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Mapping 3-D mantle electrical conductivity using Swarm, Cryosat-2 and ground observatory data

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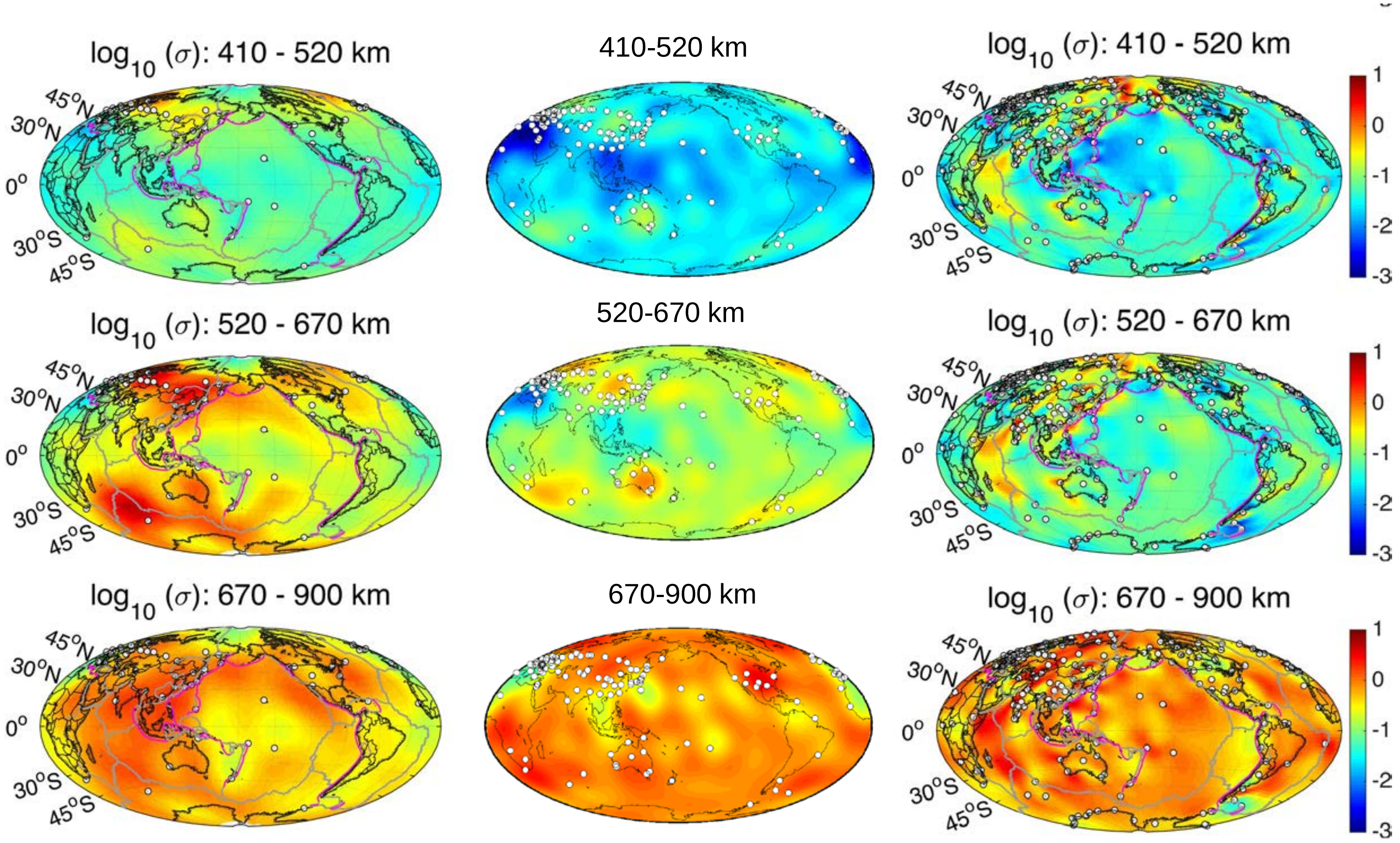
Existing global 3-D models of mantle conductivity

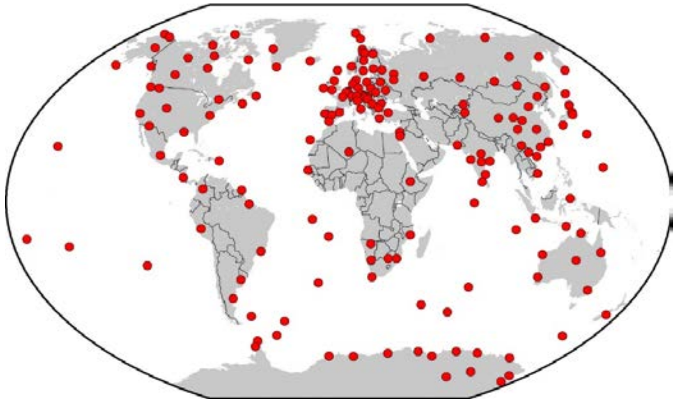


Kelbert et al, 2009 Semenov & Kuvshinov, 2012 Sun et al, 2015

- Models differ much
- Lateral variations of conductivity are (unrealistically?) large

Some details on obtaining the models

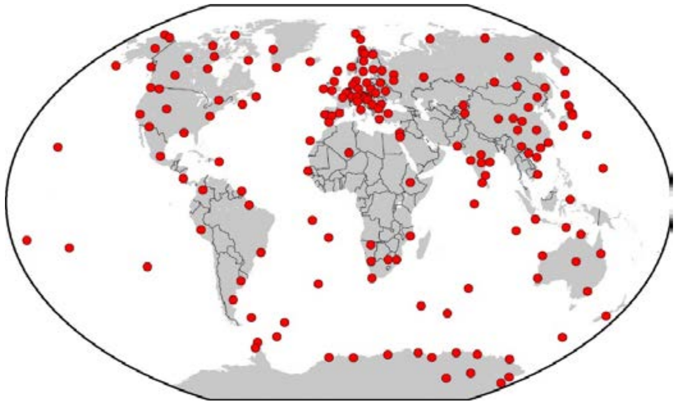
- Variations of magnetic field from global net of geomagnetic observatories are used
- Work in frequency domain
- Variations are in period range 2 days – 100 days; “mid” mantle (400 – 1600 km) is probed
- Assumption on the source – magnetospheric ring current, described near the Earth via first zonal harmonic
- This assumption allows researchers to exploit local *C*-response concept (Banks, 1969) to obtain 3-D conductivity models



Challenges

- Spatial irregularity of observatories precludes obtaining trustworthy 3-D conductivity distributions beneath the regions with the scarcity of observations (in particular, beneath the oceans)
- The ring current source is not so simple; ignoring it’s more complex spatial structure might lead to artefacts in the recovered 3-D models

Potential solution: complement observatory data with satellite data



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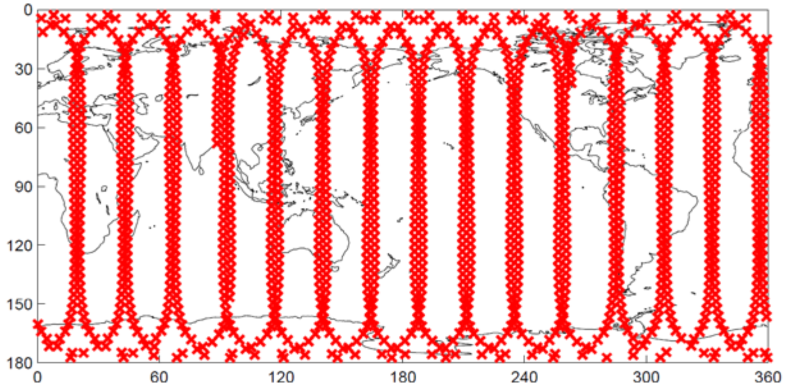


Significantly improves spatial coverage with the data. This allows: a) to specify the source more accurately; b) to obtain information about mantle structures beneath oceans

Work with satellite data (in frequency domain)

- Standard concept based on local *C*-responses does not work since satellites move in space
- Matrix Q-responses* overcome the above problem

$$\iota_k^l(\omega) = \sum_{n=1}^{N}\sum_{m=-n}^{n} Q_{kn}^{lm}(\omega)\varepsilon_n^m(\omega)$$

$$\delta\mathbf{B}(\bar{r},\omega) = -grad\left\{a\sum_{n=1}^{N}\sum_{m=-n}^{n} \varepsilon_n^m(\omega)\left(\frac{r}{a}\right)^n Y_n^m(\vartheta,\varphi) + \sum_{k=1}^{K}\sum_{l=-k}^{k} \iota_k^l(\omega)\left(\frac{a}{r}\right)^{k+1} Y_k^l(\vartheta,\varphi)\right\}$$

$\delta\mathbf{B}$ - field due to magnetospheric ring current

ε_n^m - external coefficients (due to the source)

ι_k^l - induced coefficients (due to 3-D EM induction)

Q_{kn}^{lm} - matrix Q-responses; note that for 1-D Earth: $\iota_n^m(\omega) = Q_n(\omega)\varepsilon_n^m(\omega)$

* Püthe C., A. Kuvshinov, 2014, Mapping 3-D mantle electrical conductivity from space: a new 3-D inversion scheme based on analysis of matrix *Q*-responses, *Geophys. J. Int.*, 197, 768–784.

General scheme to estimate matrix Q-responses

- Subtract from the data core, lithospheric and ionospheric fields
- Take (residual) data only from mid latitudes
- Specify *N* and *K* (maximum degrees in SH expansion of external and induced contributions)
- Estimate time series of external and induced coefficients (by least squares)
- Estimate matrix Q-responses

$$\iota_k^l(\omega) = \sum_{n=1}^{N} \sum_{m=-n}^{n} Q_{kn}^{lm}(\omega)\varepsilon_n^m(\omega)$$

$$\delta\mathbf{B}(\bar{r},\omega) = -grad\left\{a\sum_{n=1}^{N}\sum_{m=-n}^{n} \varepsilon_n^m(\omega)\left(\frac{r}{a}\right)^n Y_n^m(\vartheta,\varphi) + \sum_{k=1}^{K}\sum_{l=-k}^{k} \iota_k^l(\omega)\left(\frac{a}{r}\right)^{k+1} Y_k^l(\vartheta,\varphi)\right\}$$

Actual implementation

- Data are 5 years of Swarm (A/B/C) + Cryosat-2 + observatory measurements
- Subtract from the data the CM* core, lithospheric and ionospheric fields
- Take (residual) data from $\pm 55^0$ geomagnetic latitudes
- Specify *N = 2* and *K = 3*
- Estimate time series of external and induced coefficients with cadence of 6 hours
- Estimate matrix Q-responses at 16 periods between 2 and 31 days

$$\iota_k^l(\omega) = \sum_{n=1}^{2} \sum_{m=-n}^{n} Q_{kn}^{lm}(\omega)\varepsilon_n^m(\omega)$$

$$\delta\mathbf{B}(\bar{r},\omega) = -grad\left\{ a\sum_{n=1}^{2}\sum_{m=-n}^{n} \varepsilon_n^m(\omega)\left(\frac{r}{a}\right)^n Y_n^m(\vartheta,\varphi) + \sum_{k=1}^{3}\sum_{l=-k}^{k} \iota_k^l(\omega)\left(\frac{a}{r}\right)^{k+1} Y_k^l(\vartheta,\varphi)\right\}$$

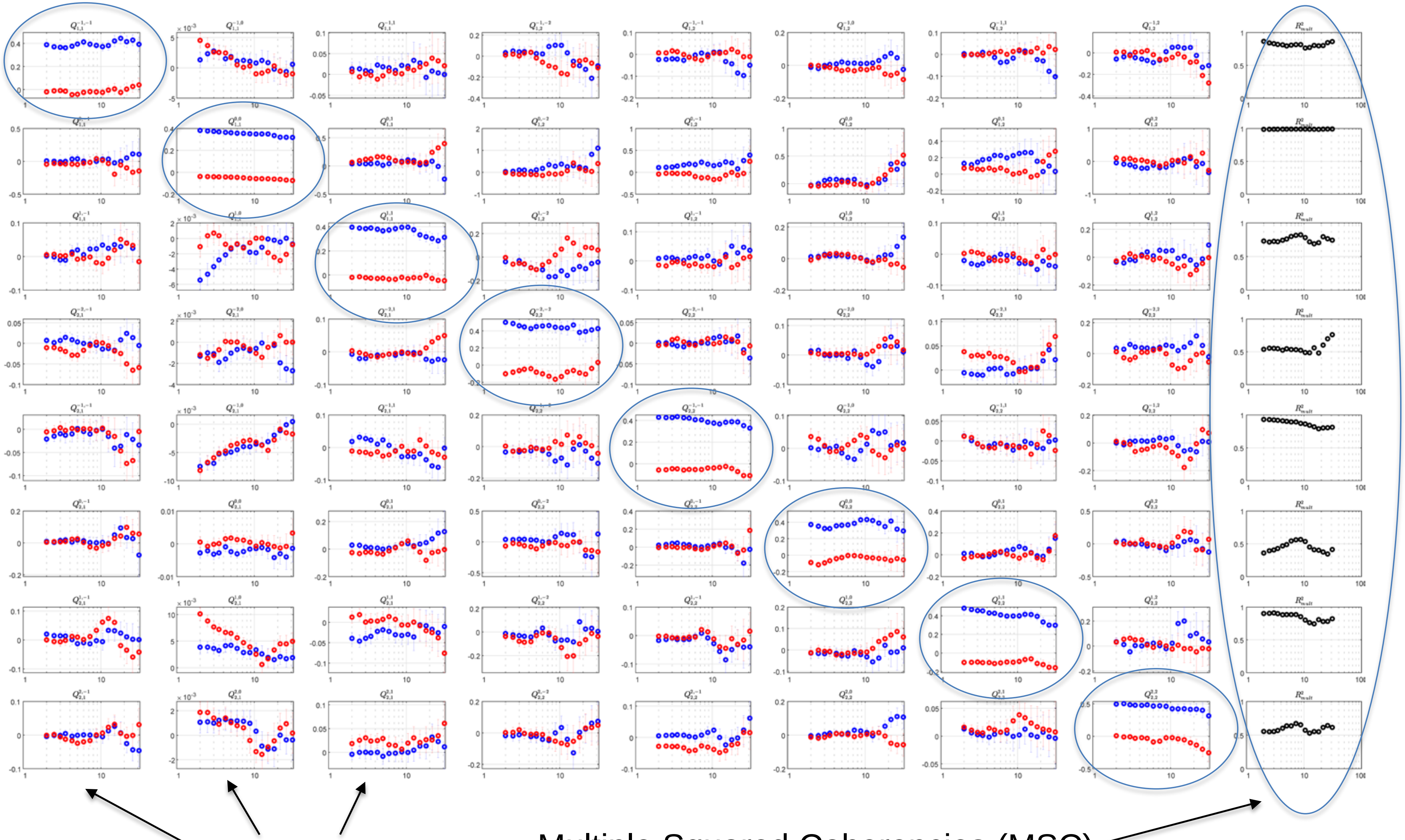
* Sabaka T., L. Tøffner-Clausen, N. Olsen, and C. C. Finlay. 2018. "A comprehensive model of Earth’s magnetic field determined from 4 years of Swarm satellite observations." Earth, Planets and Space, 70 (1).

As a result we estimate N(N+2) x K(K+2) = 8 x 15 elements of Q-matrices at 16 periods (N =2; K = 3)

$$\text{3 x 5 = 15}\left\{\begin{pmatrix} \iota_1^{-1}(\omega_j) \\ \iota_1^{0}(\omega_j) \\ \dots \\ \iota_3^{3}(\omega_j) \end{pmatrix}\right. = \overbrace{\begin{pmatrix} Q_{11}^{-1-1}(\omega_j) & Q_{11}^{-10}(\omega_j) & \dots & Q_{12}^{-12}(\omega_j) \\ Q_{11}^{0-1}(\omega_j) & Q_{11}^{00}(\omega_j) & \dots & Q_{12}^{02}(\omega_j) \\ & \dots & & \\ Q_{31}^{3-1}(\omega_j) & Q_{31}^{30}(\omega_j) & \dots & Q_{32}^{32}(\omega_j) \end{pmatrix}}^{\text{2 x 4 = 8}} \left.\begin{pmatrix} \varepsilon_1^{-1}(\omega_j) \\ \varepsilon_1^{0}(\omega_j) \\ \dots \\ \varepsilon_2^{2}(\omega_j) \end{pmatrix}\right\}\text{2 x 4 = 8}$$

$$j = 1, 2, \dots, 16$$

The best resolved are diagonal elements



Multiple squared coherencies (MSC) is a measure that estimates the extent to which the output (time series of induced coefficients of a given degree k and order l) can be predicted from the input (time series of external coefficients up to degree 2) using a linear model

The closer MSC to 1 the better determination/separation of external and induced coefficients

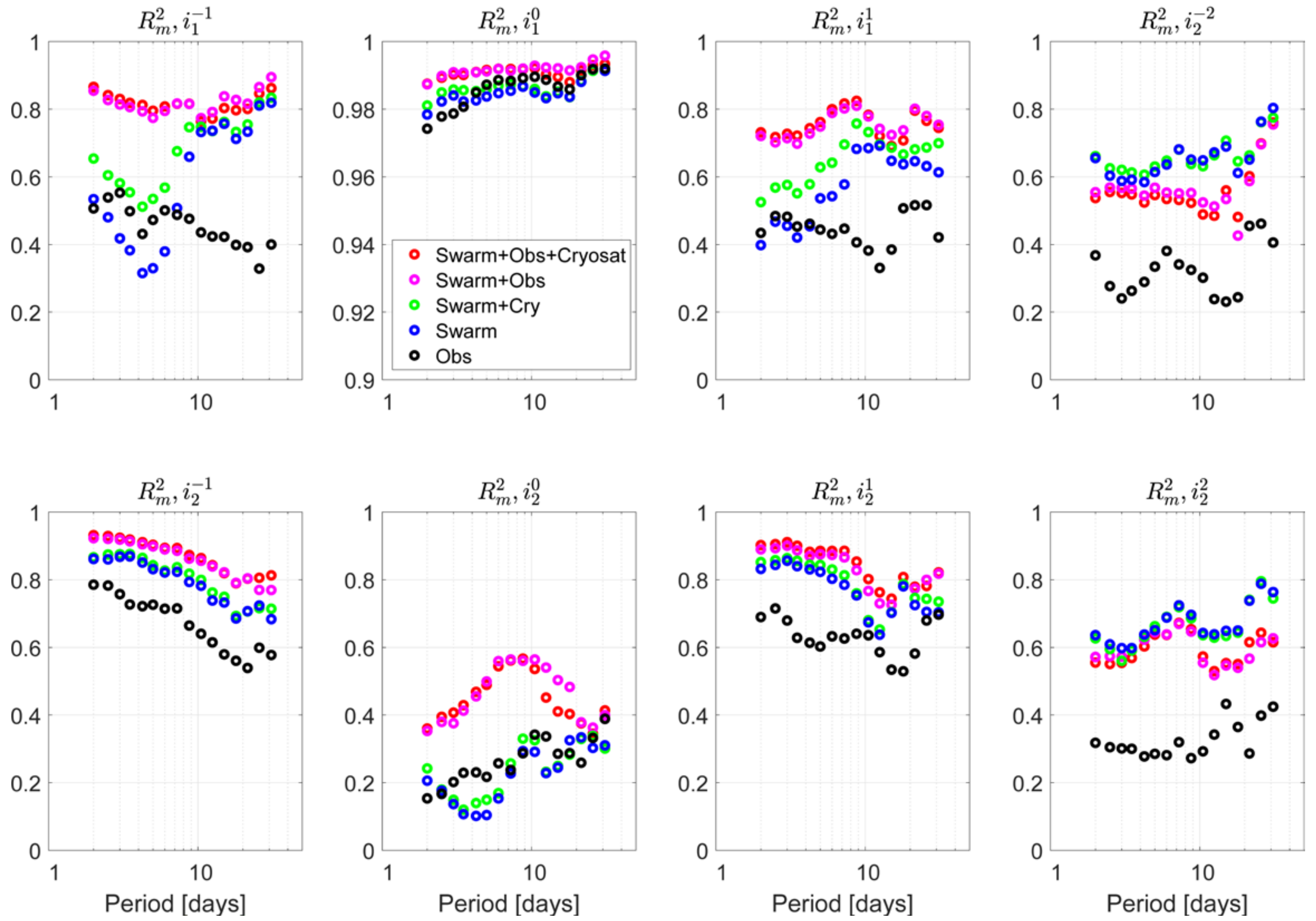
MSC for different data sets

Multiple squared coherencies (MSC) for induced coefficients up to k = 2 and up to l = 2 for different data sets

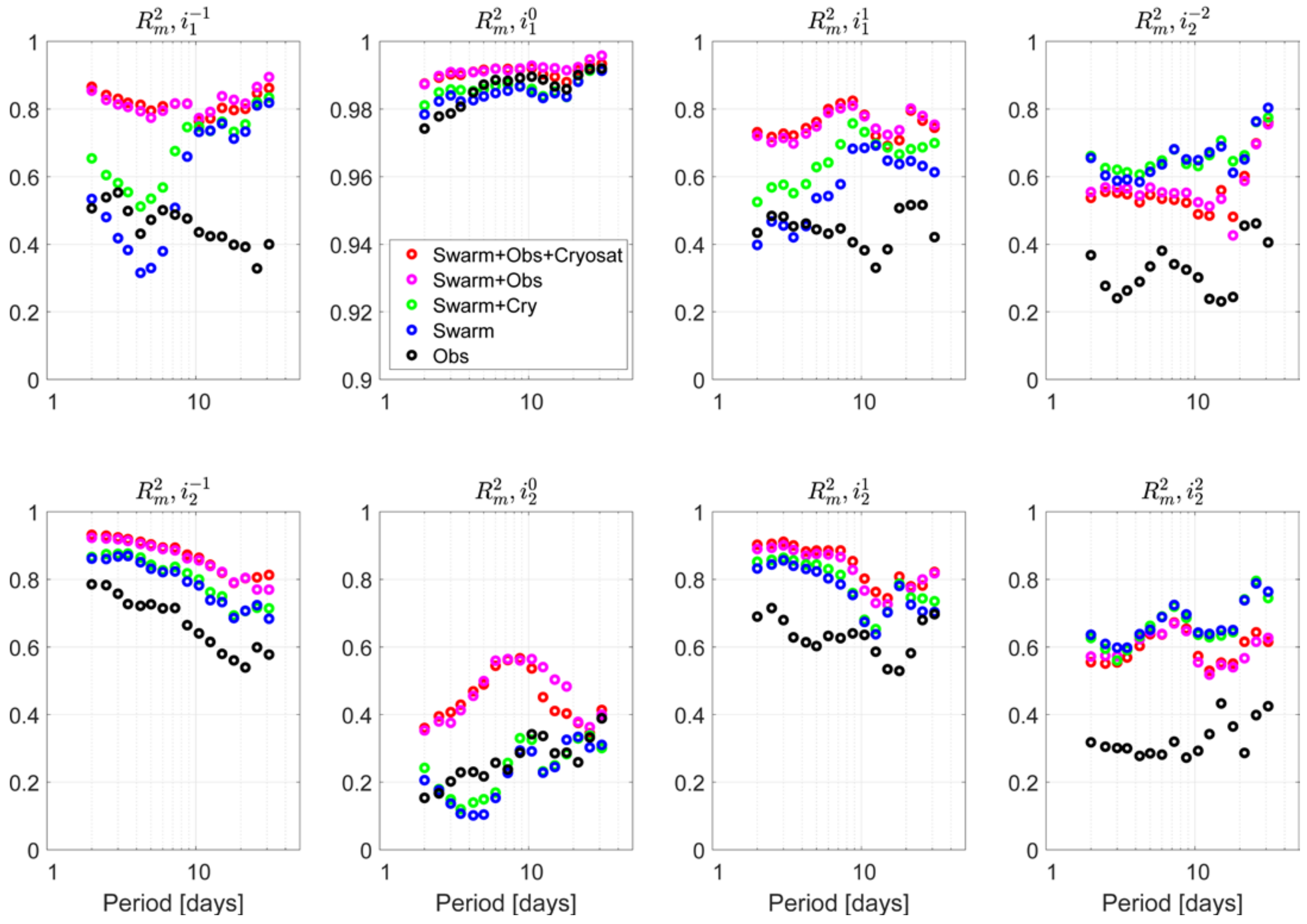
$$\iota_k^l(\omega) = \sum_{n=1}^{2}\sum_{m=-n}^{n} Q_{kn}^{lm}(\omega)\,\varepsilon_n^m(\omega)$$

$$k \le 2,\; l \le 2$$

Note different scale for $\iota_1^0(\omega)$



More on these plots



- The smallest MSC is for the scenario when only observatory data are used to separate external and induced coefficients
- The largest MSC is for the scenario when both observatory and satellite data are used
- If observatory data are not included, adding Cryosat to *Swarm* data improves MSC
- If observatory data are included, adding Cryosat to *Swarm* and observatory data does not improve MSC

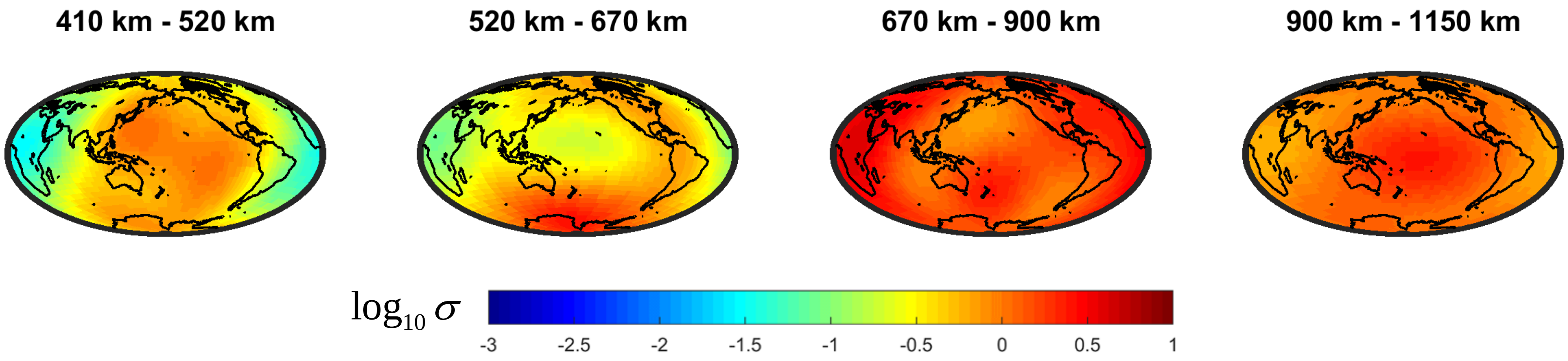
Exploiting satellite data significantly improves separation of non P10 external and induced coefficients

Inverting matrix Q-responses in terms of 3-D mantle conductivity

- Matrix Q-responses are estimated using Swarm + Cryosat + observatory data
- Responses are inverted using quasi-Newton optimization algorithm applied to penalty function which consists of misfit and regularization terms (note that the inverse problem is highly non-linear)
- Conductivity is recovered in layers: 410-520, 520-670, 670-900 and 900-1150 km
- Conductivity in these layers is parameterized by spherical harmonics up to degree 3
- Forward modellings are performed on lateral grid of 5^0 x 5^0
- Outside target depth range 410-1150 km, conductivity model is fixed to 1-D distribution similar to Grayver et al (2017)*; it also includes thin layer of known, laterally-varying, conductance. This layer approximates nonuniform oceans and continents

* Grayver A., F. Munch, A. Kuvshinov, A. Khan, T. Sabaka, L. Toffner-Clausen, 2017. Joint inversion of satellite detected tidal and magnetospheric signals constrains electrical conductivity and water content of the upper mantle and transition zone, Geophys. Res. Lett., 44, 6074–6081

Results of inversion



- Recovered anomalies are concentrated in/around Pacific Ocean
- Inversion results are rather robust with respect to different inversion set ups (parameterization, regularization, number of Q-matrix elements involved into inversion)
- The model is of low-resolution, since with existing data only low-degree and order external and induced coefficients can be determined, and thus low-degree and order Q-matrices can be estimated and inverted

Two comments: a) in considered period range (2 – 30 days) 670 – 900 and 900 – 1150 km layers should be better resolved

b) since non-polar data is not involved in the analysis, results in polar regions should be taken with caution

Summary and outlook

- Exploiting satellite data significantly improves separation of non-P10 external and induced coefficients
- If observatory data are not included into analysis, adding Cryosat to Swarm data improves separation
- The presented satellite-based 3-D conductivity model is of very low (ocean scale) resolution and is confined to the depths 400 – 1150 km
- More (well-calibrated) multi-platform (Grace, Iridium, ….) data would potentially improve separation of external and induced contributions and give an opportunity to resolve higher-degree and order terms, and thus to refine satellite-based 3-D models
- Longer measurements by Swarm will allow for probing deeper structures (> 1150 km)
- Natural way forward is to build/compile global multi-resolution 3-D conductivity model for the whole depth column (0 – 1500 km) using multi-source, multi-data and multi-response approach