Multifractal analysis of velocity and temperature fluctuations in the atmospheric boundary layer over the Hyytiälä forest

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Outline I

Atmospheric Boundary Layer

- Fluid flow in the atmospheric boundary layer is affected by various factors such as the topography, roughness elements, and thermal stratification.
- Thus, boundary-layer turbulence is significantly different from turbulence that is statistically homogeneous and isotropic.

Deviations from K41 I

- The Kolmogorov 1941 (K41) phenomenological theory applies to statistically homogeneous and isotropic turbulence.
- This does not apply to turbulent flows in the atmospheric boundary layer, because they are not homogeneous and isotropic.
- Furthermore, K41 phenomenology leads to mono-fractal or single-exponent scaling of velocity structure functions.
- We use measurements of the flow velocity in the canopy sublayer above the Hyytiälä forest to examine the multifractality and anisotropy of velocity structure functions in this sublayer.

Methods: MF-DFA and MMA

- We have employed the multifractal detrended fluctuation analysis (MF-DFA) and its extension, the multiscale multifractal analysis (MMA), to examine the nature of the multiscaling of turbulent fluctuations of velocity and temperature time series.
- In particular, we calculate the Hurst exponent $h(q)$ (also a scale-dependent version $h(q, s)$) and the singularity spectrum $f(\alpha)$, the Legendre transform of $\tau(q) = qh(q) - 1$; here, α is the singularity strength or the Hölder exponent.

MF-DFA algorithm

Given a time-series x_k, $k = 1, \ldots N$ that has a compact support, we carry out the following calculations:

1. calculate *profile*: $Y(i) = \sum_{k=1}^{i} (x_k - \langle x \rangle)$
2. partition the time series into $N_s = \lfloor N/s \rfloor$ intervals of length s and calculating the order-m detrended fluctuation: $F^2(\nu, s) = \frac{1}{s} \sum_{i=1}^{s} \left\{ Y(i + (\nu - 1)s) - y_\nu^m(i) \right\}^2, \nu = 1, \ldots N_s$
3. calculate the averaged q-th order fluctuation for scale s: $F_q(s) = \left[\frac{1}{N_s} \sum_{\nu=1}^{N_s} F^2(\nu, s)^{q/2} \right]^{1/q}$, where q can take any real value.
4. calculate the Hurst exponent $h(q)$ from the scaling form: $F_q(s) \sim s^{h(q)}$.

Once we have $h(q)$, we can calculate the singularity strength or Hölder exponent α and the singularity spectrum $f(\alpha)$ as the Legendre transform of $\tau(q) = qh(q) - 1$:

$$f(\alpha) = q\alpha - \tau(q); \; \alpha = \frac{\mathrm{d}\tau(q)}{\mathrm{d}q}. \tag{1}$$

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MMA Algorithm

In the MF-DFA, the (best-fit) slope of the following line in the log-log plot of $F_q(s)$ versus s, over the entire scale range of scales s, yields $h(q)$:

$$\ln F_q(s) = h(q) \ln s + c; \tag{2}$$

However, different ranges of scales are often seen to have different scaling exponents, so we can, alternatively, calculate the exponents over moving windows:

$$\ln F_q(s) = h(q, s) \ln s + c, s \in [s_{lower}(s), s_{upper}(s)] \tag{3}$$

over a window $[s_{lower}(s), s_{upper}(s)]$ determined by the scaled s.

$h(q)$ and $f(\alpha)$

- $h(q)$ depends on the nature of correlations between the fluctuations of x_k. For a monofractal time series, $h(q)$ is a constant and independent of q.
- For persistent (long-range correlated) time series, $1 > h(q) > 0.5$, whereas, for an anti-persistent (long-range anti-correlated) time series, $h(q) < 0.5$. For an uncorrelated, white-noise-type time series, $h(q) = 0.5$.
- The width of the plot of $f(\alpha)$ versus α, $\delta\alpha = (\alpha_{max} - \alpha_{min})$, and the range of $h(q)$, $\delta h = (h(q_{min}) - h(q_{max}))$, serve as measures of the degree of multifractality.

MF-DFA results I

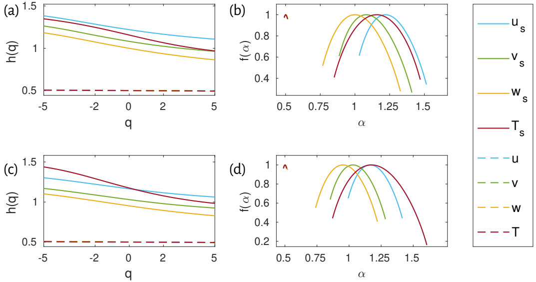

Figure 1: Representative plots of $h(q)$ versus q (a,c) and corresponding plots of $f(\alpha)$ versus α (b,d) for the original and shuffled series (dashed) from MF-DFA for at heights $z = 23.3$m (a,b) and $z = 33$m (c,d).

The shuffled series (subscript *shuff*) is monofractal, with $h(q,s) = 0.49 \pm 0.01$. We see that $\delta\alpha_{shuff} << \delta\alpha, \delta h_{shuff} << \delta h \simeq 0.3$.

This is because random shuffling destroys the long-range correlations that are present in the original time series.

MF-DFA results II

z	δh_{shuff}	δh_u	δh_v	δh_w	δh_T
$23.3m$	0.01	0.28	0.30	0.32	0.38
$33m$	0.01	0.24	0.24	0.27	0.45

Table: $\delta h = h(q_{min}) - h(q_{max})$, for different velocity components $u,\ v$, and w and the temperature T. For all these variables, the shuffled time series yields $\delta h_{shuff} \simeq 0.01$. Note that $h_u < h_v < h_w < h_T$.

Note that $f(\alpha)$ is not symmetrical about $\alpha_0 \equiv \max_\alpha[f(\alpha)]$. This is related to the levelling of Hurst surfaces in Fig. 1 (a,c), for positive q.

The multifractality of velocity and temperature time series is related to long-range correlations.

MMA results I

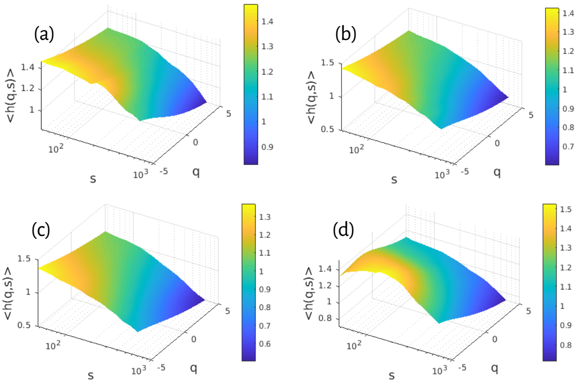

Figure 2: Representative Hurst-surface plots $h(q, s)$ for the velocity components u, v, w (a-c) and temperature (d).

MMA results II

The anisotropy of the flow is evident from the cross-sections of the Hurst surfaces.

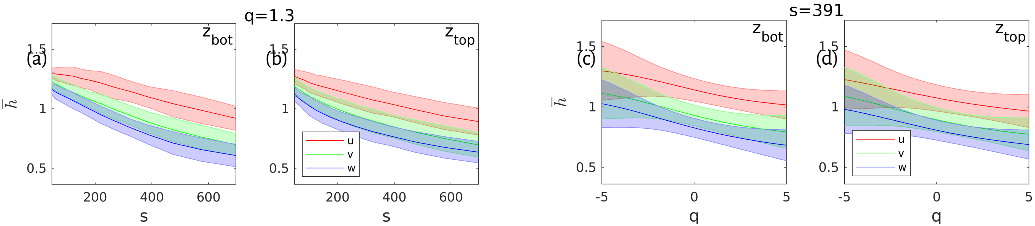

Figure 3: Cross sections of constant scale $q = 1.3$ (a,b) and $s = 391$ (c,d) of averaged Hurst surfaces (with one standard deviation error bars) corresponding to different velocity components ($h_u(q,s)$ (red), $h_v(q,s)$ (green) $h_w(q,s)$ (blue)) at different heights z_{bot}(a,c) and z_{top} (b,d) for stable stratification.

Conclusions

- We have used the MF-DFA and MMA to quantify the multifractal corrections to simple K41 scaling of structure functions in turbulent flows in the canopy sublayer over the Hyytiälä forest.
- Our multifractal analysis shows quantitatively that the anisotropy in the correlation of fluctuations in the roughness sublayer is more pronounced close to the canopy than higher up, away from the top of the canopy.
- Our plots of $h(q)$, $f(\alpha)$, and $h(q, s)$ uncover clearly the anisotropy of the multiscaling of velocity and temperature structure functions.

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Thank you for your attention.