Reducing the Memory Requirements of Parameter Estimation using Model Order Reduction

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Introduction

Previous development of a parameter estimation scheme for a Global Tide and Surge Model (GTSM) showed that accurate estimation of the parameters is currently limited by the memory use of the analysis step. Singular values decomposition (SVD) is a useful technique to reduce the high dimension system with a smaller linear subspace. In this study, we focus on the application of SVD in time patterns to reduce the dimension of model output and observations. As expected, the time patterns show a strong resemblance to the tidal constituents, which indicates that the memory requirements can be reduced dramatically by projection of the model output and observations onto the time-SVD patterns.

Parameter Estimation Scheme

A parameter estimation scheme is proposed and applied to GTSM to reduce parameter uncertainty. Dud ^[1], an iterative least-squares algorithm available in OpenDA, is applied with time-series derived from the FES2014 database to calibrate the bathymetry. However, previous experiments showed that with an increase of model simulation time, the required memory storage grows beyond the available resources because the estimation algorithm solver requires storage of the model output matching each observation for each parameter (or ensemble member).

Therefore, SVD^[2] is proposed to reduce the dimensions in model output and observations, as Figure 1 shows. In this parameter estimation procedure, model is first simulated with the initial parameters and once with each parameter perturbed. For each run the output is replaced by the smaller projected output.



Figure 1: Flowchart for Parameter estimation procedure

Accuracy of Projection and Reconstruction

SVD is generally applied for spatial patterns, but here we consider temporal patterns. Tests are performed to check the accuracy of reconstructed model output and observation. Figure 2 shows the RMSE for the projected and then reconstructed time-series for various numbers of modes. It illustrates that the error is small to approximate represent the matrix with a smaller size by truncated projection matrix.



Figure 2: Error [m] for reconstructed model output and observations with truncated projection U in different size

Table 1: Experiment set-up	for parameter	estimation
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.Simulation	•Output	•Time	 Number of	.Total output Size
time	locations	steps	parameters	
•1 - 14 Jan. 2014	. 1973	. 2016	. 110	•3.5Gb- >0.35Gb

Experiments and Results

Experiments are performed here to assess the accuracy of model order reduction. The set-up is shown in Table 1. A truncation size of 200 is selected to reconstruct model output and observations, which is about 10 times smaller than the original output. Results for two experiments are shown in Figure 3 to compare the estimation accuracy with and without SVD. Estimation with SVD shows a similar cost function in each iteration as without SVD. Small differences can be seen in the upper right panel for the final correction factor applied to the bathymetry. The RMSE for both experiments are decreased from 4.85 cm to 3.3 cm and the results are nearly the same, while still reducing the memory needed by a factor of 10 in this case.



Figure 3: Cost function (upper left); Final correction factor difference in bathymetry for estimation without and with SVD (upper right); RMSE difference between initial model and estimated model without (lower left) and with (lower right) SVD. Color blue shows the improvement.

Conclusion

We present a complete application of model order reduction in a parameter estimation scheme for the high-resolution GTSM to estimate bathymetry. We can conclude that:

- Reducing parameter uncertainties such as bathymetry can greatly improve the prediction accuracy for tide models.
- The memory requirements can be significantly reduced by model order reduction with time patterns.
- Bathymetry estimation with SVD is as accurate as without SVD.
- Further work will continue on bathymetry calibration for a longer simulation time, which has now become possible.

Reference

[1] Ralston, M. L., & Jennrich, R. I. (1978). Dud, A Derivative-Free Algorithm for Nonlinear Least Squares. Technometrics, 20 (1), 7-14.

[2] Y.C. Liang, H.P. Lee, S.P. Lim, W.Z. Lin, K.H. Lee, and C.G. Wu. Proper orthogonal decomposition and its applications - part i: Theory. Journal of Sound and Vibration, 252(3):527 544, 2002.

