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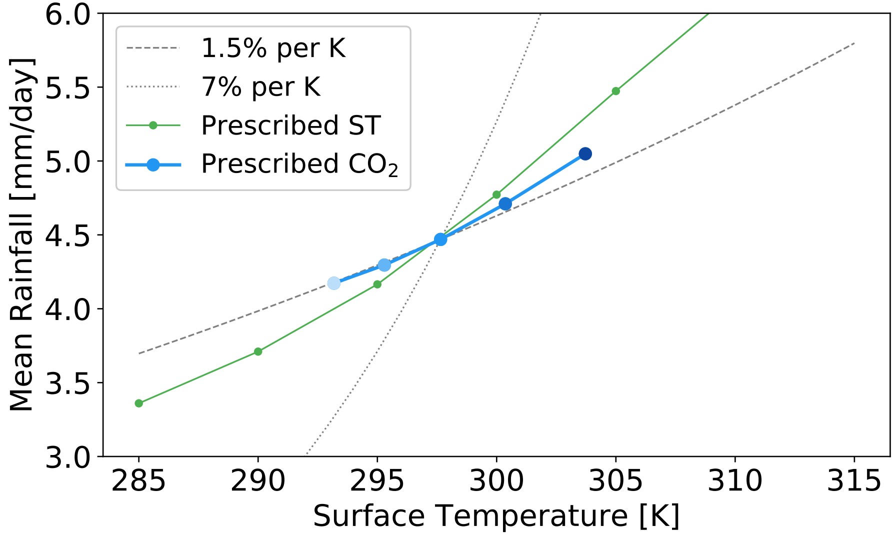
International Max Planck Research School on Earth System Modelling

A simple derivation of tropical mean precipitation and circulation changes

Method overview

Using output from konrad and a few simple assumptions such as water and energy balance, we try to explain why rainfall increases in a warming climate. Further, by thinking of the atmosphere as two distinct regions, a saturated moist region of upward motion and a subsidence region, we derive tropical mean circulation and predict how it changes to accommodate the changes in precipitation.

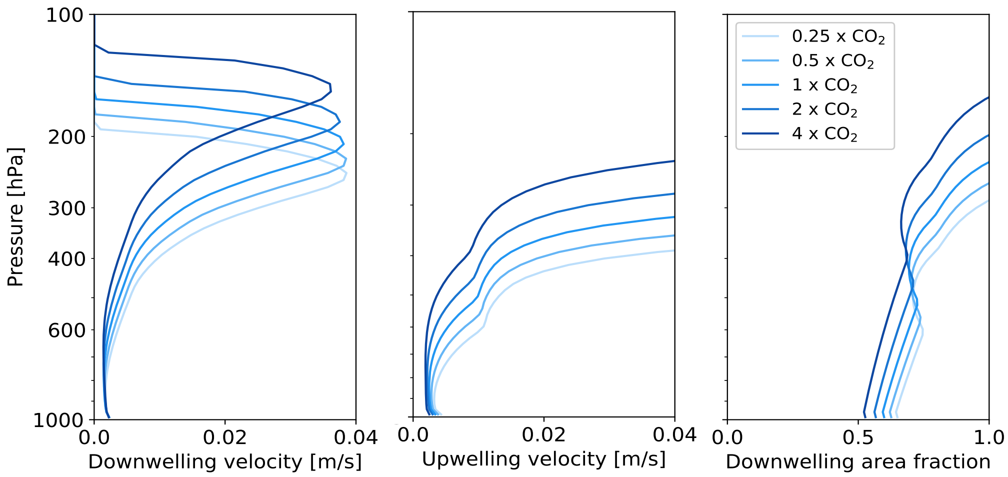
Changes in precipitation



We find precipitation increases of 2.0 to 2.7 % per Kelvin increase in surface temperature, but only 1.4 to 2.0 % /K when the increase in surface temperature is caused by CO_2.

Changes in circulation

In a warming climate we find a slowdown of the mean circulation and a decrease in subsidence area.



However, the upper troposphere is not well represented, suggesting that our conceptual picture needs to be revised.

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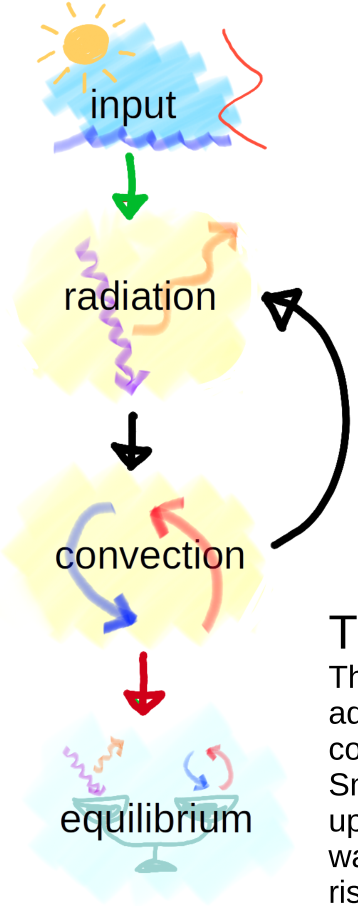
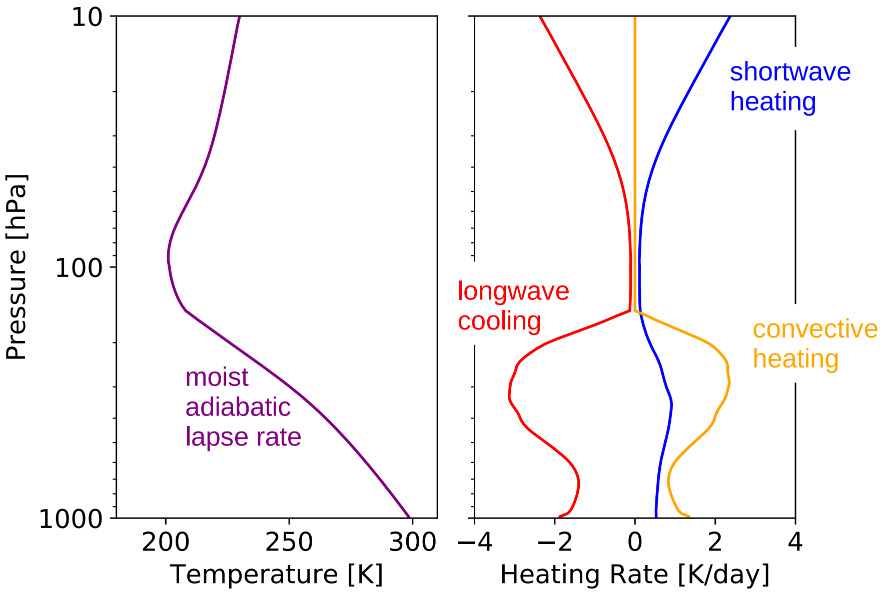
Konrad

Konrad is a 1D radiative convective equilibrium model, much like those used 50 years ago by *e.g.* Manabe and Wetherald (1967). Only two processes change the temperature in konrad, namely radiative heating rates calculated by RRTMG and our convective adjustment, which simply sets the tropospheric temperature profile to the moist adiabat. The model is run iteratively until the system reaches equilibrium and at this point the net radiative cooling rate profile is exactly balanced by the convective heating rate profile in the troposphere.

Konrad is written in python and is available on our github page. For a simple overview and to look at some output from konrad for different experiments see our konrad website.





The moist adiabat

The convective adjustment in konrad fixes the temperature profile to the moist adiabat. This assumes that the mean temperature profile is set by regions of moist convection according to the weak temperature gradient hypothesis (Bretherton and Smolarkiewicz 1989). The moist adiabat is calculated from energy balance in the upwelling regions. There, it is assumed that air is saturated and condensation of water vapour provides thermal energy as well as the energy needed for the air to rise. The air rises quickly, such that radiative heating is negligible.

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Derivation of precipitation rates

In the global mean at equilibrium, net radiative cooling of the atmosphere must be balanced by convective heating and net condensation must produce precipitation. We assume the same is true for the tropics, *i.e.* that there is no atmospheric transport of water vapour to mid latitudes. Ignoring the sensible heat flux, in the mean the convective heating rates must be provided by condensational heating. The condensation rate can then be calculated as:

$$C=\frac{Q\rho c_p}{L}$$

where Q is the heating rate, ρ air density, c_p heat capacity and L the latent heat of vapourisation of water. Then the precipitation rate is given as the integral over the troposphere of the condensation rate.

Note that because the energy used to heat the atmosphere in the convective adjustment of konrad is equal to that which cools the surface, this method is equivalent to that used in the introduction of Jeevanjee and Romps (2018), which assumes that surface evaporation is the only process providing surface cooling and that the mean evaporation rate equals the precipitation rate.

Derivation of mean circulation

We first consider the upward mass flux required to provide the water vapour for condensation.

$$C=-w_{up}A_{up}\frac{d\rho_v^*}{dz}=-w_{down}A_{down}\frac{d\rho_v^*}{dz}$$

where w_{up} is the upward velocity, A_{up} the area fraction of the upwelling region, ρ_v* the saturated water vapour mass density and z height. Here we have also assumed that mass is conserved in the tropics, *i.e.* the upward mass flux equals the downward mass flux. At this point, we need to make another assumption. The simplest is to assume that the area fractions do not change. Alternatively, we can assume that the net radiative cooling is homogeneous, such that the mean convective heating over the whole tropics is the same as that in the downwelling regions. Then we can calculate the downwelling velocity needed to provide the convective heating from adiabatic motion.

$$Q=-w_{down}\frac{dT}{dz}\frac{g}{c_p}$$

where g is Earth's gravity and the other symbols are as previously defined. Given that we take Q as the convective heating rate from konrad, we can rearrange to find the downwelling velocity and from the condensation equation above we can also find the downwelling area fraction. Then, knowing that the total area must be covered by either downwelling or upwelling regions, we can find the upwelling area fraction and from mass conservation also the upwelling velocity.

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Precipitation increase

Per Kelvin increase in surface temperature, we find precipitation increases of 2.0 to 2.7 %. When the surface temperature change is caused by CO_2 this increase is reduced to 1.4 to 2.0 % /K, in agreement with many previous studies (*e.g.* Allen and Ingram 2002, Yang et al. 2003, Romps 2020) and explained below.

CO_2 traps outgoing longwave radiation in the troposphere, making longwave cooling of the lower troposphere less efficient. If there was no change in surface temperature, at equilibrium a reduction in longwave cooling would enforce a reduction in convective heating, which in turn would lead to a reduction in condensation rate and a decrease in mean precipitation. With an increase in temperature, the expansion of the troposphere leads to an increase in precipitation (Jeevanjee and Romps, 2018). When the troposphere expands, longwave cooling occurs over a larger depth, which leads to convective heating and therefore condensation to occur over a larger depth. The mean precipitation rate, given by the integral of the condensation rate, increases. The direct radiative impact of an increase in CO_2 is a decrease in precipitation, but at equilibrium the effect of the increased surface temperature is larger than the direct radiative effect of CO_2.

Changes in circulation

Associated with the warming and increase in precipitation, the convective mass flux decreases, in agreement with Held and Soden (2006). The warmer atmosphere contains more water vapour and more water vapour condenses per unit volume of rising air, leading to the decrease in convective mass flux.

Under the fixed area assumption, this decrease is actualised by decreases in the upwelling and downwelling velocities. Under the homogeneous radiation assumption, the downwelling velocity calculated from adiabatic warming also decreases with warming throughout most of the troposphere (in pressure coordinates), as the temperature lapse rate becomes less negative with warming. The decrease in downwelling velocity alone does not cause a sufficient decrease in the downward mass flux, so the downwelling area fraction also decreases with warming. As such, the upwelling area fraction must increase and the combination of an increased upwelling area fraction and a decrease in mass flux implies a much decreased upwelling velocity (particularly in terms of fractional decrease).

Using temperature as the vertical coordinate, we find a slight increase in the density normalised mass flux (area fraction multiplied by velocity). Under the homogeneous radiation assumption, this is associated with an increase in downwelling velocity, a decrease in downwelling area, an increase in upwelling area and a slight decrease in upwelling velocity.

The methodology performs poorly in the upper troposphere, where the convective heating and thus condensation rates are large, but the mean atmosphere is cold and dry. It would likely be better represented by rare strong convective events involving warmer and moister than average air.

References

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