Sensitivity Analysis and challenges posed by multiple approaches: a multifaced mess

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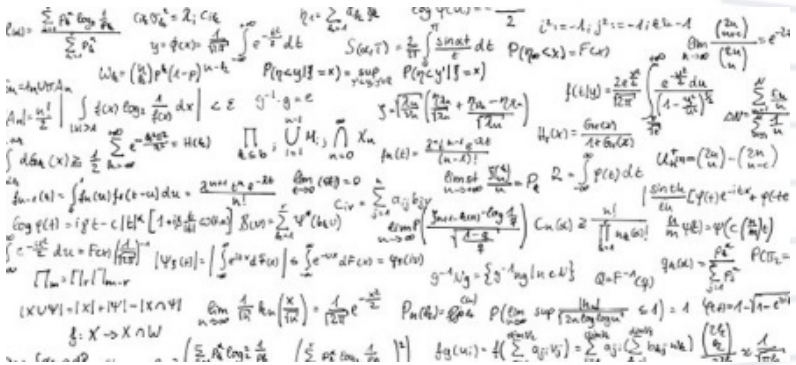
May 4th, 2020 – EGU2020 - Vienna

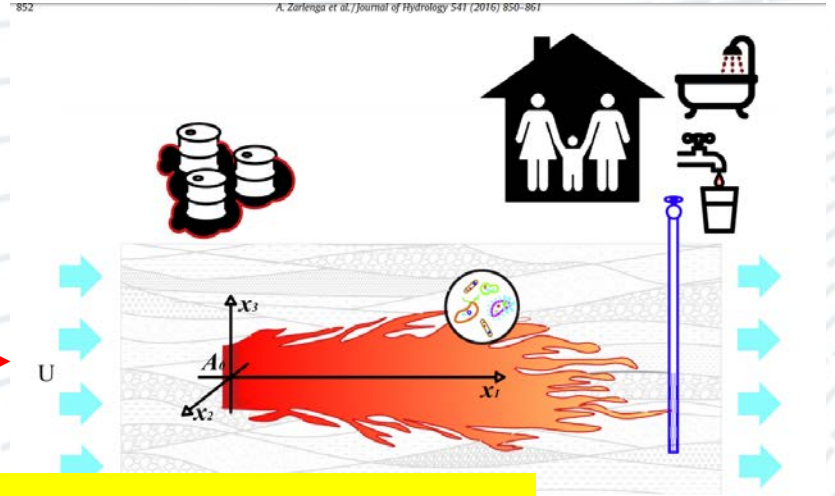
EGU General Assembly 2020
Sharing Geoscience Online

INPUTs

COMPLEX MODEL

OUTPUTs





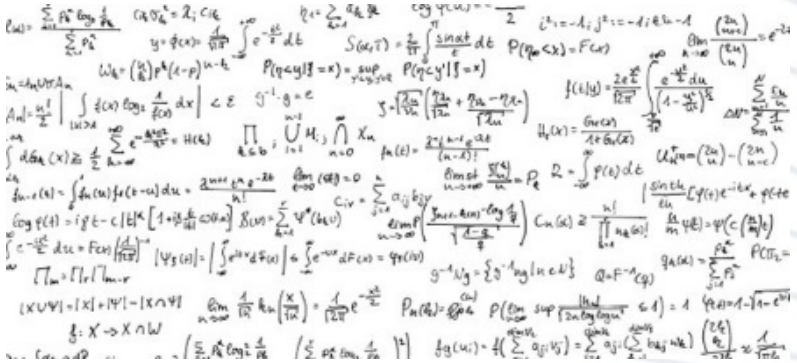
It becomes challenging to understand the relationships between INPUTs and OUTPUT(s)!

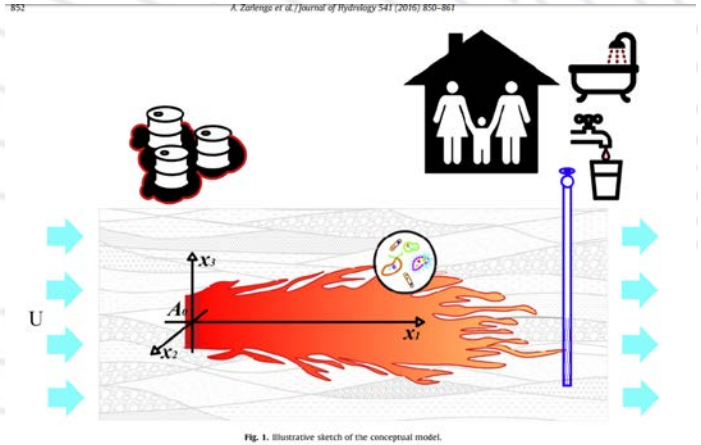
INPUTs → COMPLEX MODEL → OUTPUTs

It becomes challenging to understand the relationships between INPUTs and OUTPUT(s)!

KEY QUESTIONS

How does the ***model act***?
Which are the most ***relevant/influential*** INPUTs? Why?
Which INPUTs provide the most ***relevant contribution*** to OUTPUT(s) variability/uncertainty?
……..





Sensitivity Analysis

Sensitivity Analysis

'**Sensitivity**': despite being an **intuitive** concept, it is also a **general** concept!

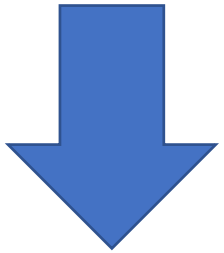
What is **your** definition of Sensitivity?

Sensitivity Analysis

Which questions do we want to address with a S.A.?

Which are our objectives/scopes?

A parameter could be relevant with respect to a *sensitivity metric*, but not for another one.



Definition of a '*Sensitivity Metric*'

❑ A key distinction is in the *nature* of the S.A., i.e., *Local* or *Global*

- ***Local*** S.A.: the sensitivity is measured around a single value of the model input (typically grounded on the derivative concept).

- ***Global*** S.A.: the sensitivity considers variations over a space/range of variability for the model input (this enables us to naturally account for input uncertainty).

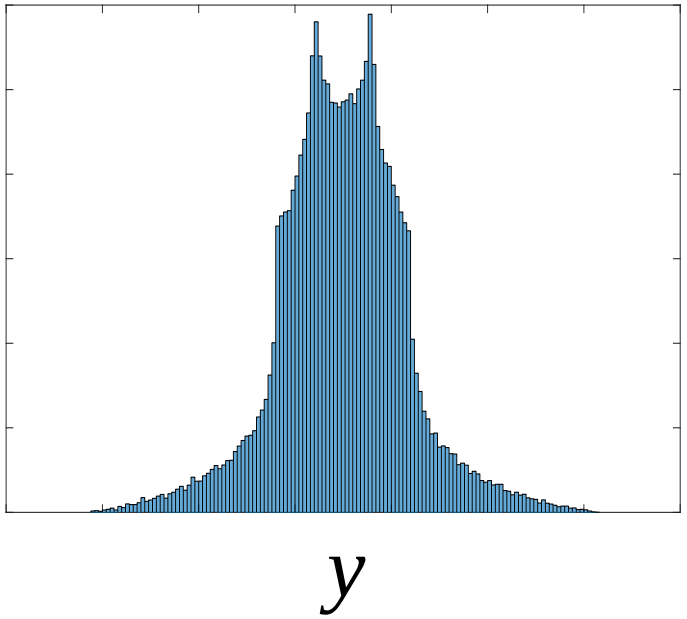
Another distinction is between *qualitative* and *quantitative* (here, a S.A. metric is usually involved)

Sobol' Indices

$$S_{x_i} = \frac{\mathrm{E}\left[V_y - V\left[y \mid x_i\right]\right]}{V_y}$$

- High value: most of the variance due to the variability in a parameter

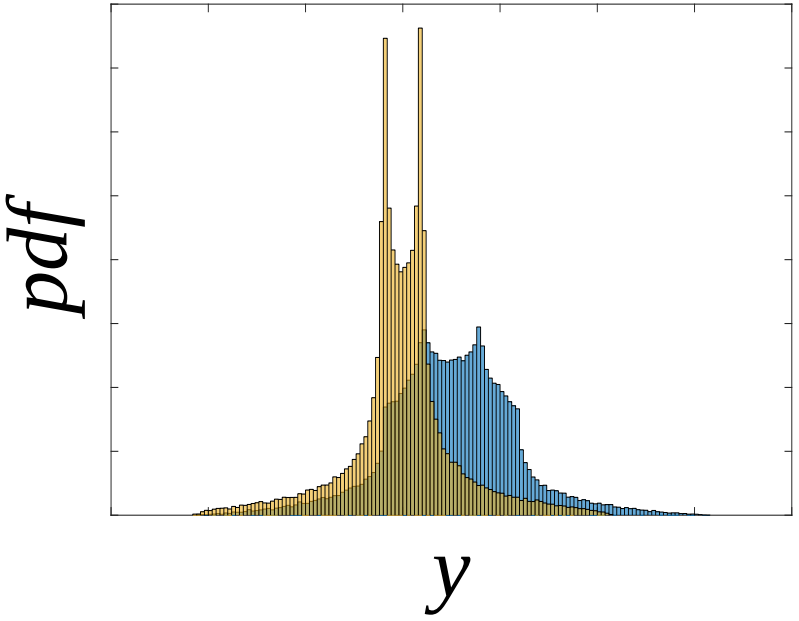
- If I know a parameter chances are that the variance-based output uncertainty be reduced



- Is the variance always a good proxy for uncertainty?

Sobol' Indices

$$S_{x_i} = \frac{\mathrm{E}\left[V_y - V\left[y \mid x_i\right]\right]}{V_y}$$



- Is the variance always a good proxy for uncertainty?
- How does the entire *pdf* change if I know an input?

Moment-Based Global Sensitivity Analysis

Expected **variation** of a given statistical moment M of f due to knowledge in p_i

Hydrol. Earth Syst. Sci., 21, 6219–6234, 2017
https://doi.org/10.5194/hess-21-6219-2017

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$$\mathrm{AMAM}_{p_i} = \begin{cases} \dfrac{1}{\left|M[f]\right|} \displaystyle\int_{\Gamma_{p_i}} \left|M[f] - M[f \mid p_i]\right| \rho_{\Gamma_{p_i}} dp_i & \text{if } M[f] \neq 0 \\ \displaystyle\int_{\Gamma_{p_i}} \left|M[f] - M[f \mid p_i]\right| \rho_{\Gamma_{p_i}} dp_i & \text{if } M[f] = 0 \end{cases}$$

Moment-based metrics for global sensitivity analysis of hydrological systems

Aronne Dell'Oca[1], Monica Riva[1,2], and Alberto Guadagnini[1,2]

$$\mathrm{AMAM}_{p_1,\ldots,p_s} = \begin{cases} \dfrac{1}{\left|M[f]\right|} \displaystyle\int_{\Gamma_{p_1,\ldots,p_s}} \left|M[f] - M[f \mid p_1,\ldots,p_s]\right| \rho_{\Gamma p_1,\ldots,p_s} dp_1 \ldots dp_s & \text{if } M[f] \neq 0 \\ \displaystyle\int_{\Gamma_{p_1,\ldots,p_s}} \left|M[f \mid p_1,\ldots,p_s]\right| \rho_{\Gamma p_1,\ldots,p_s} dp_1 \ldots dp_s & \text{if } M[f] = 0 \end{cases}$$

Moment-Based Global Sensitivity Analysis

$$\mathrm{AMA}E_{p_i} = \frac{1}{\left|E[f]\right|_{\Gamma_{p_i}}} \int \left|E[f] - E[f \mid p_i]\right| \rho_{\Gamma_{p_i}} dp_i$$

Expected **variation** of the Mean Value of f due to knowledge in p_i

which quantifies the average relative distance between the unconditional expected value of f and its counterpart conditional to the diverse values of p_i.

It answers the question 'Is a parameter influential on the expected value of the model output?'

Moment-Based Global Sensitivity Analysis

$$AMAE_{p_i} = \frac{\mathrm{E}[|\,\mathrm{E}_f - \mathrm{E}[\,f \mid p_i\,]\,|]}{|\mathrm{E}_f|}$$

Expected **variation** of the Mean Value of f due to knowledge in p_i

$$AMAV_{p_i} = \frac{\mathrm{E}\left[\left|V_f - V\left[\,f \mid p_i\,\right]\right|\right]}{V_f}$$

Expected **variation** of Variance of f due to knowledge in p_i

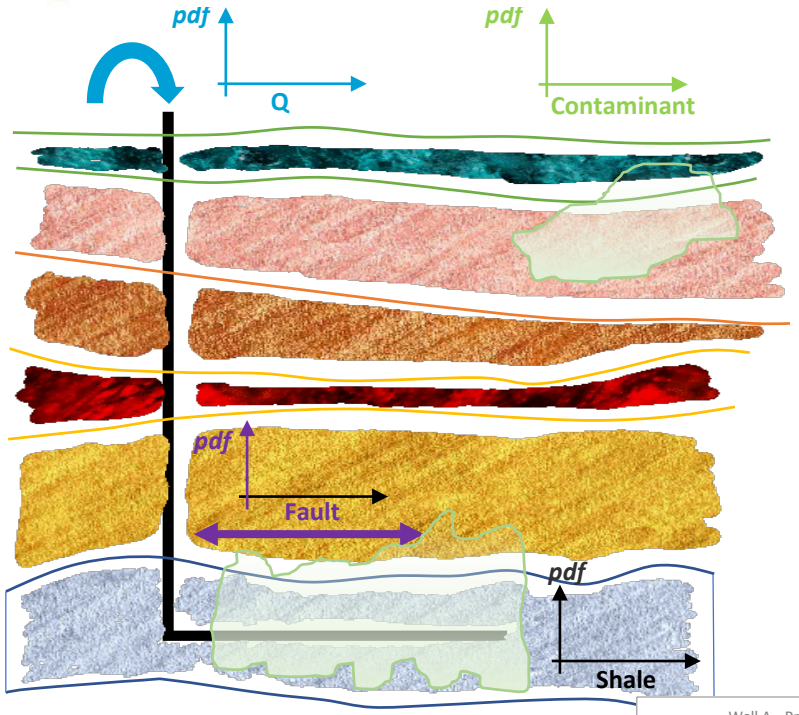
$$AMA\gamma_{p_i} = \frac{\mathrm{E}\left[\left|\gamma_f - \gamma[\,f \mid p_i\,]\right|\right]}{|\gamma_f|}$$

$$AMAk_{p_i} = \frac{\mathrm{E}\left[\left|k_f - k[\,f \mid p_i\,]\right|\right]}{|k_f|}$$

Expected **variation** of Skewness and Kurtosis of f due to knowledge in p_i

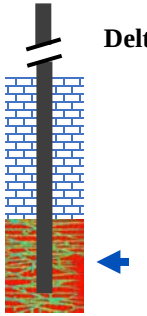
Some applications

Furthering the Knowledge Base for Reducing the Environmental Footprint of Shale Gas Development

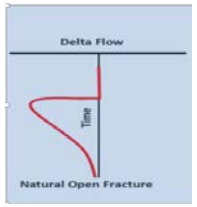


Stochastic inverse modeling and global sensitivity analysis to assist interpretation of drilling mud losses in fractured formations

A. Russian · M. Riva · E. R. Russo · M. A. Chiaramonte · A. Guadagnini



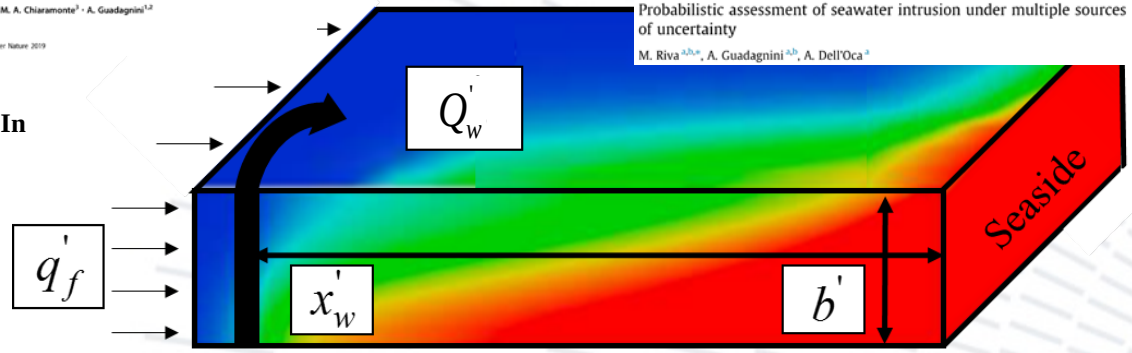
Delta Flow = Flow Out – Flow In



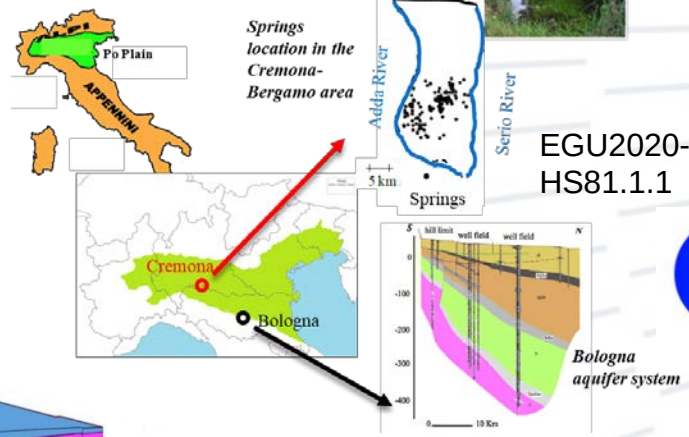
Advances in Water Resources

Probabilistic assessment of seawater intrusion under multiple sources of uncertainty

M. Riva, A. Guadagnini, A. Dell'Oca



Pumping Well



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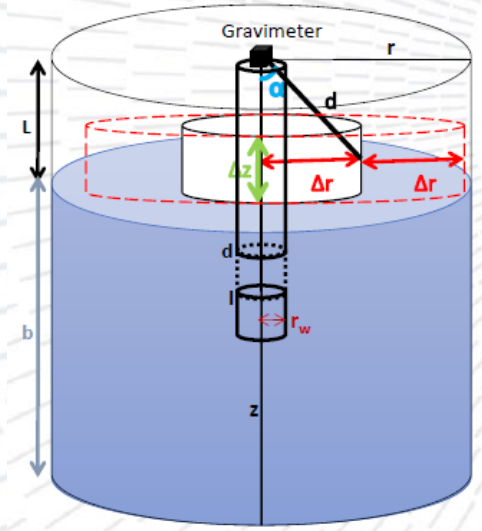
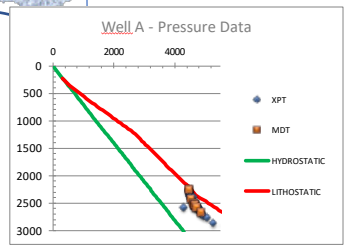
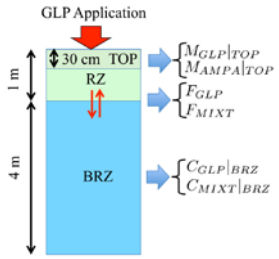


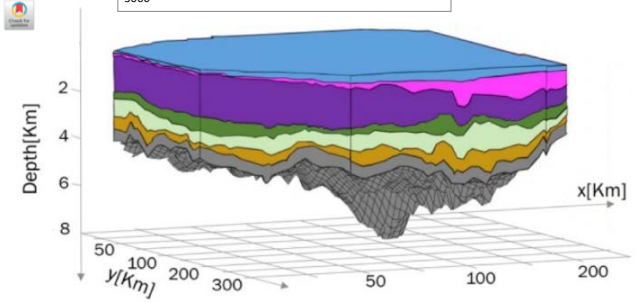
Ecological Indicators

Probabilistic indicators for soil and groundwater contamination risk assessment

Daniele la Cecilia, Giovanni M. Porta, Fiona H.M. Tang, Monica Riva, Federico Maggi







Basin scale compaction

Journal of Hydrology

Research papers

Global sensitivity analyses of multiple conceptual models with uncertain parameters driving groundwater flow in a regional-scale sedimentary aquifer

Emanuela Bianchi Janetti, Laura Guadagnini, Monica Riva, Alberto Guadagnini

AGU PUBLICATIONS

Water Resources Research

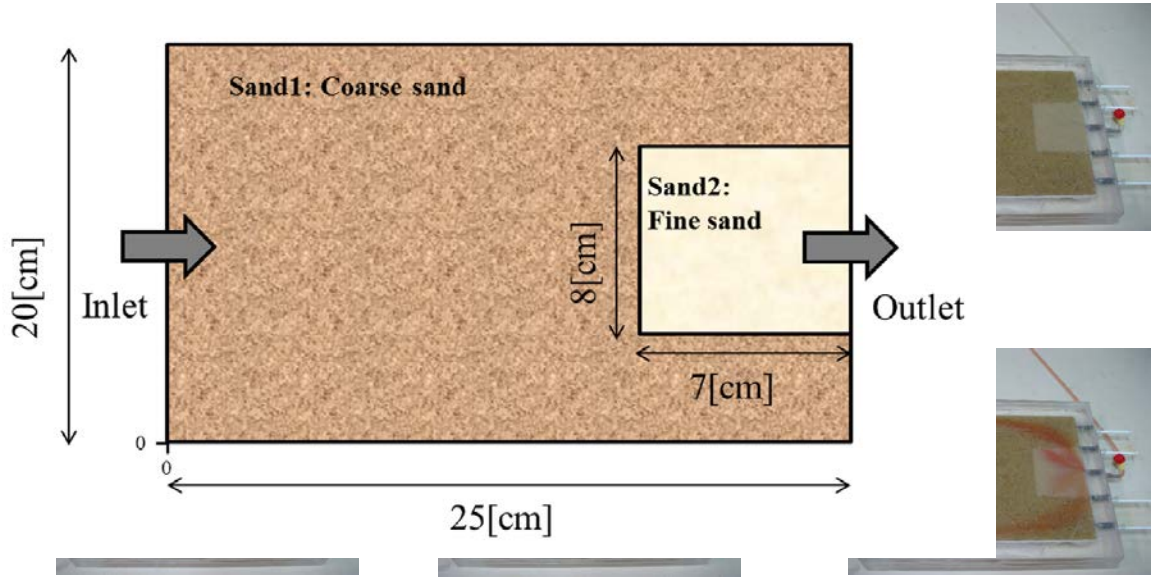
RESEARCH ARTICLE

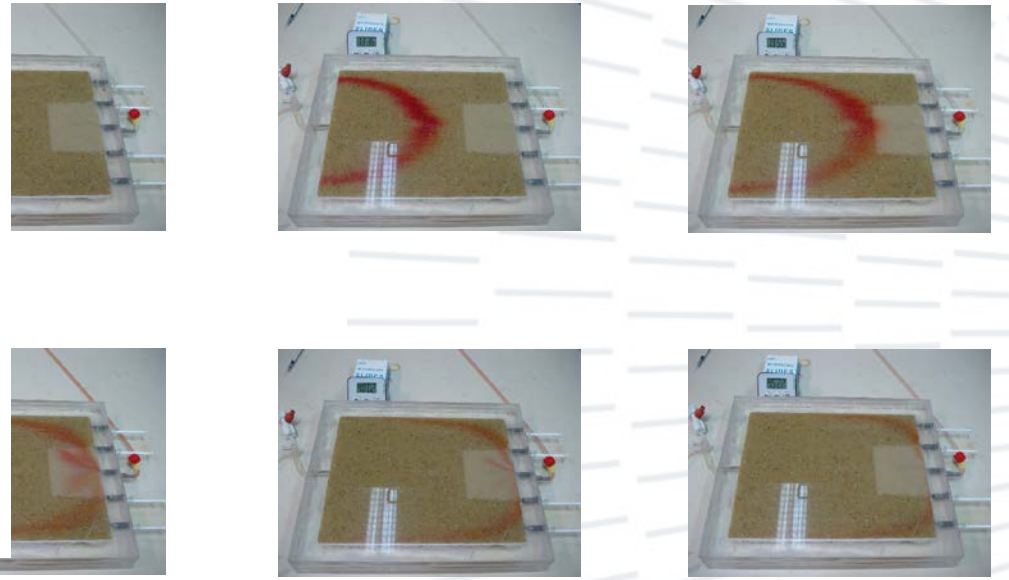
Uncertainty Quantification and Global Sensitivity Analysis of Subsurface Flow Parameters to Gravimetric Variations during Pumping Tests in Unconfined Aquifers

Fadji Zaouna Maina and Alberto Guadagnini

Application Example

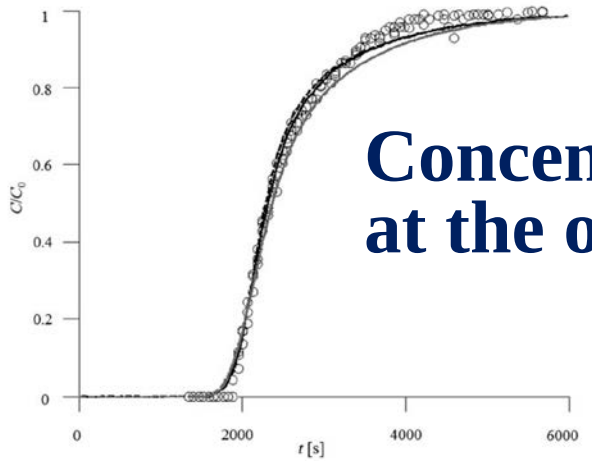
➢ Solute Transport at the laboratory scale



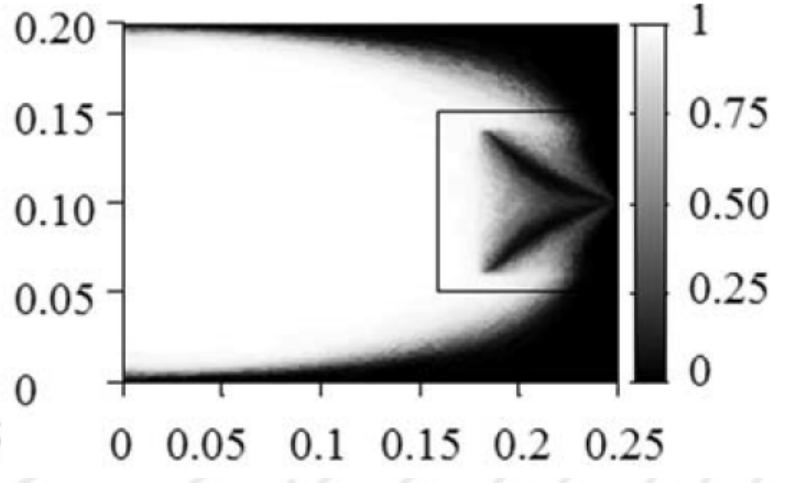


Data-set: provided to us courtesy of Prof. Brian Berkowitz, Weizmann Institute of Science.

$\overline{C}(t)$

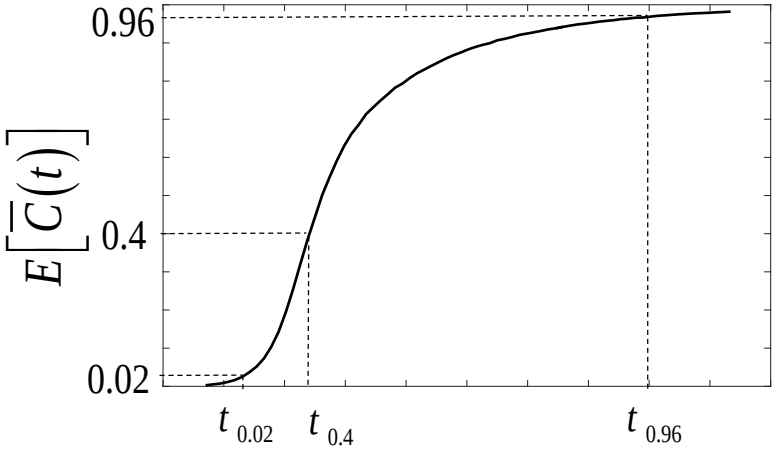


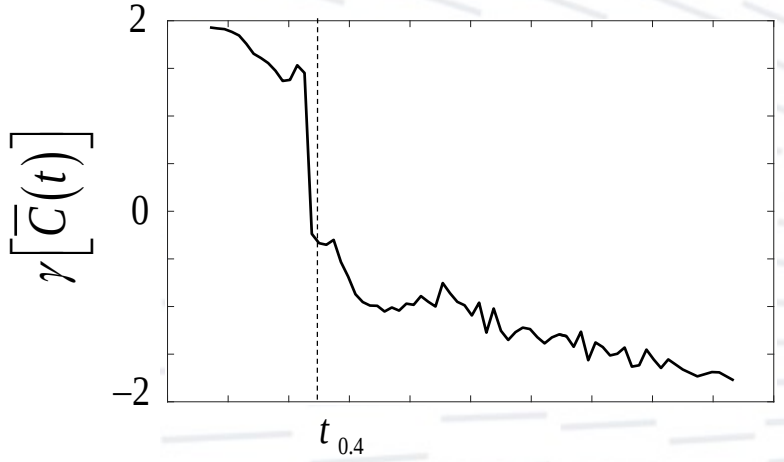
Concentration at the outlet



Application

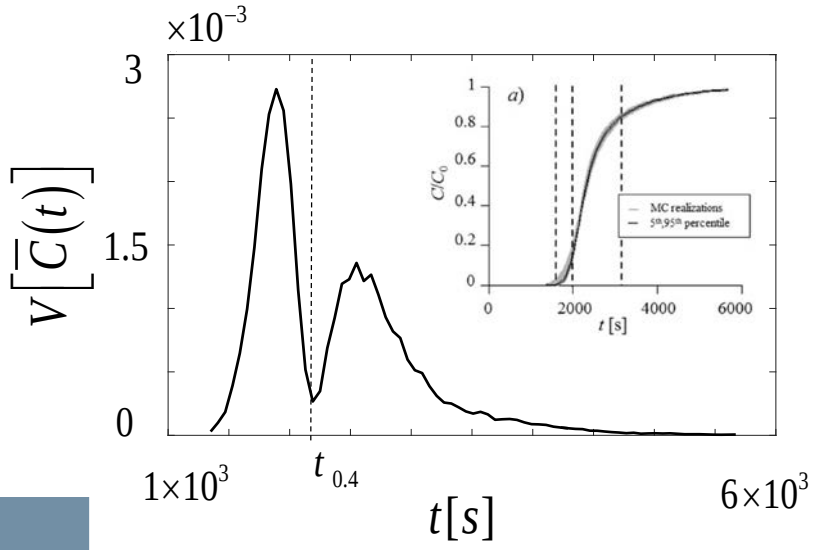
✓ First Four Unconditional Statistical Moments

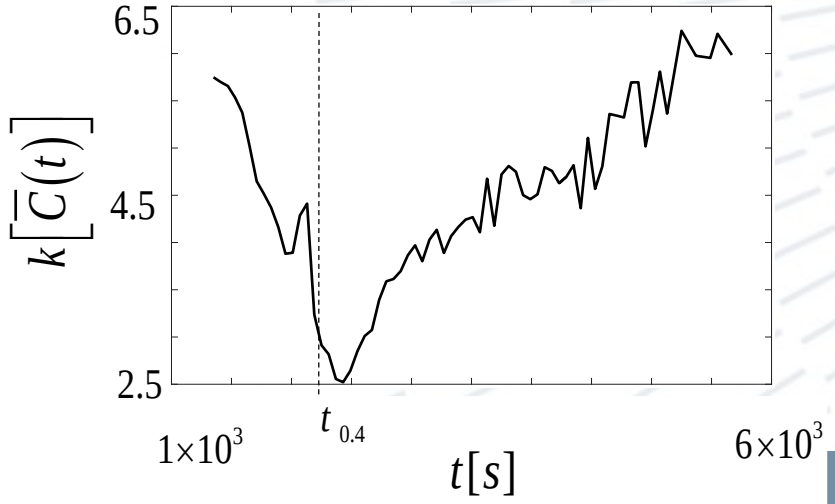




$\log_{10}(a_{L,i}) \in [-6,-2]$

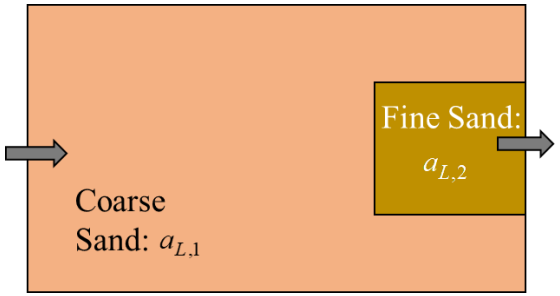
Dispersivities of the two media

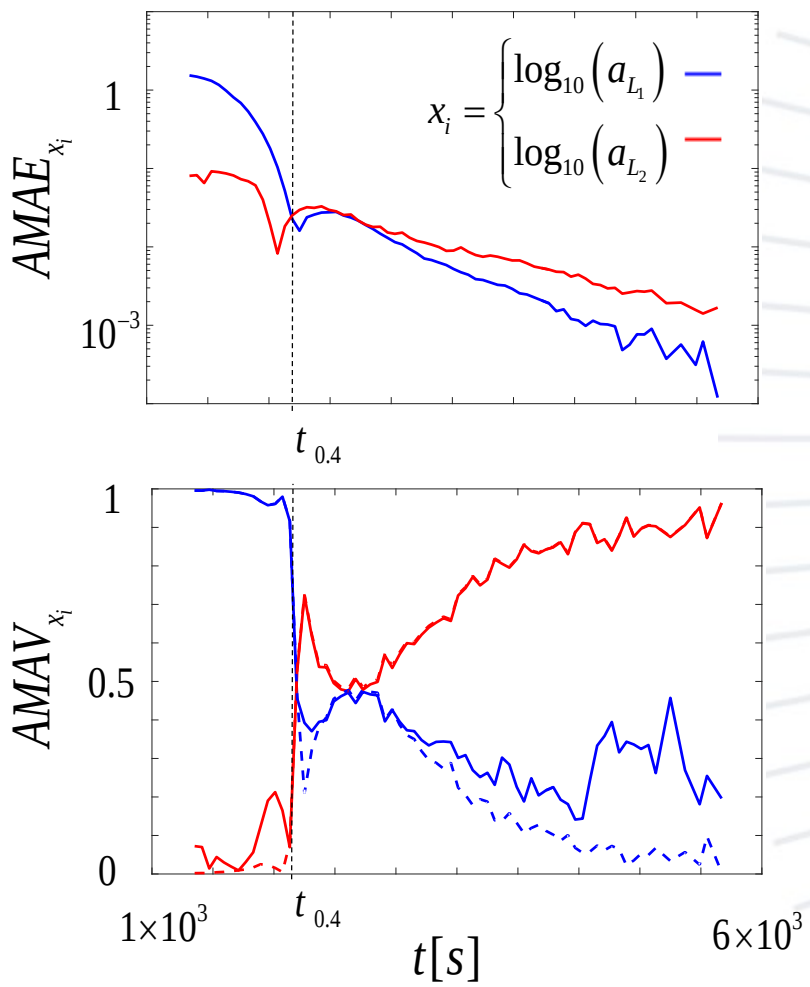


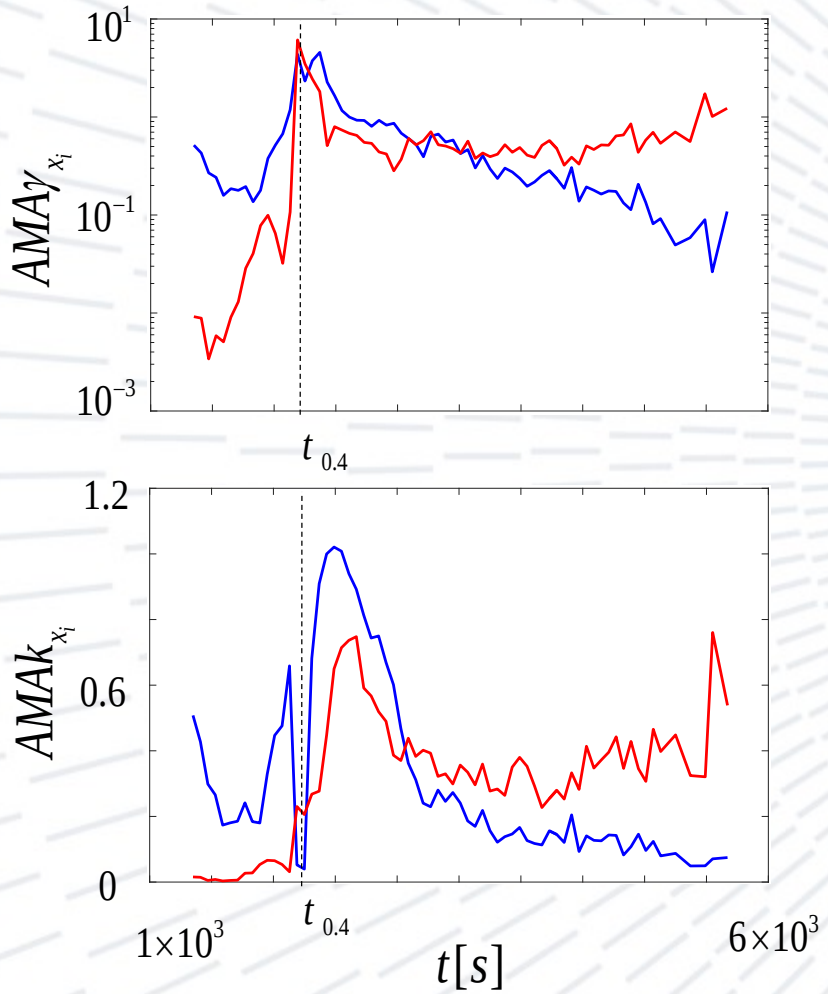


Application

- AMA indices



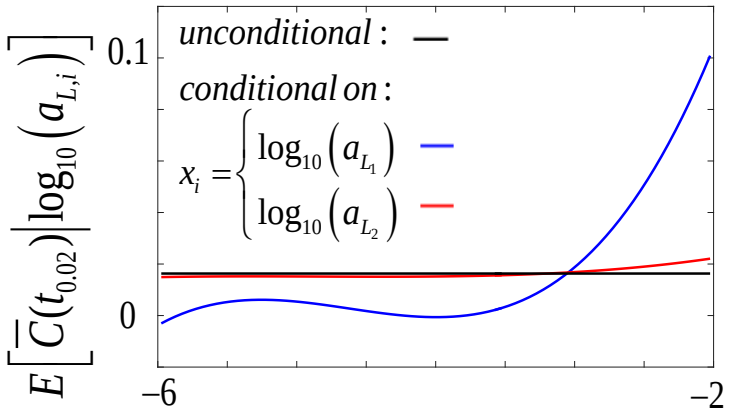


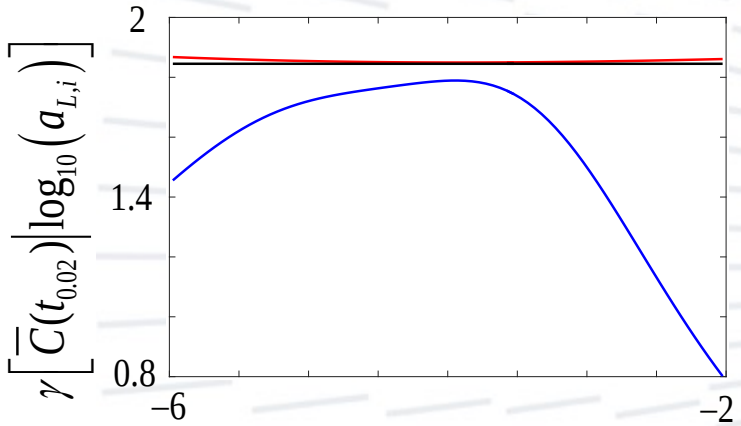


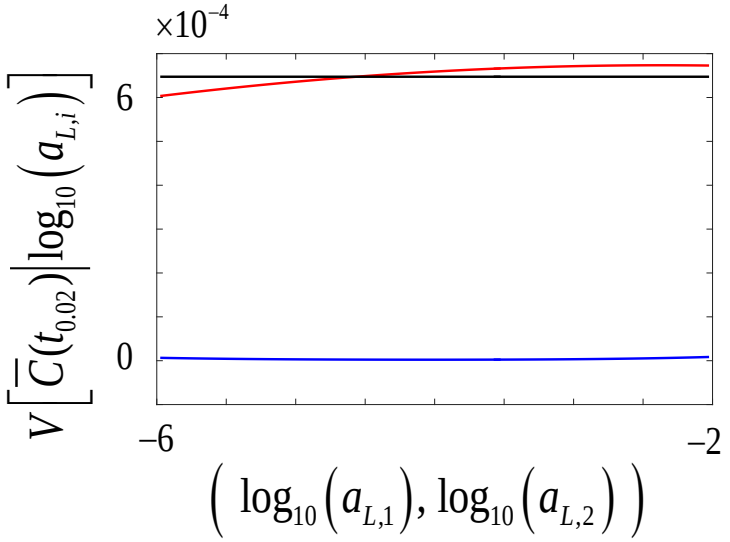
Application

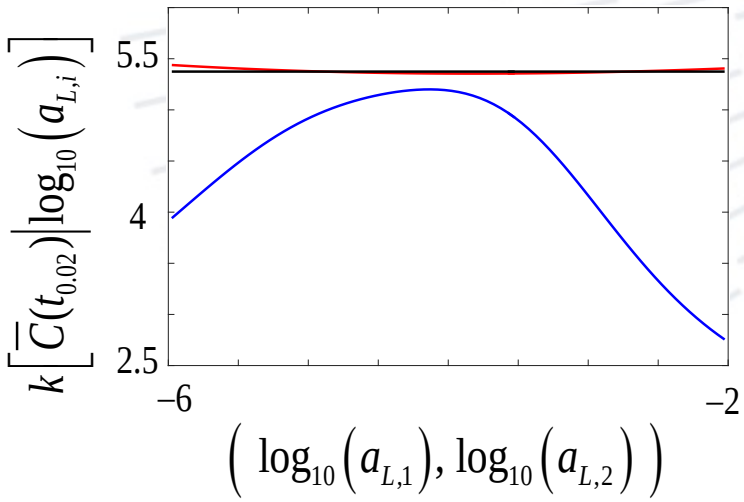
- Early times, $t_{0,02}$:

$\alpha_{L,1}$ is the main source of variability ➲ Calibration, UQ, Risk



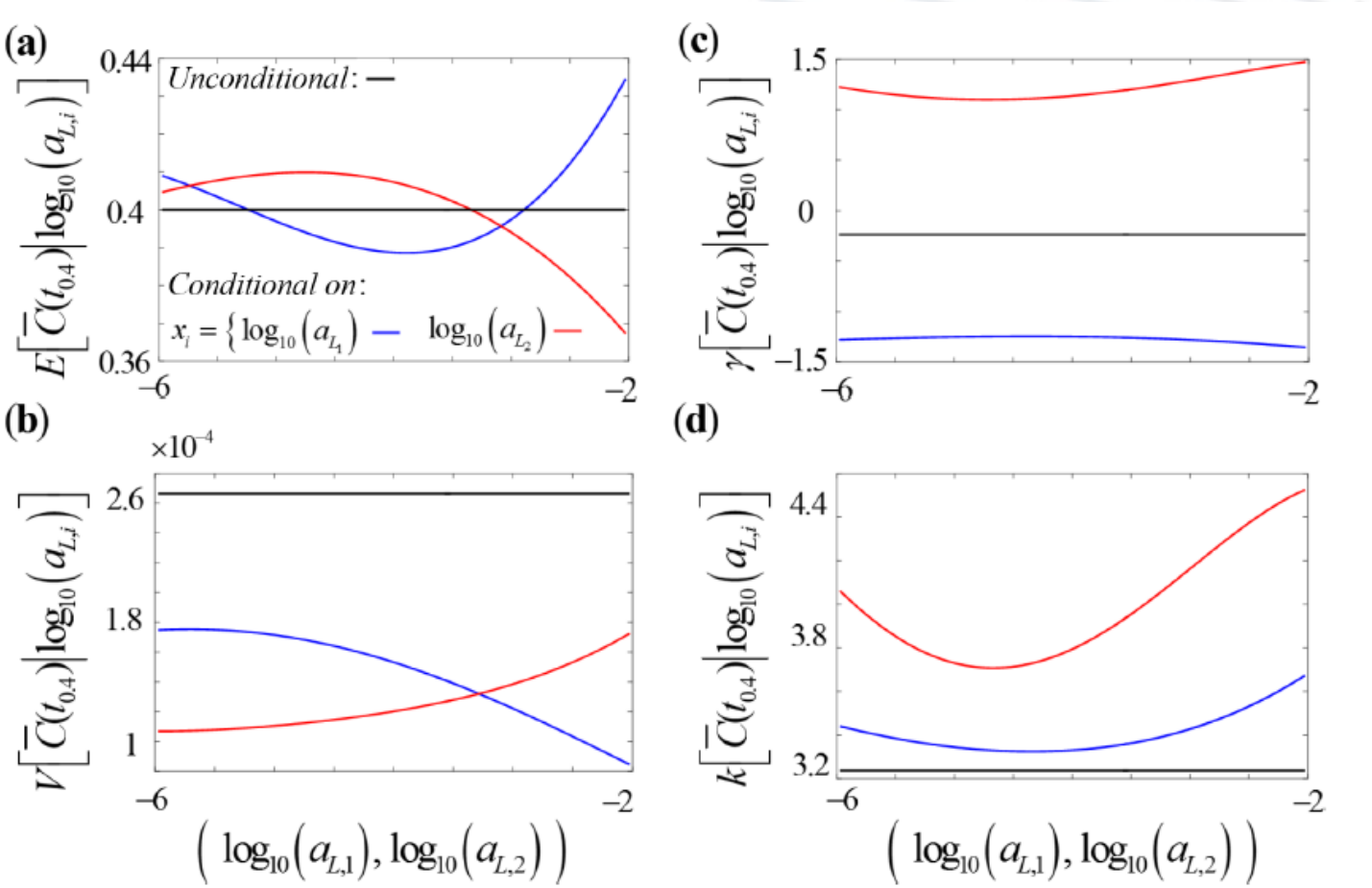




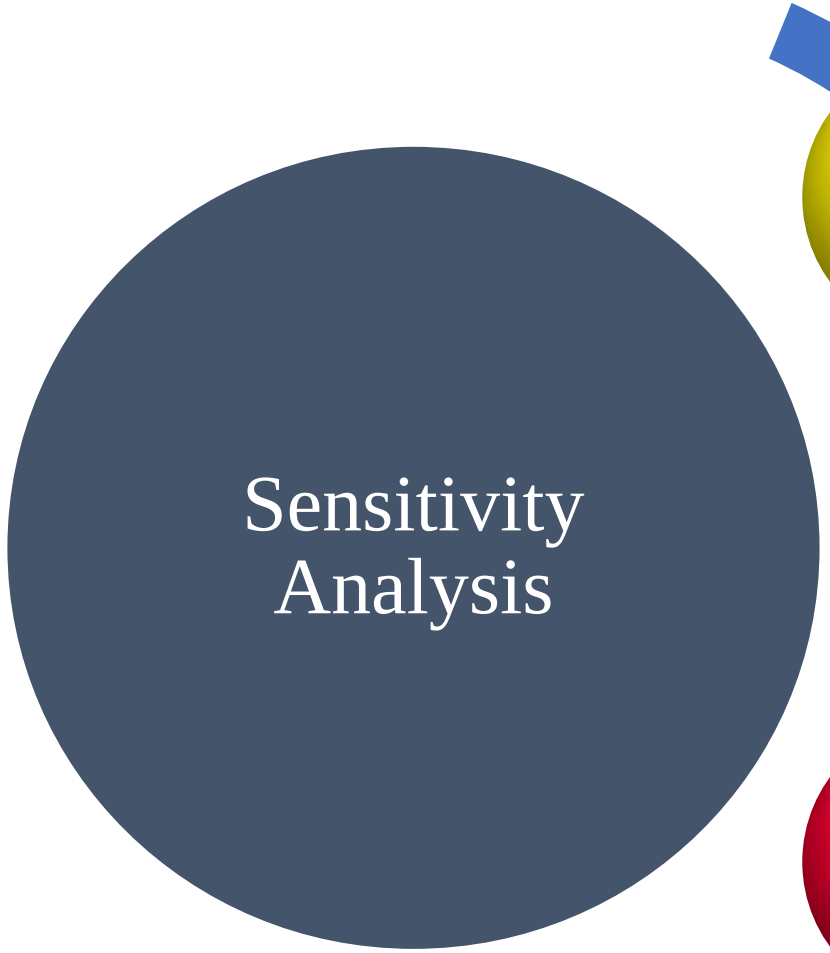


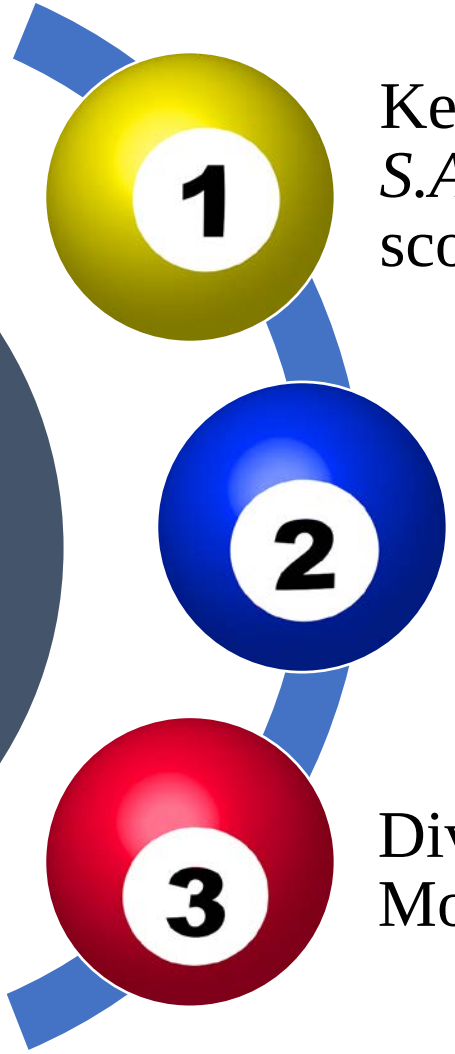
Application

- Intermediate time, $t_{0,4}$:



Observations





1. Keep in mind the scope! It defines the *S.A. metric*. There could be multiple scopes at the same time.

2. Global Sensitivity Indices based on the Statistical Moments of model output - Easy to interpret
   Uncertainty is not just the variance

3. Diverse metrics can be key for, e.g., Inverse Modeling, Uncertainty Quantification

Thank you

Water JPI
WaterWorks 2014

WE-NEED
WatEr NEEDs, Availability, Quality and Sustainability

Horizon 2020
Research and Innovation
programme

FRAC RISK
Furthering the Knowledge Base for Reducing the Environmental Footprint of Shale Gas Development

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