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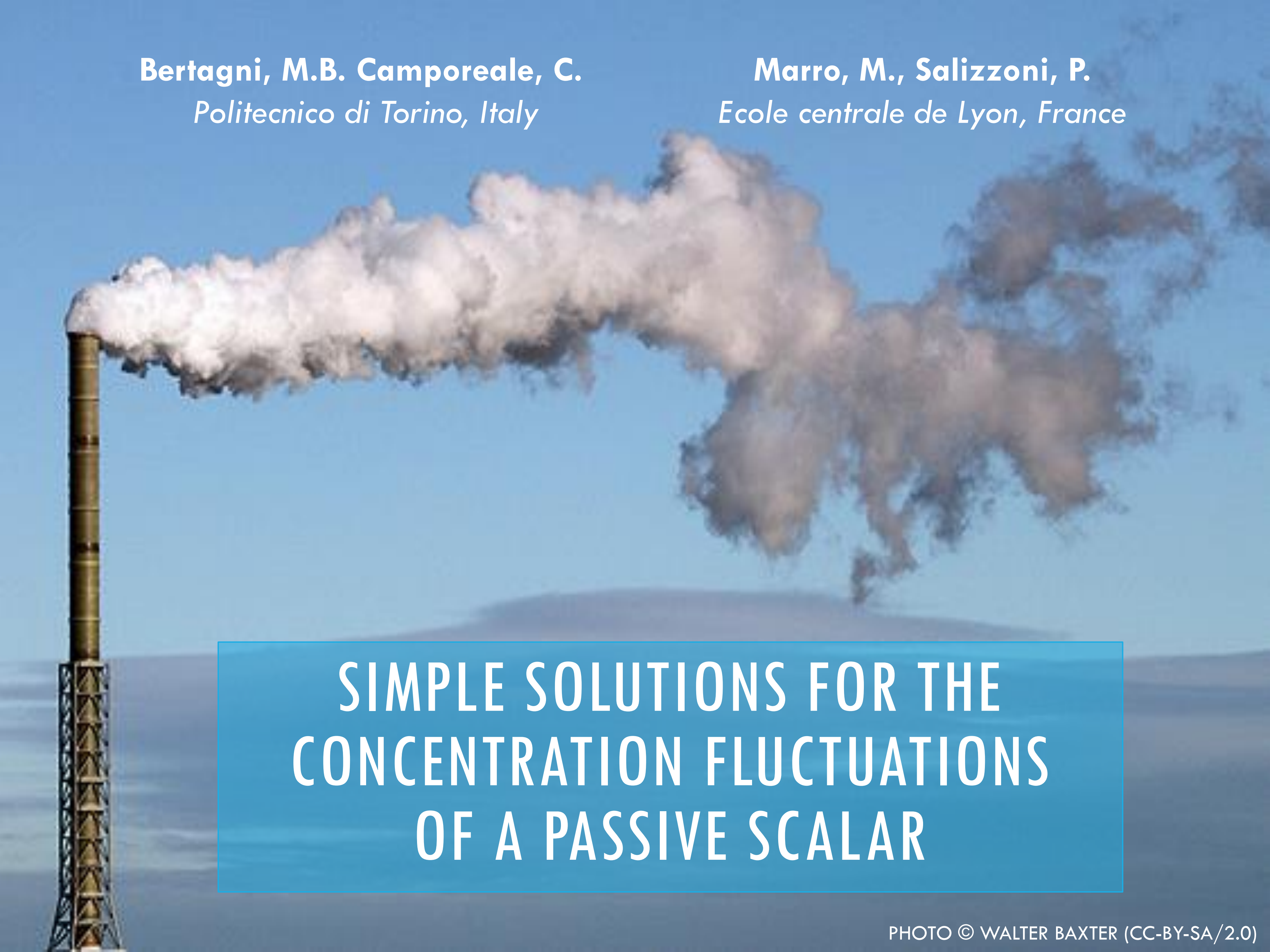

SIMPLE SOLUTIONS FOR THE CONCENTRATION FLUCTUATIONS OF A PASSIVE SCALAR

ATMOSPHERIC DISPERSION

ERUPTION OF VOLCANO ETNA, PHOTO ESA

Every day, contaminants and nuisance odors are released into the atmosphere from a variety of anthropic and natural sources. To predict their dynamics is very important for risk assessment and environmental analyses.

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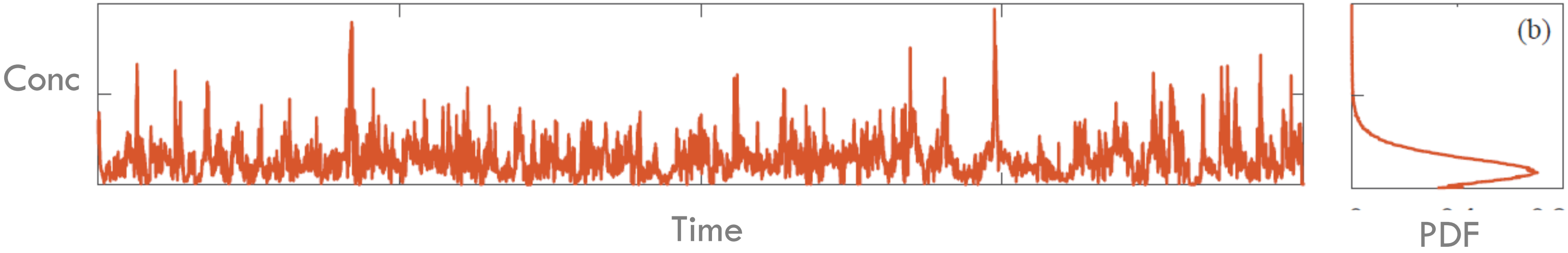
CONCENTRATION STATISTICS?



DATA FROM WIND-TUNNEL EXPERIMENTS (*NIRONI ET AL, BLM 2015)*

Contaminants released from localized sources are then dispersed by the wind in a complex turbulent dynamics, whose statistics (mean, standard deviation, PDF, level-crossings…) are difficult to address.

MATHEMATICAL APPROACH

Bertagni et al, PRF, 2019

We deal with problem starting from the transport equation of the PDF of concentration. We then derive and solve the associated equations for the statistical moments μ_n.

1. Transport equation for the PDF of concentration (Pope, 2000)

$$\underbrace{U\nabla f}_{\text{Advection}} = \underbrace{\nabla\cdot(\boldsymbol{K}\nabla f)}_{\text{Turbulent dispersion}} + \frac{1}{t_m}\underbrace{\partial_\psi\left[f\cdot(\psi - \langle C\rangle)\right]}_{\text{Mixing (IEM)}}$$

2. Dimensionless equations for the statistical moments μ_n

$$\left(\partial_X - \partial_Y^2 - \partial_Z^2 + n\,A\right)\mu_n = n\,A\,\mu_1\mu_{n-1}$$

3. The solution for the mean μ_1 = classical Gaussian model

4. A novel solution for the higher moments μ_n through a Green function G

$$\mu_n = \iiint G\,n\,A\,\mu_1\mu_{n-1}\,d\boldsymbol{X}$$

SEE BERTAGNI ET AL (2019) FOR THE NOMENCLATURE

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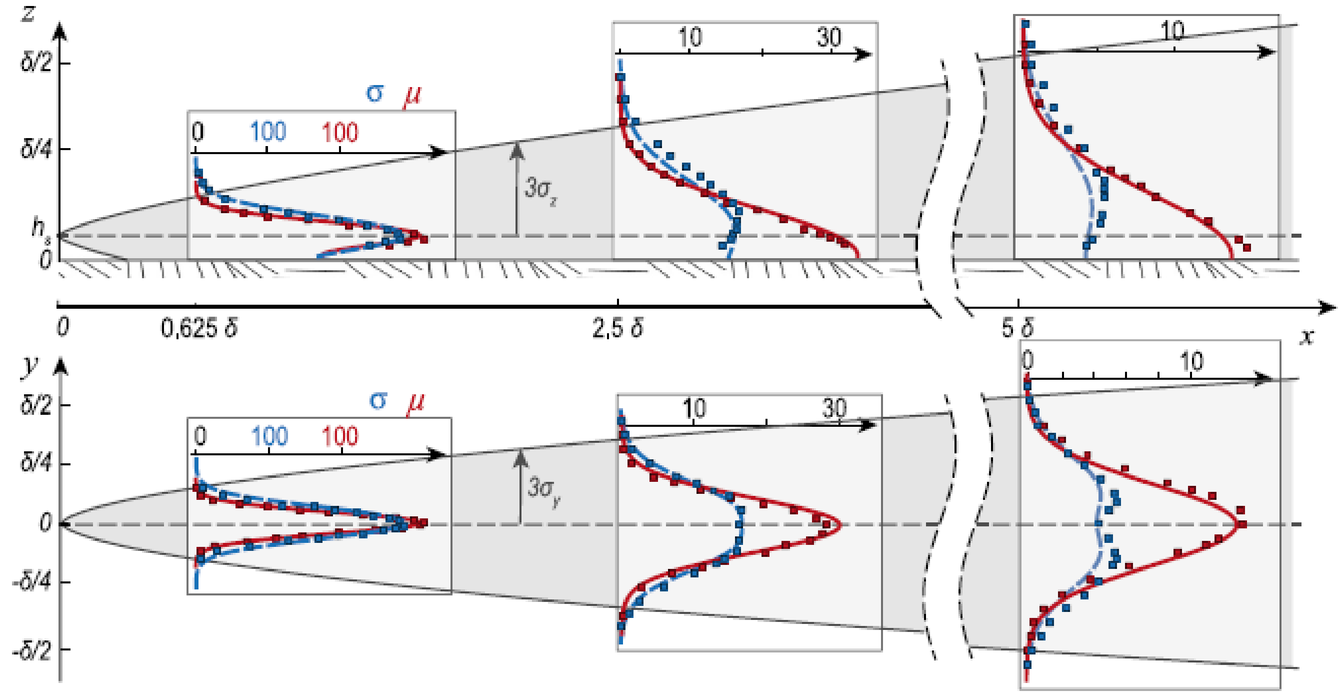
AN ANALYTICAL SOLUTION FOR μ_2

BERTAGNI ET AL, PRF, 2019

$$\mu_2 = \mu_1^2 + \sigma^2 = \int_{\xi}^{X} \left(\frac{2A\,m^2}{X_0(2X - X_0)} \exp\left[-2A(X - X_0) - \frac{Y^2 + Z^2}{2(2X - X_0)} \right] + R(\mathbf{X}, X_0) \right) \mathrm{d}X_0$$

THEORY – EXPERIMENT COMPARISON FOR THE MEAN μ_1 (GAUSSIAN MODEL) AND THE STANDARD DEVIATION σ (NOVEL SOLUTION).



The analytical solution for the second statistical moment is the first of its kind.

The assumptions are the same that characterize the Gaussian solution for the mean field, and an Interaction by Exchange with the Mean model for the mixing.

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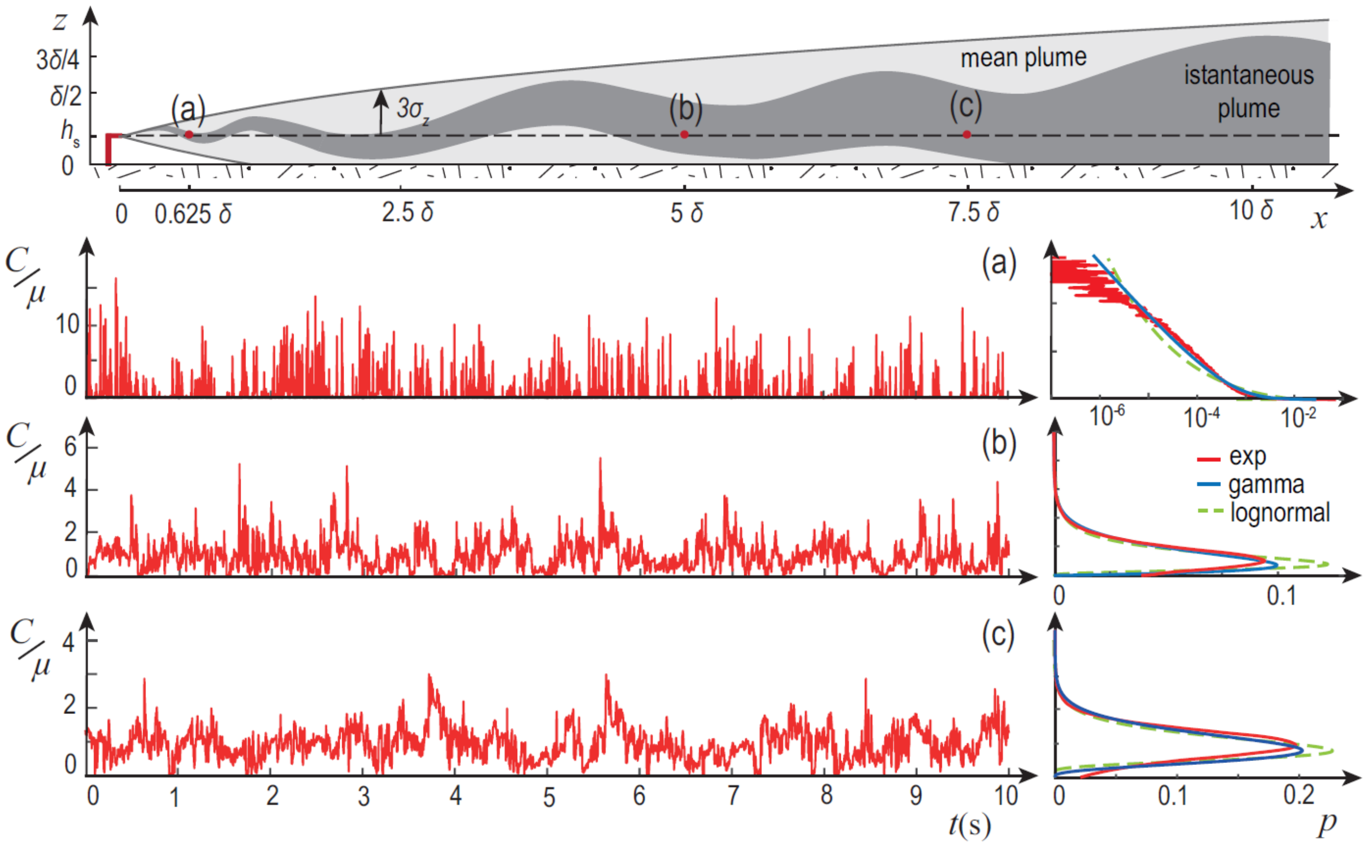
FROM μ_2 TO THE PDF (GAMMA)



In the last decades, several two-parameters distributions have been compared with laboratory and field data. The Gamma distribution has been usually verified as the best fit. With our relationships for μ_2, it is possible to define the Gamma in a completely analytical way.

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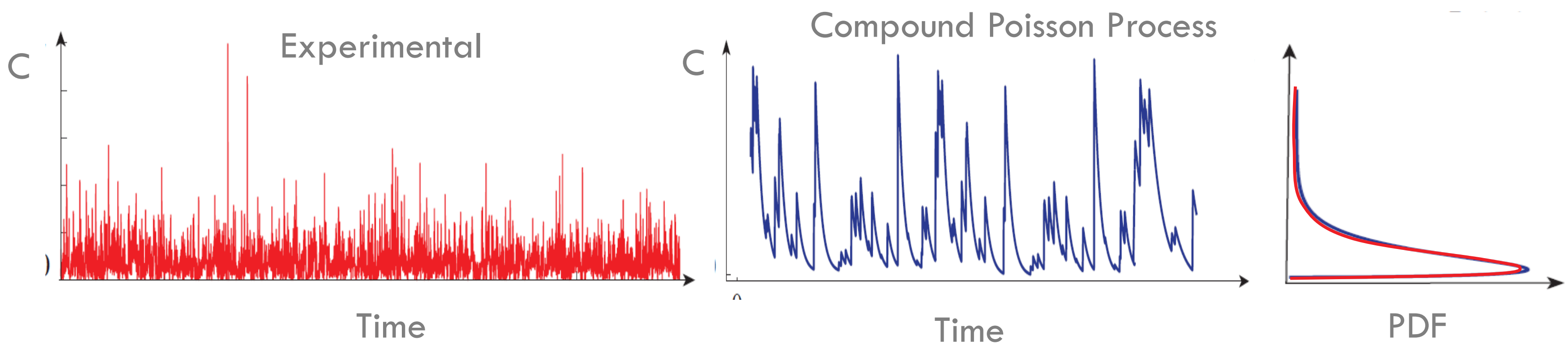
LEVEL-CROSSING STATISTICS

BERTAGNI ET AL, ATM. ENV., 2020



We define a stochastic model (CPP: Compound Poisson Process) that has a Gamma distribution as steady-state PDF. In this way, we may use the analytical relationships of the CPP for the level-crossing statistics:

$$T_C^+ = \tau e^{\lambda C/\mu_1} E\left[1-\lambda, \lambda C/\mu_1\right]$$

UPCROSSING TIME: the average time the concentration stays above the level C

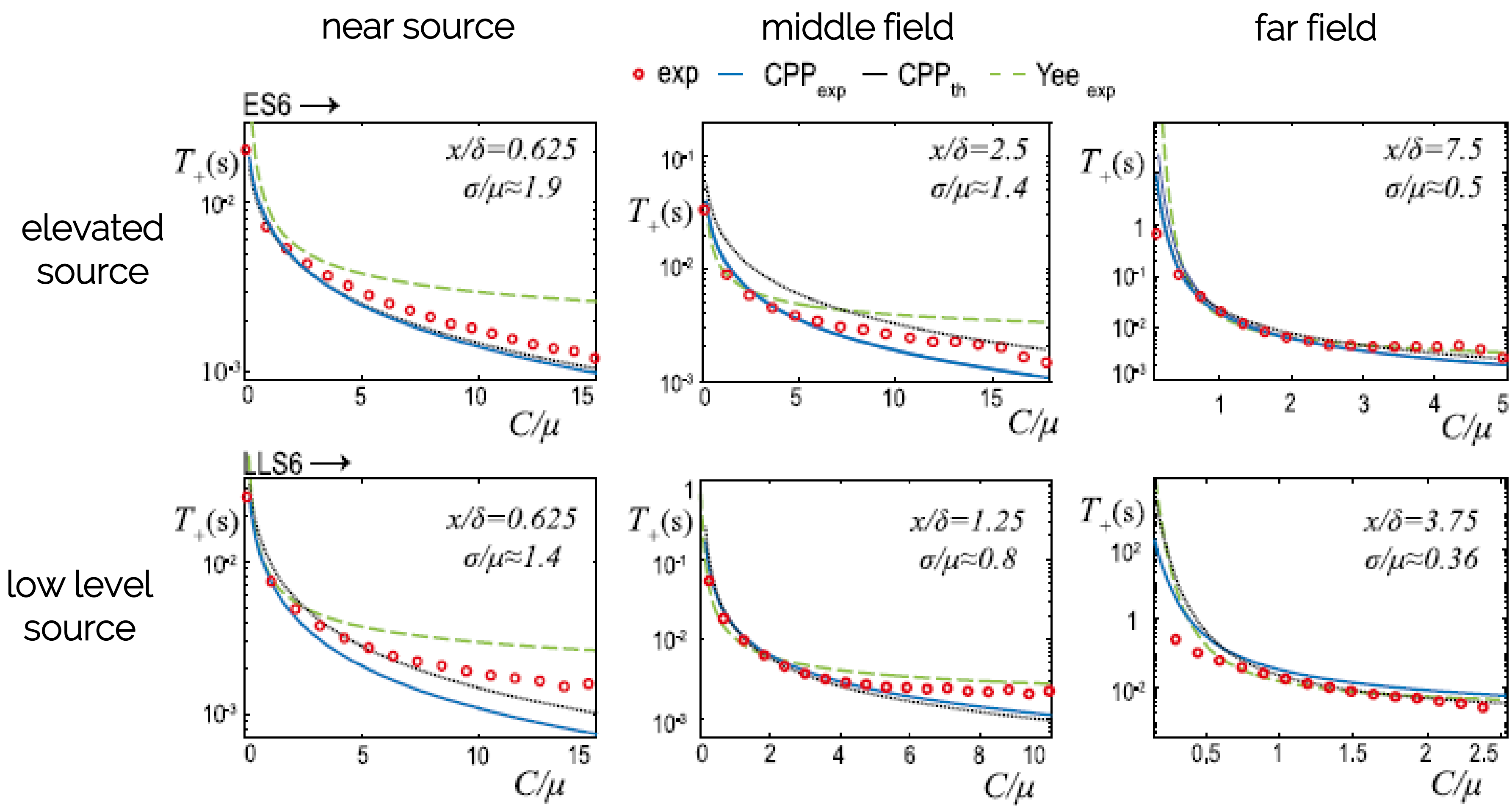
$$N_C^+ = P_C^+ / T_C^+$$

UPCROSSING RATE: the average frequency of upcrossing the level C

$\lambda = (\mu_1/\sigma)^2$ $\quad$ τ – integral scale $\quad$ P_C^+ – Probability of being above C

LEVEL-CROSSING STATISTICS

BERTAGNI ET AL, ATM. ENV., 2020



THEORY – EXPERIMENT COMPARISON FOR THE UPCROSSING TIMES

RED SYMBOLS –EXPERIMENTAL VALUES, BLUE LINES ARE THE PRESENT THEORY, DASHED GREEN LINES ARE FROM THE MODEL BY YEE (2000).

CONCLUSIONS

σ^2

Passive-scalar variance

PDF

Gamma distribution

T_C^+

Level-crossing statistics

We provide simple relationships to characterize the statistics of passive-scalar dispersion in the atmosphere from point sources. These have been verified with wind-tunnel experiments. A comparison with field data could provide useful insights.

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