Analysis of Geomagnetic Variability by Empirical Orthogonal Functions

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Introduction

Given the Gauss coefficients from the IGRF-13 model, we: (i) analyze the geomagnetic variations using Empirical Orthogonal Functions (EOF); and (ii) study possible relations between the obtained spatio-temporal EOF modes on the core-mantle boundary (CMB) and Earth's core dynamics.

Data & Procedure

1. Data: **IGRF-13** 1900-2020 (25 5-year epochs) / degree $N \le 13$

- core field dominant

$$V(r,\theta,\lambda,t) = a\sum_{n=1}^{N}\sum_{m=0}^{n}\left(\frac{a}{r}\right)^{n+1}\left[\boxed{g_n^m(t)}\cos(m\lambda)+\boxed{h_n^m(t)}\sin(m\lambda)\right]\tilde{P}_n^m(\cos\theta)$$

Gauss coefficients

2. Downward Continuation to CMB

$$V_{CMB}(r,\theta,\lambda,t) = V_{surface}(a,\theta,\lambda,t)\left(\frac{a}{r}\right)^{n+1}$$

$$\rightarrow \quad \mathbf{B}_{CMB} = -\nabla V_{CMB} = \mathbf{B}_{surface}(n+1)\left(\frac{a}{r}\right)^{n+2}$$

3. EOF and CEOF analysis

EOF analysis solves the eigenfunctions of the data covariance matrix to extract spatial-temporal standing-wave coherent signals as orthogonal EOF modes that represent major dynamics in descending order. The complex EOF (CEOF) is extended to solve the eigenfunctions of complex data (augmented by its Hilbert transform) to represent propagating-wave phenomenon.

- **EOF for Large-Scale Field (n=1-6) – Stationary Signal**

$$X(t,s) = \sum_{i=1}^{i=p} A_i(t)\cdot S_i(s)$$

$$\Lambda = S^{-1}\left(X^T X\right)S, \quad XS_i = A_i$$

$$E_i = \Lambda_i / \sum_{i=1}^{p}\Lambda_i \times 100\%$$

$$\Lambda = \begin{pmatrix} \sigma_{11}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{22}^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{pp}^2 \end{pmatrix}$$

X: data / i: mode / E: Variance
S: spatial pattern / A: temporal function

$$\sigma_{11}^2 > \sigma_{22}^2 > \cdots > \sigma_{pp}^2$$

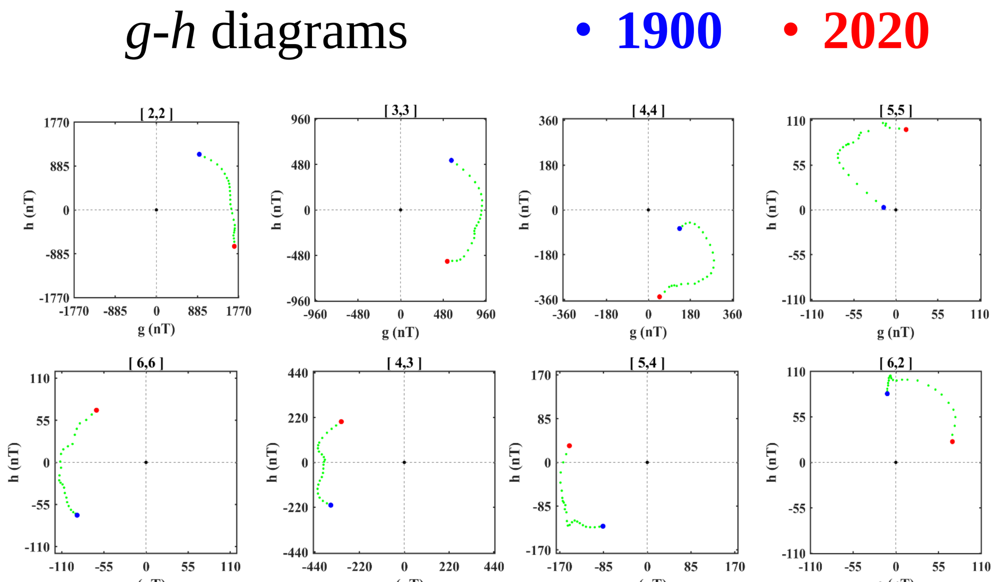
- **CEOF for Westward Drifting Field – Propagating Signal**

$$X_a(t,s) = X(t,s) + iH\{X(t,s)\}, \quad H\{x(t)\} = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}d\tau = x(t) * \frac{1}{\pi t}$$

➢ **Combining 8 *g-h* pairs to construct the westward drifting field**

$$V_{WD}(\Omega,t) = \sum_{WD} a\,\tilde{P}_n^m(\cos\theta)\left|\,g_n^m(t)+ih_n^m(t)\,\right|\cos\left(\frac{2\pi}{T_{period}}t - m\lambda + \psi_n^m\right)$$

g-h diagrams • 1900 • 2020

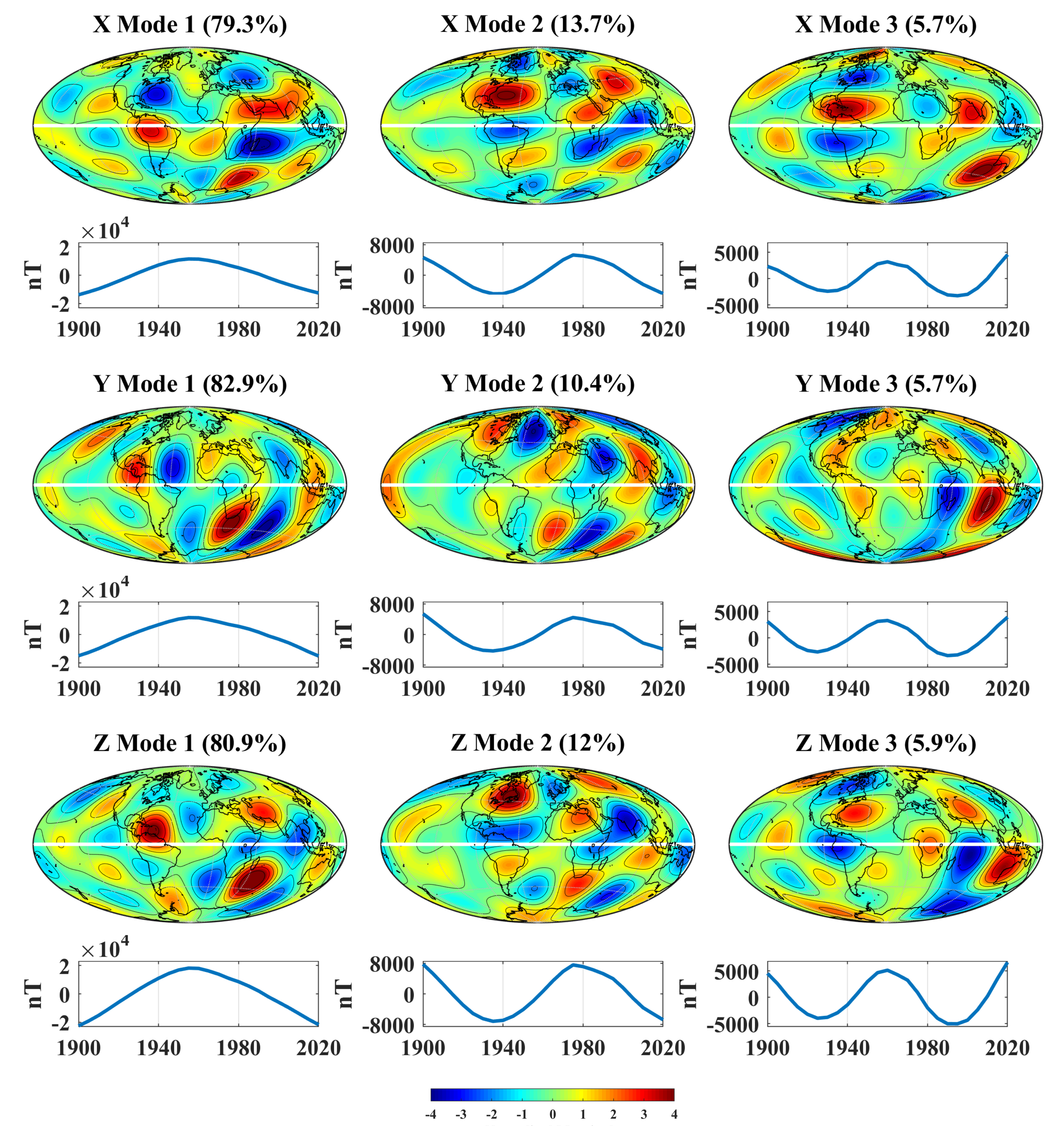


n	m	Period (year)	Phase Speed (degrees/year)
2	2	582.53	0.31
3	3	488.76	0.25
4	4	815.89	0.11
5	5	495.68	0.15
6	6	514.64	0.12
4	3	685.42	0.18
5	4	641.76	0.14
6	2	549.97	0.33

Results

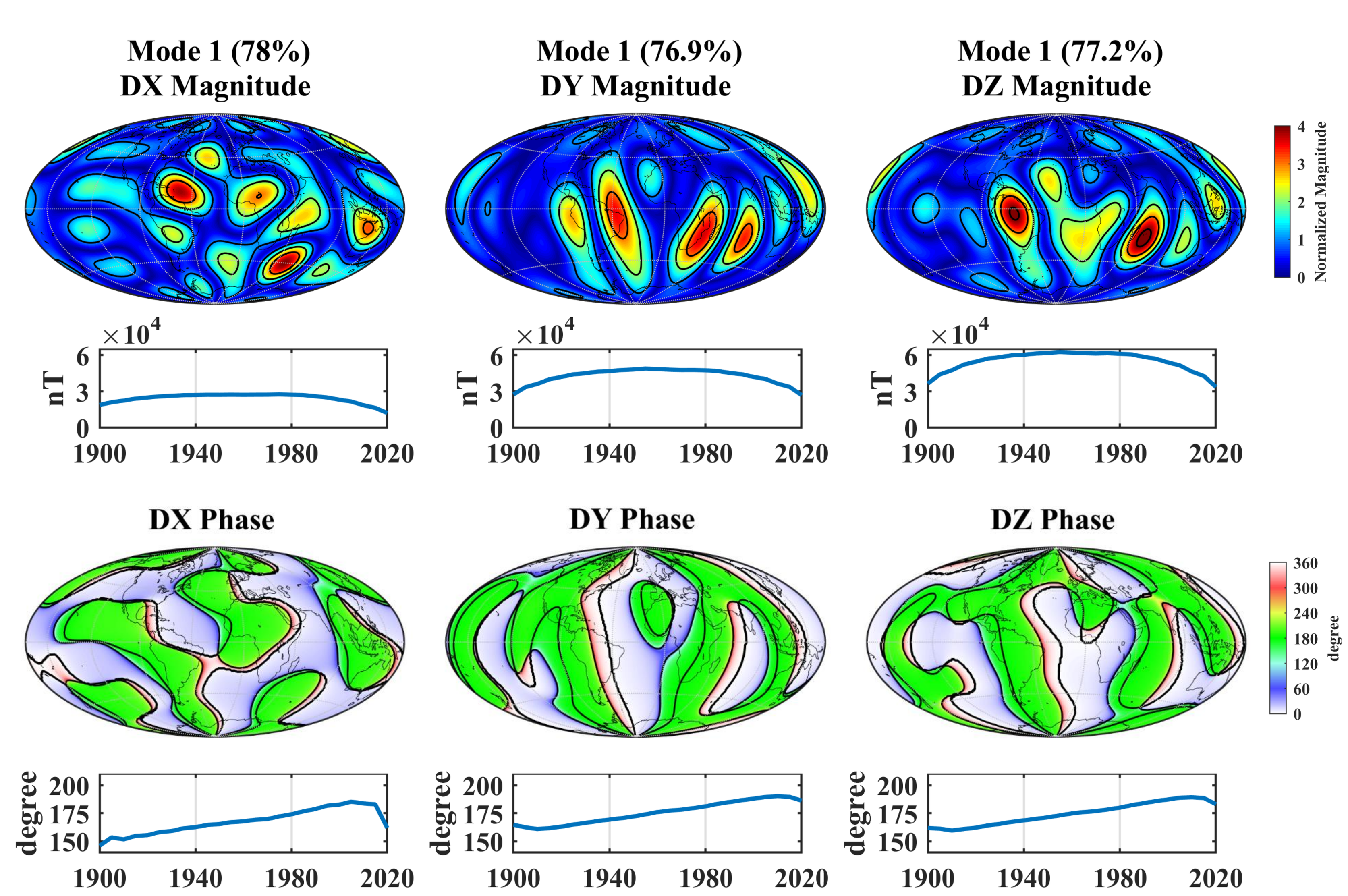
◆ EOF Modes

- The large-scale field data has been removed of the mean and linear trend.
- The spatial pattern is a snapshot. Each grid point undulates with time according to the temporal function.
- The percentage variance of the first three modes are 80%, 12% and 6%, with respective periods of about 120, 75 and 65 years.



◆ CEOF Modes

- The spatial pattern magnitude of CEOF is interpreted similar to EOF; its phase indicates the propagating direction from white → red → green → blue →white.
- The percentage variance of the first CEOF mode is ~80%; its phase indicates the ~30° westward drift.



Discussion and Conclusions

- The EOF mode temporal functions are consistent with the proposed MAC waves in the stably stratified layer of the upmost outer core (Buffett, 2014).
- We propose that the EOF mode spatial patterns are associated with: (i) **regional stratification** beneath CMB; (ii) **heat heterogeneity** and overall **convection behavior** including **chemical contribution** at CMB under the flux expulsion mechanism; (iii) **conductivity** in the lower mantle; and (iv) the CMB **topography**.
- The CEOF mode temporal phases successfully capture the westward drifting field of phase speed ~0.27°/yr (Finlay & Jackson, 2003).
- Longer data can yield better determination of westward drift so the regional non-dispersive and dispersive behavior can be better discerned distinguished w.r.t. corresponding flow and wave mechanisms, which are associated with core-mantle coupling mechanisms and perturbations (e.g., slow MC-Rossby waves).

Reference: