

Does lateral mixing really take place along neutral surfaces in the double-diffusive regions of the ocean?

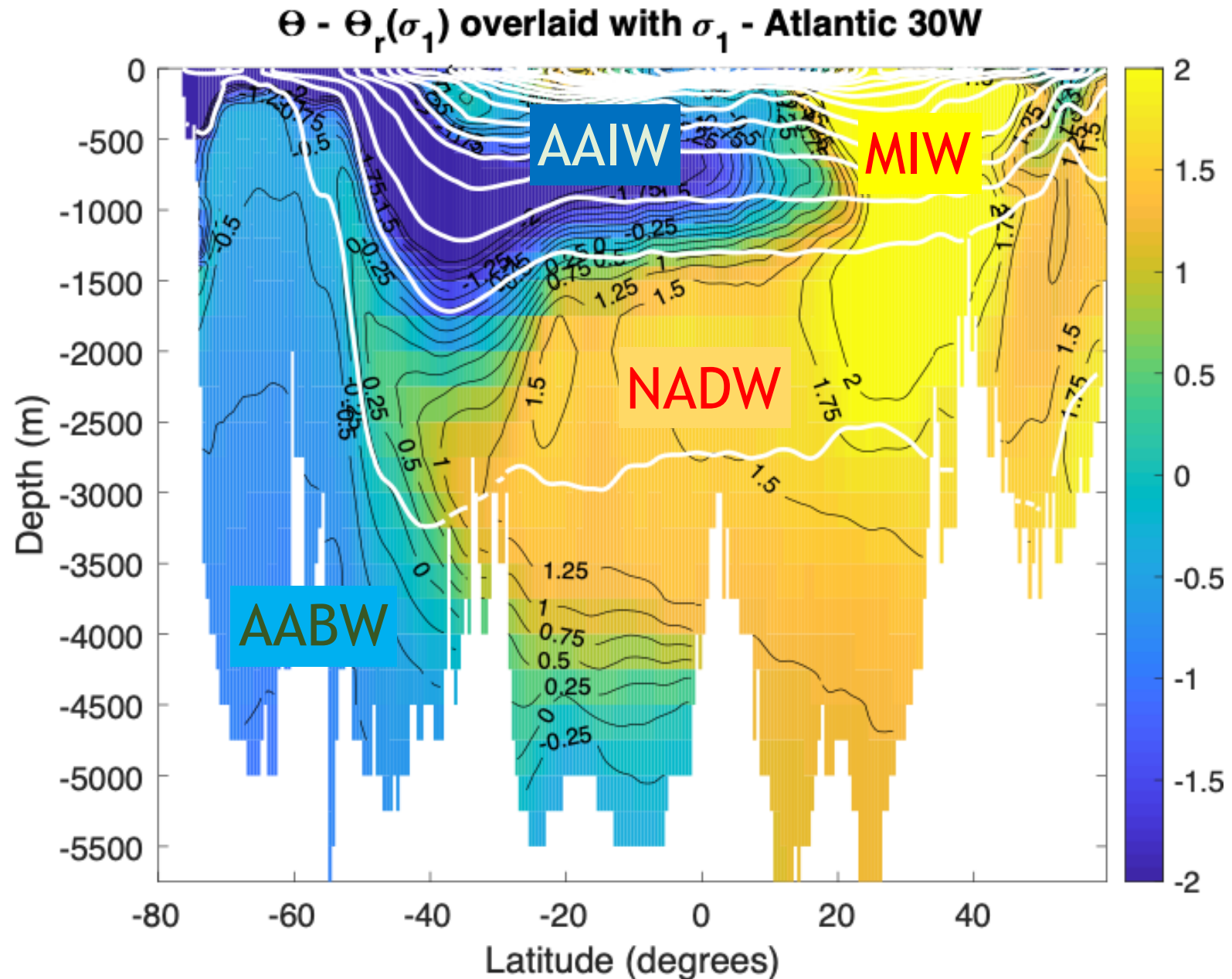
Rémi Tailleux, Gabriel Wolf, Till Kuhlbrodt, David Ferreira
Department of Meteorology, University of Reading, UK

EGU online, 04 May 2020

Evidence for
isopycnal control of
ocean water masses

Antarctic Intermediate
Water (AIW) seems to
follow isopycnal
surfaces (defined here
in terms of σ_1)

Idea dating back to
Iselin , Montgomery
etc...



Consequence for mixing and stirring of tracers

$$\overline{v'c'} = -\mathbf{K} \nabla \bar{c}$$

Redi (1982) key assumption: Symmetric part of mixing tensor \mathbf{K} is diagonalized by the isopycnal and diapycnal directions

$$\frac{1}{2}(\mathbf{K} + \mathbf{K}^T) \nabla_i \bar{c} = K_i \nabla_i \bar{c}$$


Turbulent
Isopycnal mixing
coefficient

$$\frac{1}{2}(\mathbf{K} + \mathbf{K}^T) \nabla_d \bar{c} = K_d \nabla_d \bar{c}$$


Turbulent diapycnal
mixing coefficient

Consequence for mixing and stirring of tracers

$$\overline{v'c'} = -\mathbf{K} \nabla \bar{c}$$

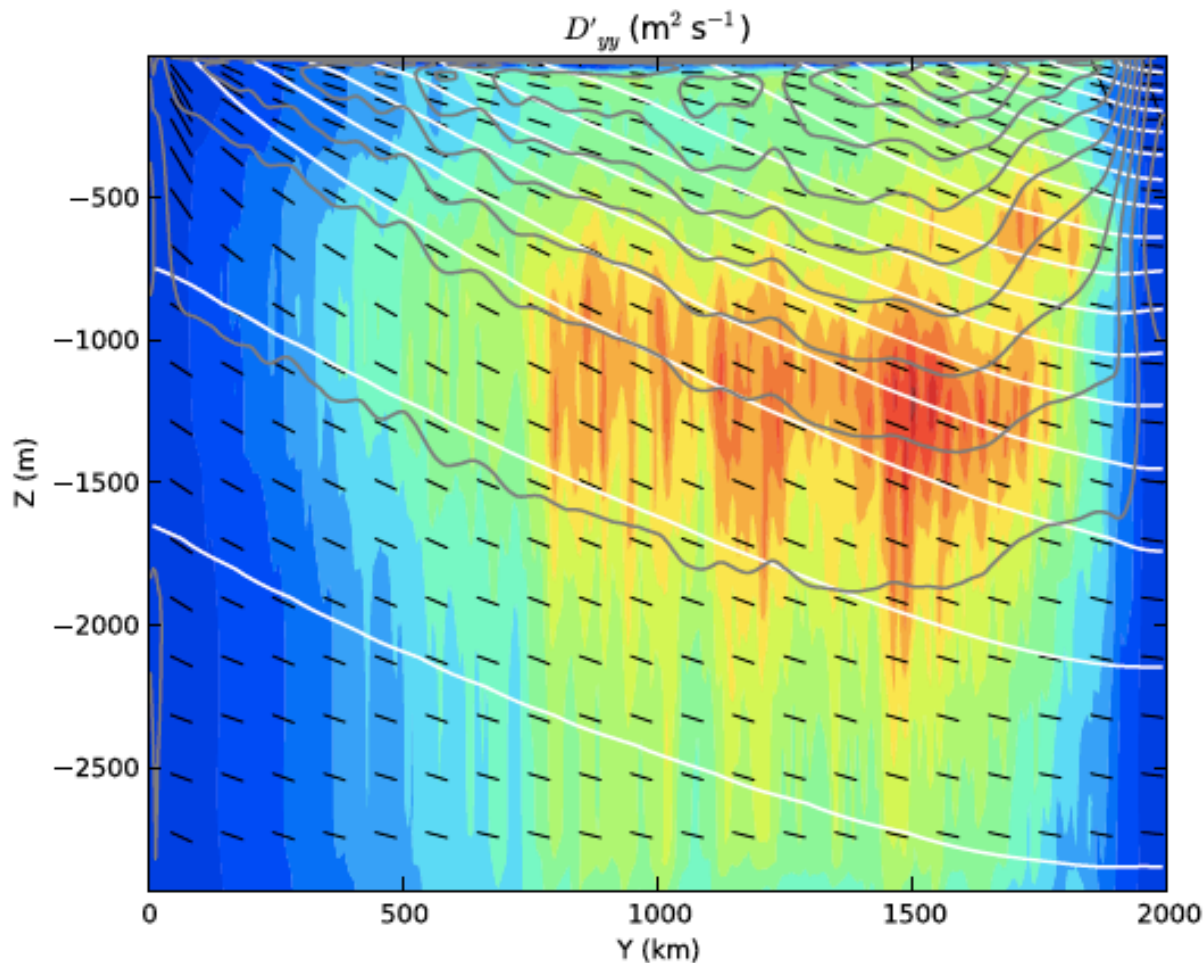
Redi (1982) key assumption: Symmetric part of mixing tensor \mathbf{K} is diagonalized by the isopycnal and diapycnal directions

$$\frac{K_i}{K_d} = O(10^7)$$

Essential to get the isopycnal directions right to avoid spurious diapycnal mixing!!!

Numerical evidence for isopycnal mixing

Abernathy, Ferreira and Klocker, 2013



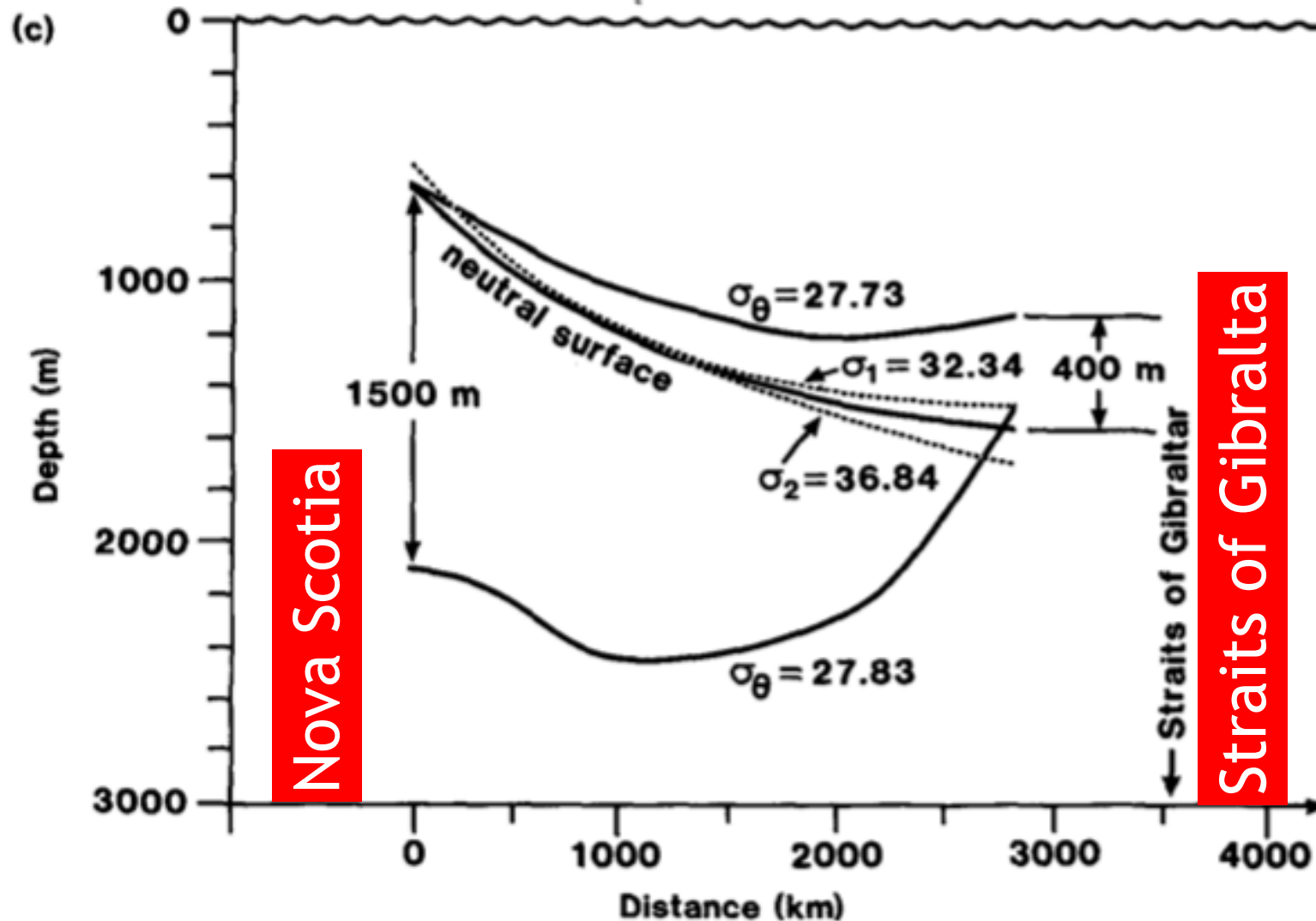
MITgcm, Linear equation of state, function of T only, no salinity

Mixing directions, obtained by diagonalizing the mixing tensor (black)

$$\begin{bmatrix} \overline{v'c'_1} & \overline{v'c'_2} & \dots & \overline{v'c'_6} \\ \overline{w'c'_1} & \overline{w'c'_2} & \dots & \overline{w'c'_6} \end{bmatrix} = - \begin{bmatrix} K_{yy} & K_{yz} \\ K_{zy} & K_{zz} \end{bmatrix} \begin{bmatrix} \partial \overline{c_1} / \partial y & \partial \overline{c_2} / \partial y & \dots & \partial \overline{c_6} / \partial y \\ \partial \overline{c_1} / \partial z & \partial \overline{c_2} / \partial z & \dots & \partial \overline{c_6} / \partial z \end{bmatrix}$$

Isopycnal directions (white)

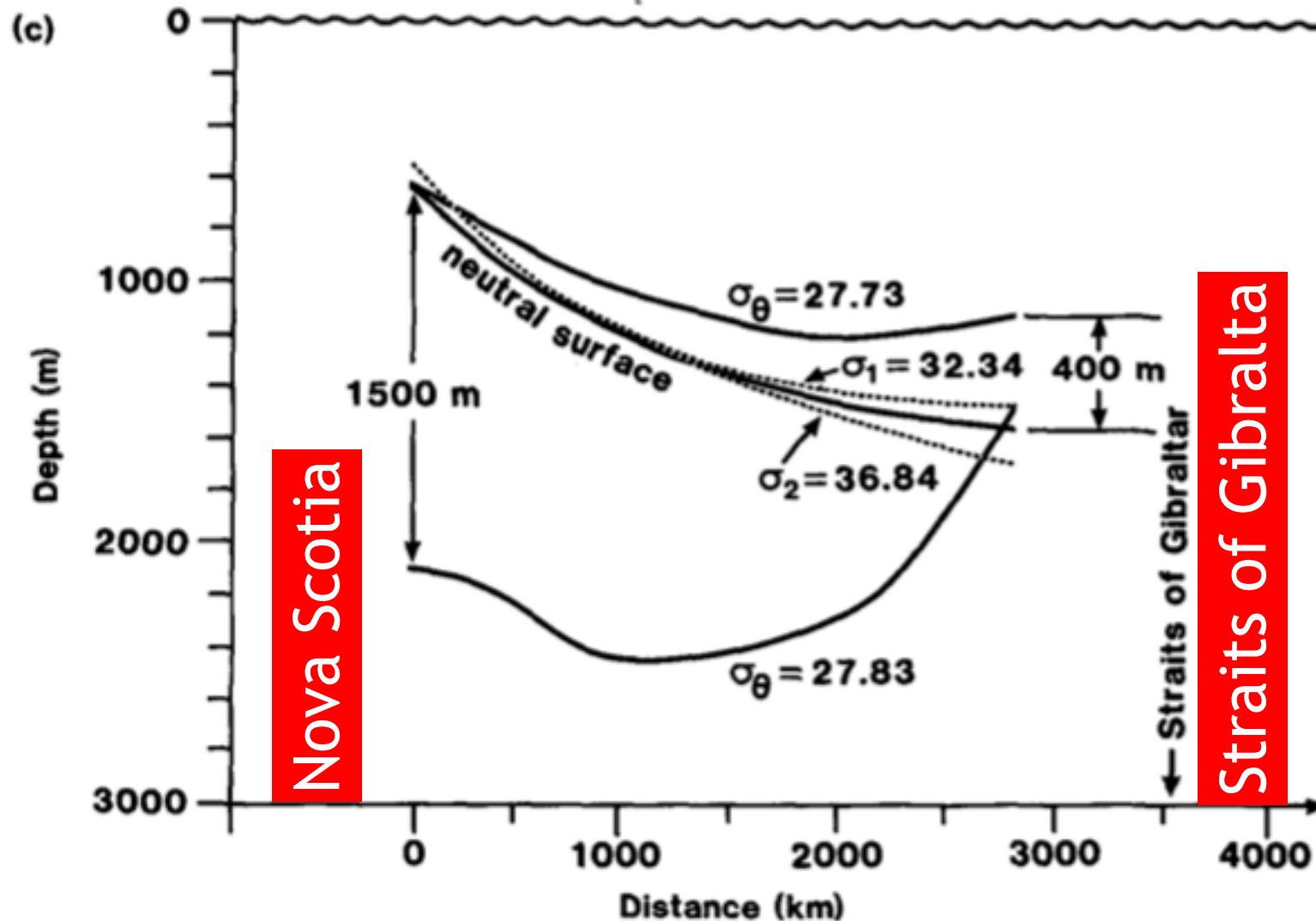
McDougall (1987): Potential density surfaces are very sensitive to choice of reference pressure and may exhibit considerable differences



Cause:
Thermobaricity

(Pressure
dependence of
thermal
expansion
coefficient)

McDougall (1987): Potential density surfaces are very sensitive to choice of reference pressure and may exhibit considerable differences



Cause:
Thermobaricity

Solution:
Neutral Density?

Neutral density: What is it?

General problem: Density-spiciness (γ, ξ) re-mapping of (S, θ) space

$$\rho = \rho(S, \theta, p) = \hat{\rho}(\gamma, \xi, p)$$

Neutral density: What is it?

General problem: Density-spiciness (γ, ξ) re-mapping of (S, θ) space

$$\rho = \rho(S, \theta, p) = \hat{\rho}(\gamma, \xi, p)$$

Neutral Density $\gamma^N \approx \gamma^N(S, \theta)$ is only density-like variable satisfying

$$\rho \approx \hat{\rho}(\gamma^N, p) \qquad \frac{\partial \hat{\rho}}{\partial \xi} \approx 0$$

Neutral density: What is it?

General problem: Density-spiciness (γ, ξ) re-mapping of (S, θ) space

$$\rho = \rho(S, \theta, p) = \hat{\rho}(\gamma, \xi, p)$$

Neutral Density $\gamma^N \approx \gamma^N(S, \theta)$ is only density-like variable satisfying

$$\rho \approx \hat{\rho}(\gamma^N, p) \quad \frac{\partial \hat{\rho}}{\partial \xi} \approx 0$$

γ^N makes seawater look like a single component fluid with all attendant benefits: materially conserved PV, perfect predictor of vertical stability and shear, zero buoyancy surface, geostrophic streamfunction, etc...

Looking for observational evidence for superiority of neutral density over potential density (Pingree 1972)

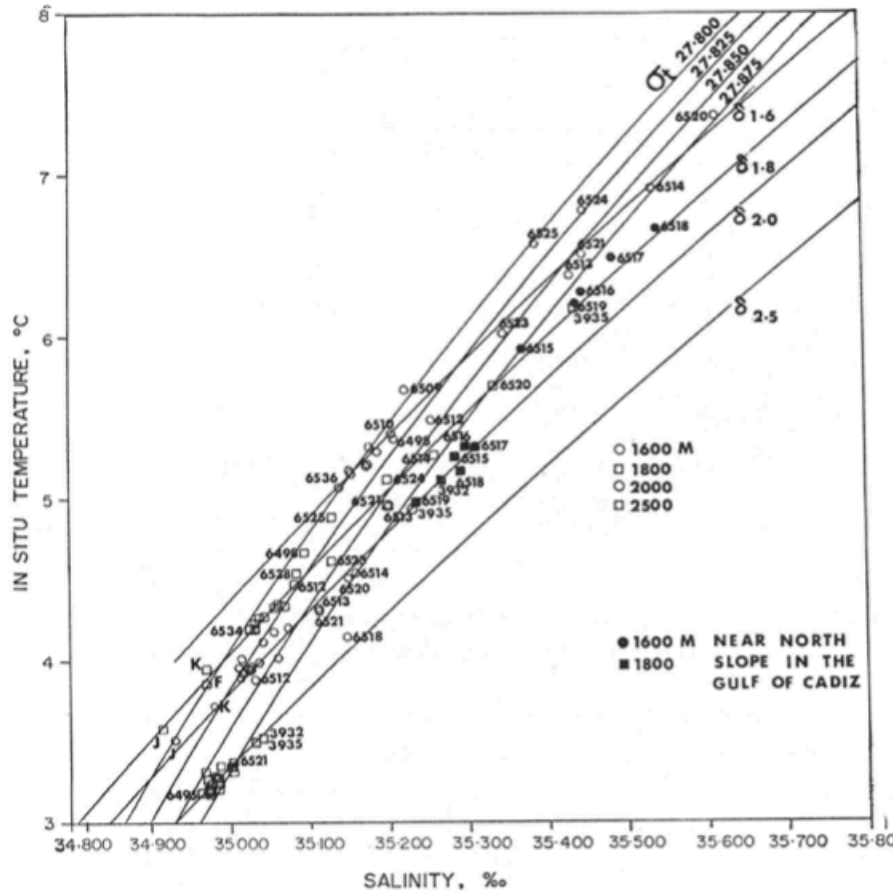


Fig. 4. T, S values for all stations shown in Fig. 1, illustrating the incorrectness of using σ_θ in these regions.



Fig. 1. Chart of the area showing the positions of *Discovery* Stations.

Spread in (θ, S) properties
reduced on potential
density surfaces referenced
to reference pressures
close to actual pressures

Question: Are neutral
surfaces minimizing spread
in (θ, S) properties?

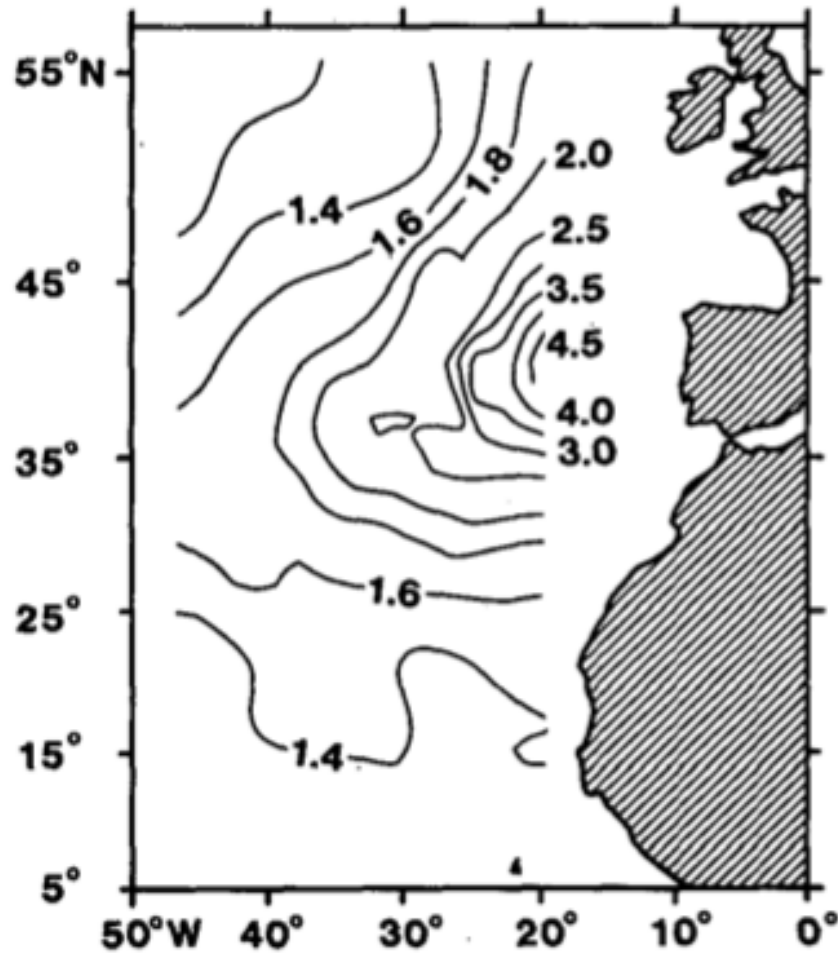


FIG. 8. Contour plot in neutral surface NSa of $c[R_p - 1]/[R_p - c]$, the ratio of the lateral gradient of θ in a potential density surface, $\nabla_\sigma \theta$, to that in the neutral surface, $\nabla_n \theta$.

McDougall (1987) analyzed the problem and discussed many examples where spread in (θ, S) properties is less on neutral surfaces than on potential density surfaces

Isopycnal gradients can be as much as 4 times larger on potential density surfaces than on neutral surfaces

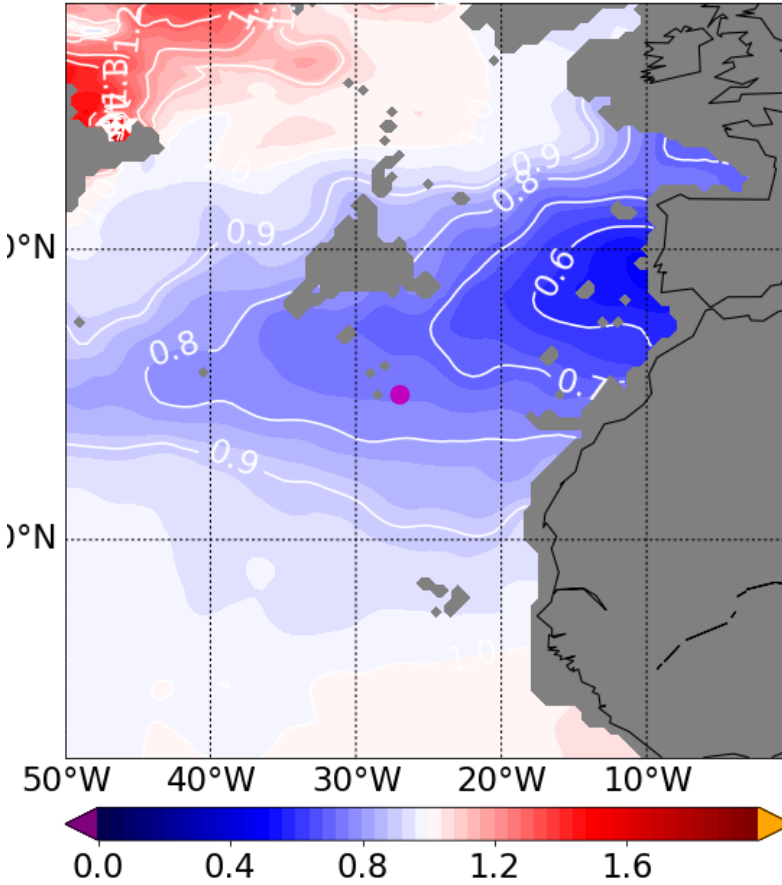
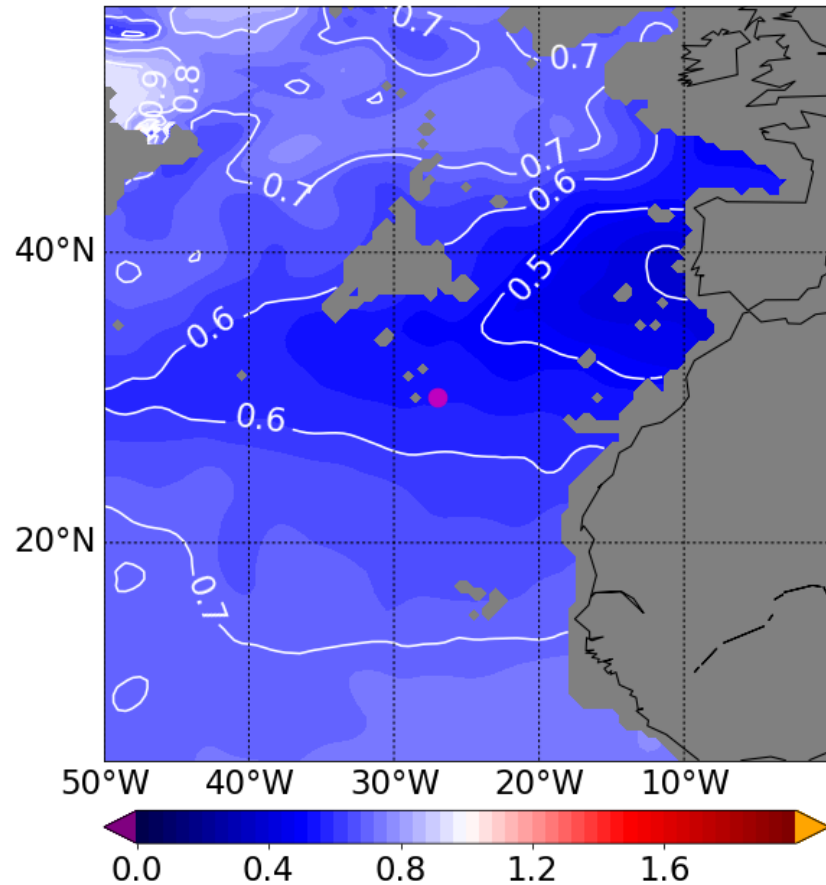
IF ONLY LIFE WAS THAT
SIMPLE...

$$\frac{\nabla_{\sigma_4} \theta}{\nabla_h \theta}$$

$$\frac{\nabla_{\sigma_4} S}{\nabla_h S}$$

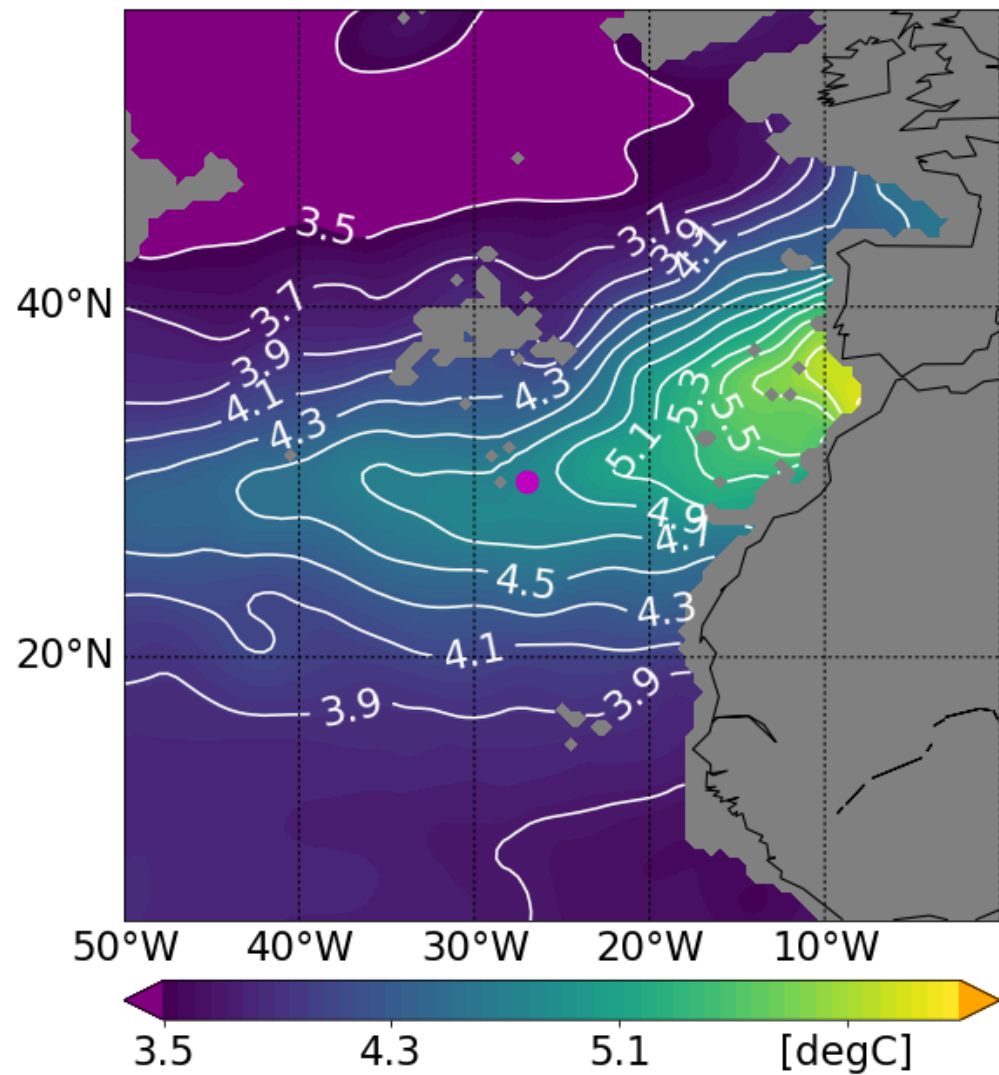
$$\gamma^N = 27.9366$$

27W, 30N, 1750 m

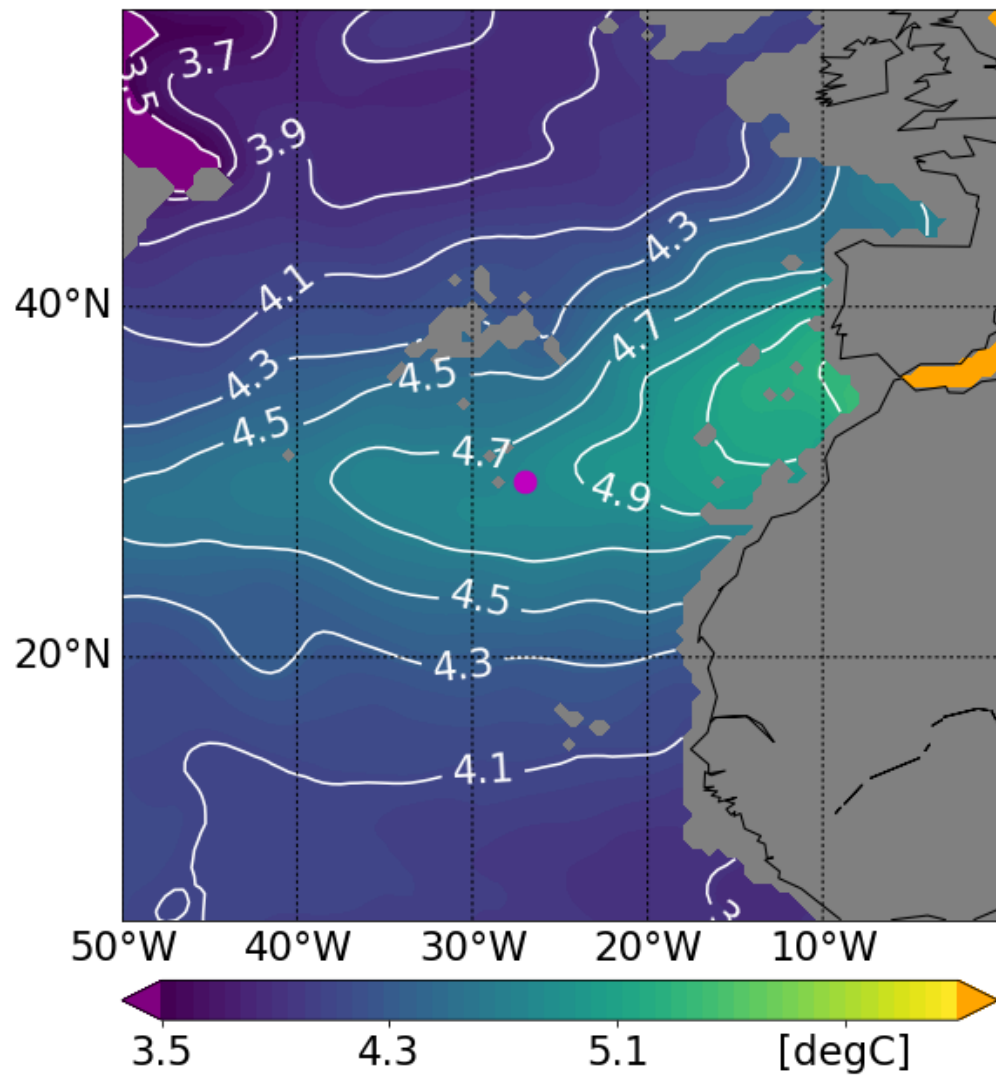


Isopycnal gradients (using σ_4) of θ and S can be reduced by as much as a factor of 2 relative to isoneutral gradients about 2000 dbar deeper than actual pressure

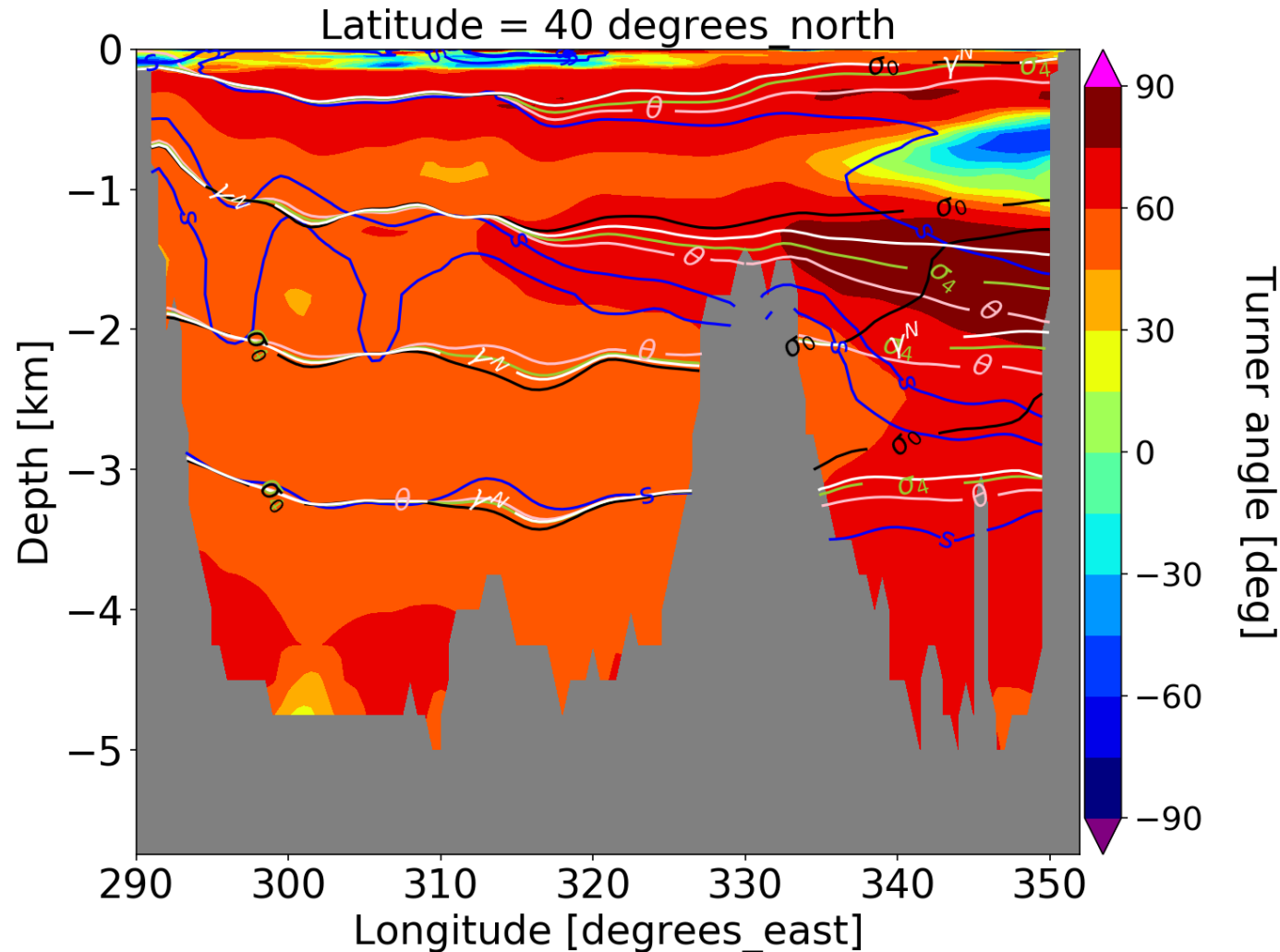
θ on $\gamma^N = 27.9366$



θ on $\sigma_4 = 45.5302$



Density surfaces and Turner angle



- Differences between density variables exacerbated in salt finger ($45 < T_u < 90$)
- Density differences minimal in doubly-stable regions ($-45 < T_u < 45$)

Theory: generalization of McDougall (1987a,b)

$$\frac{\nabla_{\gamma}\theta}{\nabla_n\theta} = \frac{c(R_{\rho} - 1)}{R_{\rho} - c}$$

$$\frac{\nabla_{\gamma}S}{\nabla_nS} = \frac{R_{\rho} - 1}{R_{\rho} - c}$$

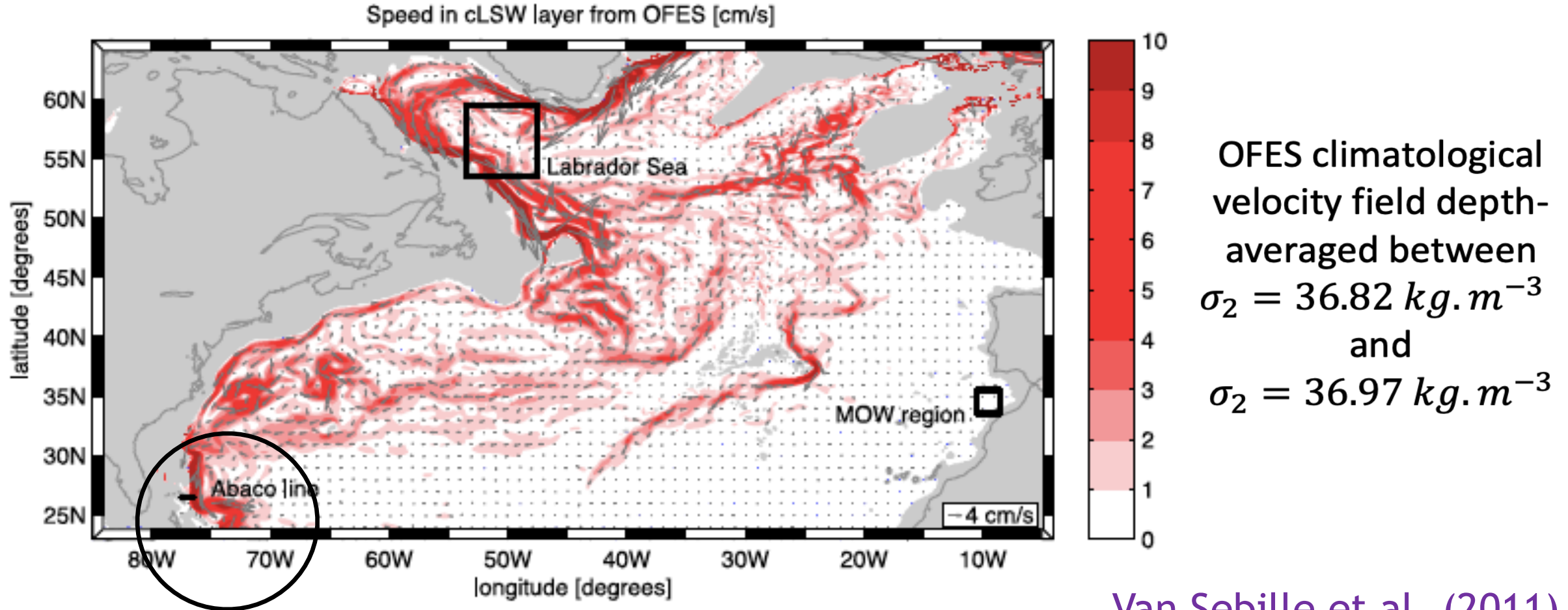
$$R_{\rho} = \frac{\alpha\theta_z}{\beta S_z}$$

$$c = \frac{\gamma_S}{\gamma_{\theta}}(S, \theta) \frac{\rho_{\theta}}{\rho_S}(S, \theta, p)$$

Doubly stable:
 γ^N better than
 $\sigma(S, \theta, p_r)$ for S but not
for θ or vice versa

Doubly diffusive:
There is always a
 $\sigma(S, \theta, p_r)$ variable
better than γ^N

Explanation for van Sebille et al. (2011) finding that σ_2 outperforms γ_N for tracing Labrador Sea Water to Abaco Line in Western Boundary Current?

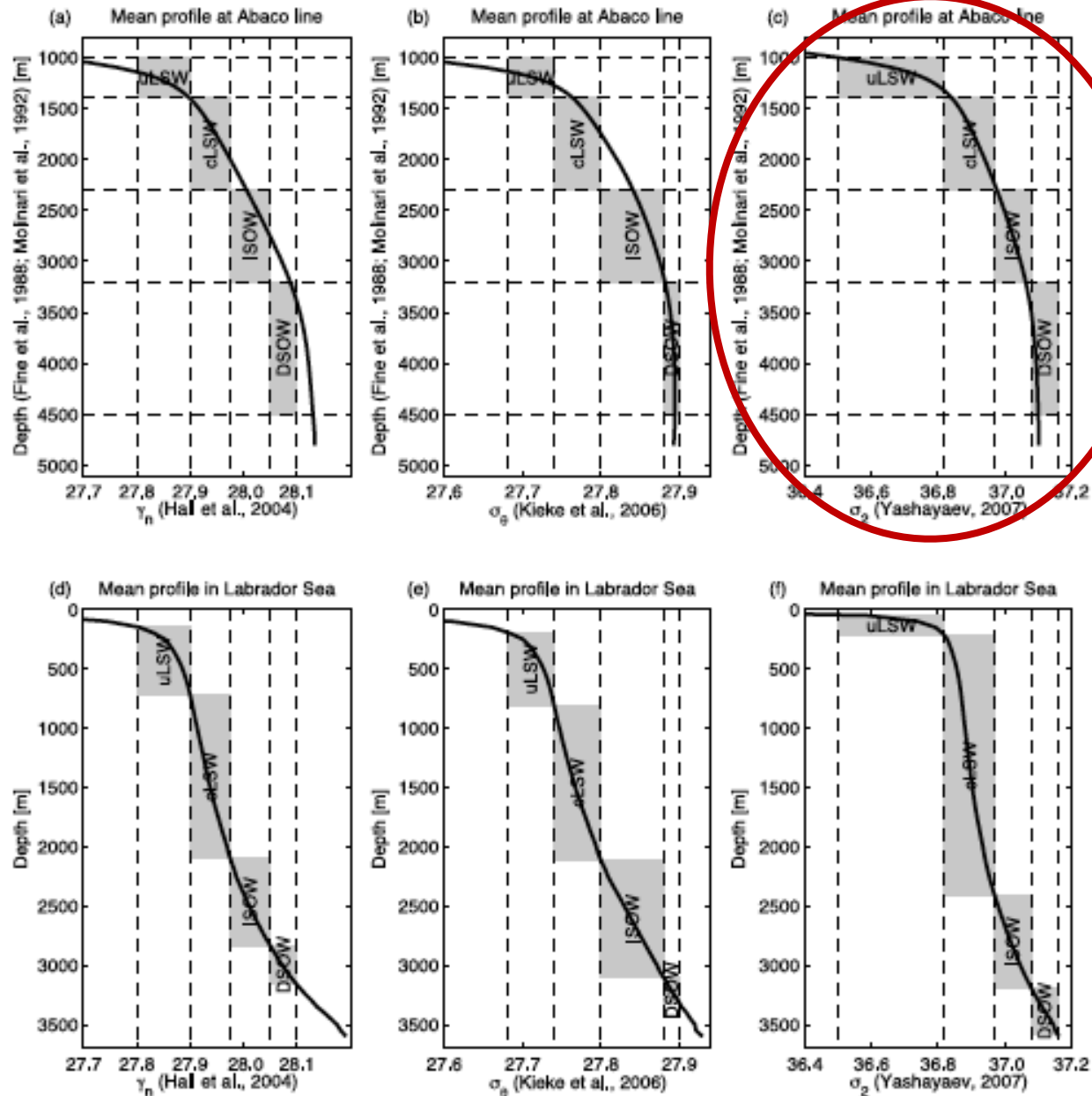


Van Sebille et al. (2011)

γ_N σ_θ σ_2

Van Seville et al. (2011)

Abaco Line



Labrador Sea

Summary

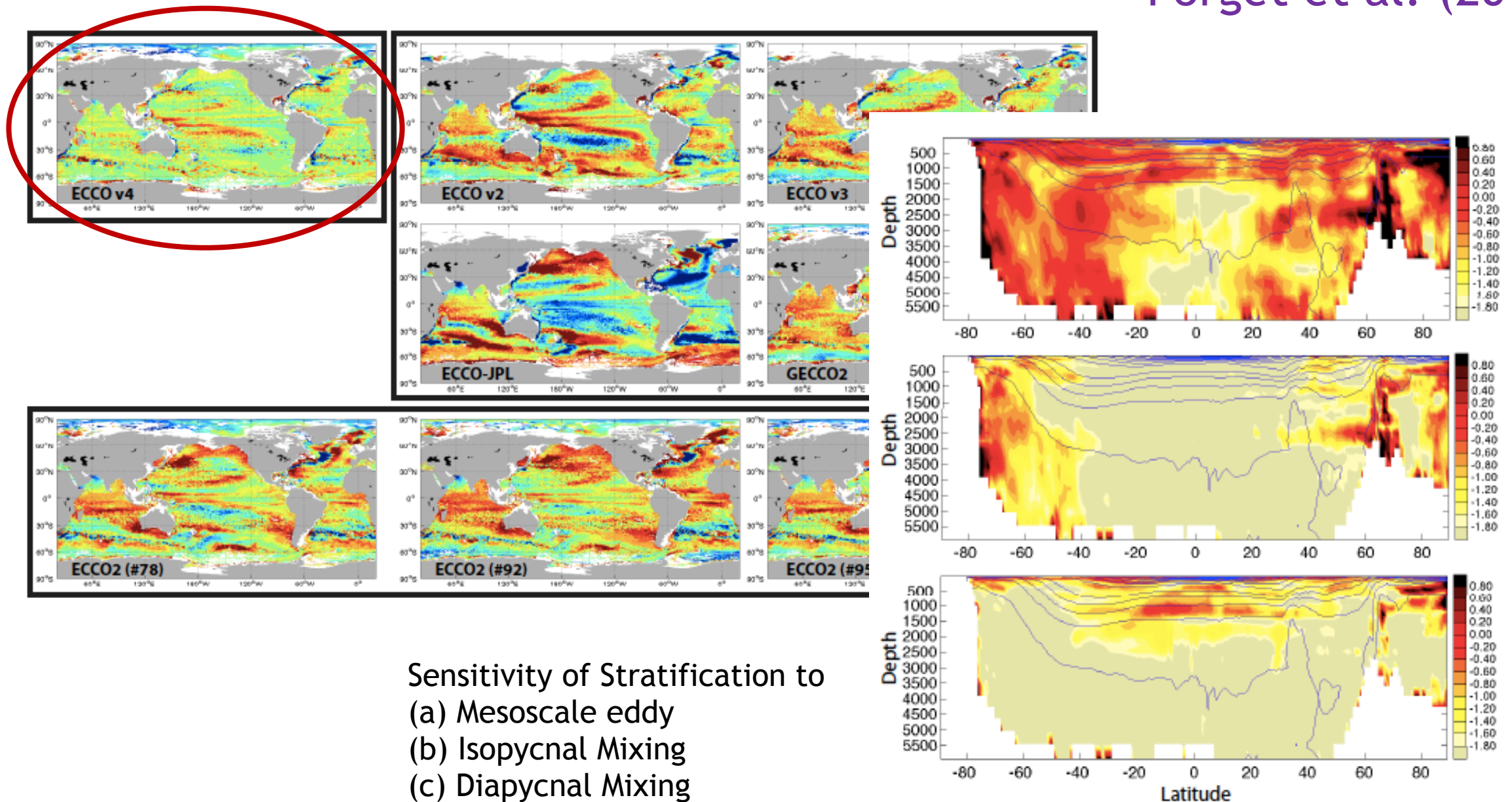
- Neutral density is optimal for making seawater look like a simple component fluid with all attendant benefits. Optimality for predicting vertical stability, vertical shear, conservative PV, etc...
 - => Justify the construction of globally defined density variable maximizing neutrality as discussed by Trevor McDougall and Geoff Stanley in next 2 talks

Summary

- Neutral density is optimal for making seawater look like a simple component fluid with all attendant benefits. Optimality for predicting vertical stability, vertical shear, conservative PV, etc...
- However, neutral density is not optimal for the following properties that are more relevant to stirring and mixing:
 - Minimising the spread in (θ, S) properties
 - Minimising the energy cost of adiabatic and isohaline parcel exchange (material surfaces exist for which energy cost is negative (Tailleux, 2016))
- Implications: Assumption that neutral rotated diffusion tensor is diagonal questionable
- Double-diffusive region: testbed for mixing parameterisations

Data/Ocean State Estimate misfit for T at 300 m

Forget et al. (2015)





“It does not matter how beautiful your theory or how smart you are. If it does not agree with experiments, it’s wrong.”

Richard P. Feynman



I would rather have questions that
can't be answered than answers that
can't be questioned.

— *Richard P. Feynman* —

AZ QUOTES