A MODEL FOR THE LONGITUDINAL PATTERNS SHAPED BY WATER ON SOLUBLE ROCKS

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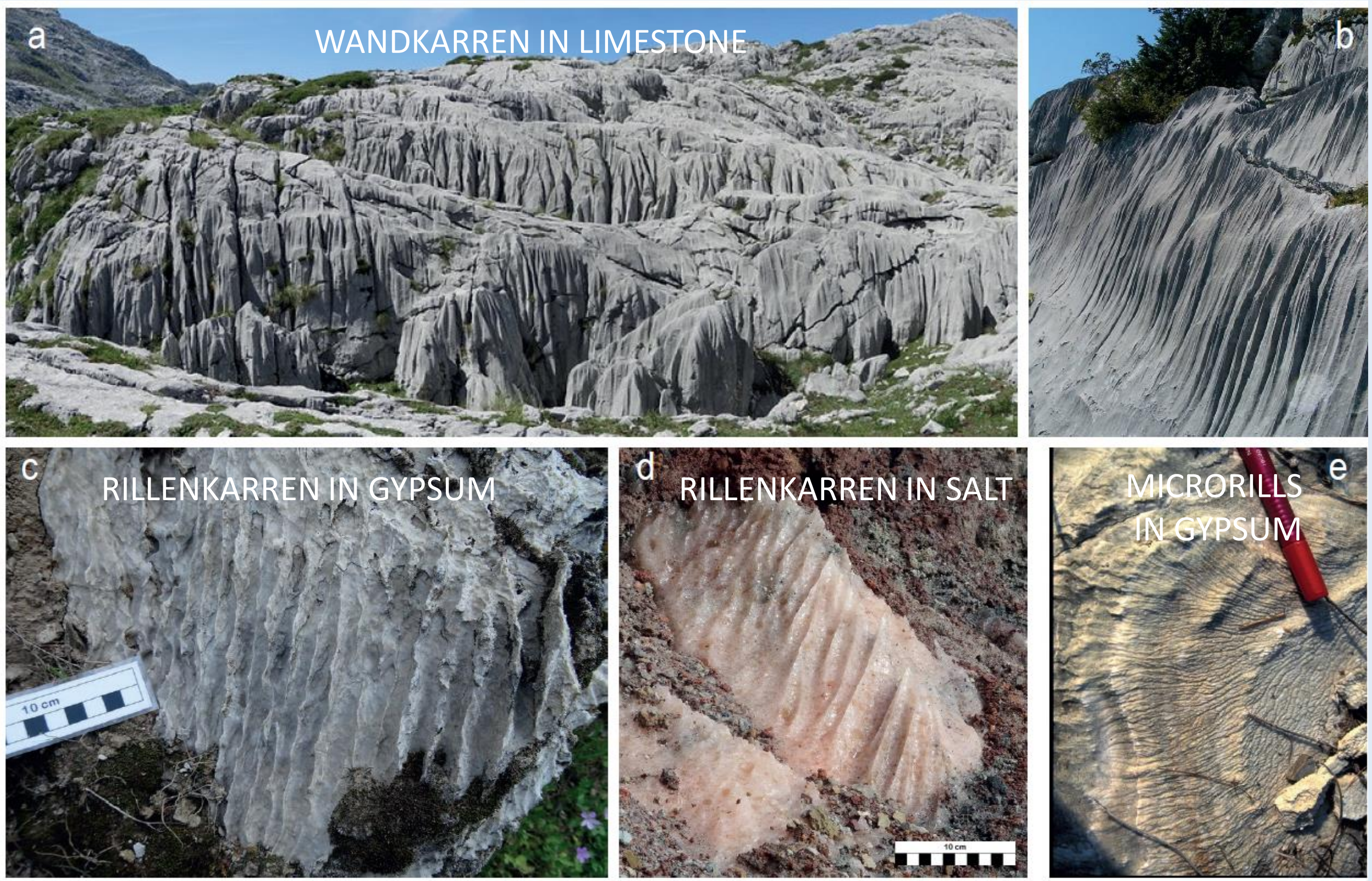
LINEAR KARREN FORMS



In karst environments, flowing water dissolves rocks and create regular longitudinal patterns, whose size can span on different order of magnitudes.

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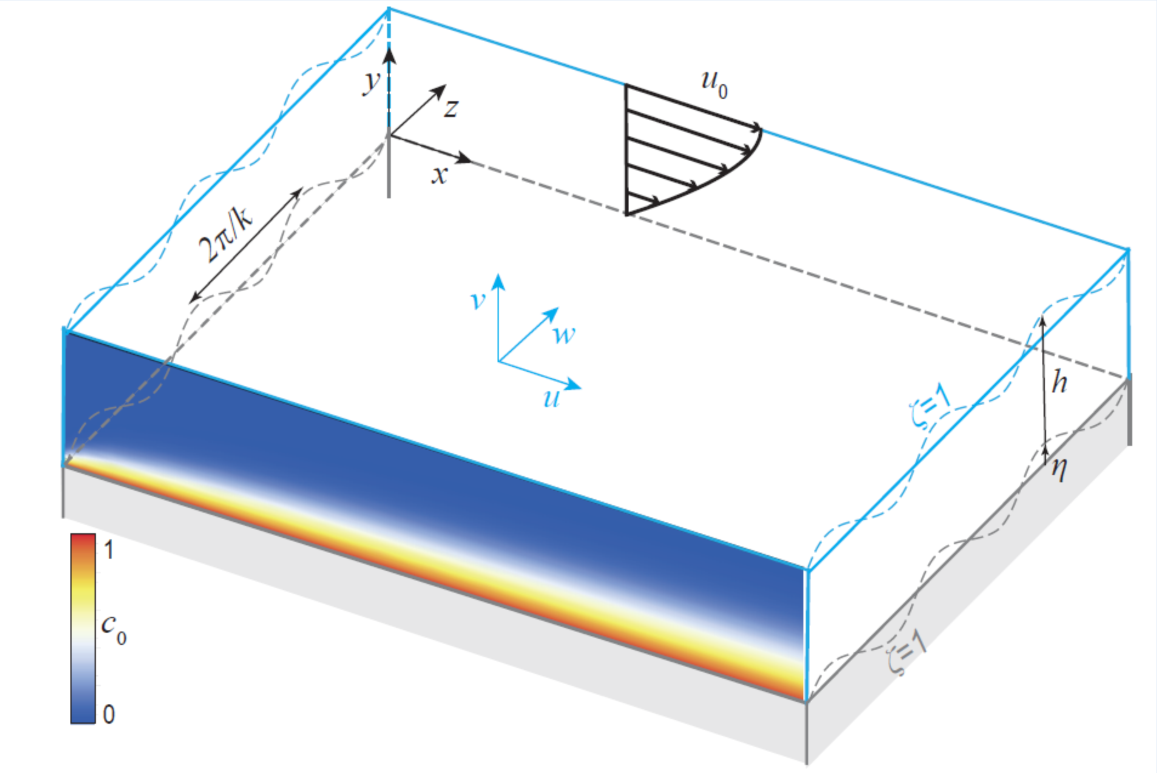
MODEL SKETCH



GOVERNING EQUATIONS

$$\nabla \cdot \mathbf{u} = 0,$$
$$Re(\mathbf{u} \cdot \nabla \mathbf{u} + \nabla p) = \nabla^2 \mathbf{u} + \boldsymbol{f},$$
$$Pe\, \boldsymbol{u} \cdot \nabla c = \nabla^2 c,$$

NAVIER-STOKES +
ADVECTION –DIFFUSION

We model a water laminar flow on a soluble rock that is dissolving. *c*(*x*,*y*,*z*) is the concentration of solute within the water film. We then study the stability of the flat solution to a transverse perturbation (karren formation).

FLAT ROCK-BASE STATE

$$u_0 = (2 - \zeta)\zeta,$$

SEMI-PARABOLIC VELOCITY

$$Pe\, u_0\, \partial_x c_0 = c_0'',$$

ADVECTION –DIFFUSION

$$c_0|_{x=0} = 0,$$

INITIALLY PURE WATER

$$c_0|_{\zeta=0} = 1,$$

DIFFUSION BOUNDARY LAYER AT THE ROCK INTERFACE

$$c_0'|_{\zeta=1} = 0.$$

NO FLUX OF SOLUTE TO THE AIR

$$c_0 = 1 - \sum_{m=0}^{\infty} A_m \exp\left(-\alpha_m^2 X\right) G_m(\zeta),$$

SOLUTION BY POLYANIN ET AL (2001)

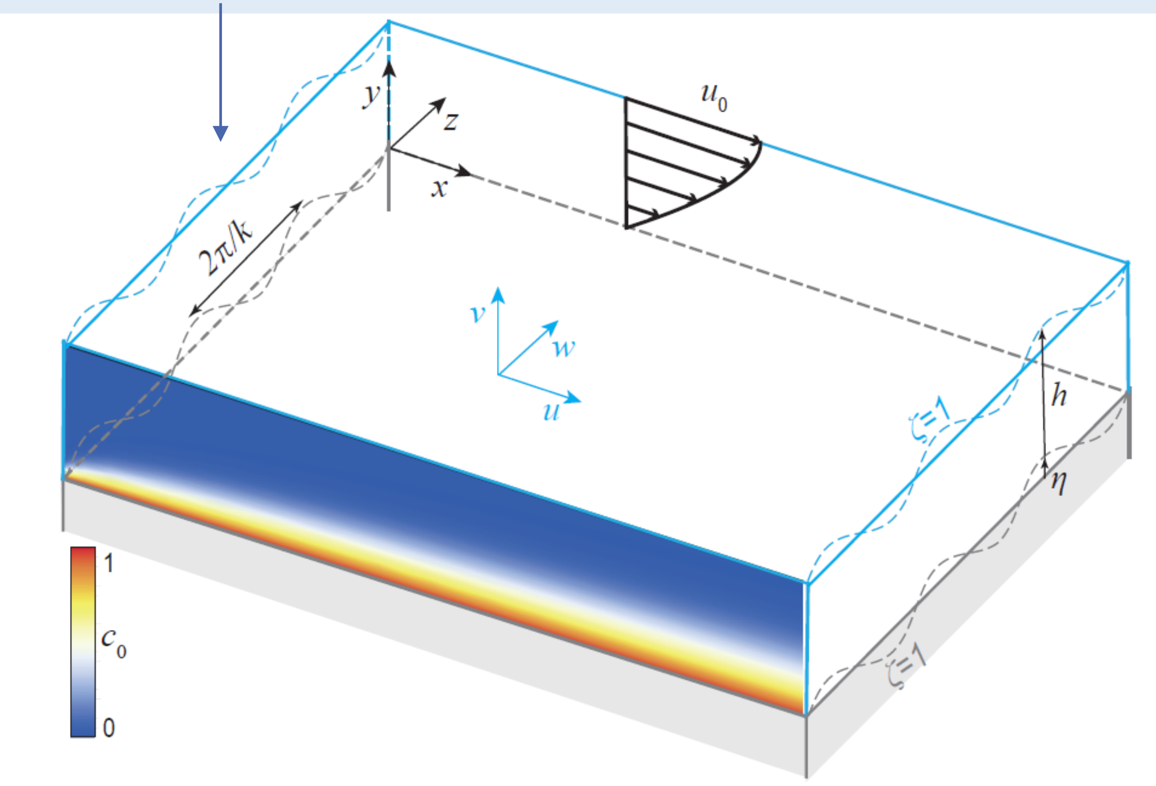
In flat conditions, the water has a semi-parabolic velocity profile u_0 (ζ is the rectangularized vertical coordinate). The solute concentration c_0 (x, ζ) has a complex solution (Polyanin et al, 2001) that takes into account that the water is nearly saturated very close to the rock surface (Diffusion Boundary Layer).

LINEAR STABILITY ANALYSIS

$$(h, \eta) = (1, 0) + \varepsilon(h_1, \eta_1)e^{\omega t + ikz}$$

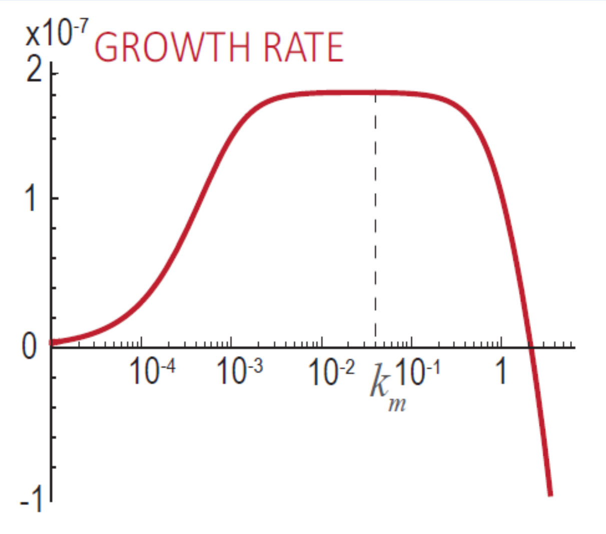
$$(\boldsymbol{u}, p, c) = (\boldsymbol{u}_0, p_0, c_0) + \varepsilon(\boldsymbol{u}_1, p_1, c_1)e^{\omega t + ikz}$$

TRANSVERSE PERTURBATION

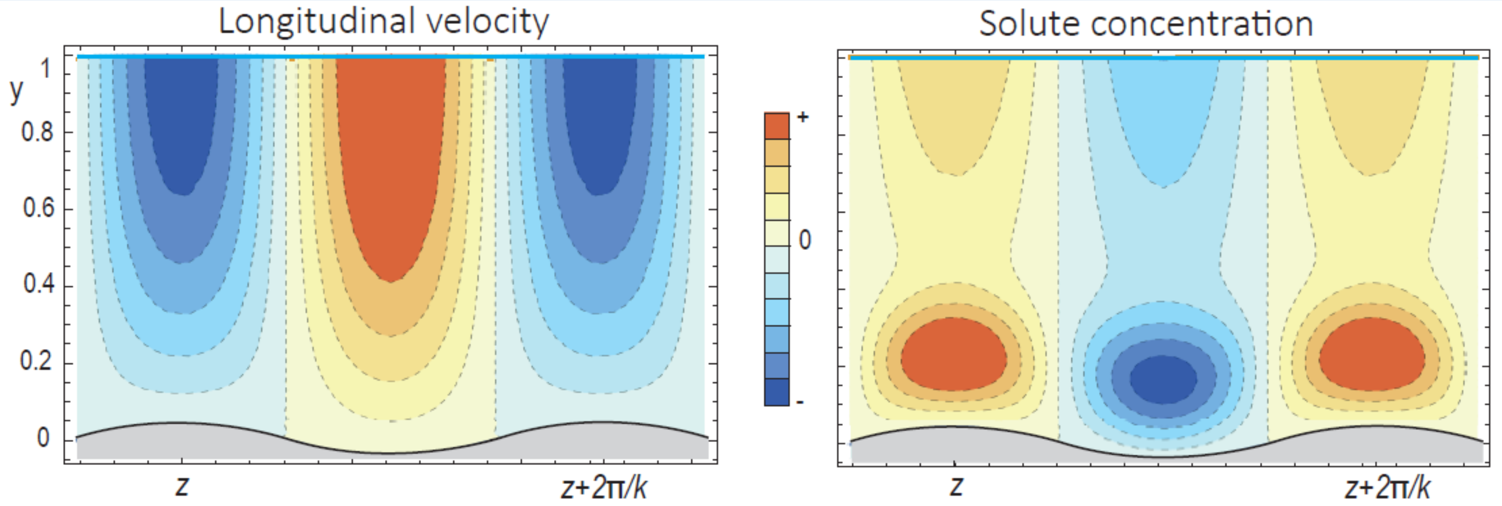


We perform a linear stability analysis of the flat solution by introducing a small transverse perturbation.

KARREN INSTABILITY



The flat solution is unstable to a band of wavenumbers (left figure). The instability arise as in correspondence of the rock through -> the velocity is higher -> the solute concentration within the film is lower->the dissolution is higher (lower figure).



TRANSVERSE SECTIONS SHOWING THE PERTURBATION FIELDS

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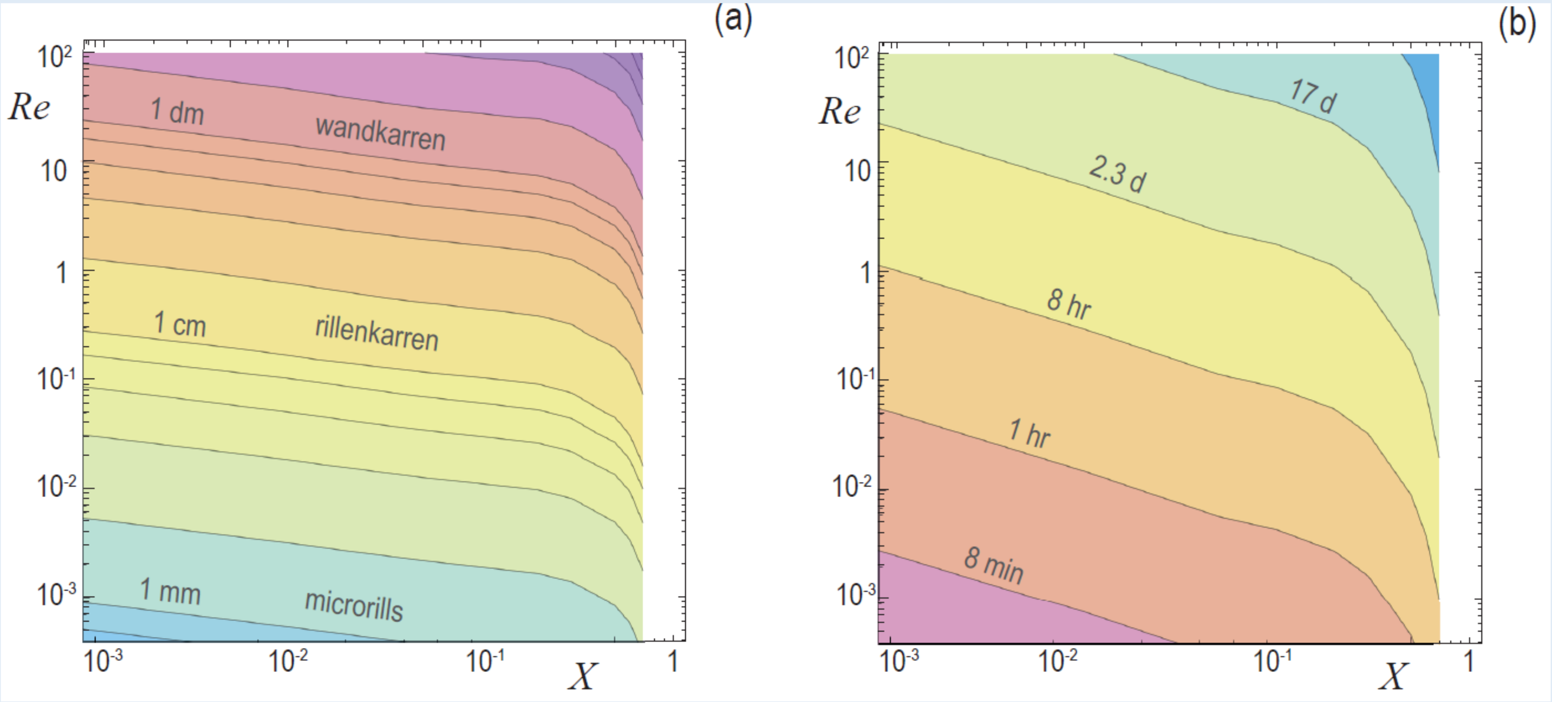
DIMENSIONAL RESULTS



Dimensional wavelength (a) and timescale (b) of karren instability as a function of *Re* and the longitudinal coordinate X. The results show that the flow intensity (*Re*) could be the discriminating factor in the different sizes of similar karren observed in nature (microrills, rillenkarren, wandkarren...).

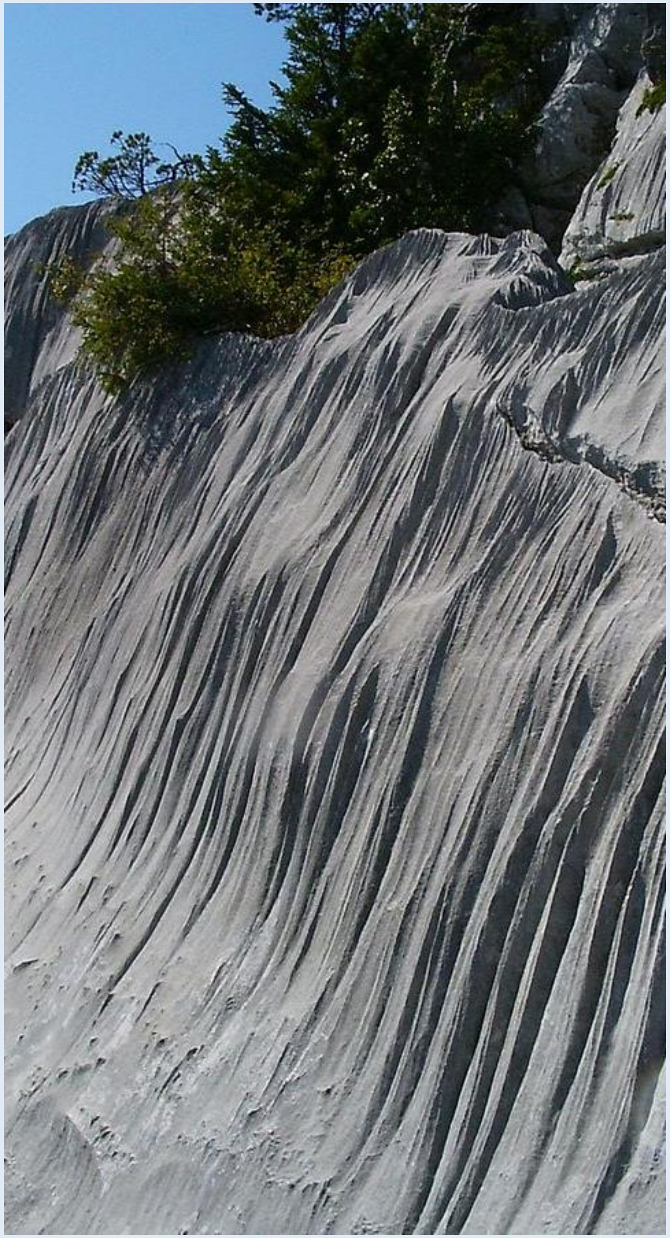
CONCLUSIONS

We have developed a simple model that may explain the genesis of linear karren formation. Numerical simulations of the full equations may reveal further insights that are lost with the approach of the linear stability analysis.

BIBLIOGRAPHY

1. Polyanin, Andrei D, Kutepov, AM, Kazenin, DA & Vyazmin, AV2001Hydrodynamics,mass and heat transfer in chemical engineering, , vol. 14. CRC Press
2. Sauro, Ugo2009 I Karren: le forme scolpite sulla roccia.

ANY SUGGESTION OR QUESTION IS WELCOMED!



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