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GEOL Geothermal Energy & Geofluids

ETH zürich

Fictitious Domain Methods for Simulating Thermo-Hydro-Mechanical Processes in Fractures

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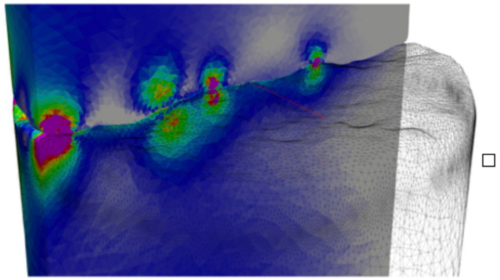
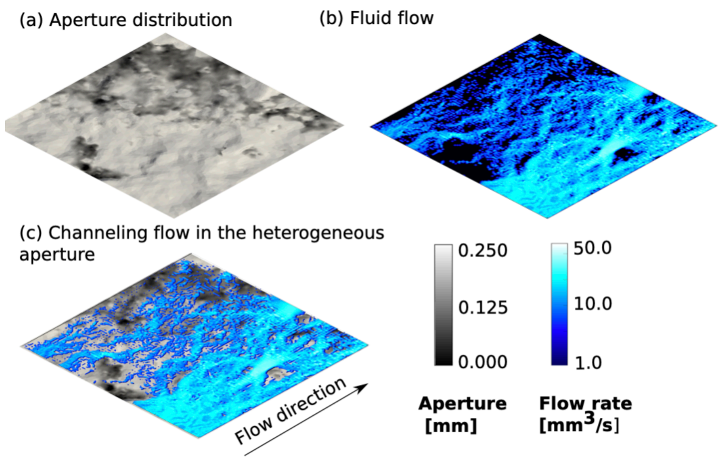
We are interested in coupled processes in rough rock geometries, which deal with *complex geometries. They* are *highly non-linear* and therefore *computationally expensive*.

Conceptually our methods are based on the *fictitious domain* method and *L^2-projections*.

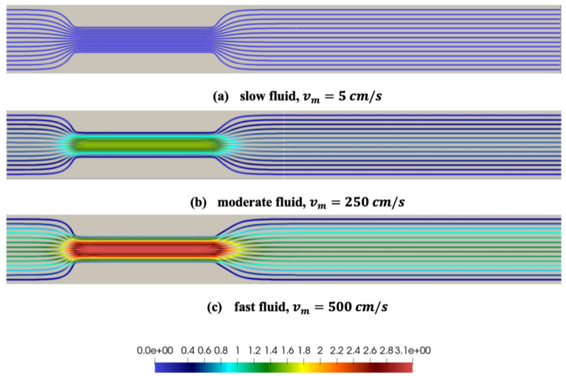
Contact problems,
Dual mortar method



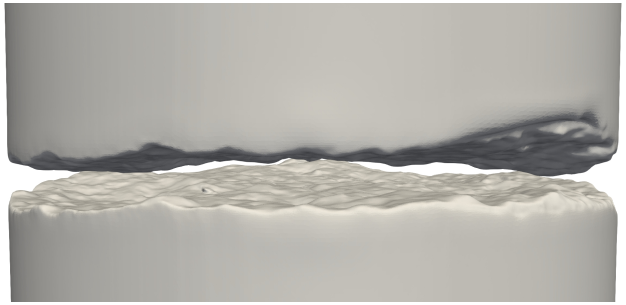
Fluid structure interaction (FSI)



Thermo-fluid-structure interaction (TFSI)

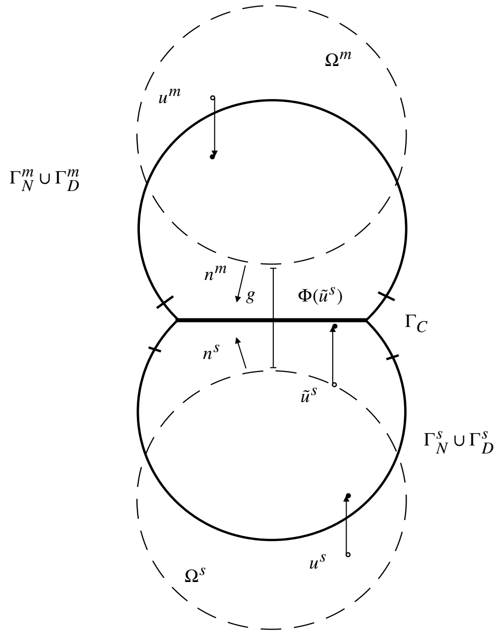


Contact problem



Jump:

$$[u] := u^s \cdot n^s + u^m \cdot n^m$$



Linear elasticity $(\alpha \in \{m, s\})$:

$$\sigma_{ij}(u^\alpha) = C^\alpha_{ijml} u^\alpha_{l,m} \text{ on } \Omega^\alpha$$

$$-\mathrm{div}\, \sigma(u) = f \text{ on } \Omega^s \cup \Omega^m$$

$$u_i = 0 \quad \text{on } \Gamma^\alpha_D, \alpha \in \{m, s\}$$

$$\sigma_{ij}(u) \cdot n_j = p_i \quad \text{on } \Gamma^\alpha_N, \alpha \in \{m, s\}$$

$$\sigma_n \leq 0 \quad \text{on } \Gamma_C$$

Contact conditions:

$$\sigma_n(u^m \circ \Phi) = \sigma_n(u^s)$$

$$[u] \leq g$$

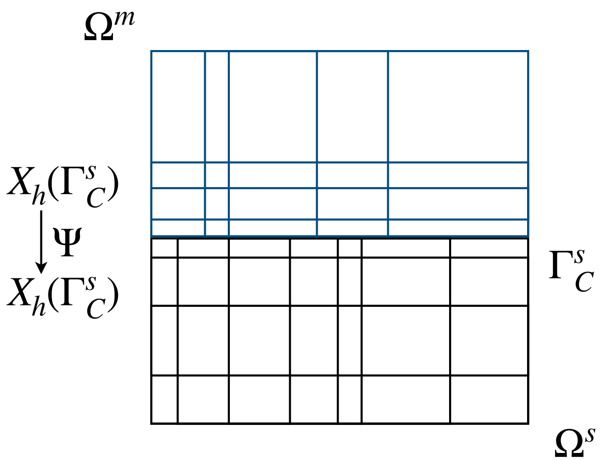
$$\big([u] - g\big)\sigma_n(u^s) = 0$$

$$\sigma_{\mathsf{T}}(u) = 0 \quad \text{(no friction)}$$

Dual Mortar Method for Contact

The dual mortar method allows us to solve contact problems for geometries with nonmatching surface meshes.

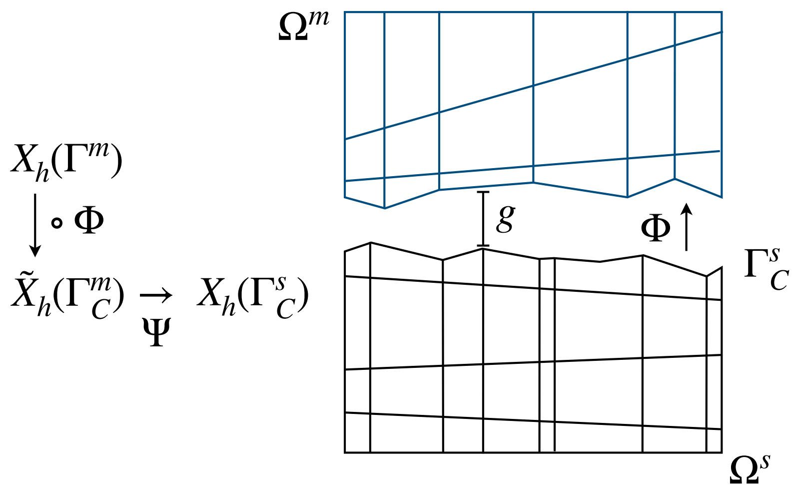
Mortar



$$\int_{\Gamma^s_C} [u]\,\tilde{\mu}\,d\gamma = 0,\ \tilde{\mu} \in \tilde{M}$$

$$\int_{\Gamma^s_C} (\Psi(u^s_h) - u^m_h)\,\tilde{\mu}_h\,d\gamma = 0,\ \tilde{\mu}_h \in \tilde{M}_h$$

dual mortar for Contact



$$\int_{\Gamma^s_C} [u]\,\mu\,d\gamma \leq 0,\ \mu \in M^+$$

$$M^+ = \left\{ \mu \in M \,\middle|\, \int_{\Gamma^s_C} \mu\,\lambda^s_+\,d\gamma \geq 0,\ \forall \lambda^s_+ \in X(\Gamma^s_C)^+ \right\}$$

$\Rightarrow \mathbf{T}$ discrete mortar operator

Belgacem et al. 1999, *The mortar finite element method for contact problems*.
Wohlmuth 2000. *A mortar finite element method using dual spaces for the Lagrange multiplier.*

Transformation of Contact Problem

Instead of solving a saddle point problem, we transfer the problem using the discrete mortar operator $\mathbf{T}$,and then solve it with a semismooth Newton method.

$$\mathbf{A} := \begin{bmatrix} \mathbf{A}_{\mathcal{I}\mathcal{I}} & \mathbf{A}_{\mathcal{I}\mathcal{M}} & \mathbf{A}_{\mathcal{I}\mathcal{S}} \\ \mathbf{A}_{\mathcal{M}\mathcal{I}} & \mathbf{A}_{\mathcal{I}\mathcal{I}} & 0 \\ \mathbf{A}_{\mathcal{S}\mathcal{I}} & 0 & \mathbf{A}_{\mathcal{I}\mathcal{I}} \end{bmatrix} \qquad \mathbf{u} := \begin{bmatrix} \mathbf{u}_{\mathcal{I}} \\ \mathbf{u}_{\mathcal{M}} \\ \mathbf{u}_{\mathcal{S}} \end{bmatrix}$$

Householder transformation: $\mathbf{O} := o_{pq} = \begin{cases} \mathbf{Id} - \frac{2}{(\mathbf{n}_p^s)^t \mathbf{n}_p^s} \mathbf{n}_p^s (\mathbf{n}_p^s)^t & , p = q \text{ and } p \in \mathcal{S} \\ \mathbf{Id} & , p = q \text{ and } p \in \mathcal{I} \cup \mathcal{M} \\ 0 & , \text{else.} \end{cases}$

Mortar transformation: $\mathbf{Z} := \begin{bmatrix} \mathbf{Id}_{\mathcal{I}} & 0 & 0 \\ 0 & \mathbf{Id}_{\mathcal{M}} & \mathbf{T}^t \\ 0 & 0 & \mathbf{Id}_{\mathcal{S}} \end{bmatrix}$

$$\hat{\mathbf{A}} := (\mathbf{OZ})\,\mathbf{A}\,(\mathbf{OZ})^t$$

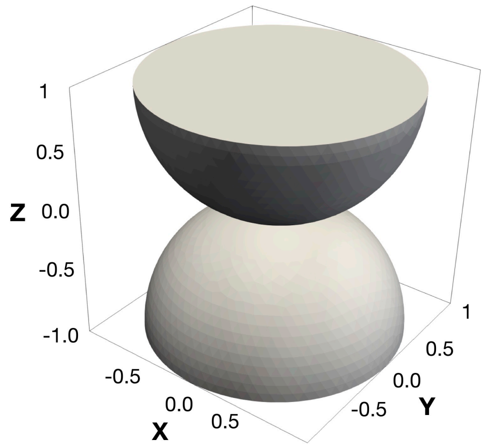
$$\hat{\mathbf{f}} := (\mathbf{OZ})\,\mathbf{f}$$

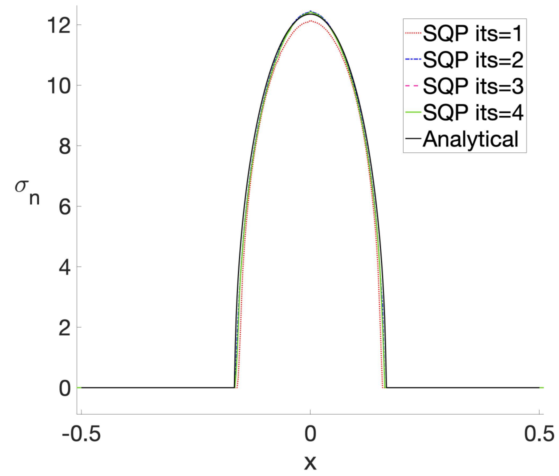
Solve $\hat{\mathbf{u}} = \underset{\mathbf{u}}{\operatorname{argmin}} \frac{1}{2} \mathbf{u}^t \hat{\mathbf{A}} \mathbf{u} - \hat{\mathbf{f}}^t \mathbf{u}$ with: $\mathbf{u} \leq \mathbf{g}$

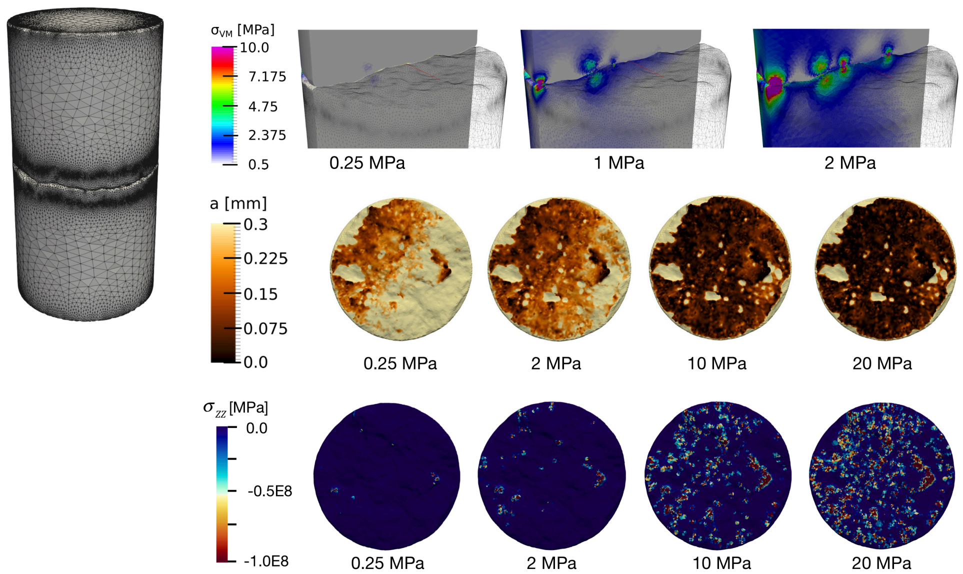
Dickopf and Krause 2009. *Efficient simulation of multi-body contact problems on complex geometries: A flexible decomposition approach using constrained minimization.*

Contact Problem - Results

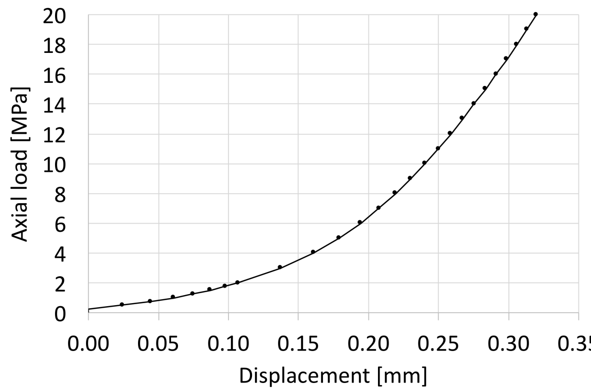
Replication of Hertzian contact stresses validates the method.







Contact simulation with rough rocks from Grimsel test site Switzerland.

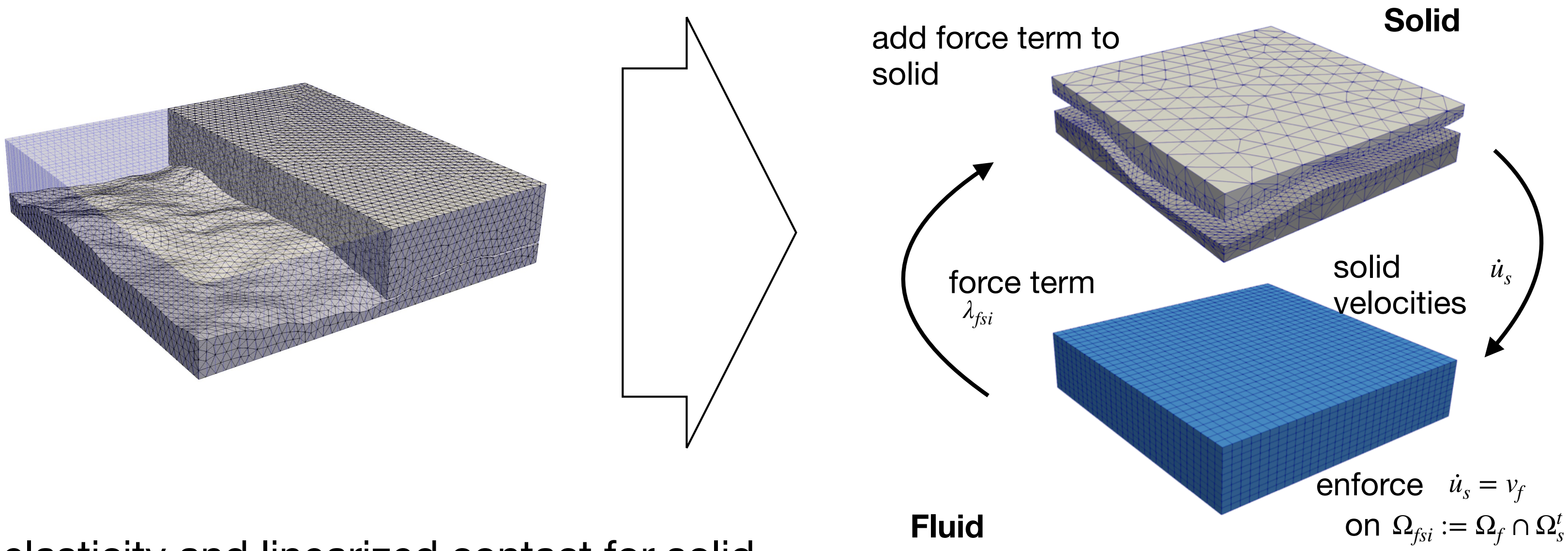


Replication of nonlinear loading curve for Grimsel sample.

Planta et al. 2019, *Solution of contact problems between rough body surfaces with non matching meshes using a dual mortar method.*

Fluid-structure interaction (FSI)

To simulate FSI we use a fictitious domain method, where the solid is immersed into the fluid. To map quantities between the solid and the fluid we need L2-projections, as the two meshes are nonmatching. The system is solved iteratively.

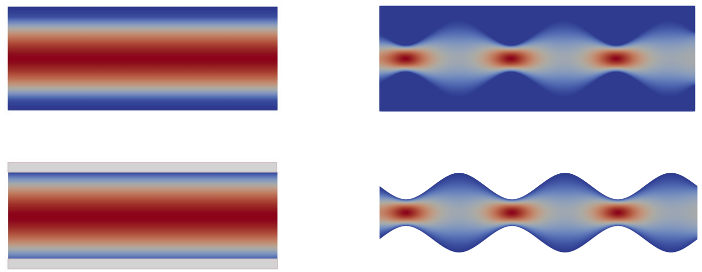


- Linear elasticity and linearized contact for solid
- Dynamic formulation
- Incompressible Navier Stokes equations for fluid
- Contact and transfer with L^2-projections
- P_1-P_1 for discretization of fluid problem
- P_1 for discretization of solid problem

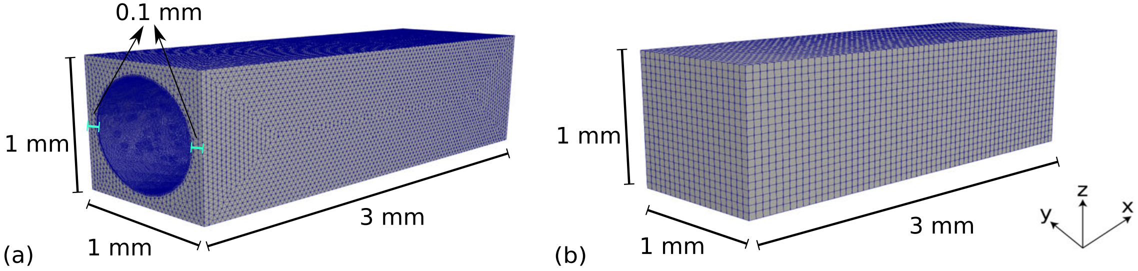
Planta et al. 2020. *Modelling of hydro-mechanical processes in heterogeneous fracture intersections using a fictitious domain method with variational transfer operators*

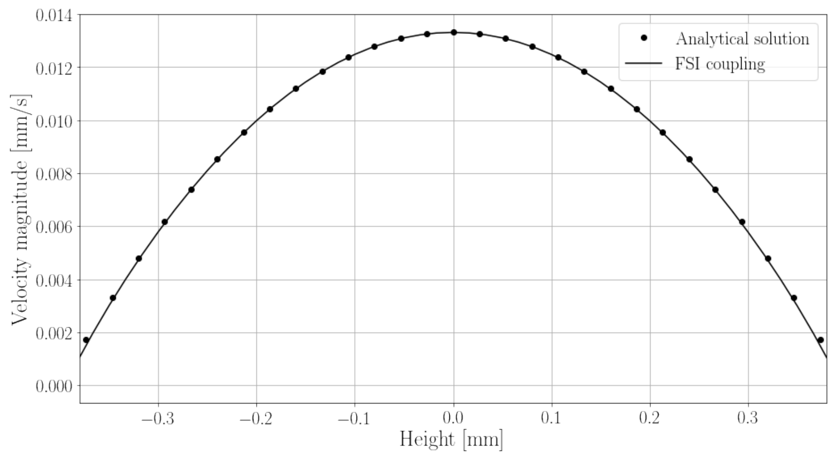
Fluid-structure interaction (FSI)

We conducted benchmark experiments in 2D and 3D. In particular Poiseuille flow to validate the method.



2D channels: Top: Solution with FSI method. Bottom: Solution with standard Navier-Stokes FEM method.

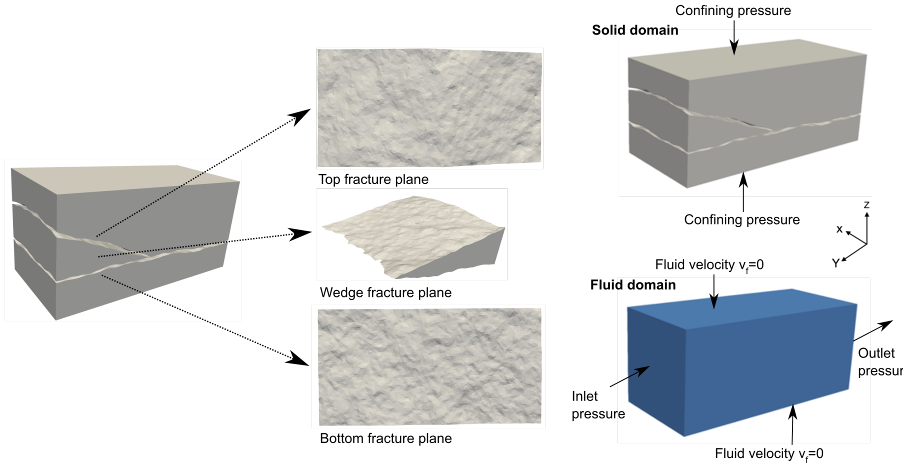


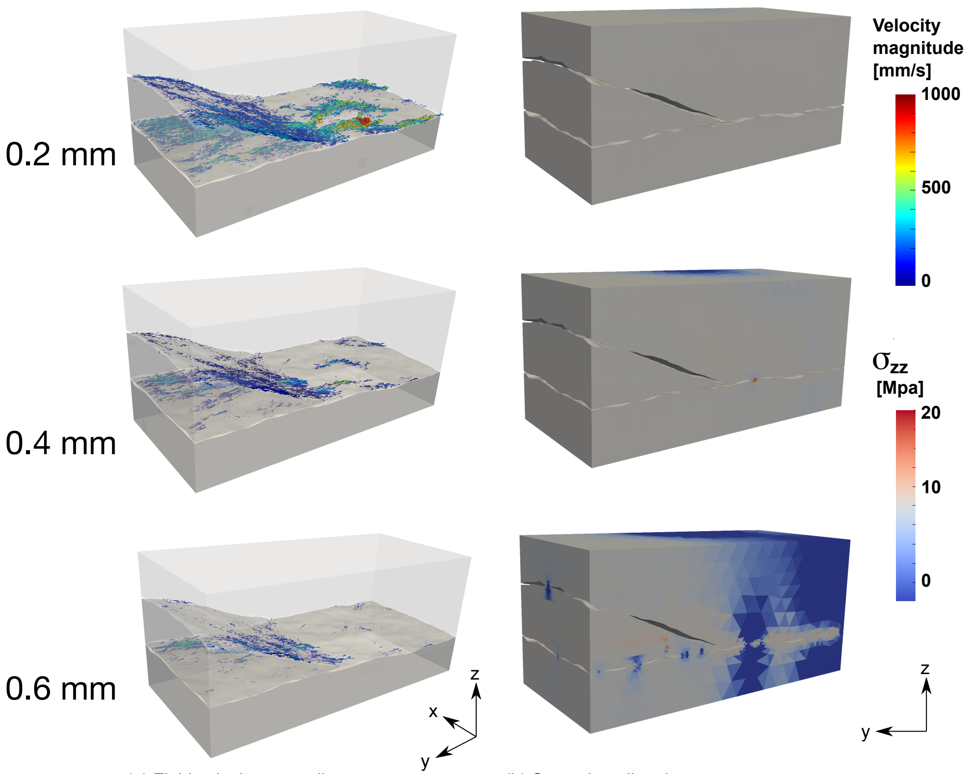


Comparison of Poiseuille Flow with FSI method versus standard FEM solution.

Fluid-structure interaction (FSI)

Intersected fracture with contact, setup of solid and fluid geometry:



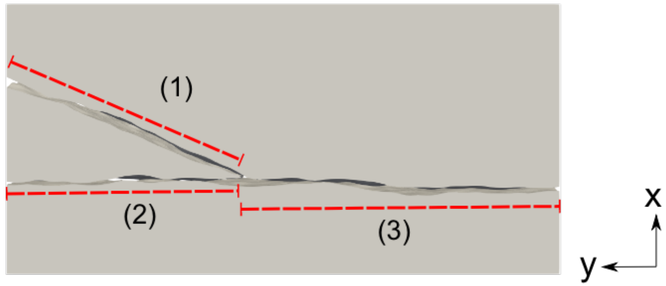


(a) Fluid velocity streamline (b) Stress in z direction

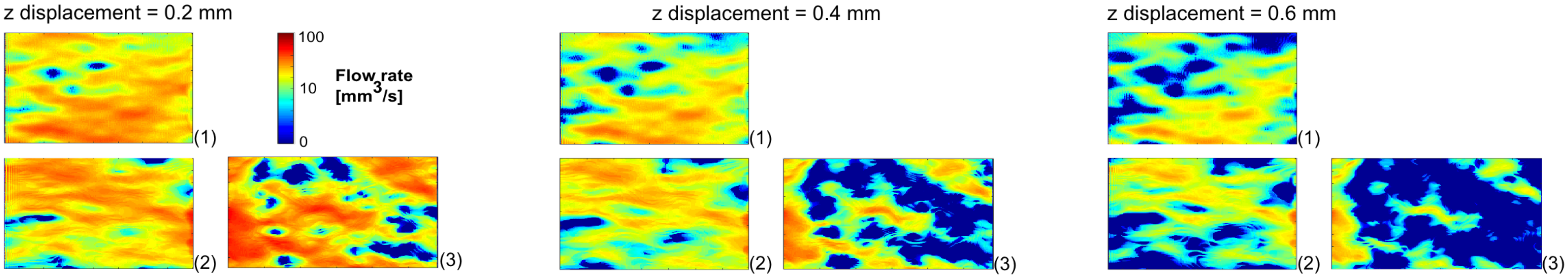
Fluid flow in closing fracture with evolving stresses in the rock.

Fluid structure interaction (FSI)

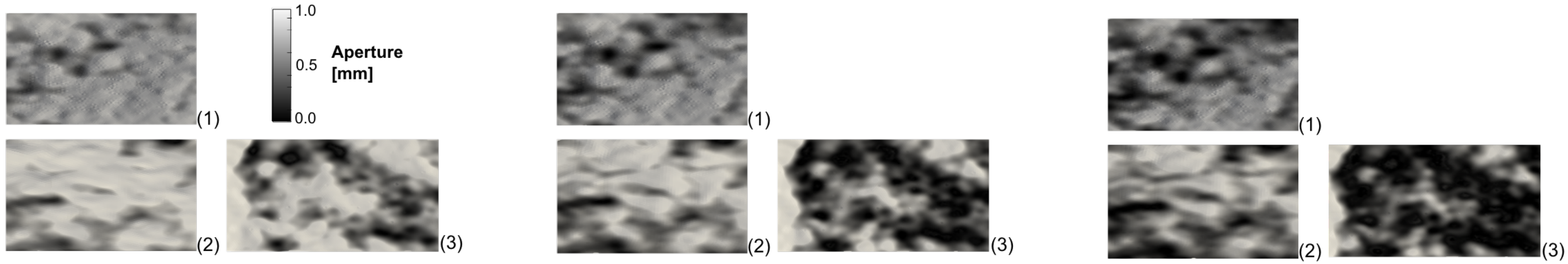
Closing fracture with aggregated flow rates. Fluid flow within fracture shows increased channeling under increased closure.



(a) Fracture planes



(b) Flow rate



Planta et al. 2020. *Modelling of hydro-mechanical processes in heterogeneous fracture intersections using a fictitious domain method with variational transfer operators*

Thermo-Fluid-Structure Interaction (TFSI)

For TFSI we introduce in addition heat in the solid and the fluid, and we alter the definition of the stress tensor of the solid.

$$\rho_s c_s - k_s \nabla \cdot T_s = \rho_s q_s$$

$$\rho_S \ddot{u} - \textbf{div}\sigma_s(u_s) = f_s$$

T_s : temperature q_s: heat source/sink σ_s: stress tensor
ρ_s: density u: displacement f_s: body forces
k_s : heat conductivity c_s: heat capacity

The stress tensor then has an additional term to account for the temperature:

$$\sigma_s(u) = 2\mu\epsilon + \lambda \textbf{tr}(\epsilon)I - (2\mu + 3\lambda)\alpha I(T_s - T_s^*)$$

μ, λ : Lame parameters ϵ: strain T_s^*: stress free temp.
α: coefficient of linear thermal expansion

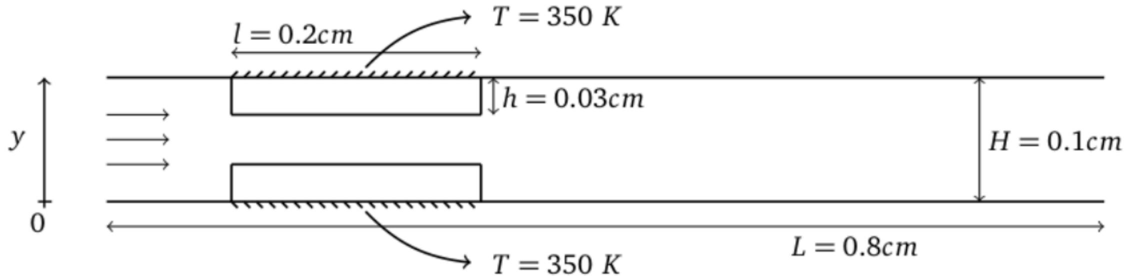
In the fluid, the temperature evolves according to: $\rho_f c_f(\dot{T}_f + v\nabla T_f) - k_f \nabla^2 T_f - \sigma_f : \nabla v = 0$
Where in general the subscript $_f$ denotes fluid quantities, and v the fluid velocity.

We then set the following condition on the interface between the solid and the fluid to allow for temperature exchange: $T_f = T_s$ on $\Omega_{fsi} := \Omega_f \cap \Omega_s^t$

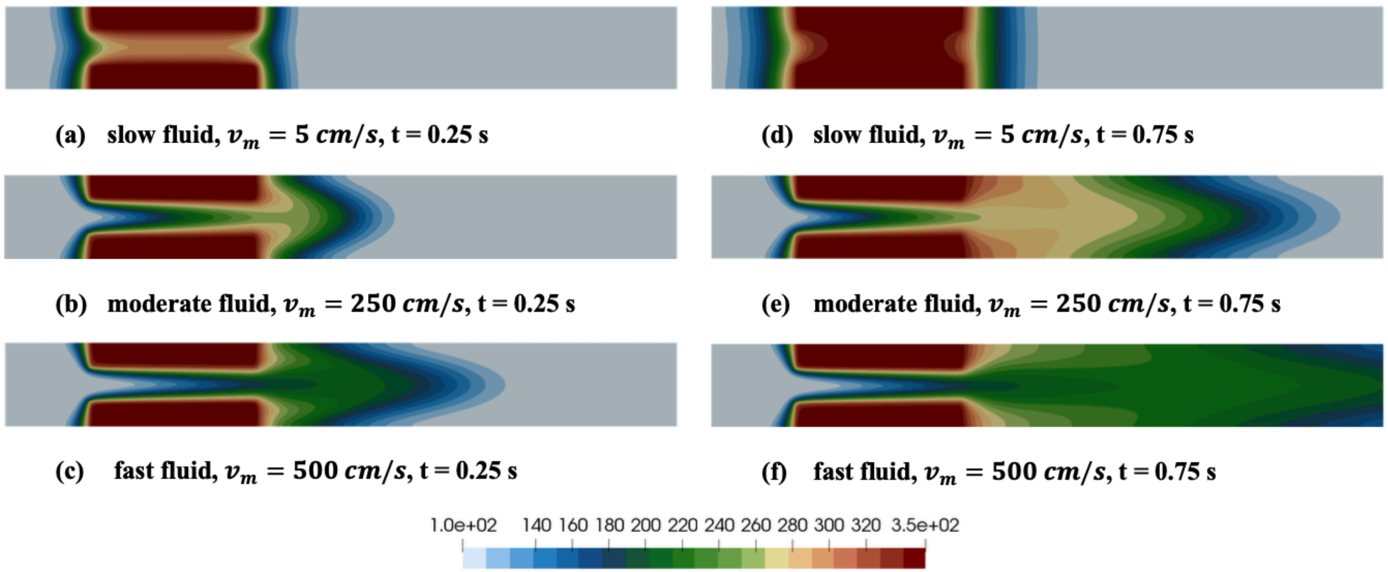
\Hassanjanikhoshkroud et al. 2020. *Thermo-Fluid-Structure Interaction Based on the Fictitious Domain Method: Application to Dry Rock Simulations*

Thermo-Fluid-Structure Interaction (TFSI)

We applied the method to a 2D channel and simulated heat transfer at different fluid velocities.



The results show, that the TFSI method can replicate the transfer from a diffusive to a convective transfer regime.



(a) slow fluid, $v_m = 5\ cm/s$, t = 0.25 s

(b) moderate fluid, $v_m = 250\ cm/s$, t = 0.25 s

(c) fast fluid, $v_m = 500\ cm/s$, t = 0.25 s

(d) slow fluid, $v_m = 5\ cm/s$, t = 0.75 s

(e) moderate fluid, $v_m = 250\ cm/s$, t = 0.75 s

(f) fast fluid, $v_m = 500\ cm/s$, t = 0.75 s

Hassanjanikhoshkroud et al. 2020. *Thermo-Fluid-Structure Interaction Based on the Fictitious Domain Method: Application to Dry Rock Simulations*

L²-Projections: how they are defined

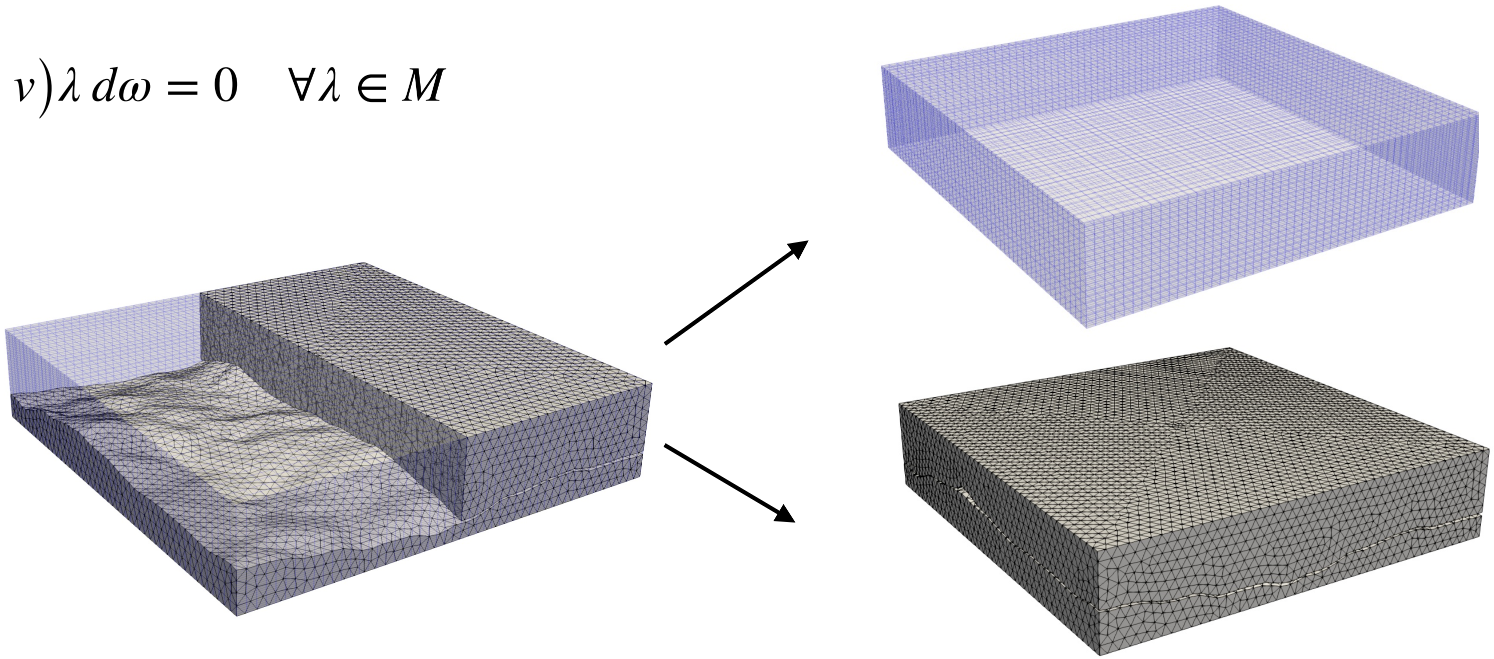
Definition:

V, W, M finite dimensional functions spaces on Ω, with dimensions n^V, n^W, n^M,

bases $(\lambda_i^V)_{i=1,\dots,n^V}, (\lambda_j^W)_{j=1,\dots,n^W}, (\lambda_k^M)_{k=1,\dots,n^M}$,where M is a multiplier space with $n^W = n^M$.

The L²-projection $\Pi : V \to W$ is defined such that

$$\int_\Omega \big(\Pi(v) - v\big)\lambda \, d\omega = 0 \quad \forall \lambda \in M$$



L^2-Projections: how the discrete operator $\mathbf{T}$ is assembled using the basis representation of the surrounding space:

Using the basis representations of $v, w := \Pi(v), \lambda$, the previous Eq. becomes:

$$\int_{\Omega} \sum_{i=1}^{n^W} w_i \lambda_i^W \, \phi_k^M \, d\omega = \int_{\Omega} \sum_{j=1}^{n^V} v_j \lambda_j^V \, \lambda_k^M \, d\omega, \quad k = 1,....,n_M$$

Using

$$\mathbf{D} := (d_{ik})_{i,k=1,\ldots,n^{W_h}},\ d_{ik} := \int_{\Omega} \phi_i^{W_h} \phi_k^{M_h} \, d\omega \qquad \mathbf{B} := (b_{jk})_{j=1,\ldots,n^{V_h},k=1,\ldots,n^{M_h}},\ b_{jk} := \int_{\Omega} \phi_i^{V_h} \phi_k^{M_h} \, d\omega,$$

$\mathbf{w} := (w_i)_{i=1,\ldots,n^{W_h}}, \quad \mathbf{v} := (v_j)_{j=1,\ldots,n^{V_h}}$,we get: $\mathbf{w} = D^{-1} B \mathbf{v} := T\mathbf{v}$

Zulian and Krause 2016. *A Parallel Approach to the Variational Transfer of Discrete Fields between Arbitrarily Distributed Unstructured Finite Element Meshes.*

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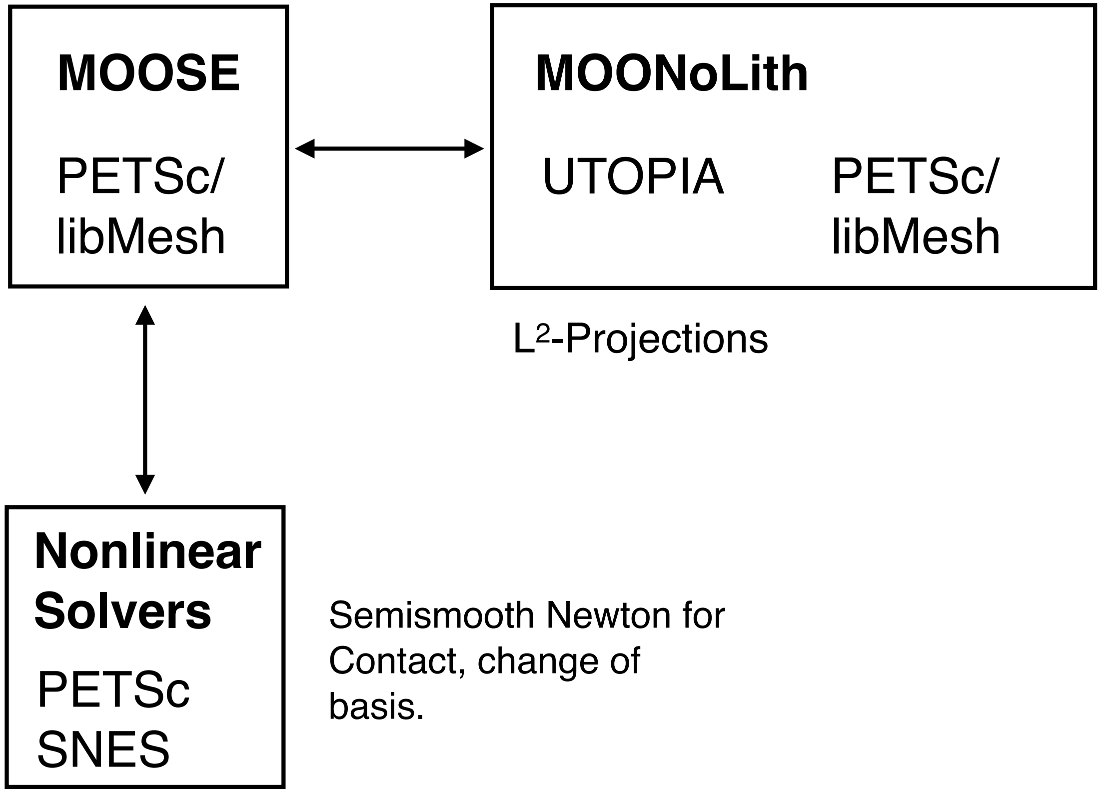
Implementation/Software

User Interface
FEM discretization
Test harness
Thermoelasticity



L^2-Projections

Semismooth Newton for
Contact, change of
basis.

Summary

- Can replicate nonlinear closing behaviour of fracture
- Can replicate channeling in closing fracture
- Can replicate transition from diffusive to convective heat transfer
- L^2-projections were used for (1) coupling in FSI, TFSI, (2) contact problem
- Approach is adaptable and extendable

Future work

- Augment contact formulation with friction
- Augment contact formulation with stress free deformations
- Nonlinear solid
- 3D TSFI simulations

References

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6. Krause R, Zulian P, 2016, “A parallel approach to the variational transfer of discrete fields between arbitrarily distributed unstructured finite element meshes.” SIAM Journal on Scientific Computing 38(3):C307-C333.