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Session HS-8.1.6: Handling Uncertainties in Model Concepts, Parameters, Forcings and Forecasts: Diagnostics, Sensitivity, Inversion and Uncertainty Analysis

A comprehensive global sensitivity analysis using generic sampling designs by means of a combination of variance- and distribution-based approaches

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Outline

A) **Motivations:** why and how to improve current best practices in global sensitivity analysis (GSA)?

B) A new **C**ombined **V**ariance- and **D**istribution-based global sensitivity analysis **CVD - GSA**: how does it work?

C) **Tests** to three analytic functions and one hydrological model

D) **Conclusions** and Outlook

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A) Motivation

1. **global sensitivity analysis (GSA) is an important tool for**
   - supporting model developments
   - processes understanding

2. State-of-the-art **variance-based approach** (Saltelli et al., 2010)
   - Identify important parameters (main effect) and interactions (total effect – main effect)
   - It works also on non-scalar factors (e.g., Baroni and Tarantola, 2014)

3. **Limitations**
   - Specific sampling design
   - Relatively high number of simulation runs
   - Issues with non-gaussian distributions (distribution-based approaches)
   - how to improve?

$$S_i = \frac{V[E(Y|X_i)]}{V(Y)} \qquad T_i = \frac{E[V(Y|X_{\sim i})]}{V(Y)}$$

Where:

S_i main effect of factor i
T_i total effect of factor i
E mean operator
V variance operator
$Y\,|\,X_i$ output Y conditioned to X_i
$Y\,|\,X_{\sim i}$ output Y conditioned to all but not X_i

Interaction $I_i = T_i - S_i$

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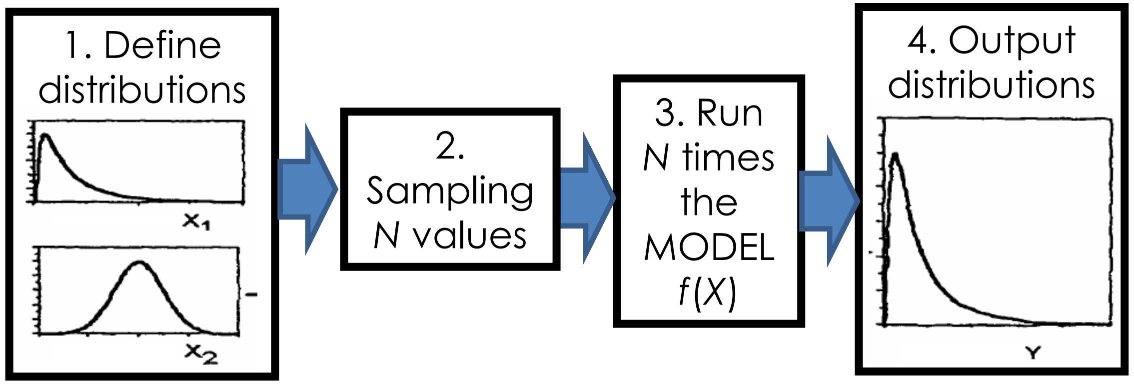
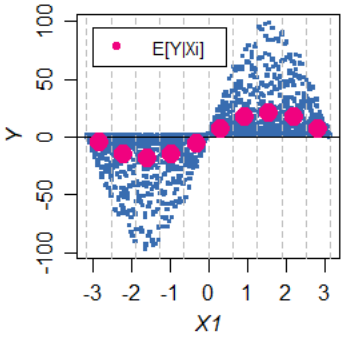
B) An effective Combined Variance- and Distribution-based strategy (CVD)

1. Run generic Monte Carlo simulations



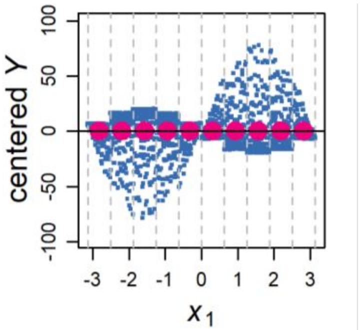
2. Main effect is estimated based on a filtering approach (Kucherenko et al., 2017)



- X_i is divided in m intervals
- $E[Y|X_i]$ is calculated in each m
- Main effect S_i is estimated

$$S_i = \frac{V[E(Y|X_i)]}{V(Y)}$$

3. The main effect is removed from the input-output space



By removing the conditional mean $E(Y|X_i)$, we obtain m centralized conditional distributions

$$P(Y|X_i) = P(Y|X_i) - E(Y|X_i)$$

4. Distribution-based measure (e.g., Kolmogorov-Smirnov test - KS) is used to estimate the interaction term I_i by comparing the m centralized conditional distributions

$$I_i = median\{KS[P(Y|X_{i=j}), P(Y|X_{i=j+1})]\}$$

With $j = [1\ldots m]$

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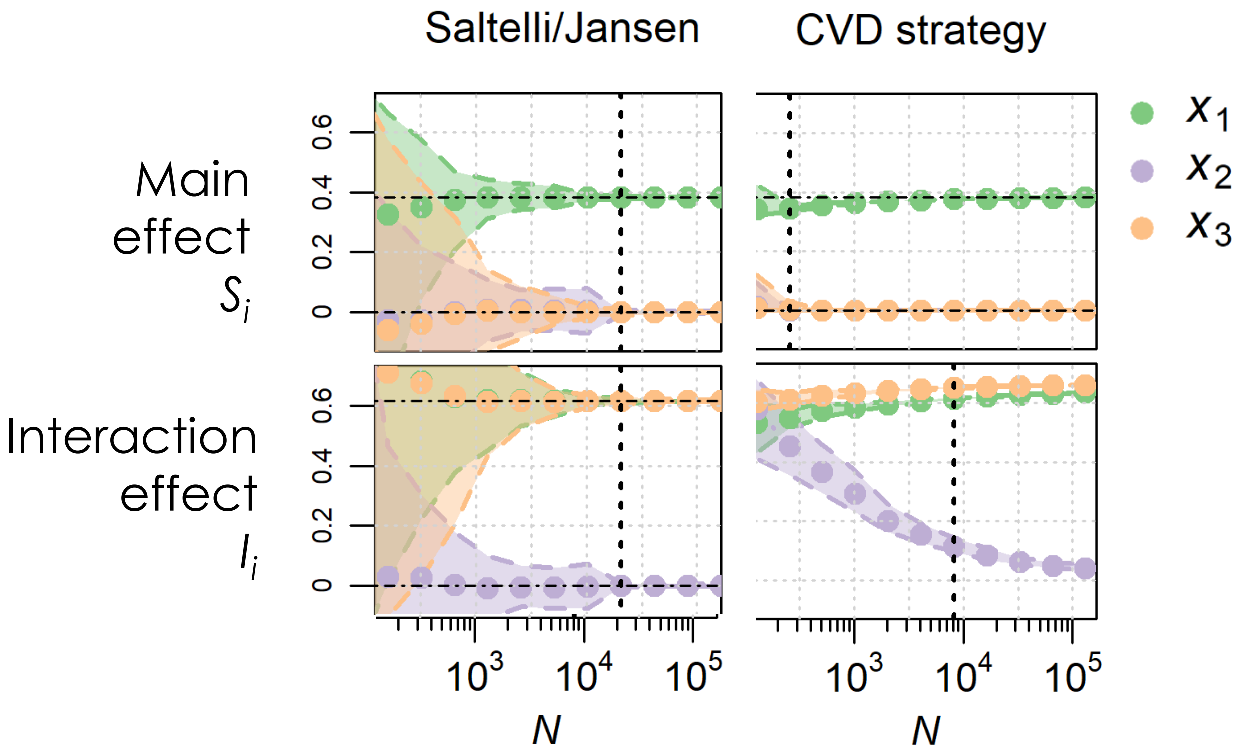
C) Tests to three analytic functions and one hydrological model

e.g., Ishigami-Homma function

$$y = \sin(x_1) + a \cdot \sin(x_2)^2 + b \cdot x_3^4 \cdot \sin(x_1)$$

with x_i in [-π, +π]
and a = 2 and b = 1

Number of simulations N ~ 10^5
Number of repetitions r = 100



CVD well estimates main and interaction with lower sample size

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C) Tests to three analytic functions and one hydrological model

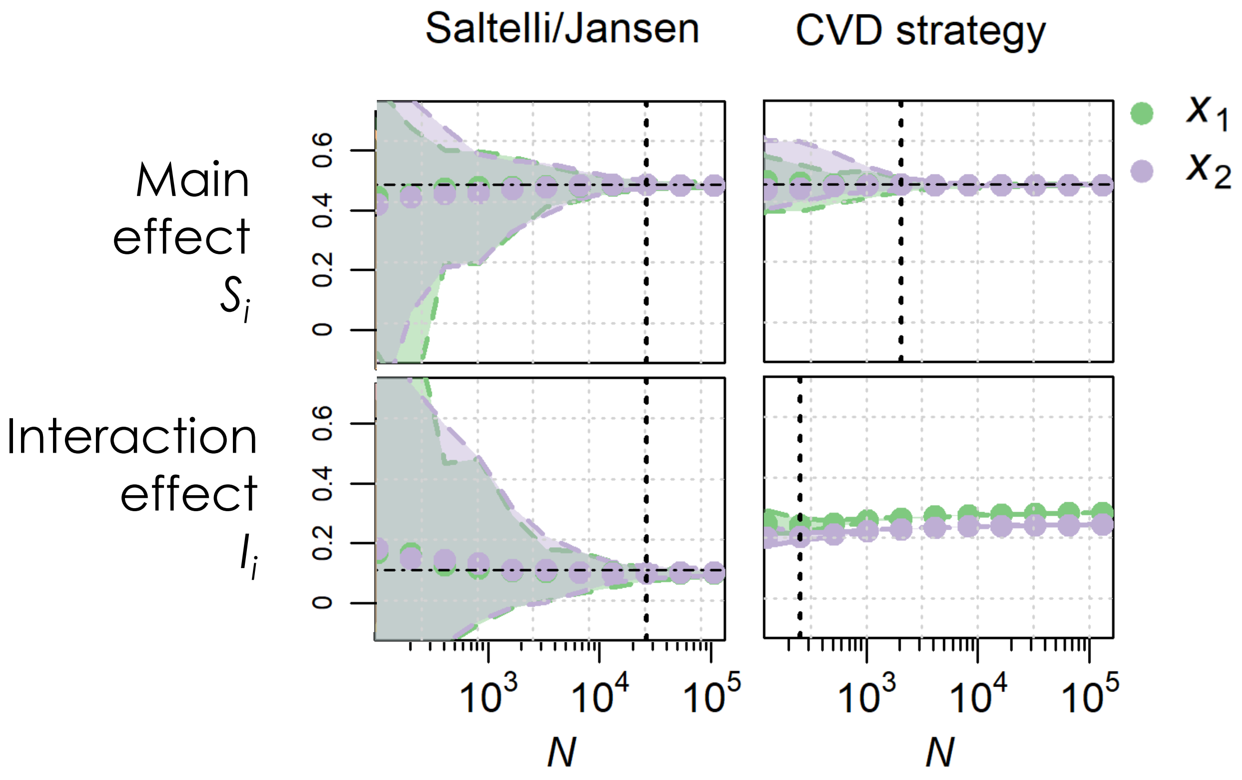
e.g., Skewed function

$$y = \frac{x_1}{x_2}$$

With x_1 and x_2 both follow χ2 distributions with degrees of freedom of 10 and 13.978, respectively

Number of simulations N ~ 10^5

Number of repetitions r = 100



Variance-based measures do not identify differences between the two parameters. The interaction terms *I* based on CVD identify differences between the two factors

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D) Conclusions and Outlook

Conclusions

- Main and interactions effects are estimated from a generic sampling design. CVD strategy can be easily integrated in any modelling framework
- The new approach converges faster than Saltelli/Jansen formula and combines the strength of variance and distribution-based approaches in exploring input-output space

Repository and document

- GitHub: https://github.com/baronig/GSA-cvd
- Manuscript under review: Baroni and Francke, An effective strategy for combining variance- and distribution-based global sensitivity analysis

Outlook

- Alternatives estimation of main and interaction term can be tested e.g., spline interpolation, δ-measure (Borgonovo et al., 2007)
- Testing on highly skewed modelling output

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Thank you for the attention

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