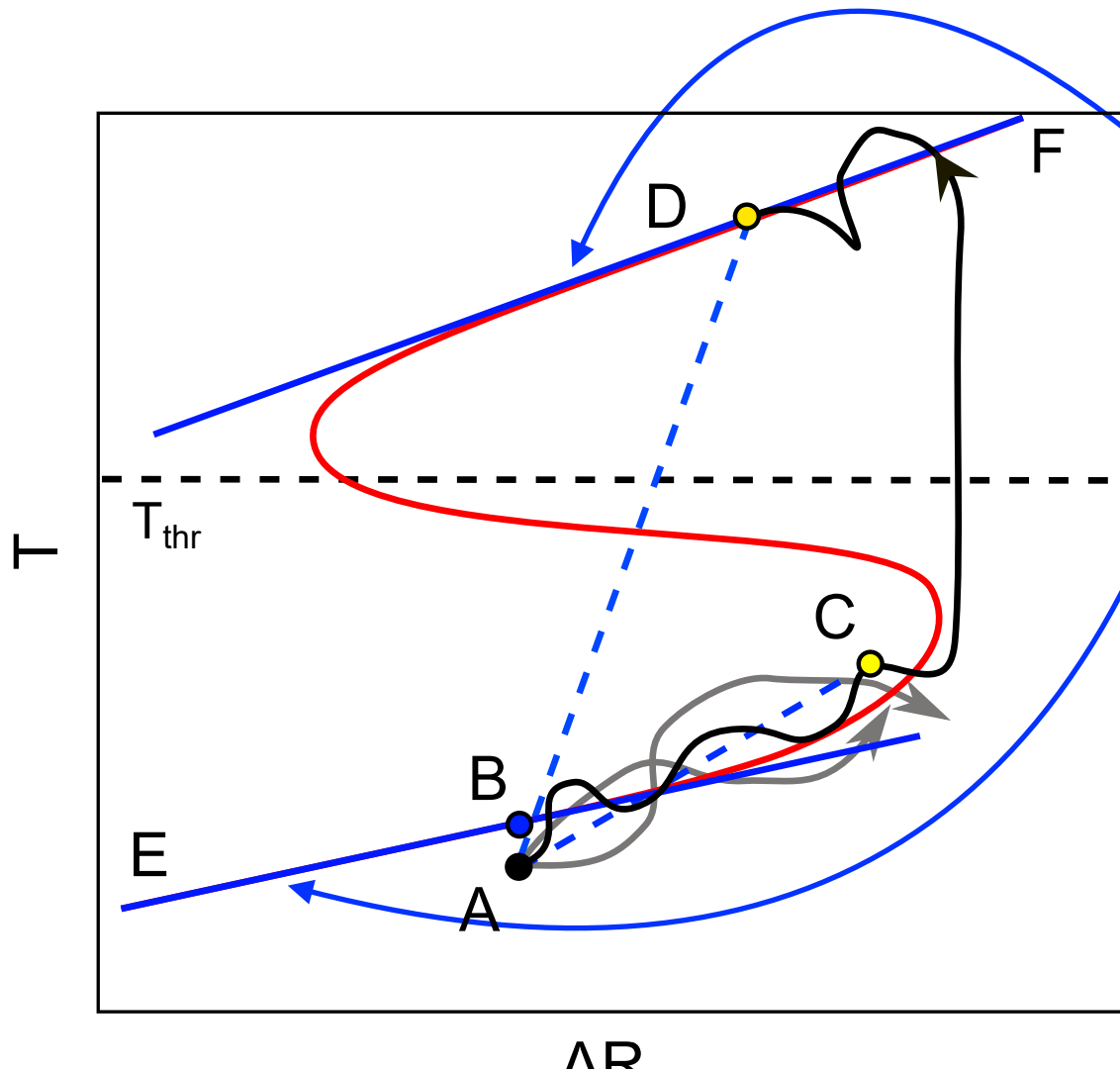
von der Heydt & Ashwin, Dyn. Stat. Clim. Syst. **1** dzx001 [\(2016\)](#) 

The 'climate attractor'



Ashwin & von der Heydt, J. Stat. Phys. (2019)

The 'climate attractor'

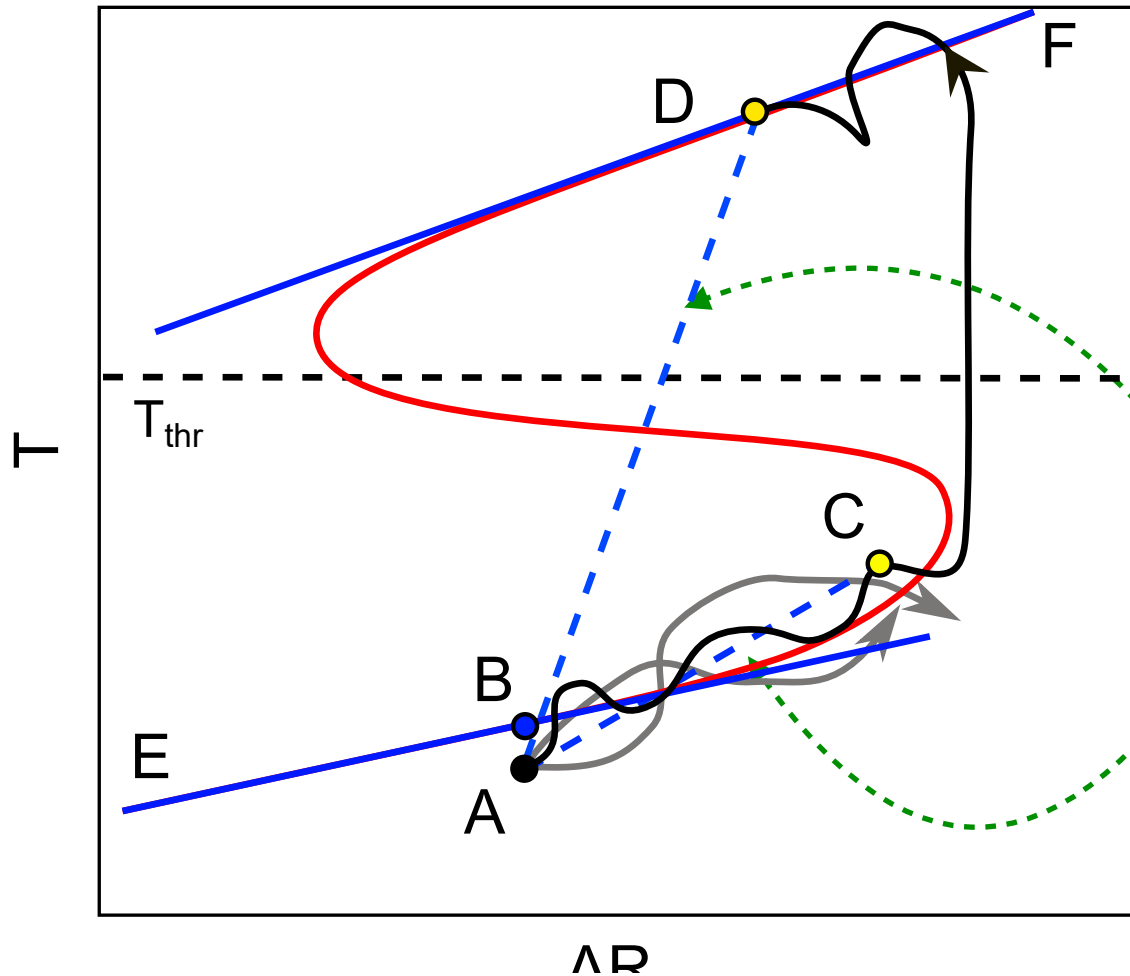


Instantaneous/Local
slope sensitivity

$$S = \left[\frac{d}{dT} \Delta R_{[CO_2]} \right]^{-1}$$

Ashwin & von der Heydt, J. Stat. Phys. (2019)

The 'climate attractor'



Instantaneous/Local
slope sensitivity

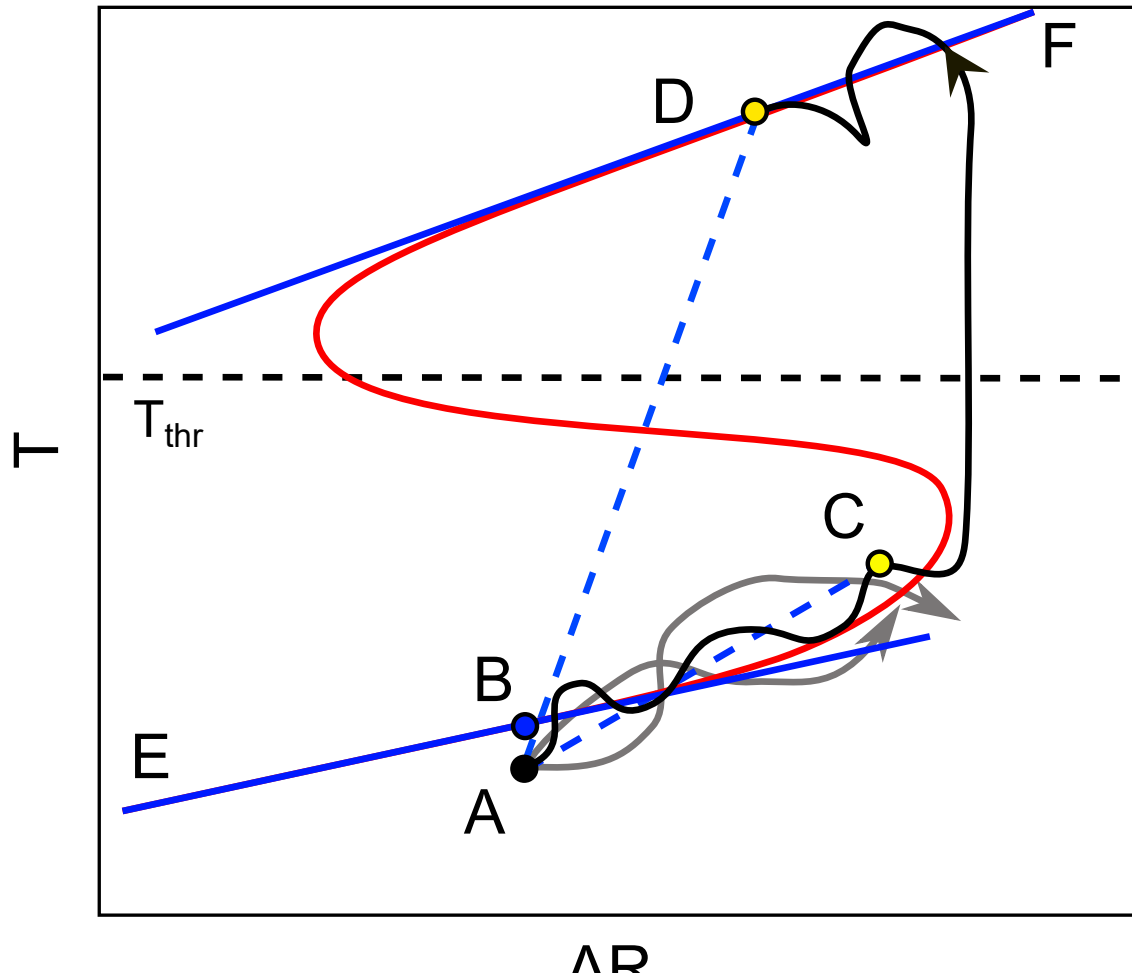
$$S = \left[\frac{dT}{dR} \Delta R_{[CO_2]} \right]^{-1}$$

Incremental
sensitivity (fixed Δt)

$$S_0^{\Delta t}(x) = \frac{T(\varphi_{\Delta t}(x)) - T_0}{R(\varphi_{\Delta t}(x)) - R_0}$$

Ashwin & von der Heydt, J. Stat. Phys. (2019)

The 'climate attractor'



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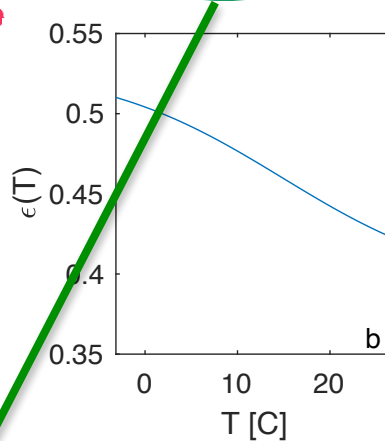
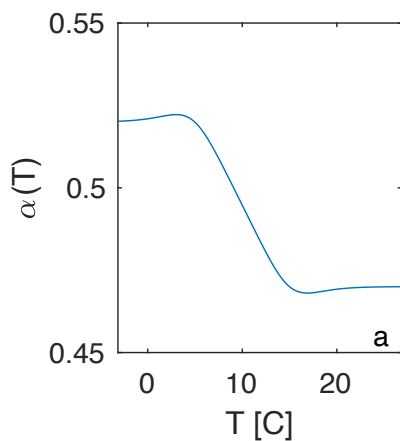
Two-point
sensitivity (all Δt)

$$S_{0,1}^{\infty} = \frac{T_1 - T_0}{R_1 - R_0}.$$

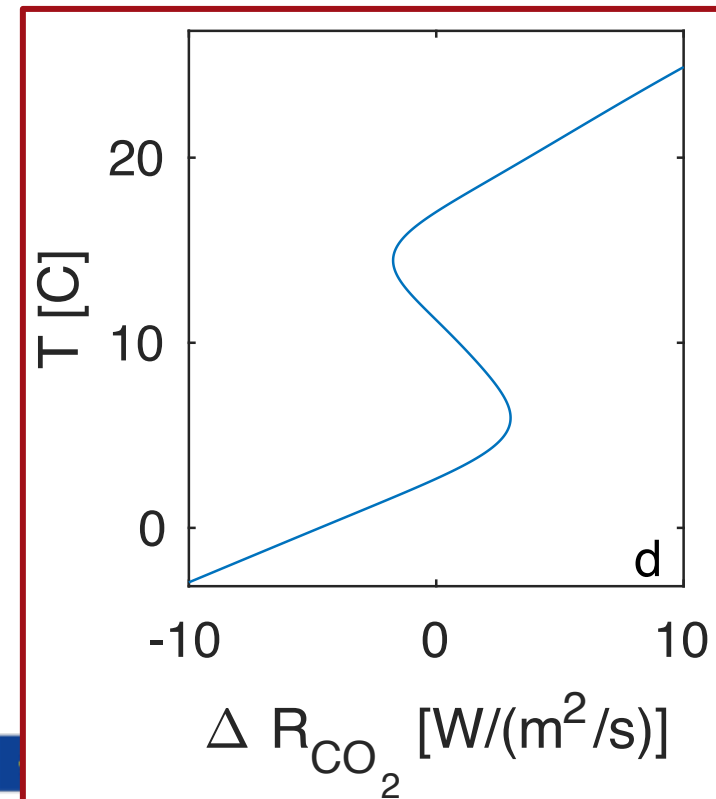
Ashwin & von der Heydt, J. Stat. Phys. (2019)

Energy Balance model

$$C_T dT = \left[Q_0 (1 - \underbrace{\alpha(T)}_{\text{albedo}}) + A \ln \left(\frac{C}{C_0} \right) - \underbrace{\epsilon(T)}_{\text{emissivity}} \sigma T^4 \right] dt + \underbrace{\eta_T \omega_T}_{\text{noise}}$$



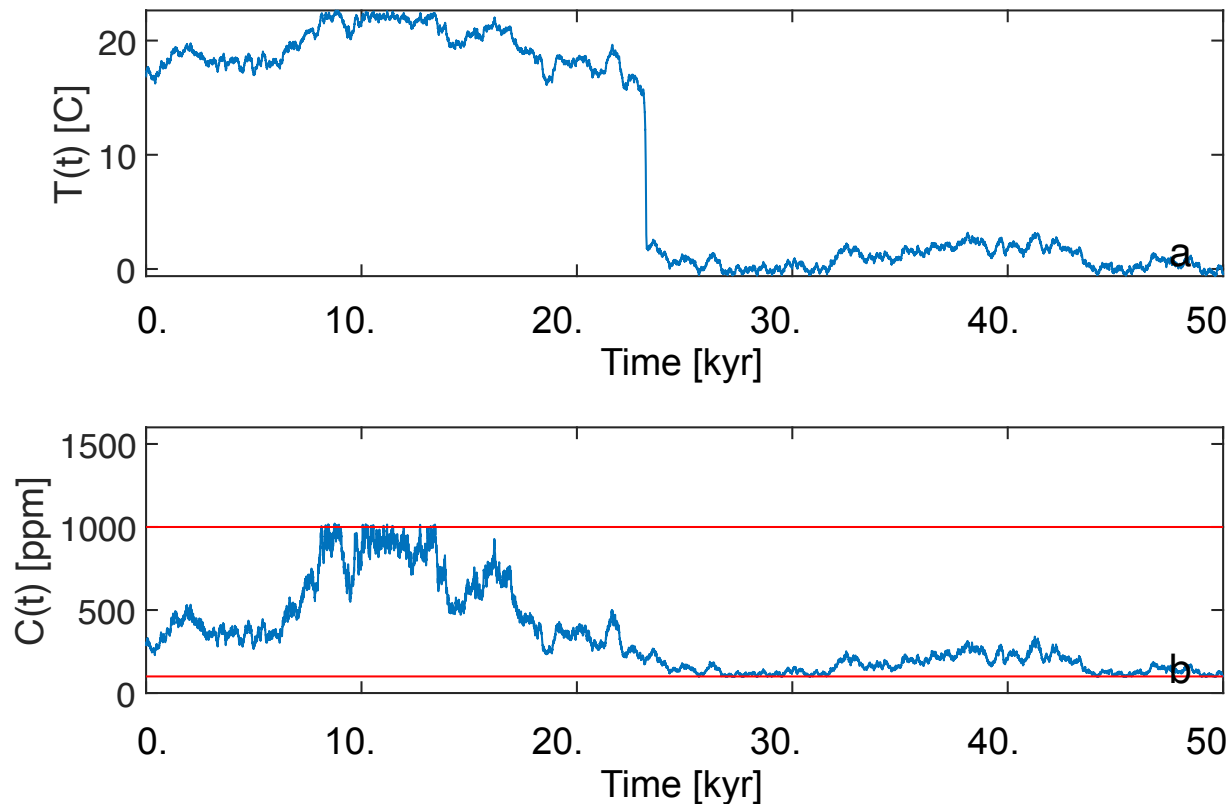
wandering CO_2 = additional noise



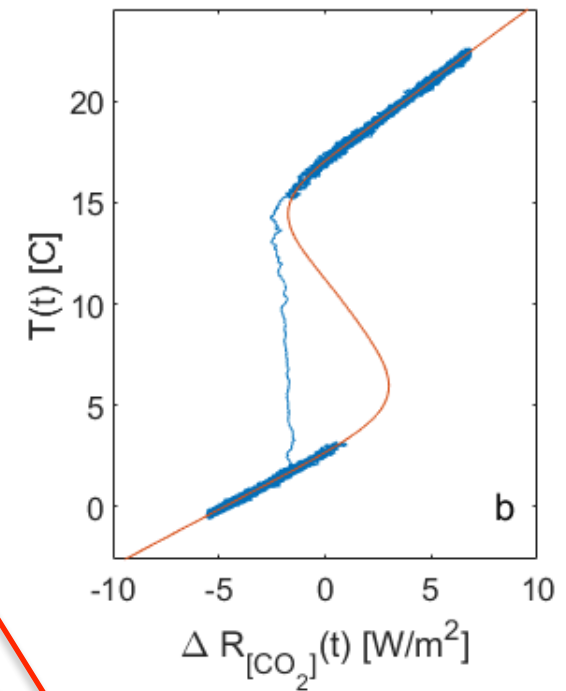
Ashwin & von der Heydt, J. Stat. Phys. (2019)

Energy Balance model with noise

Typical time series under 'wandering' CO₂



Climate 'attractor'



limits of wandering CO₂


Ashwin & von der Heydt, J. Stat. Phys. (2019)

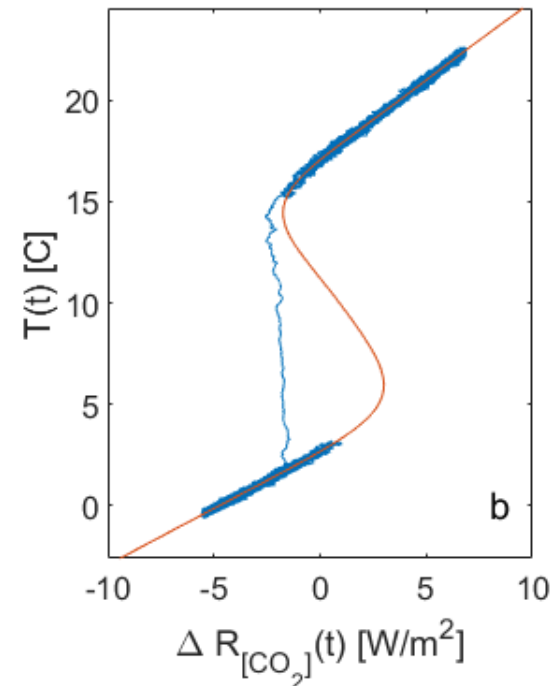
Instantaneous sensitivity

$$S = 1/\lambda$$

$$\lambda = \frac{d}{dT} \Delta R_{[CO_2]} = [\epsilon'(T)T + 4\epsilon]\sigma T^3 + Q_0 \alpha'(T)$$

- Different S on each branch: state-dependence
- Near saddle-nodes: runaway climate

Ashwin & von der Heydt, J. Stat. Phys. [\(2019\)](#) 

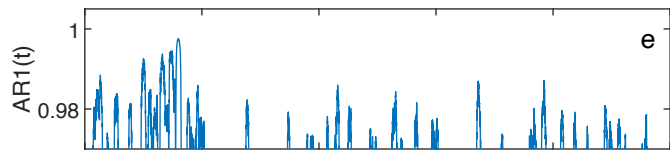
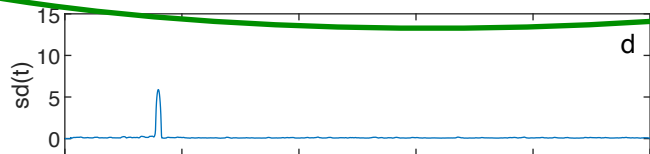
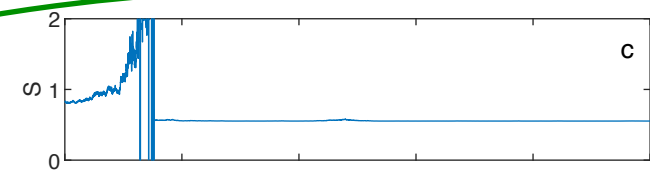
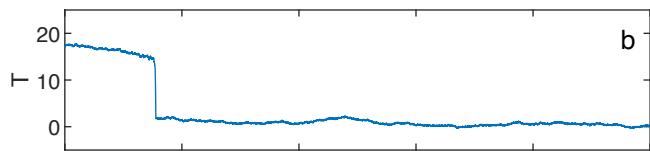
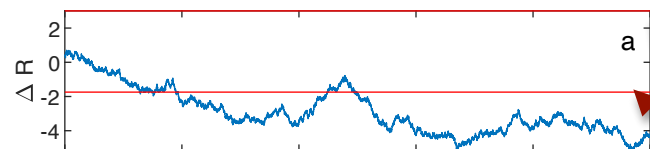


Instantaneous sensitivity and early warning

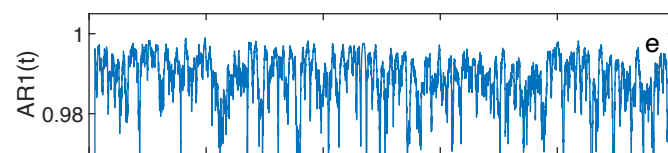
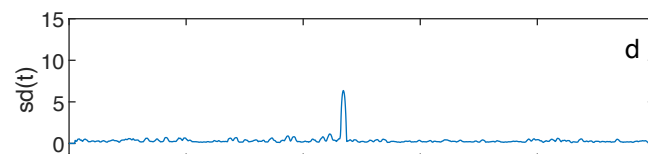
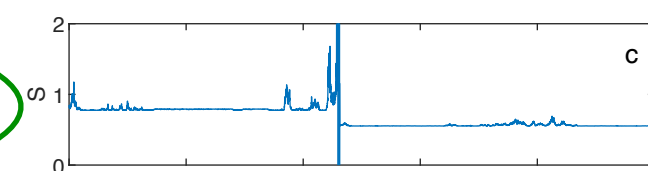
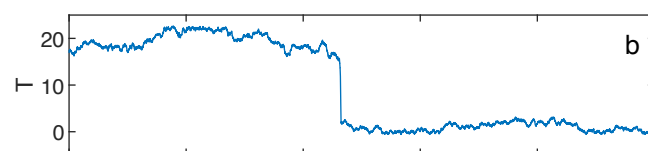
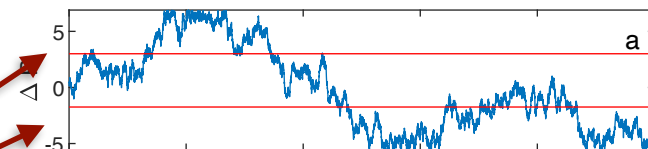
$$S = 1/\lambda$$

$$\lambda = \frac{d}{dT} \Delta R_{[CO_2]} = [\epsilon'(T)T + 4\epsilon]\sigma T^3 + Q_0 \alpha'(T)$$

slow evolution of CO₂ ($\sim T$)



fast evolution of CO₂



Saddle node points

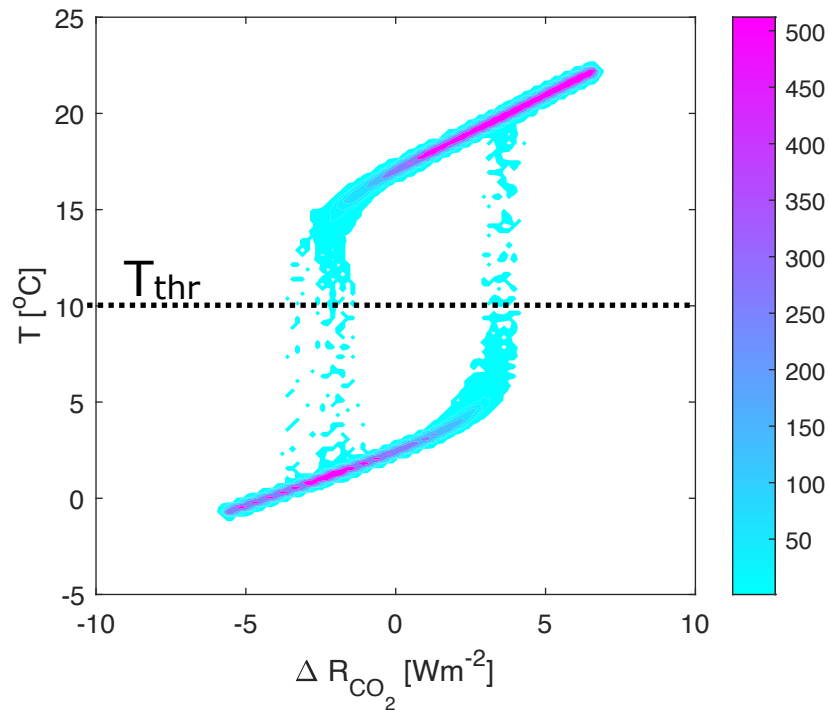
Early warning of regime shift?

0. 10. 20. 30. 40. 50
Time [kyr]

Ashwin & von der Heydt,
J. Stat. Phys. (2019)

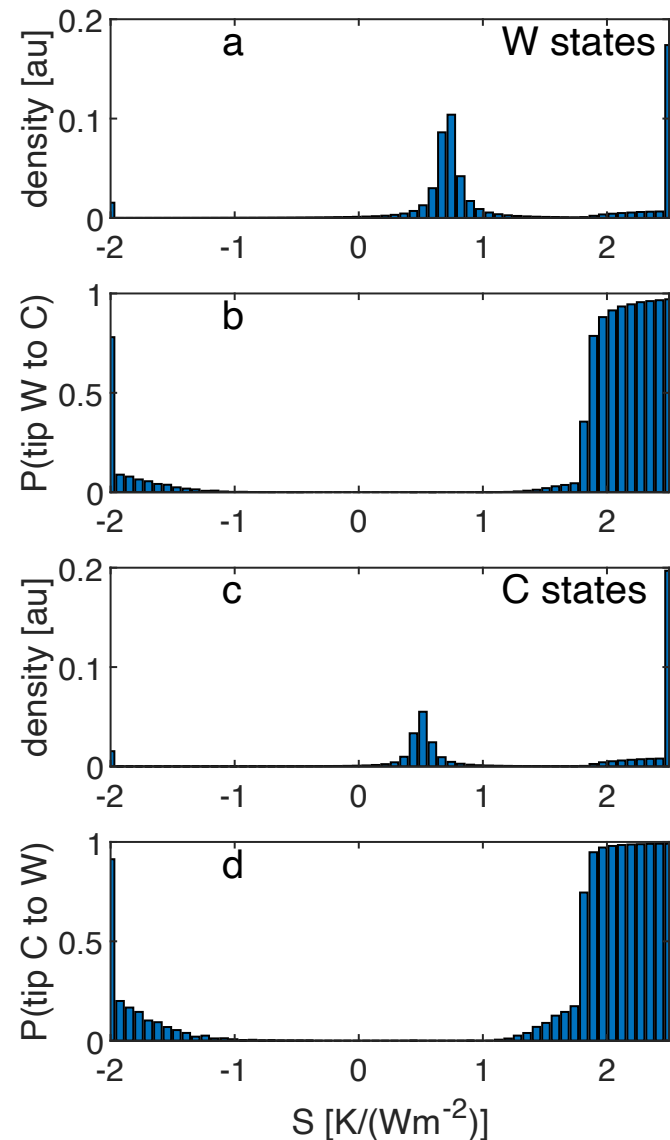
0. 10. 20. 30. 40. 50
Time [kyr]

Two-point sensitivity



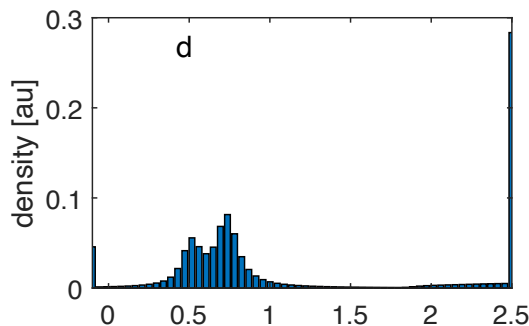
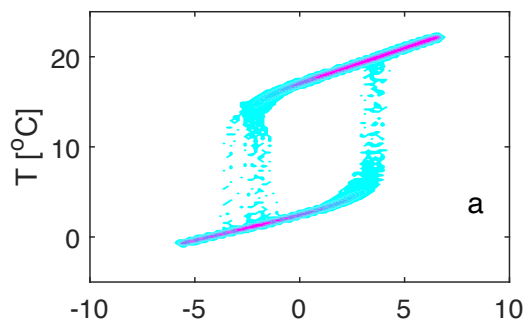
Conditional, incremental sensitivity
for all delays up to 20kyr

Ashwin & von der Heydt, J. Stat. Phys. (2019)



Skewed PDFs of two-point sensitivity

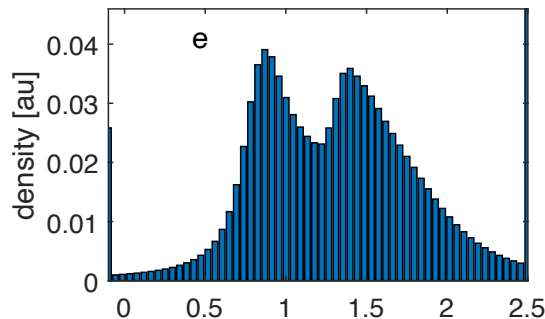
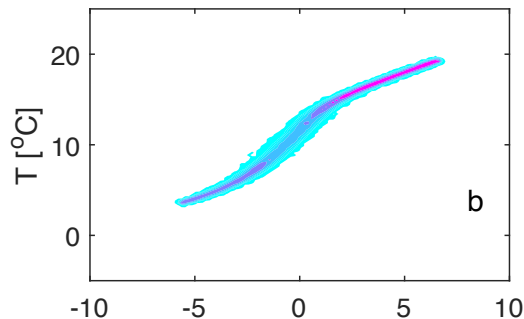
Standard albedo contrast



Tipping and state-dependence of feedbacks

non-constant feedback factors

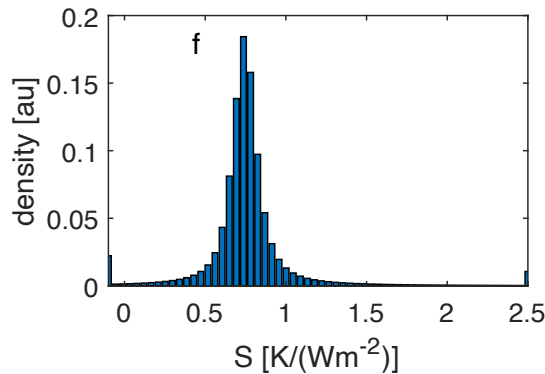
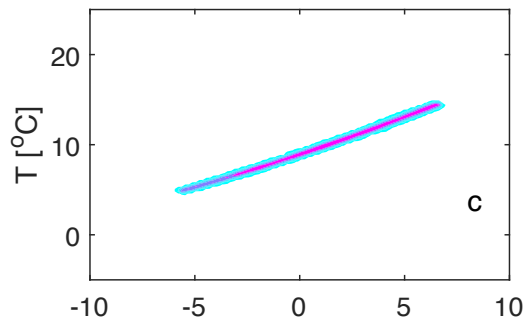
Low albedo contrast



Only state-dependence of feedbacks

non-constant feedback factors

NO albedo contrast



No state-dependence of feedbacks

constant feedback factors
~ Gaussian PDF

Conclusions

- Climate sensitivity depends on the background climate state:
 - ★ non-constant fast feedback processes,
 - ★ multiple equilibrium or oscillatory states ('tipping').
- 'Flavours' of (palaeo)climate sensitivity on the 'climate attractor':
 - ★ instantaneous S: available from underlying model ('nearest equilibrium'),
 - ★ incremental S: fixed delay Δt ,
 - ★ two-point S: all delays, two points on attractor.
- Nonlinearities lead to skewed PDFs of measured climate sensitivity
 - ★ Extremes of climate sensitivity seem to relate to high probability of tipping.