Cumulant lattice Boltzmann approach: an application to hydraulic risk

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Topographic data

External force data

manning's coefficient
wind velocity

GIS LB ROUTINE

interchange data

Boundary conditions

inlet,
outlet,
obstacles position

LB
shallow water
CODE

Initial conditions

water height
velocity field
temperature
Pollutants concentration

Thematic maps
(Hydraulic Risk maps)

GIS LB ROUTINE

interchange data

semi-automatic procedure for modelling flood events

- **CumLB** method: numerical technique for the solution of the hydrodynamic problem
- **parallelization**
- The procedure includes the preparation of the input data and the visualizations of the results (**Open source Qgis platform**)

SILVIA DI FRANCESCO, SARA VENTURI, MARTIN GEIER

Presentation Overview

- Shallow water – lattice Boltzmann (LB) **innovative models**
- **Wet - dry approach**
- Case study: **Malpasset dam-break**

SHALLOW WATER EQUATIONS

continuity

$$\frac{\partial h}{\partial t} + \frac{\partial (hu_j)}{\partial x_j} = 0$$

momentum

$$\frac{\partial (hu_i)}{\partial t} + \frac{\partial (hu_i u_j)}{\partial x_j} = -g \frac{\partial}{\partial x_i}\left(\frac{h^2}{2}\right) + \nu \frac{\partial^2 (hu_i)}{\partial x_j \partial x_j} + F_i$$

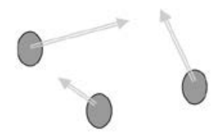
$$F_i = -gh \frac{\partial z_b}{\partial x_i} + \frac{\tau_{wi}}{\rho} + E_i$$

h, u_i

averaged along the vertical direction

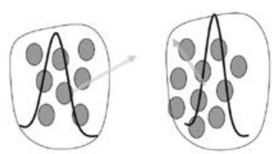
LATTICE BOLTZMANN METHOD

microscopic



Molecular Dynamics (MD)

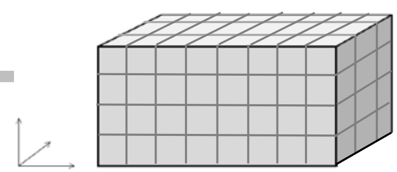
mesoscopic



LATTICE BOLTZMANN METHOD

Probability Distribution Function $f(x,c,t)$

macroscopic



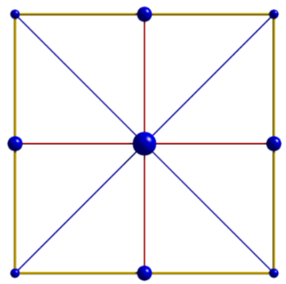
continuum

Navier Stokes Equations

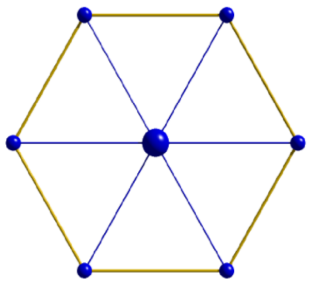
Continuum fluid macroscopic properties

LATTICE BOLTZMANN MODEL

2D models

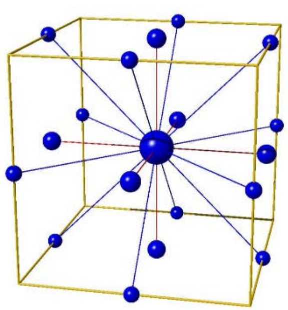


D2Q9

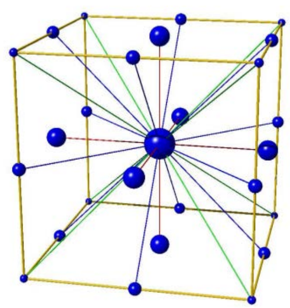


D2Q7

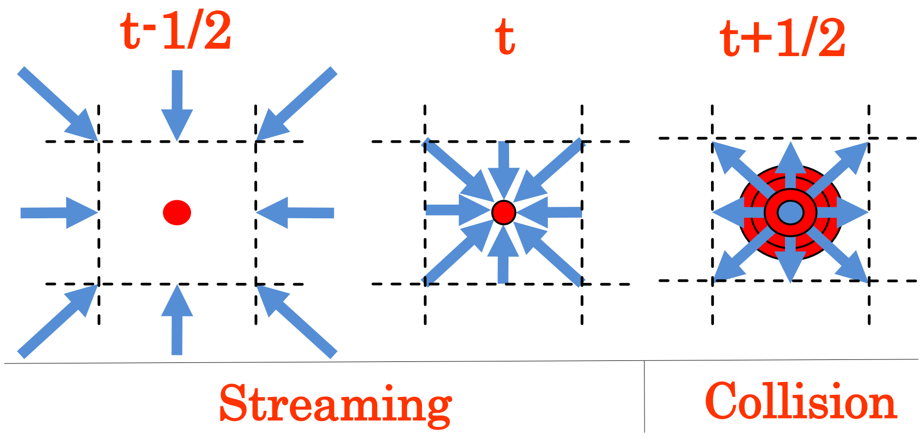
3D models



D3Q19



D3Q27



$$f_\alpha\left(x+e_\alpha\Delta t,t+\Delta t\right)=f_\alpha\left(x,t\right)+\Omega_\alpha+F_\alpha \qquad \alpha=1,\dots,n$$

F_α : *external force*

Ω_α : *collision operator*

Conservation of mass and momentum in the collision

LB MODELS FOR SWE

Speed of sound

$$P = c_s^2 h$$

equation of state

$$\partial t h + \nabla(h\mathbf{u}) = 0$$

$$\partial t(h\mathbf{u}) + \nabla(PI + h\mathbf{uu} - S) = 0$$

$$c_s^2 \Rightarrow \frac{gh}{2} \neq \frac{\mathbf{1}}{\mathbf{3}}$$

Viscosity

$$\nu = c_s^2\left(\frac{1}{\omega} - \frac{1}{2}\right)$$

$$\omega = \frac{1}{\tau}$$

relaxation time

Macroscopic Variables

$$h = \sum_{\alpha=1}^{n} f_\alpha \qquad u_i = \frac{1}{h}\sum_{\alpha=1}^{n} e_{\alpha i} f_\alpha$$

MRT CASCADED MODEL (CaLB)

Central moments

$$\kappa_{\alpha\beta} = \sum_{i,j} (i-u)^{\alpha} (j-v)^{\beta} f_{ij} \quad i,j = -1, 0, 1$$

$\kappa_{00} = h$, $\kappa_{10} = 0$, $\kappa_{01} = 0$, $\kappa_{20} = c_s^2 h$, $\kappa_{02} = c_s^2 h$, $\kappa_{11} = 0$, $\kappa_{12} = 0$, $\kappa_{22} = 0$

$\kappa_{\alpha\beta}$ = central moments **u, v = macroscopic velocities**

Collision step

$$\kappa_{\alpha\beta,post} = \kappa_{\alpha\beta} - \omega_{\alpha\beta} (\kappa_{\alpha\beta} - \kappa_{\alpha\beta,eq})$$

$$\kappa_{20+02,post} = \kappa_{20+02} - \omega_{20+02} (\kappa_{20} - \kappa_{20,eq} + \kappa_{02} - \kappa_{02,eq})$$

$$\kappa_{20-02,post} = \kappa_{20-02} (1 - \omega_{20-02})$$

Relaxation rates

$$\omega_{11} = \frac{1}{3\nu + 0.5} \qquad \omega_{20-02} = \omega_{11} \qquad \omega_{\alpha\beta} = 1$$

MRT CUMULANT MODEL (CumLB)

Theory of cumulant

$$c_{\alpha\beta\gamma} = \frac{\partial^\alpha \partial^\beta \partial^\gamma}{\partial \Xi \partial \Upsilon \partial Z} \ln\left(\frac{F(\Xi, \Upsilon, Z)}{\varrho_0}\right)\Bigg|_{\Xi, \Upsilon, Z=0}$$

Cumulants: coefficients of the Taylor expansion of the logarithm of the Laplace transform of the distribution function f (PDF)

- The cumulant CO uses multiple relaxation rates (MRT CO)
- It relaxes observable quantities (cumulants) that are both Galilean invariant and statistically independent of each other by construction

Galilean Invariance

- Break of Galilean invariance: due to the finite number of velocities and a resulting finite number of independent moments
- not Galilean invariant model: results depend on the reference frame $\rightarrow$ presence of preferential directions

MRT CUMULANT MODEL (CumLB)

cumulants

$$C_{ab} = \kappa_{ab}$$ **until 3th order**

$$C_{22} = \kappa_{22} - \left(\kappa_{20}\kappa_{02} + 2\kappa_{11}^{2}\right)/h$$

collision step

$$C_{\boldsymbol{\alpha\beta},\, \boldsymbol{post}} = C_{\alpha\beta} - \omega_{\alpha\beta}\,(C_{\alpha\beta} - C_{\alpha\beta,\boldsymbol{eq}})$$

- The PDF is transformed into cumulants.
- The collision step is performed in terms of cumulants;
- After the collision, the backward transformation is applied, from cumulants to PDF.

WET-DRY APPROACH

It was necessary to develop a procedure simulating the flow propagation over a dry bed.

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threshold value: allows to avoid the error due to the division by zero

1 **Height Limiter**

$$h \le h_{\lim} \quad \rightarrow \quad h = \boxed{h_{\lim}}$$

2 **Velocity Limiter**

$$Fr \ge Fr_{\lim} \quad \rightarrow \quad |u|' = |u| \cdot \frac{Fr_{\lim}}{Fr}$$

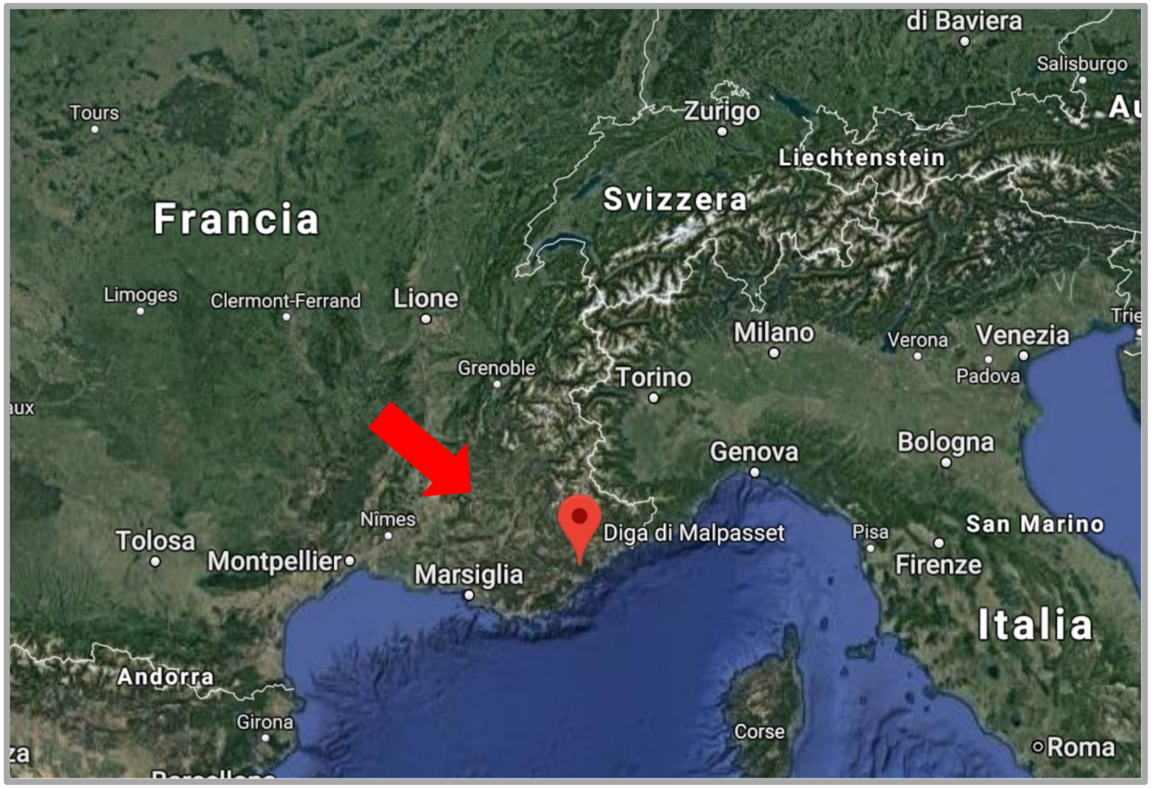
$$\frac{Fr_{\lim}}{Fr} < 1$$

The velocity is limited by means of the Froude Number, multiplying its value for the ratio between the Fr-limiter and the actual Froude Number

$$|\mathrm{u}|' = |\mathrm{u}| \cdot (1 - \delta) \quad \rightarrow \quad \delta = 1 - Fr_{\lim} \sqrt{\frac{gh}{|\mathrm{u}| \cdot |\mathrm{u}|}}$$

CASE STUDY

MALPASSET DAM-BREAK (21/12/1959)



A. Valiani et al., 'Case Study: Malpasset Dam-Break Simulation using a Two-Dimensional Finite Volume Method', 2002

C. Biscarini et al., ' On the Simulation of Floods in a Narrow Bending Valley: The Malpasset Dam Break Case Study ', 2016

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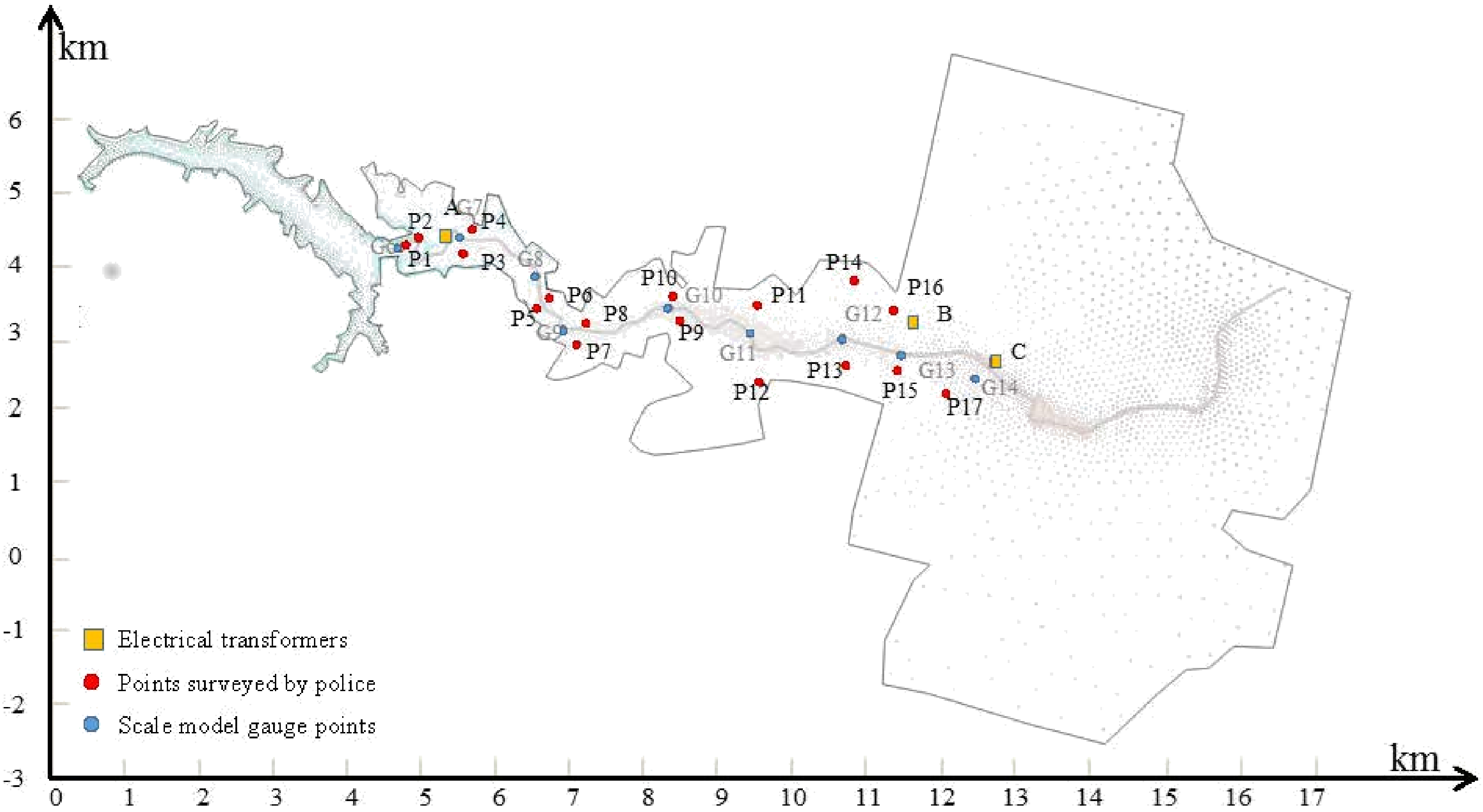


AT at hydroelectric plants **(A, B, C)**

WS and velocity at points measured by the police P_i , $\mathbf{i = 1, \ldots, 17}$

WS (water surface) and **AT (arrival time)**, Physical model, EDF, G_i $\mathbf{i = 1, \ldots, 14}$

Domain – bounding box

width: $17500\ m$
height: $10000\ m$

setup

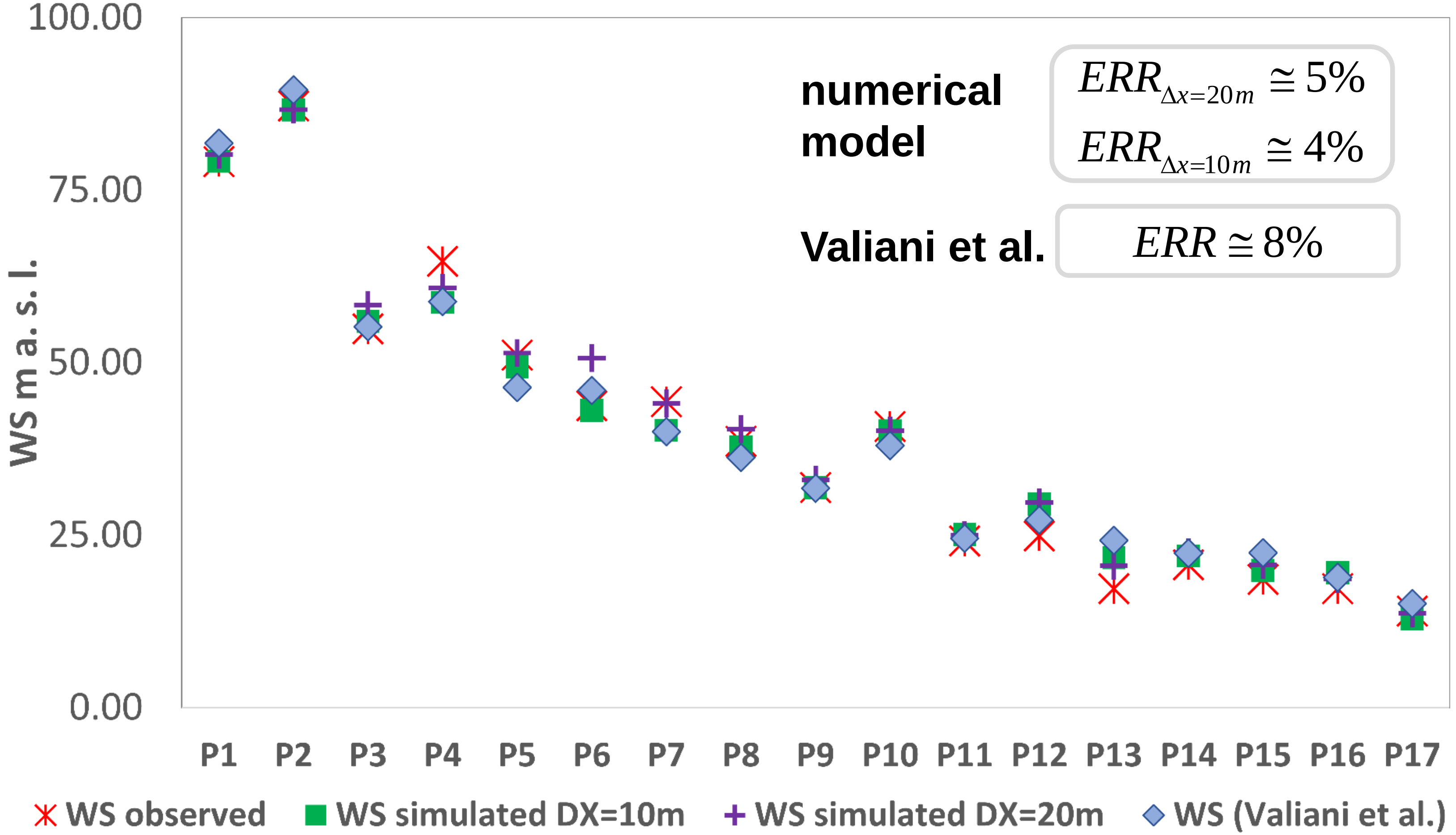
Model: $Cumulant\ model$
Grid spacing Δx: 10 m, 20 m
relaxation rate τ: 0.8
Manning coeff. n: 0.03 $s\ m^{1/3}$ *(Hervouet and Petitjean, 1999)*

Arrival time (AT) at Electrical Transformer

Comparison with simulation time

ET	**At *obs*** (s)	**At *sim*** (s)
A	100	98.6
B	1240	1314
C	1420	1465

Comparison of WS at Pi - points



numerical model

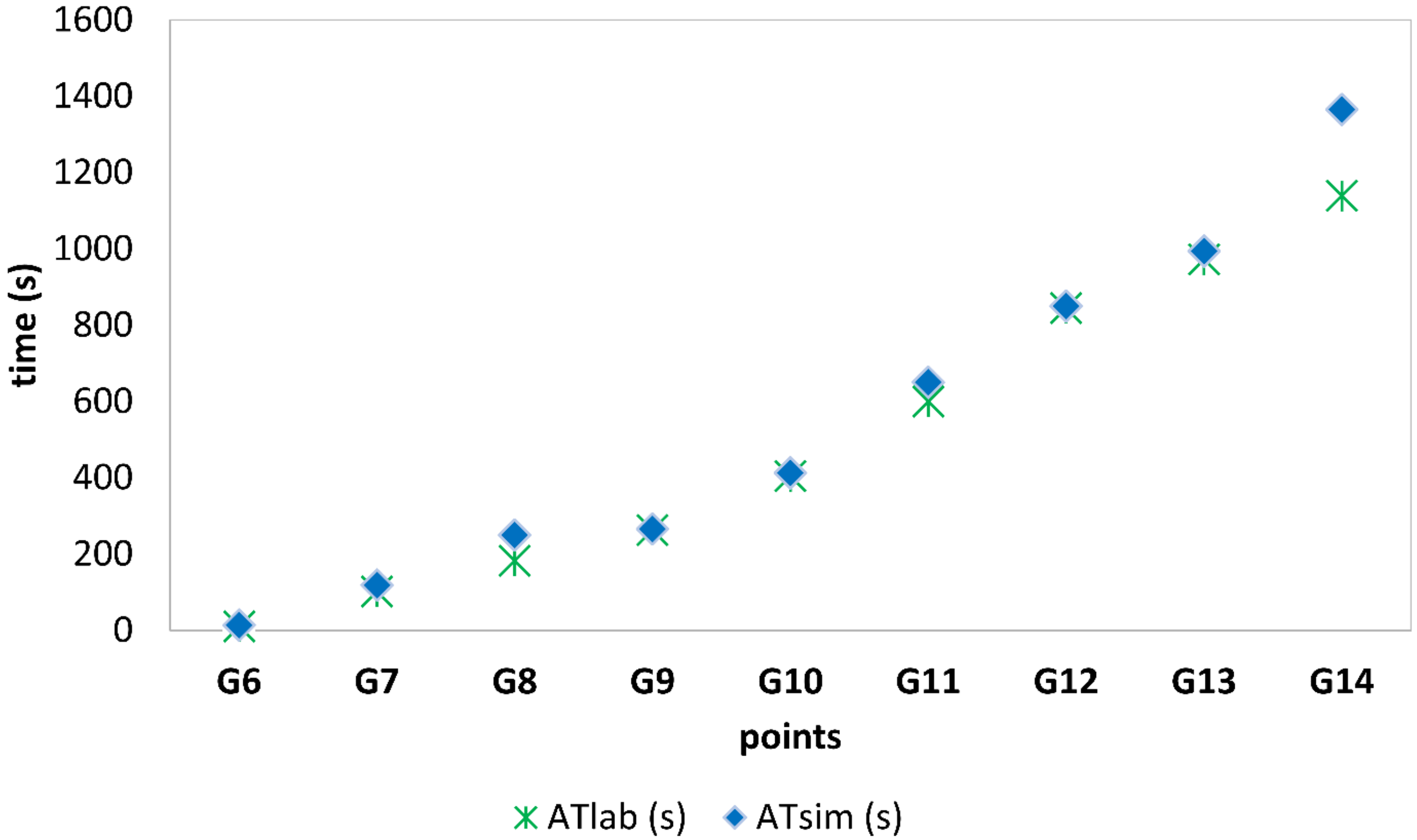
$ERR_{\Delta x=20m} \cong 5\%$

$ERR_{\Delta x=10m} \cong 4\%$

Valiani et al.

$ERR \cong 8\%$

AT comparison – Gi points



Satisfying accordance between the arrival time **(AT)** at almost all G_i points

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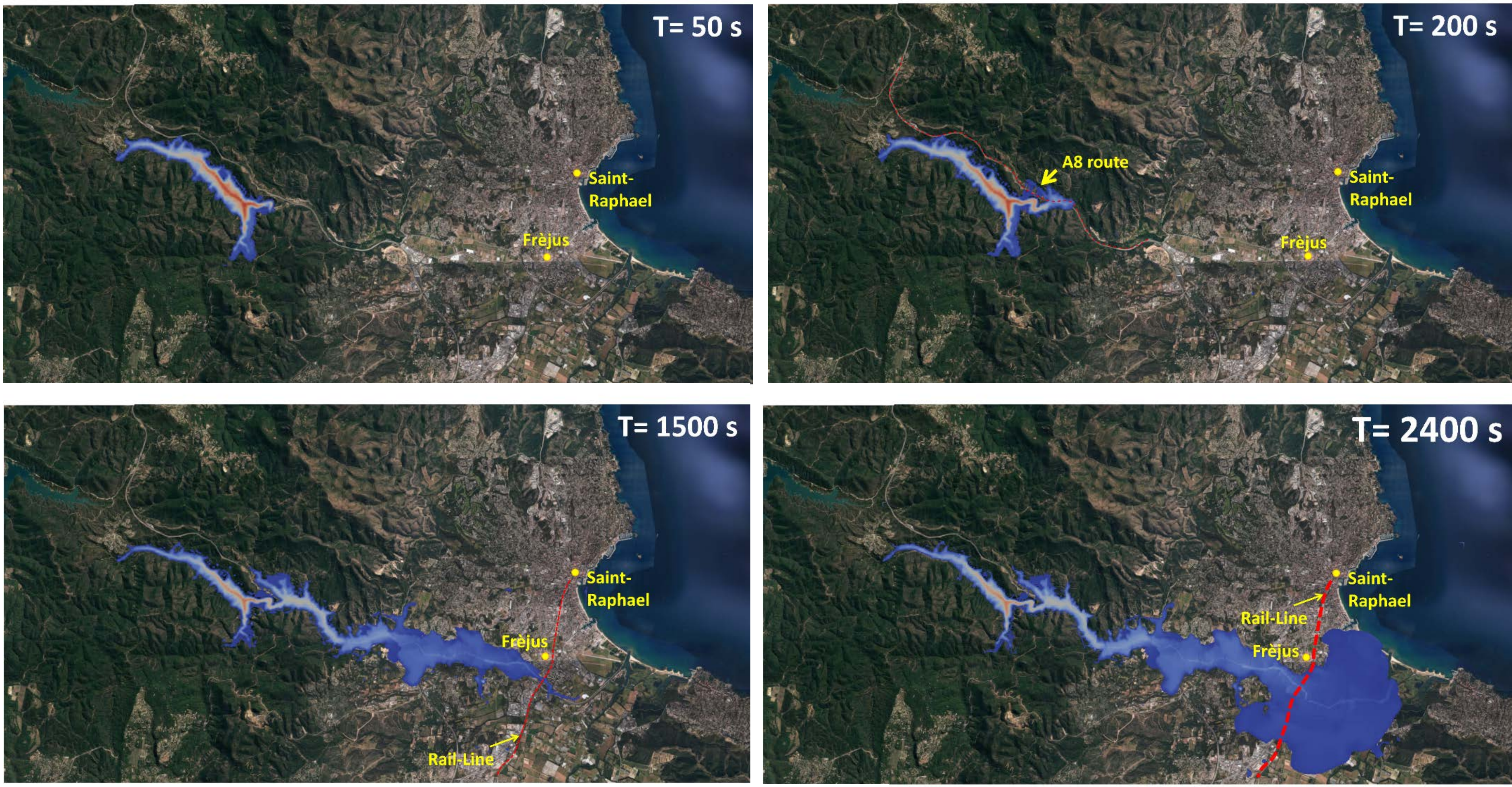

water height

0.0e+00 10 20 30 40 50 5.5e+01

Reyran Valley

CONCLUSIONS

- The applicability of **CumLB** model to the propagation of **floods** has been successfully tested;
- **Innovative models** exhibit satisfying characteristics of accuracy and stability in predicting a flood wave, introducing the possible application of the **LB Cum- GIS** routine to the assessment of the **hydraulic risk**.