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The nutations of a rigid Mars

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Abstract

The radioscience experiments **RISE** (Rotation and Interior Structure Experiment, InSight) and **LaRa** (Lander Radioscience, ExoMars2022) will determine the nutations of Mars [1,2].

Nutations are short-period oscillations in the spin axis orientation in space due to
• the periodical changes in the Solar torque acting on Mars
• the torques exerted by Phobos and Deimos
• the torques exerted by the other planets.
The **amplitude** of the largest (semi-annual) nutation is a few hundreds mas (1 mas = 1.6 cm at the surface).

The nutation amplitudes are affected by the coupling between the mantle and the core. The changes in amplitudes are computed with transfer functions [1]. Depending on the size and shape of the **liquid core** [3,4]
• the semi-annual nutation can be amplified by 5 to 30 mas
• the ter-annual nutation can be amplified by $\geq$ 10 mas [5].

The **accuracy** on the measured main nutation terms, currently not sufficient to determine the core properties [6,7], will be improved with RISE and LaRa to a level that allows to constrain the core radius [2,8]. To **avoid introducing systematic errors**, an accurate representation for the nutation of a rigid Mars is needed.

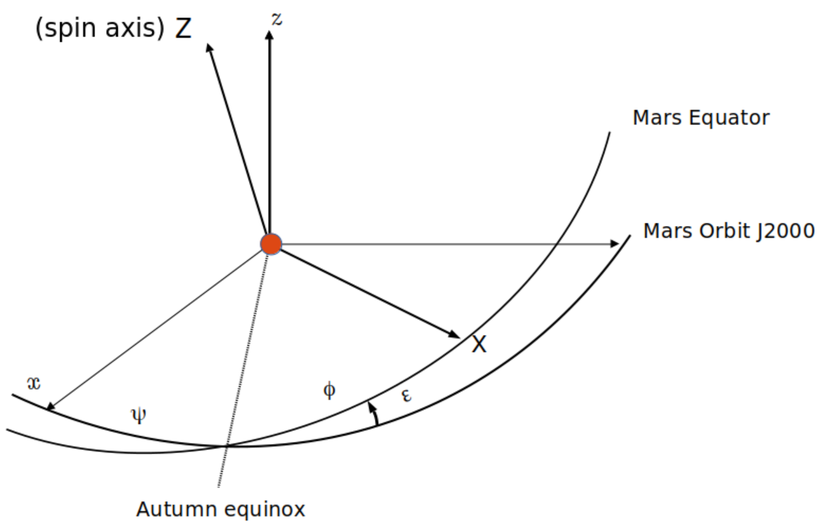
We provide an **up to date rigid nutation model** consistent with recent orbital ephemerides. We identify 43 nutation terms with an amplitude above the chosen truncation criterion of 0.025 milliarcseconds in prograde and/or retrograde nutations.

The torques by the Sun, Phobos, Deimos, and the planets also contribute to the **precession rate** of the spin axis of Mars. Given the current determination of the precession rate (7608.3 ± 2.1 mas/yr, [9]), our model predicts a dynamical flattening $H_D = 0.00538017 \pm 0.00000148$ and a normalized polar moment of inertia $C/MR^2 = 0.36367 \pm 0.00010$ for Mars.

Theory

Longitude and Obliquity

Euler angles between the Martian rotating frame (XYZ, the Z-axis is aligned with the spin axis of Mars) and the Inertial frame associated with the Martian mean orbit at epoch J2000 (axes xyz):



ψ is the node longitude, ε is the obliquity
(ϕ is the rotation angle)

Angular momentum equations

Differential equations governing the behaviour of ψ and ε:

$$\dot{\psi} = H_D \frac{3GM_B}{\sin\varepsilon_0\Omega_R}\frac{Y_BZ_B}{d_B^5} + \delta H_D^{tri}\frac{3GM_B}{2\sin\varepsilon_0\Omega_R}\frac{X_BZ_B\sin 2\phi - Y_BZ_B\cos 2\phi}{d_B^5},$$
$$\dot{\varepsilon} = H_D \frac{3GM_B}{\Omega_R}\frac{X_BZ_B}{d_B^5} + \delta H_D^{tri}\frac{3GM_B}{2\Omega_R}\frac{X_BZ_B\cos 2\phi + Y_BZ_B\sin 2\phi}{d_B^5}.$$

• (X_B, Y_B, Z_B) is the position of the perturbating body in the coordinates of a Martian frame whose x-axis is in the direction of the autumn equinox, d_B is its distance to the center of Mars, and M_B is its mass.
• Ω_R is the rotation rate of Mars, ε_0 is the J2000 obliquity, and G is the gravitational constant.

As Mars' rotation is faster than its revolution, the response to the external torques is divided into two distinct parts:

(1) The first part ($\propto H_D = (C-\bar{A})/C$) of the angular momentum equations governs the response of an axisymmetric Mars, and induces nutations at various periods (annual, semi-annual, ...). H_D is the *dynamical flattening*, also called *scaling factor*.

(2) The second part ($\propto \delta H_D^{tri} = (B-A)/C$) of the angular momentum equations is related to the triaxiality and induces quasi semi-diurnal nutations of small amplitudes, since ϕ is a fast angle.

$A < B < C$ are the principal moments of inertia of Mars, and $\bar{A} = (A+B)/2$ is the average equatorial moment if inertia.

Solution

Form of the solution

We derive a semi-analytical solution, where the node longitude and obliquity are written as:

$$\psi = \psi_0 + \dot{\Psi}t + \frac{\ddot{\Psi}}{2}t^2 + \Delta\psi,$$
$$\varepsilon = \varepsilon_0 + \dot{E}t + \frac{\ddot{E}}{2}t^2 + \Delta\varepsilon.$$

$\dot{\Psi}$ and $\dot{E}$ are the precession rate in longitude and obliquity.

The nutations are expressed as series under the form

$$\left\{\begin{array}{c}\Delta\psi\\ \Delta\varepsilon\end{array}\right\} = \sum_j \left(\left\{\begin{array}{c}\psi_j^c\\ \varepsilon_j^c\end{array}\right\}\cos\varphi_j + \left\{\begin{array}{c}\psi_j^s\\ \varepsilon_j^s\end{array}\right\}\sin\varphi_j \right),$$

with $\psi/\varepsilon_j^{c/s}$ the amplitudes and $\varphi_j = f_j t + \varphi_j^0$ a linear combination of fundamental arguments (the mean longitude of Mars, the Earth, Venus, Jupiter, and Saturn, the nodes of Phobos and Deimos, and the rotation angle).

BMAN20

The solution is named BMAN20 (*Baland Martian Analytical Nutations 2020*). We consider Solar, satelllite, and direct planetary torques. We also include the geodetic precession and nutations in the solution.

Ephemerides used to build the solution:
• VSOP2000 for the planet motion [10]
• Jacobson and Lainey (2014) [11] for Phobos and Deimos motion

Nutations: With a truncation criterion of 0.025 milliarcseconds in prograde and/or retrograde amplitude, we identify 43 nutation terms (see table below). The uncertainty on our solution (0.03%) mainly derives from the observational uncertainty on the current determination of the precession rate of Mars [9] which is used to determine H_D (see below). Uncertainties related to our modeling choices are negligible in comparison.

Longitude precession rate: The precession rate in longitude is a combination of different contributions from the Sun (S), geodetic (g), the planets (Pl), Phobos (P), and Deimos (D):

$$\dot{\Psi} = \dot{\Psi}_S + \dot{\Psi}_g + \dot{\Psi}_{Pl} + \dot{\Psi}_P + \dot{\Psi}_D$$

To obtain the observed precession rate $\dot{\Psi} = -7608.3$ mas/yr, we use $H_D = 0.00538017$. The different contributions are:
• $\dot{\Psi}_S = -7614.28$ mas/yr,
• $\dot{\Psi}_g = +6.75$ mas/yr,
• $\dot{\Psi}_{Pl} = -0.34$ mas/yr,
• $\dot{\Psi}_P = -0.23$ mas/yr,
• $\dot{\Psi}_D = -0.20$ mas/yr.

Quadratic terms in longitude, secular and quadratic terms in obliquity, as well as secular variations in the nutation amplitudes are given in the complete BMAN20 precession/nutation solution available at

`https://doi.org/10.24414/h5pn-7n71`

Polar moment of inertia

From H_D and the unnormalized gravity coefficient $J_2 = (C-\bar{A})/MR^2 = 0.00195661 \pm 2.82\times10^{-10}$ (MRO120D gravity field, [9]), we determine the normalized polar moment of inertia of Mars

$$\frac{C}{MR^2} = \frac{J_2}{H_D} = 0.36367 \pm 0.00010.$$

The uncertainty on C/MR^2 results almost entirely from $\sigma_{HD} = 2.1$ mas/y.

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	j	Sa	Ju	Ma	Ea	Ve	N_{Ph}	N_{De}	ϕ	Period (days)	ψ_j^c(mas)	ψ_j^s(mas)	ε_j^c(mas)	ε_j^s(mas)	$\mathscr{P}_j$(mas)	$\mathscr{R}_j$(mas)	$\pi_j(°)$	$\rho_j(°)$	α_j^c(mas)	α_j^s(mas)	δ_j^c(mas)	δ_j^s(mas)
semi-diurnal	1	0	0	0	0	0	0	0	2	0.513	0.000	0.110	-0.047	0.000	0.000	0.047	146.731	326.731	-0.053	0.056	0.034	0.032
	2	0	0	7	0	0	0	0	0	98.140	-0.102	0.085	0.040	0.048	0.059	0.003	187.699	6.173	-0.007	0.098	-0.059	-0.010
	3	0	0	6	0	0	0	0	0	114.497	-0.898	0.255	0.118	0.421	0.417	0.020	168.343	346.409	-0.327	0.609	-0.348	-0.232
	4	0	0	5	0	0	0	0	0	137.396	-6.292	-0.889	-0.429	2.942	2.839	0.134	148.997	326.401	-3.720	2.883	-1.523	-2.402
	5	0	0	4	0	0	0	0	0	171.745	-34.998	-21.766	-10.258	16.269	18.388	0.847	129.675	305.847	-29.631	7.290	-2.735	-18.198
	6	0	-2	4	0	0	0	0	0	186.533	-0.022	-0.126	-0.058	0.008	0.056	0.002	11.906	168.784	-0.077	-0.056	0.036	-0.043
	7	0	-3	11	-4	0	0	0	0	228.913	0.095	-0.031	-0.014	-0.044	0.044	0.002	182.583	355.828	0.033	-0.066	0.038	0.023
ter-annual (*)	8	0	0	3	0	0	0	0	0	228.993	-137.727	-201.016	-93.959	62.969	108.412	4.727	110.413	283.663	-177.457	-31.789	28.275	-104.489
	9	0	3	-5	4	0	0	0	0	229.074	0.063	-0.078	-0.036	-0.030	0.045	0.002	218.283	31.468	-0.008	-0.074	0.045	-0.001
	10	0	0	1	1	0	0	0	0	238.467	-0.007	-0.066	-0.030	0.002	0.029	0.001	190.982	353.828	-0.038	-0.031	0.020	-0.021
	11	0	-1	3	0	0	0	0	0	241.772	0.023	0.162	0.073	-0.008	0.072	0.002	228.978	20.657	0.095	0.074	-0.047	0.053
	12	0	-2	3	0	0	0	0	0	256.061	0.027	-0.158	-0.058	-0.020	0.064	0.006	353.609	49.610	-0.052	-0.103	0.050	-0.032
	13	0	0	4	-1	0	0	0	0	324.172	0.069	0.057	0.027	-0.031	0.039	0.001	201.120	0.761	0.066	-0.006	0.001	0.039
	14	0	-3	10	-4	0	0	0	0	343.309	0.309	0.028	0.016	-0.140	0.137	0.005	163.505	324.096	0.177	-0.145	0.078	0.111
	15	-6	8	-5	0	2	0	0	0	343.489	0.075	-0.077	-0.034	-0.035	0.047	0.002	46.095	206.680	0.000	-0.079	0.047	0.003
semi-annual (*)	16	0	0	2	0	0	0	0	0	343.490	-221.944	-1113.768	-509.879	88.885	500.446	18.118	91.424	252.001	-692.998	-471.366	306.673	-389.554
	17	6	-8	9	0	-2	0	0	0	343.491	0.099	0.042	0.020	-0.045	0.047	0.002	316.754	117.331	0.074	-0.029	0.014	0.045
	18	0	3	-6	4	0	0	0	0	343.671	0.274	-0.144	-0.063	-0.127	0.137	0.005	199.351	359.568	0.070	-0.218	0.126	0.050
	19	0	0	0	1	0	0	0	0	365.256	0.028	-0.072	-0.026	-0.016	0.032	0.003	164.447	235.415	-0.016	-0.055	0.027	-0.009
	20	0	-2	2	0	0	0	0	0	408.217	0.003	-0.228	0.002	0.000	0.047	0.050	11.246	11.374	0.004	-0.117	-0.001	-0.066
	21	0	2	1	0	0	0	0	0	521.576	0.050	0.064	0.030	-0.023	0.036	0.002	12.161	188.872	0.060	0.006	-0.007	0.036
	22	0	1	1	0	0	0	0	0	592.960	-0.020	-0.073	-0.038	0.008	0.035	0.004	133.478	294.976	-0.054	-0.028	0.022	-0.027
geodetic	23	0	0	1	0	0	0	0	0	686.980	0.229	0.516	0.000	0.000	0.120	0.120	289.374	289.374	0.118	0.265	0.067	0.151
annual	24	0	0	1	0	0	0	0	0	686.980	-283.834	-480.044	47.897	11.969	102.595	137.356	125.759	108.776	-91.457	-233.072	-117.664	-148.715
	25	0	0	-1	1	0	0	0	0	779.936	0.002	-0.119	0.001	0.001	0.025	0.026	195.155	193.184	0.002	-0.060	0.000	-0.035
	26	0	-1	1	0	0	0	0	0	816.435	-0.014	0.439	0.000	-0.001	0.093	0.094	229.633	228.802	-0.008	0.224	-0.004	0.129
Phobos	27	0	0	0	0	0	-1	0	0	825.688	0.000	10.127	-4.310	0.000	0.000	4.310	327.928	147.928	-4.894	5.203	3.140	2.953
	28	0	0	-3	2	0	0	0	0	901.985	0.061	-0.109	0.000	0.000	0.027	0.027	275.585	275.154	0.031	-0.056	0.018	-0.032
	29	0	-2	1	0	0	0	0	0	1006.007	0.210	-0.402	0.001	0.002	0.096	0.097	349.511	348.780	0.109	-0.205	0.061	-0.118
Jupiter	30	0	-3	1	0	0	0	0	0	1310.238	0.018	-0.079	0.037	0.009	0.002	0.036	137.888	329.101	0.051	-0.030	-0.022	-0.030
Jupiter	31	0	2	0	0	0	0	0	0	2166.295	-0.042	-0.187	-0.088	0.022	0.086	0.005	172.078	1.872	-0.122	-0.071	0.052	-0.070
	32	0	2	0	0	0	0	0	0	2166.295	-0.074	-0.046	-0.003	0.016	0.026	0.011	222.741	202.936	-0.042	-0.006	-0.019	-0.025
Earth	33	0	0	4	-2	0	0	0	0	2882.003	-0.012	-0.078	-0.029	0.006	0.032	0.002	241.009	220.170	-0.040	-0.033	0.018	-0.027
	34	0	1	0	0	0	0	0	0	4332.589	0.033	0.210	0.004	-0.007	0.048	0.043	317.343	308.964	0.021	0.100	0.007	0.067
Earth	35	0	0	2	-1	0	0	0	0	5764.006	-0.076	-0.129	0.006	0.001	0.030	0.034	14.621	7.806	-0.032	-0.065	-0.027	-0.038
	36	0	0	2	-1	0	0	0	0	5764.006	0.204	0.190	-0.014	0.004	0.053	0.066	211.750	204.127	0.089	0.102	0.070	0.053
Venus	37	0	0	-3	0	1	0	0	0	11987.226	0.034	-0.150	0.066	0.012	0.002	0.066	154.426	274.266	0.093	-0.064	-0.038	-0.052
	38	0	0	-3	0	1	0	0	0	11987.226	0.060	0.048	-0.042	-0.009	0.020	0.032	227.814	120.830	-0.017	0.014	0.048	0.020
Deimos	39	0	0	0	0	0	0	-1	0	20000.000	0.000	3.532	-1.503	0.000	0.000	1.503	78.378	258.378	-1.707	1.815	1.095	1.030
	40	5	-2	0	0	0	0	0	0	322618.691	-0.373	0.112	-0.143	0.076	0.127	0.104	339.462	68.232	-0.354	0.144	-0.004	-0.023
	41	0	-3	8	-4	0	0	0	0	651385.029	1.003	0.284	-0.002	0.015	0.215	0.229	162.473	163.028	0.513	0.162	0.294	0.072
	42	5	4	-16	8	0	0	0	0	3.41836×10^7	0.131	0.052	-0.008	-0.015	0.036	0.025	173.588	147.832	0.058	0.010	0.044	0.026
	43	-6	8	-7	0	2	0	0	0	1.32273×10^8	0.278	-0.212	-0.029	-0.024	0.093	0.056	50.142	43.218	0.110	-0.136	0.102	-0.044

Table 1: ***BMAN20 Rigid nutation series, sorted by increasing periods:***

Main solar terms
Other solar terms (terms related to planetary perturbations, geodetic nutations, semi-diurnal nutations due to triaxiality)
Terms related to the direct effects of Phobos, Deimos, Jupiter, Earth, and Venus.

Sa, Ju, Ma, Ea, Ve are the planets' mean longitude as defined in [10] (e.g. $Ma = (355.434 + 191403T)°$ with T measured in thousands of years from J2000).
N_{Ph} and N_{De} are the satellites' orbital nodes.
ϕ is Mars rotation angle.

Three possibles representations: - longitude & obliquity (ψ_j & ε_j, cosine and sine terms),
- prograde & retrograde ($\mathscr{P}_j$ & $\mathscr{R}_j$, and their associated phases π_j and ρ_j), truncature level of 0.025 mas in prograde/retrograde motion,
- right ascension & declination according to IAU conventions (α_j & δ_j, cosine and sine terms).

(*) $\mathscr{P}_{16}$ (semi-annual prograde) and $\mathscr{R}_8$ (ter-annual retrograde) are of particular interest for the determination of the core properties