

Uncertainties in χ analysis

Implications for drainage network and
divide stability

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How big is big?

- Uncertainty in χ not quantified
- No way to tell whether a difference across the divide is significant!
- Uncertainties in drainage area control uncertainties in χ

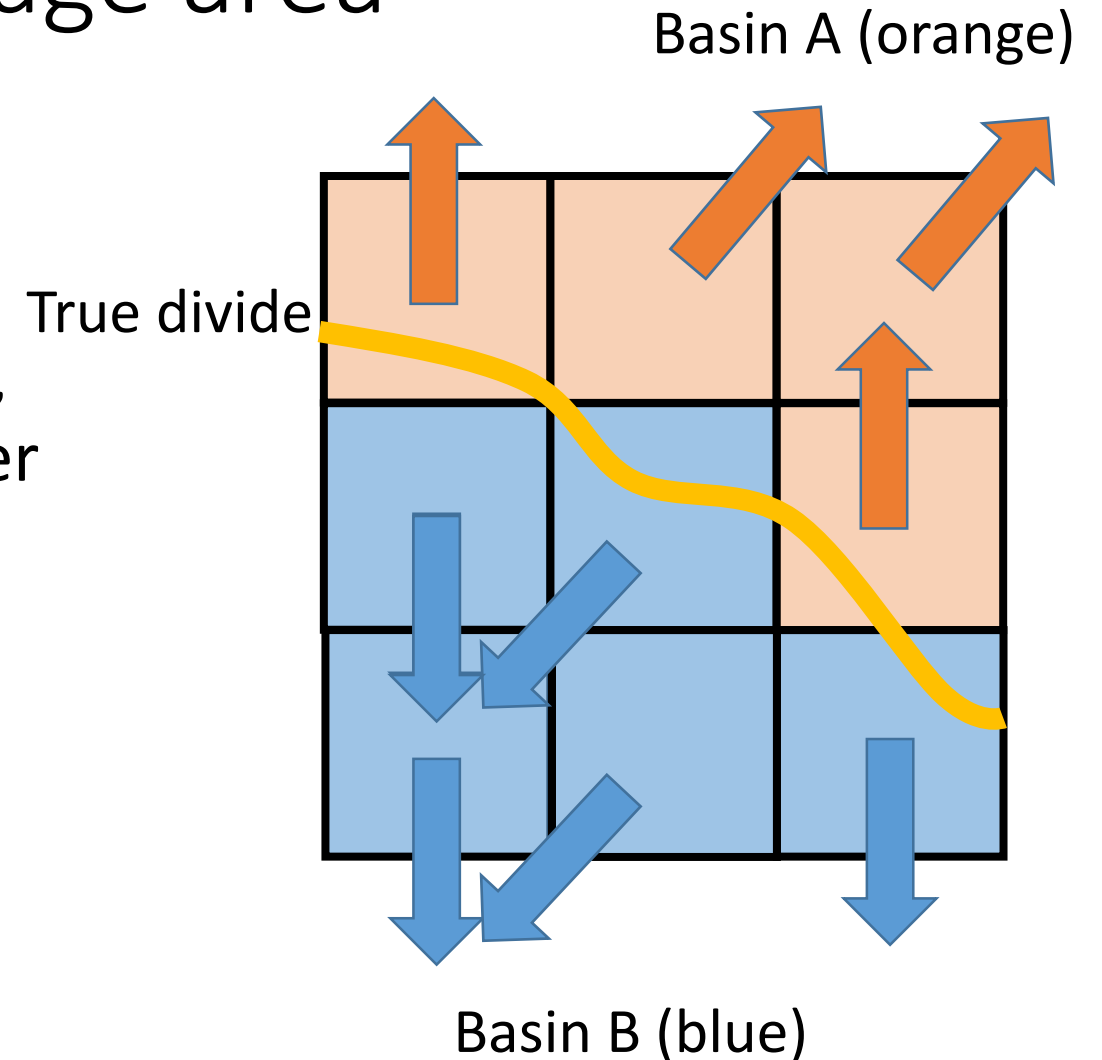
$$\chi(x) = \int_x^{x_b} \left(\frac{A_0}{A(\tilde{x})} \right)^\theta d\tilde{x}$$

Diagram illustrating the components of the equation:

- x_b : Channel length at baselevel
- A_0 : Reference area
- θ : Concavity index
- $A(\tilde{x})$: Drainage area
- x : Channel length

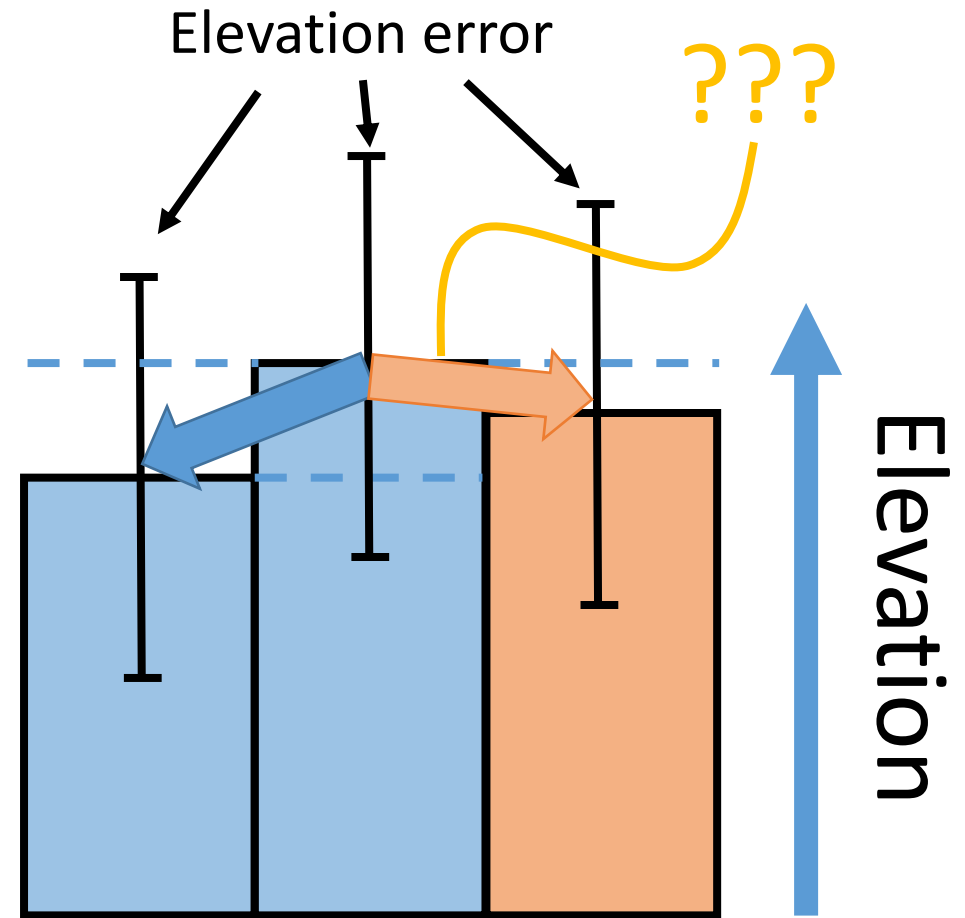
Sources of error of drainage area

- Divide follows a line, pixels are area elements
- Pixels are fully assigned to one basin, but contain area belonging to another basin



Sources of error of drainage area

- Pixels may be assigned to the wrong basin
- This can happen if the pixel elevation error exceeds the difference in elevation in the adjacent pixels at the divide

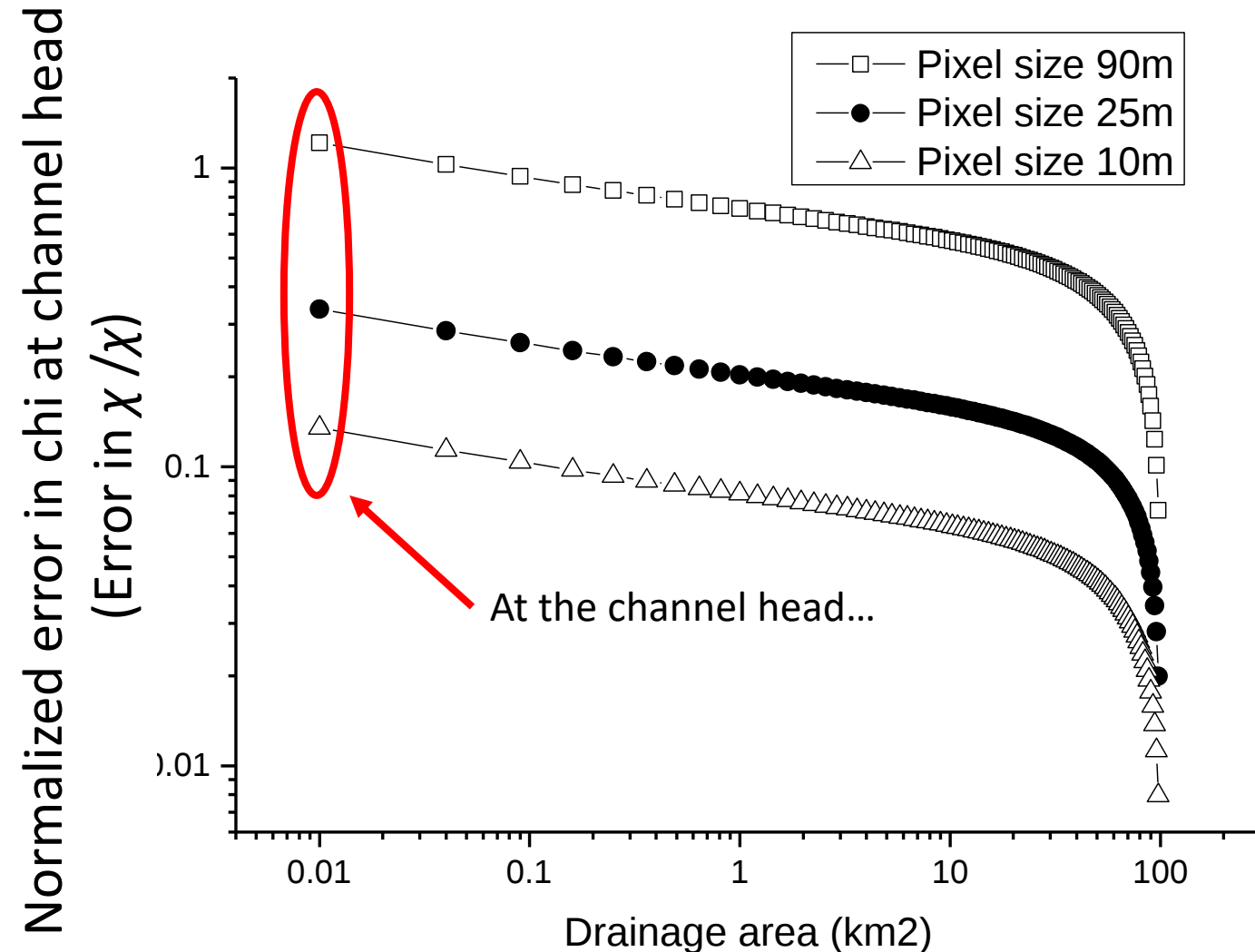


Analytical solution

- Gaussian error propagation
 - Uncorrelated errors
- Hack's law for drainage basin shape
- Simple quantification of error sources
- Arrive at analytical solution dependent on
 - Pixel size
 - Concavity index (set to 0.5)
 - Basin shape (Hack exponent, geometric shape factor)

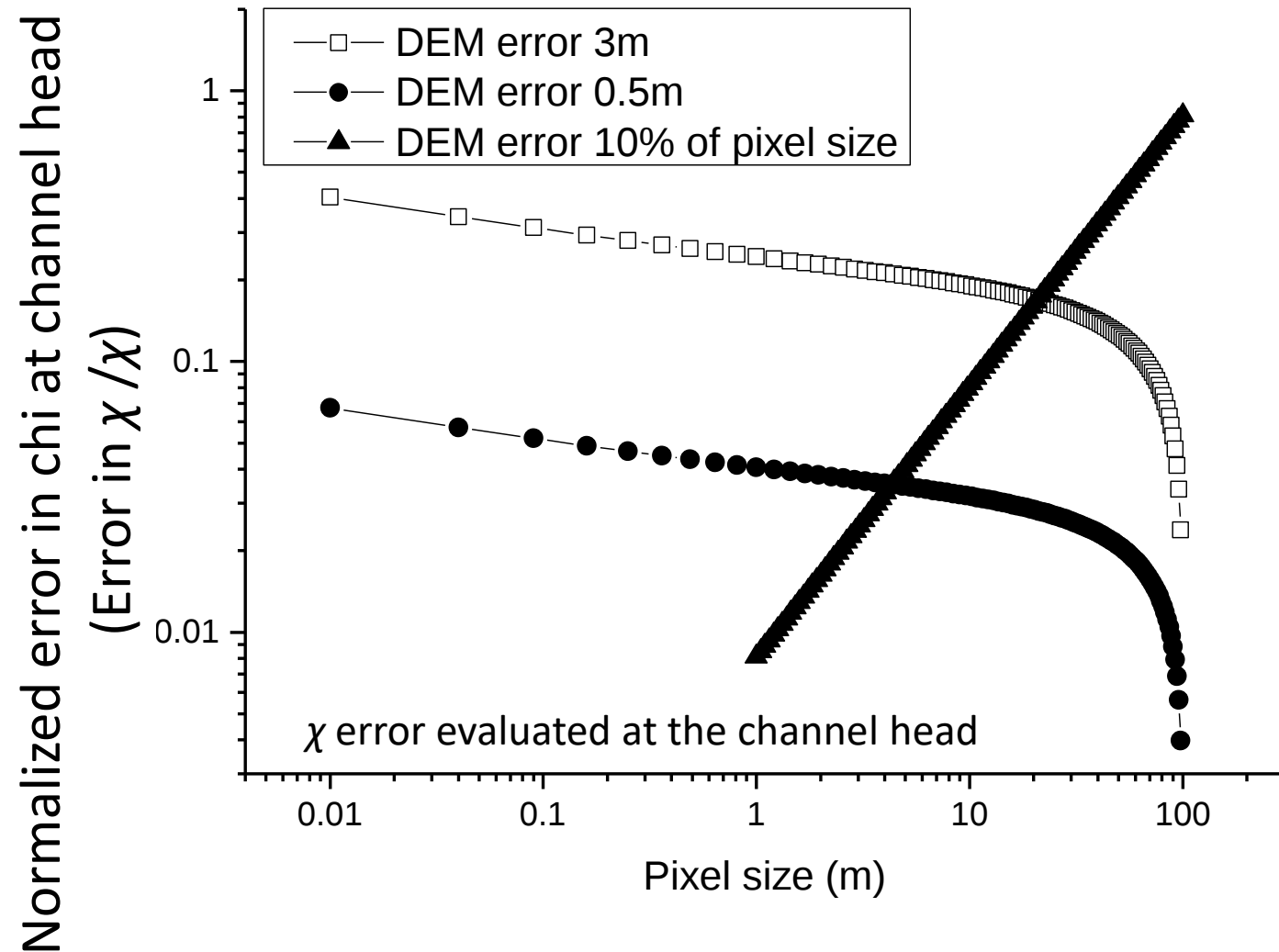
Dependence on drainage area

- Uncertainty largest close to the divide
- Depending on input parameters, 10% to >100% of the value of χ !



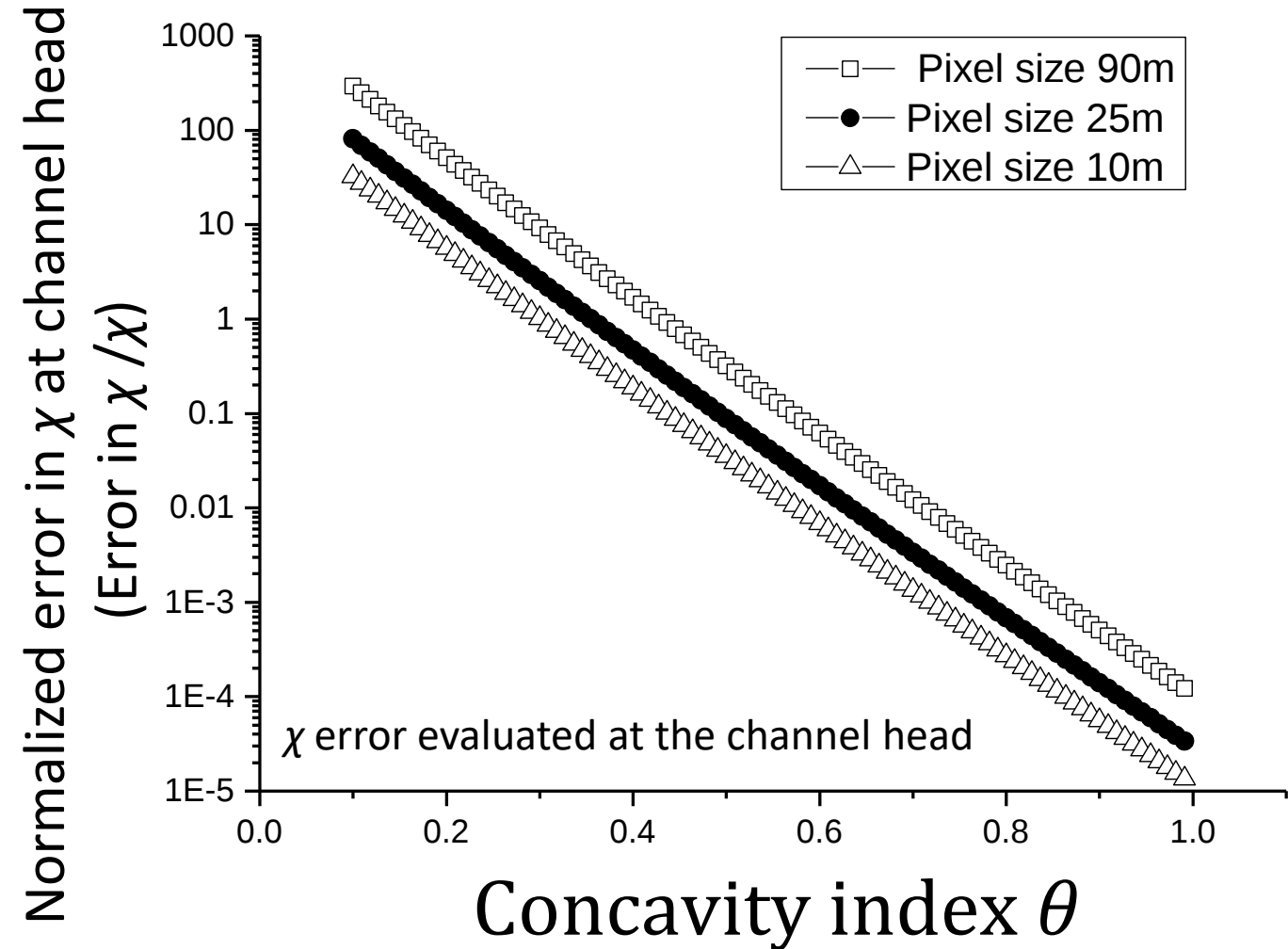
Dependence on pixel size

- For constant absolute elevation error, uncertainty decreases with pixel size
- For elevation errors that scale with pixel size (constant relative errors), positive quadratic dependence



Dependence on concavity

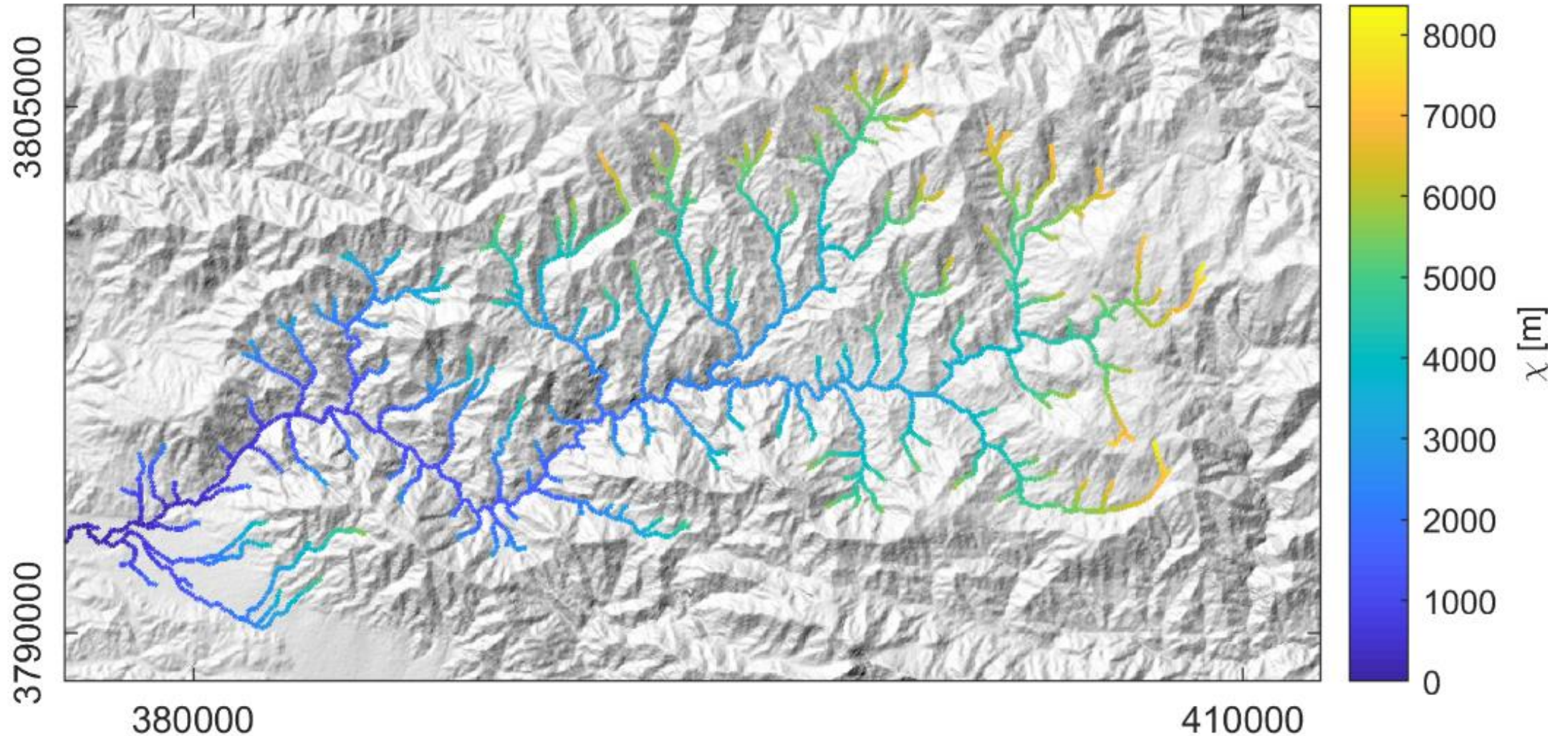
- Strong negative exponential dependence on concavity index



Alternative approach: Multiple flow directions

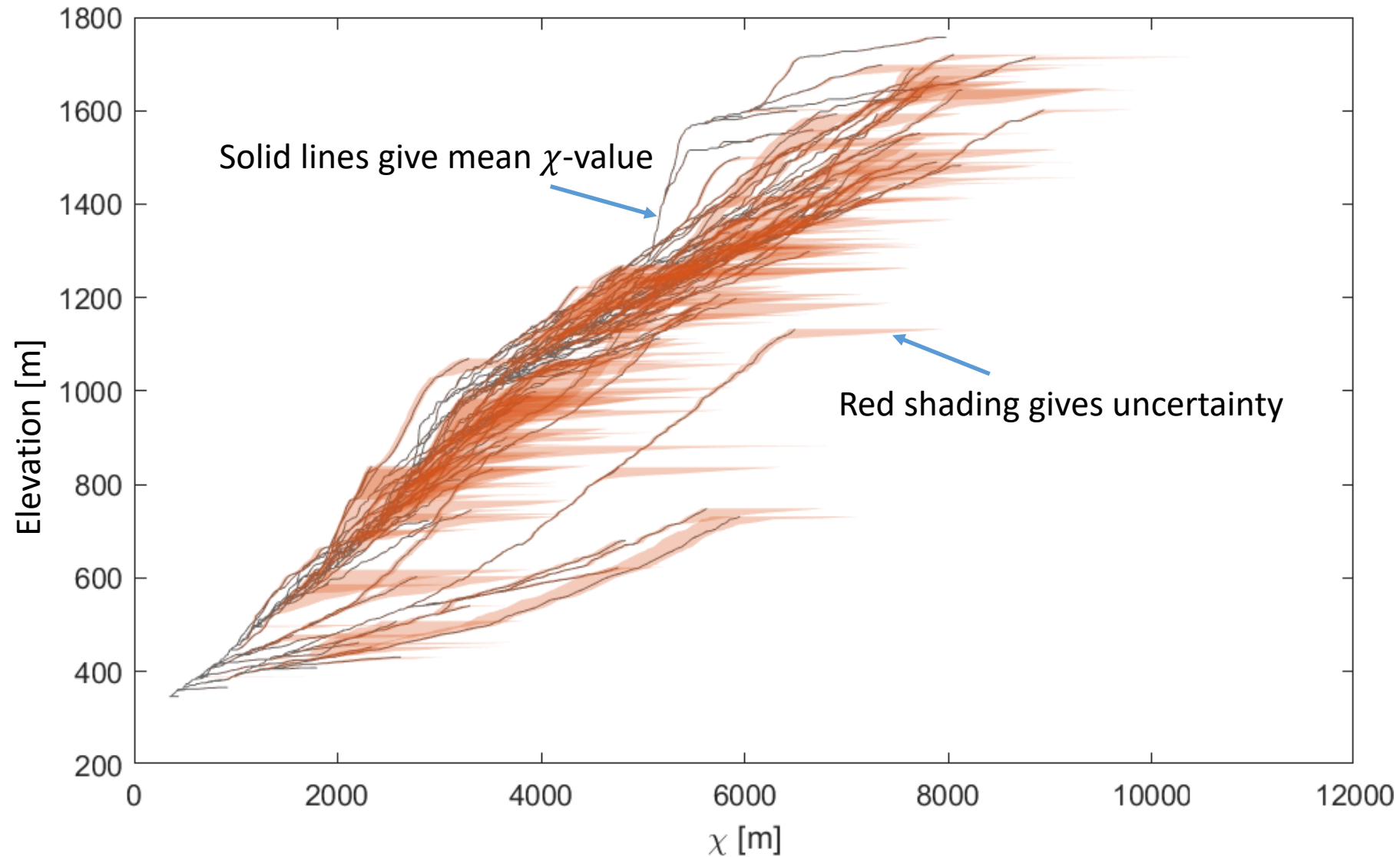
- Different method to obtain errors in drainage area
 - Relies on interpretation of fractions of multiple flow directions as probabilities
 - Multiple possible flow directions allow different flow paths
 - Probabilistic assignment of divide pixels (and adjacent pixels) to individual outlets
 - Monte-Carlo method to select a flow path realization
 - See Schwanghart & Heckmann, Environmental Modelling & Software, 2012
 - Drainage area and divide location varies with the specific flow path
 - Allows calculation of variance of drainage area and χ

Alternative approach: Multiple flow directions



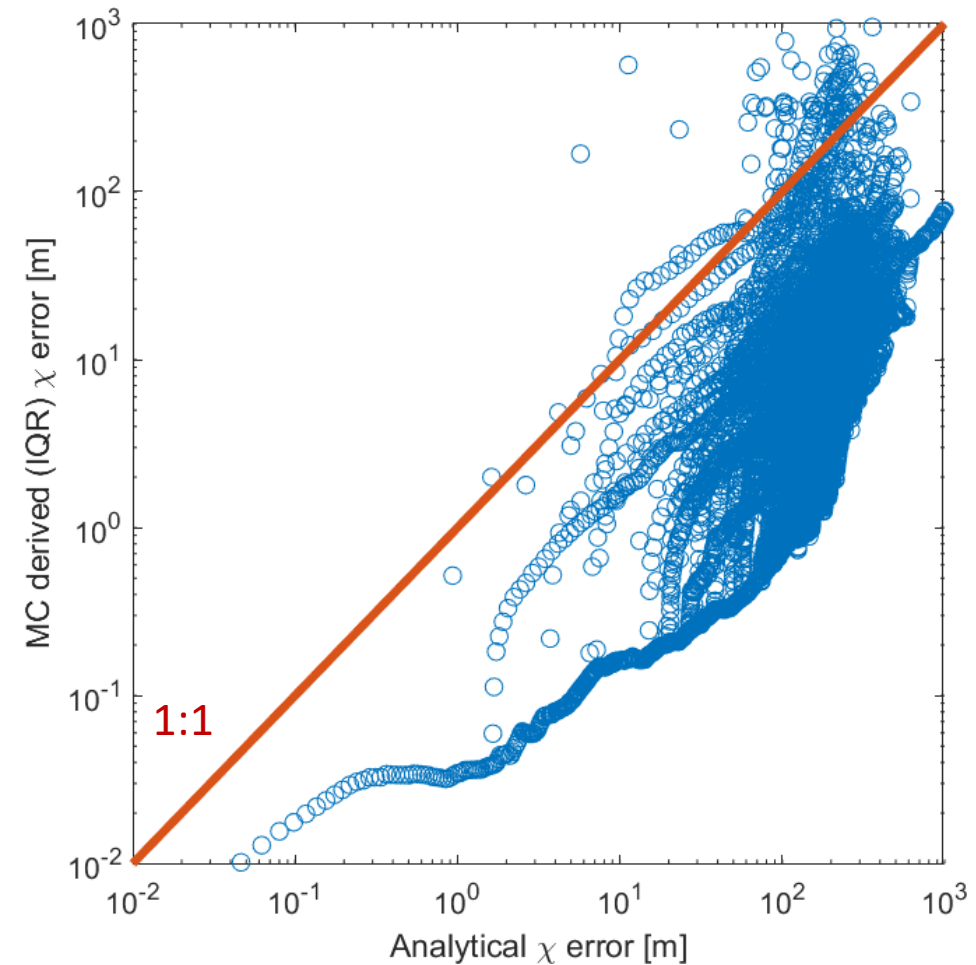
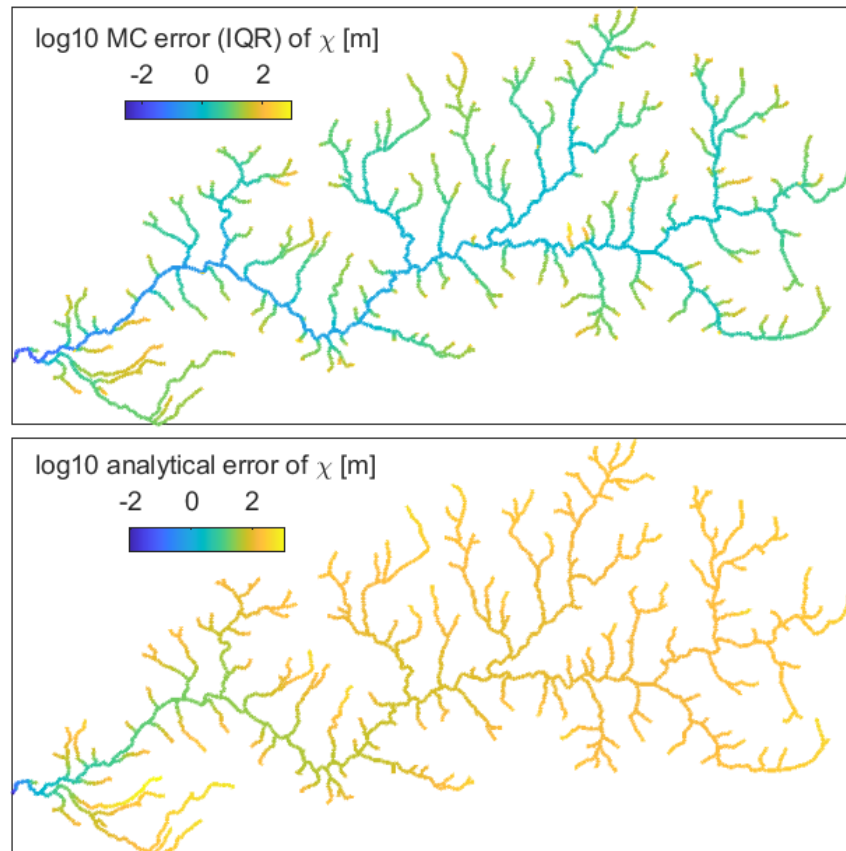
Example calculation for the Big Tujunga basin.

Alternative approach: Multiple flow directions



Comparison

- Consistency of analytical and MC derived errors



Covariance?

- Errors in χ depend on pixels at the divide
- Although drainage area is serially correlated, divide pixels are not, at least not to the same extent
- Example to the right
 - Blue basin shares $\sim 1/2$ of boundary with orange basin
 - Orange basin shares $\sim 1/3$ of boundary with blue basin
 - Yellow basin does not share any boundary pixels with the blue or orange basins

