Application of spectra-temporal analysis methods to detect common signals in length of day, global mean sea level, global mean surface temperature data, and ENSO indices

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DATA

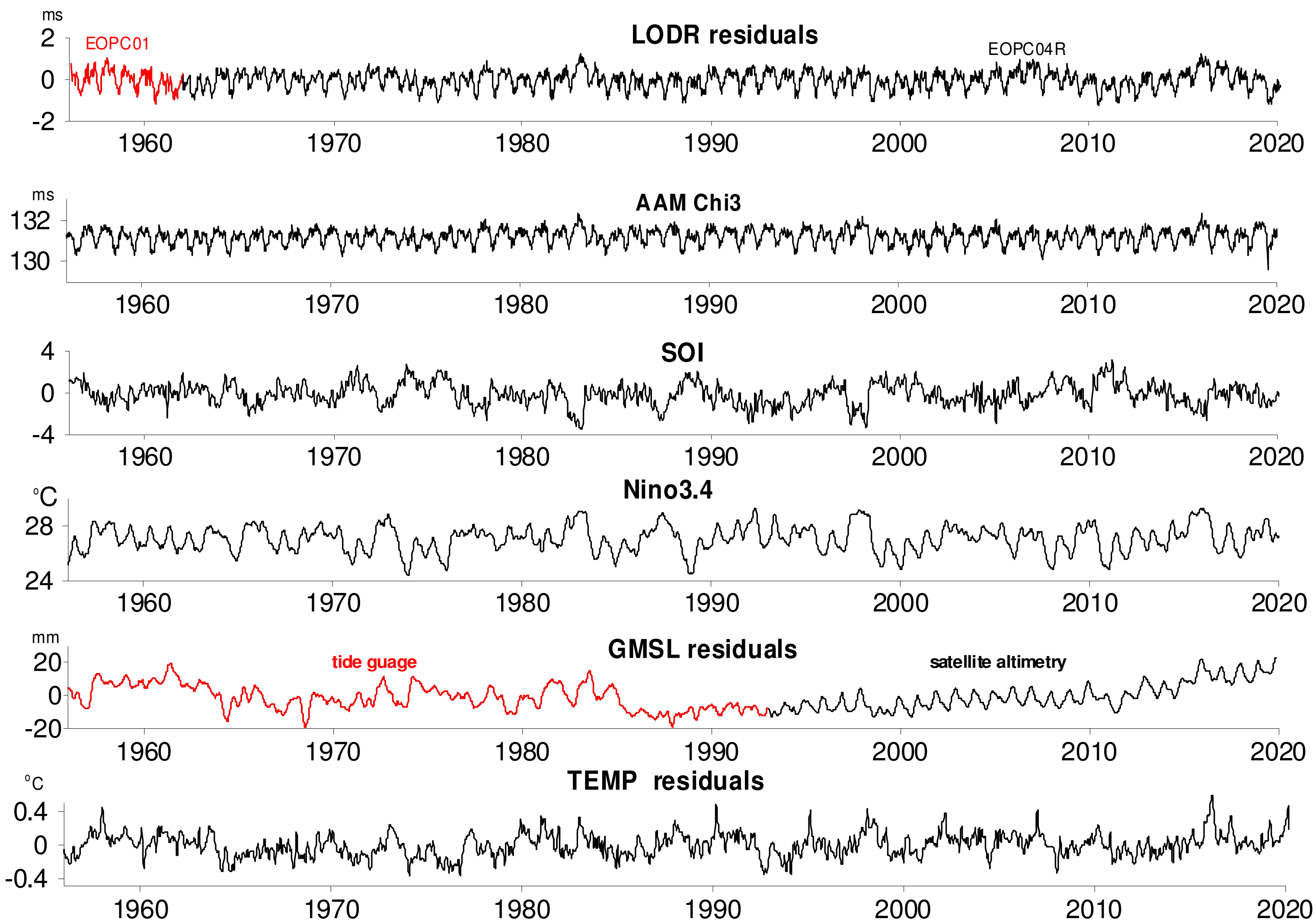
- **Length of Day** since 1962.0 till now, eopc04R_IAU2000_daily and **UT1-TAI** data from EOPC01_IAU2000_(1900-now) with 0.05 year sampling interval from the IERS (http://hpiers.obspm.fr/eoppc/series/opa/
- **The axial component of atmospheric angular momentum AAMχ_3** computed from the NCEP/NCAR reanalysis project available with 6-hour sampling interval from Jan. 1948 to Mar. 2019 http://files.aer.com/aerweb/AAM/
- **Global-mean monthly Surface Temperature** from Goddard Institute for Space Studies: 1880 – present: **GLB.Ts+dSST.csv** https://data.giss.nasa.gov/gistemp/
- **Southern Oscillation Index**: from Climate Research Unit, monthly, Jan. 1866 – Feb. 2020, **soi.dat** https://crudata.uea.ac.uk/cru/data/soi/
- **Nino3.4.long.data:** monthly, from Climate Data Guide based on HadISST from Jan. 1870 to Jan. 2020. https://psl.noaa.gov/gcos_wgsp/Timeseries/Data/nino34.long.data
- **Global Mean Sea Level Data**

  - Satellite Sea Level data from NASA Goddard Space Flight Center which combines Sea Surface Heights from TOPEX/Poseidon, Jason-1, OSTM/Jason-2, and Jason-3: 1993.01-2019.94: **GMSL_TPJAOSV4.2*.txt**

  https://podaac.jpl.nasa.gov/dataset/MERGED_TP_J1_OSTM_OST_ALL_V42

  - Tide Gauge Sea Level Data from CSIRO Marine and Atmospheric Research: smoothed (60-day Gaussian type filter) GMSL (GIA applied)) with respect to 20-year mean: Jan 1880 - Dec 2013: **CSIRO_Recons_gmsl_mo_2015.csv** http://www.cmar.csiro.au/sealevel/sl_data_cmar.html

time series prepared for further analysis



LODR - polynomial degree 4 / harmonic least squares model

GMSL - polynomial degree 2 least squares model

TEMP - polynomial degree 2 least squares model

The Fourier Transform Band Pass Filter (FTBPF)

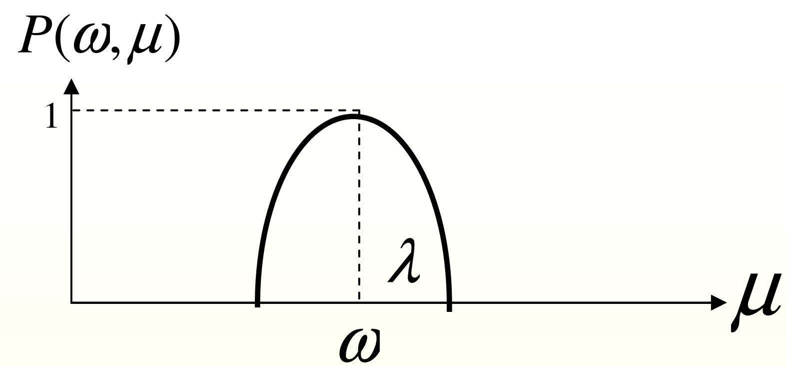
$$u(t,\omega) = FFT^{-1}\left[FFT[x(t)]P(\omega,\mu)\right]$$

where:

$x(t)$ **- real-valued time series**

$u(t,\omega)$ **- broadband oscillation with central frequency ω**

$$P(\omega,\mu) = \begin{cases} 1-\left(\dfrac{\omega-\mu}{\lambda}\right)^2 & if \quad |\omega-\mu| \le \lambda \\ 0 & if \quad |\omega-\mu| > \lambda \end{cases}$$



- parabolic transmittance function

$$\omega = \frac{\Delta t}{T}$$

Δt **- sampling interval of data**

T **– mean period of broadband oscillation**

λ **- half of the filter bandwidth**

Combination of the FTBPF and Hilbert transform (FTBPF+HT)

this combination consists in creating a complex-valued oscillation based on real-valued oscillation by inserting the Hilbert transform of the real-valued oscillation into the imaginary part

$$z(t,\omega) = u(t,\omega) + i \cdot HT\left[u(t,\omega))\right]$$

It turns out that the complex-valued oscillation can be calculated with a FTBPF with a slightly different transmittance function (see appendix)

$$z(t,\omega) = FFT^{-1}\left[FFT(x(t)) \cdot P(\omega,\mu) \cdot (sign(\omega)+1)\right]$$

The instantaneous amplitudes can be computed by:

$$A(t,\omega) = \sqrt{\left[\mathrm{Re}(z(t,\omega))\right]^2 + \left[\mathrm{Im}(z(t,\omega))\right]^2}$$

Normalized Morlet Wavelet Transform (NMWT)

$$\hat{X}(b,T)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\breve{x}(\omega)\overline{\varphi}(T\omega)\exp(ib\omega)d\omega$$

(Liu et al. 2007, Liu and Hsu 2012)

b - translation (or time) parameter and

T - dilation (or period) parameter,

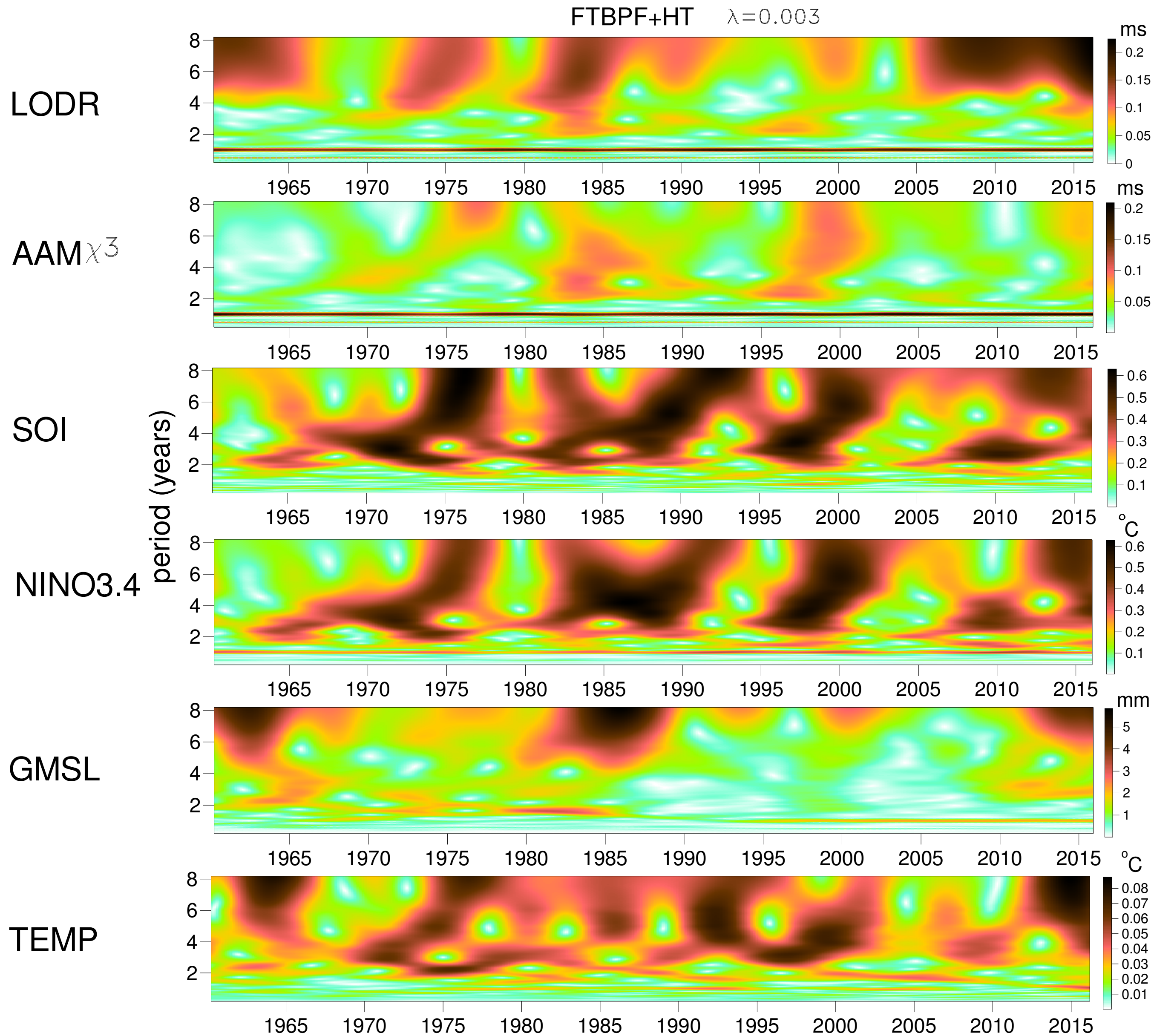
$\breve{x}(\omega)$ - DFT of $x(t)$ time series

$$\varphi(\omega)=\exp\left(\frac{-(\omega-2\pi)^2\sigma^2}{2}\right)-\exp\left(\frac{-(\omega-2\pi)^2\sigma^2}{4}\right)\exp\left(-\pi^2\sigma^2\right)$$

is the continuous FT of the complex-valued Morlet wavelet function (Schmitz-Hübsch and Schuh 1999), σ is the decay parameter which controls the frequency resolution. The instantaneous amplitude is given by the formula:

$$A(b,T)=\sqrt{\left(Re[\hat{X}(b,T)]\right)^2+\left(Im[\hat{X}(b,T)]\right)^2}$$

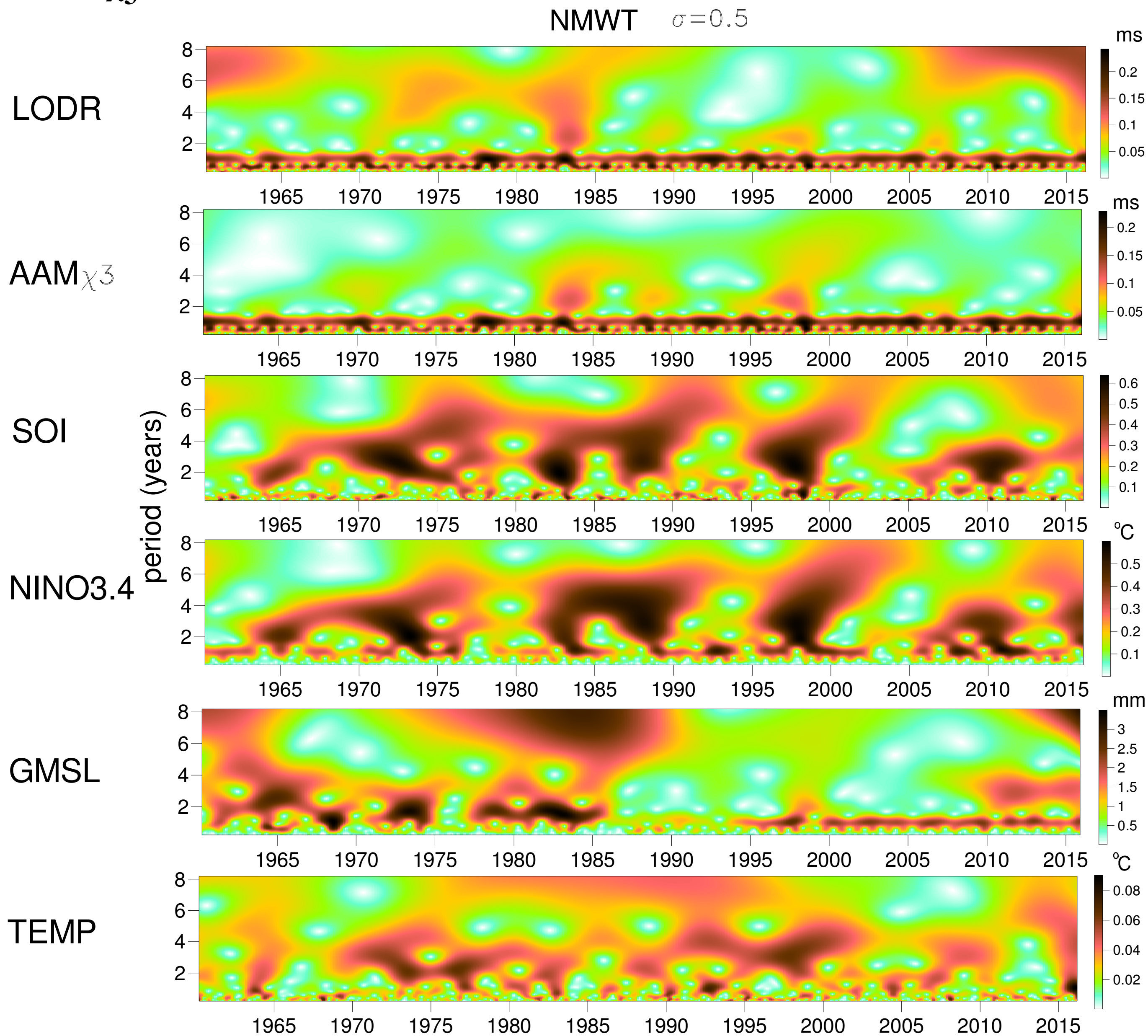
Amplitudes of oscillations computed by the FTBPF+HT (λ=0.003) in LODR residuals, AAMχ_3, SOI, NINO3.4 indices, GMSL residuals and TEMP residuals



El Niño events:

1972/73
1982/83
1997/98
2009/10
2014/16

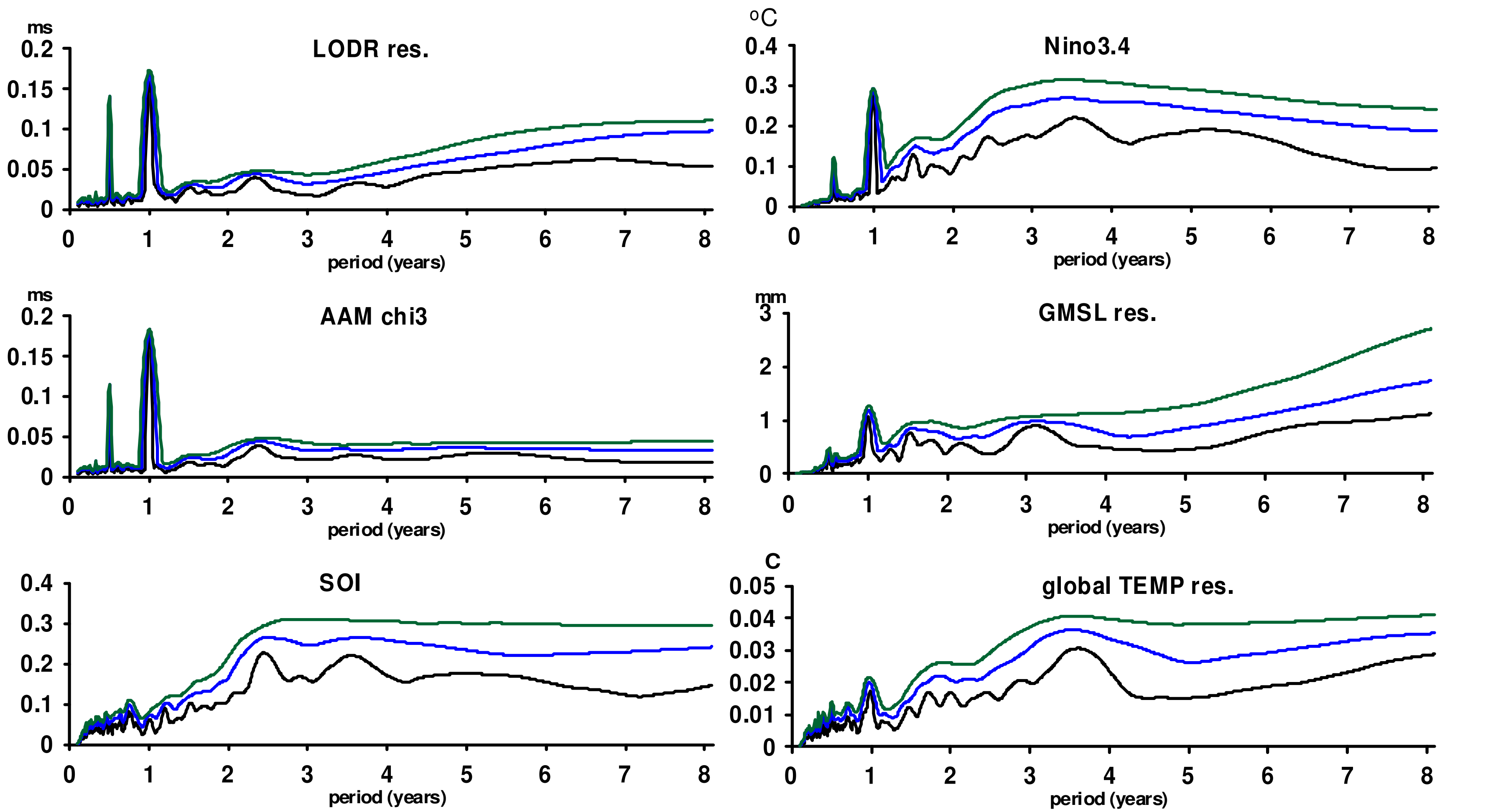
Amplitudes of oscillations computed by the NMWT (σ=0.5) in LODR residuals, AAMχ_3, SOI, NINO3.4 indices, GMSL residuals and TEMP residuals



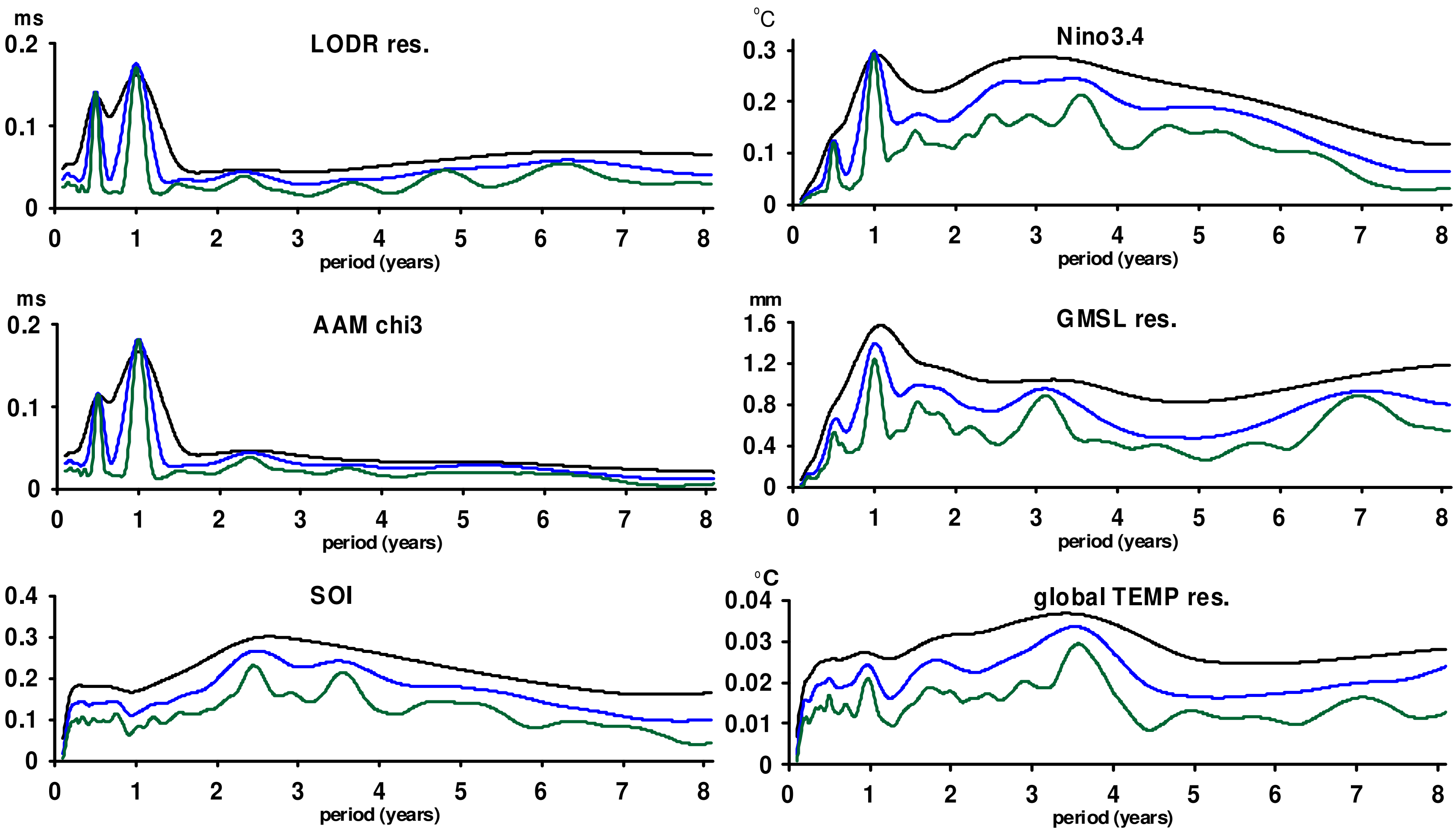
El Niño events:

1972/73
1982/83
1997/98
2009/10
2014/16

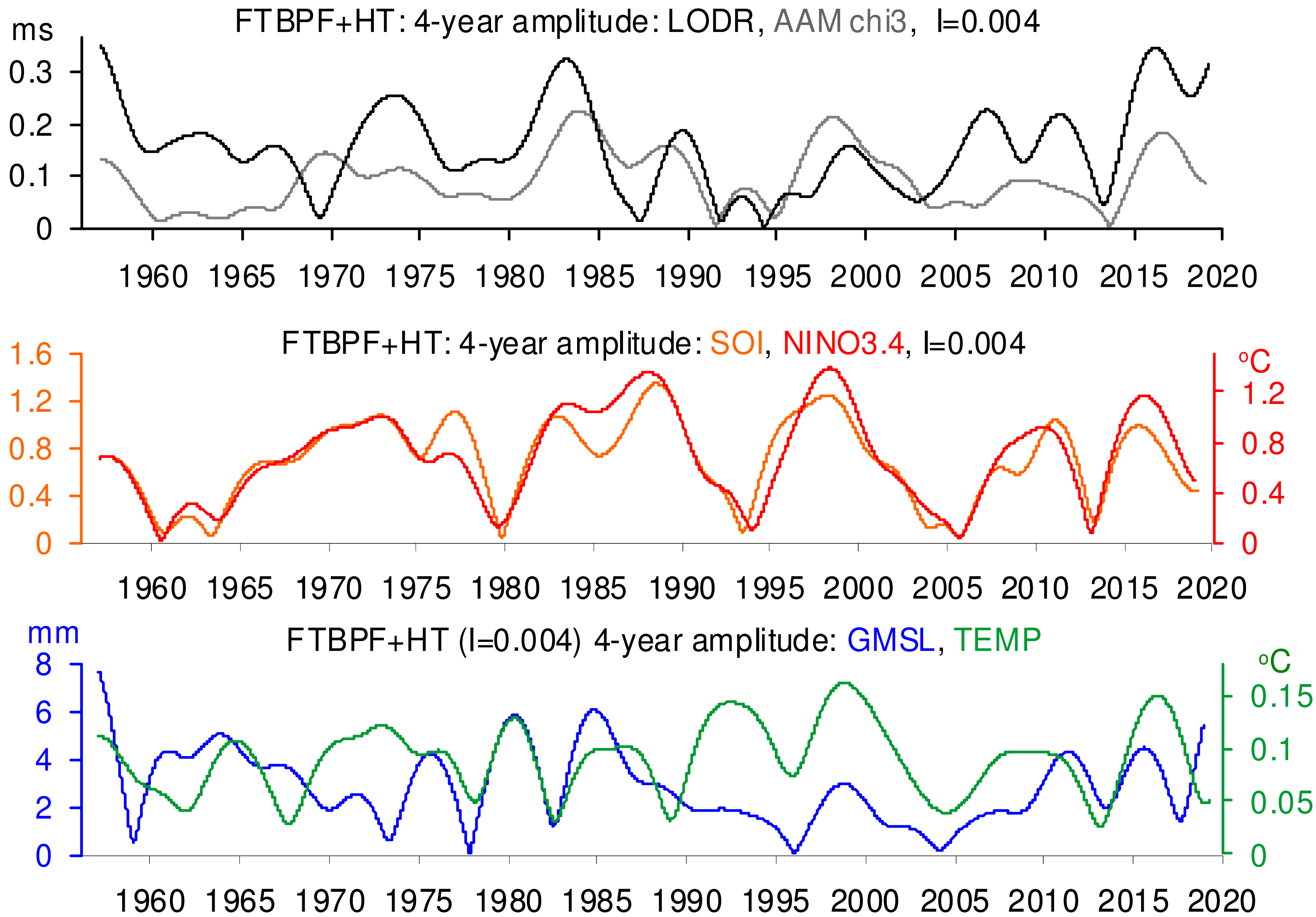
FTPBF+HT (λ=0.001, λ=0.002, λ=0.003) mean amplitude spectra of LODR residuals, AAMχ_3, SOI, NINO3.4, GMSL residuals and global TEMP residuals in 1956-2020



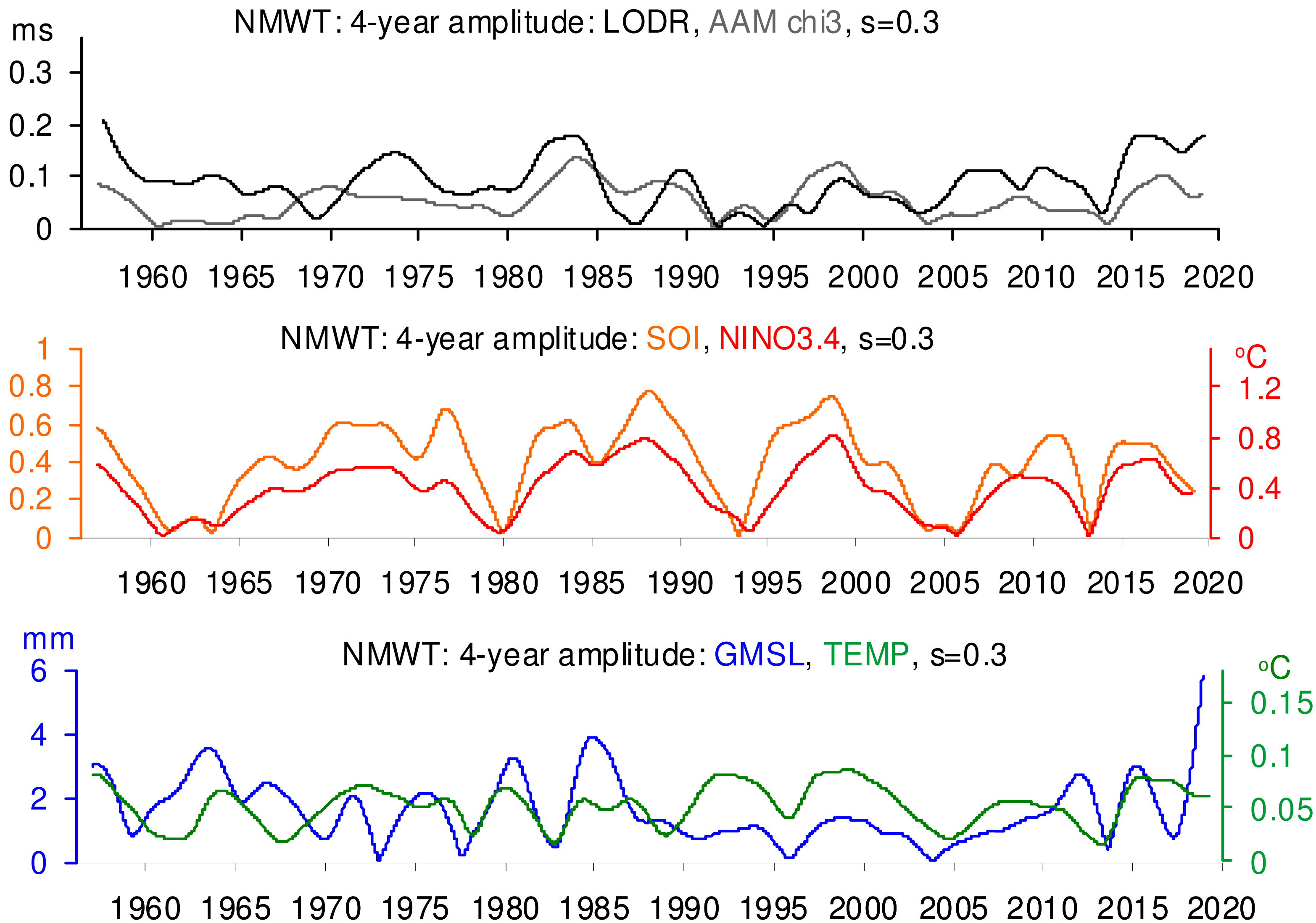
NMWT mean amplitude spectra (σ=0.5, 1.0, 2.0) of LODR residuals, AAMχ_3, SOI, NINO3.4 indices, GMSL residuals and global TEMP residuals in 1956-2020



FTPBF+HT (λ=0.004) amplitudes of oscillations with central period of 4-years (cutoff periods 2.2-20.0 years) in LODR residuals, AAMχ_3, SOI, NINO3.4 indices, GMSL and TEMP residuals



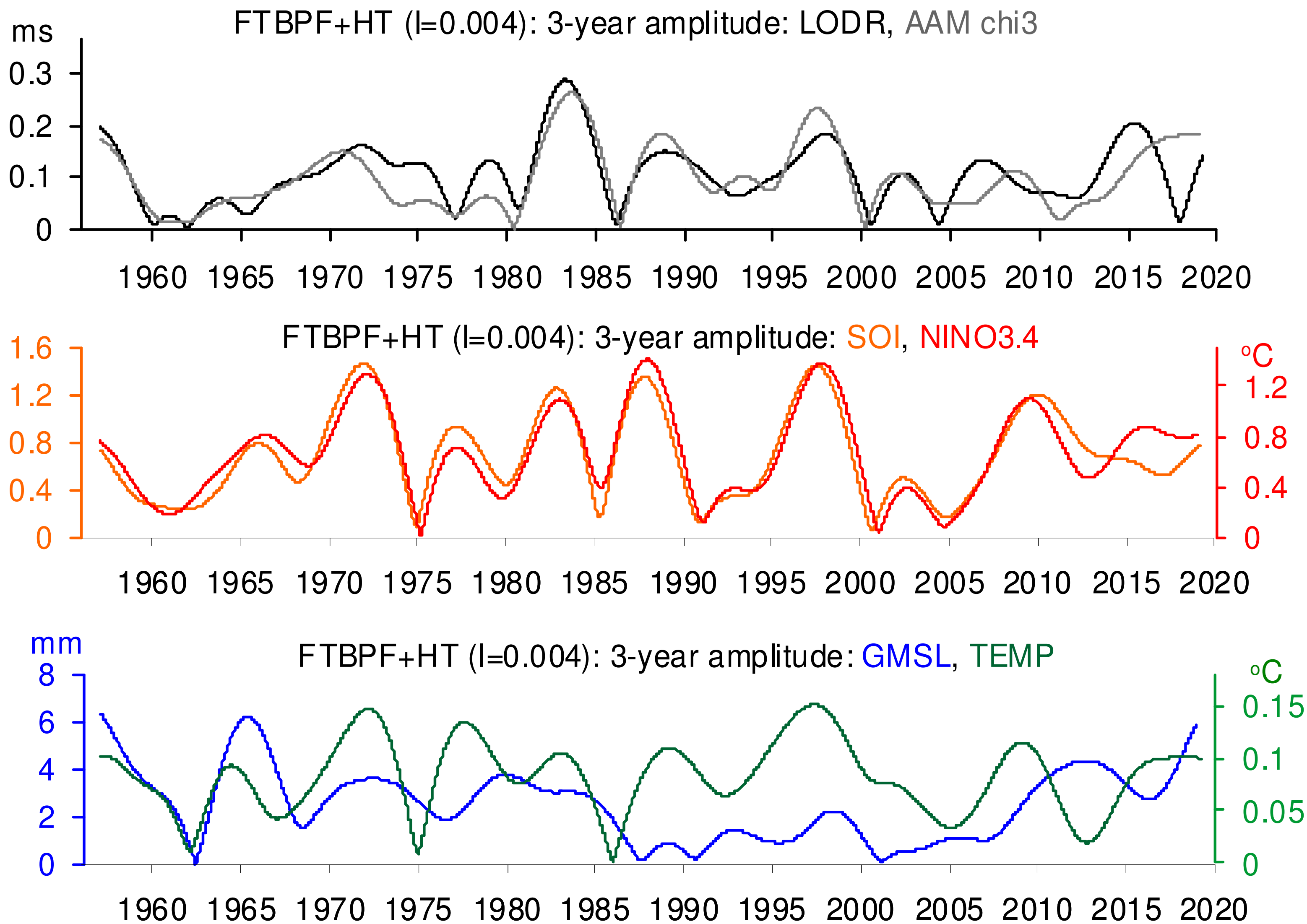
NMWT (σ=0.3) amplitudes of the 4-year oscillation in LODR residuals, AAMχ_3, SOI, NINO3.4 indices, GMSL and TEMP residuals



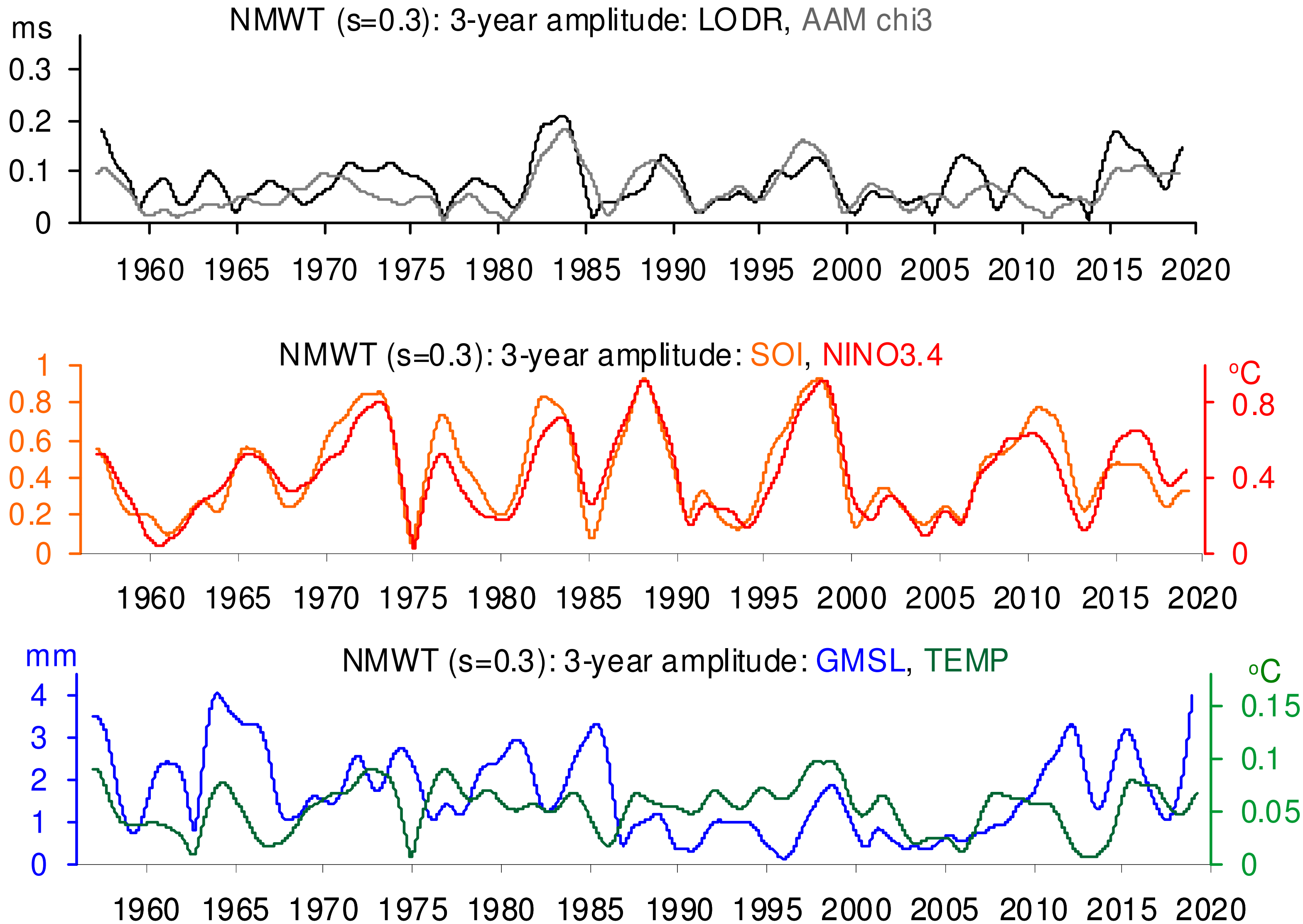
Correlation coefficients between amplitude variations in 1970-2019 of oscillations with central period of 4-years (2.2 - 20 years) in LODR residuals, AAMχ_3, SOI, Nino3.4 indices, GMSL residuals and TEMP residuals computed by the FTBPF+HT (λ=0.004) and NMWT (σ=0.3), and between results of both methods (green).

	LODR	AAMχ_3	SOI	Nino3.4	GMSL	TEMP
LODR	0.987	0.429 0.444	0.236 0.233	0.321 0.320	0657 0.510	0.005 0.010
AAMχ_3		0.988	0.646 0.664	0.828 0.829	0.644 0.574	0.283 0.247
SOI			0.986	0.880 0.887	0.644 0.569	0.176 0.166
Nino3.4				0.992	0.652 0.579	0.297 0.283
GMSL					0.904	0.552 0.584
TEMP						0.957

FTPBF+HT (λ=0.004) amplitudes of oscillations with the central period of 3-years (cutoff periods 1.9-7.5 years) in LODR residuals, AAMχ_3, SOI, NINO3.4 indices, GMSL and TEMP residuals



NMWT σ=0.3 amplitude of the 3-year oscillation in LODR residuals, AAMχ_3, SOI, NINO3.4 indices, GMSL and TEMP residuals

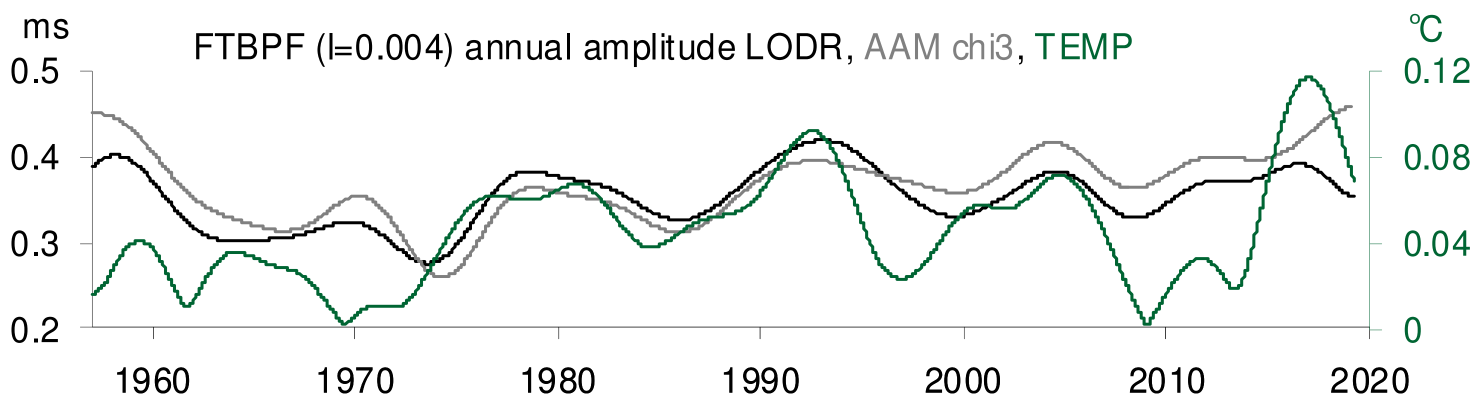


Correlation coefficients between amplitude variations in 1970-2019 of oscillations with central period of 3-years (cutoff periods 1.9 -7.5 years) in LODR, AAMχ_3, SOI, Nino3.4, GMSL and global TEMP data computed by the FTBPF+HT (λ=0.004) and NMWT (σ=0.3), and between results of both methods (green).

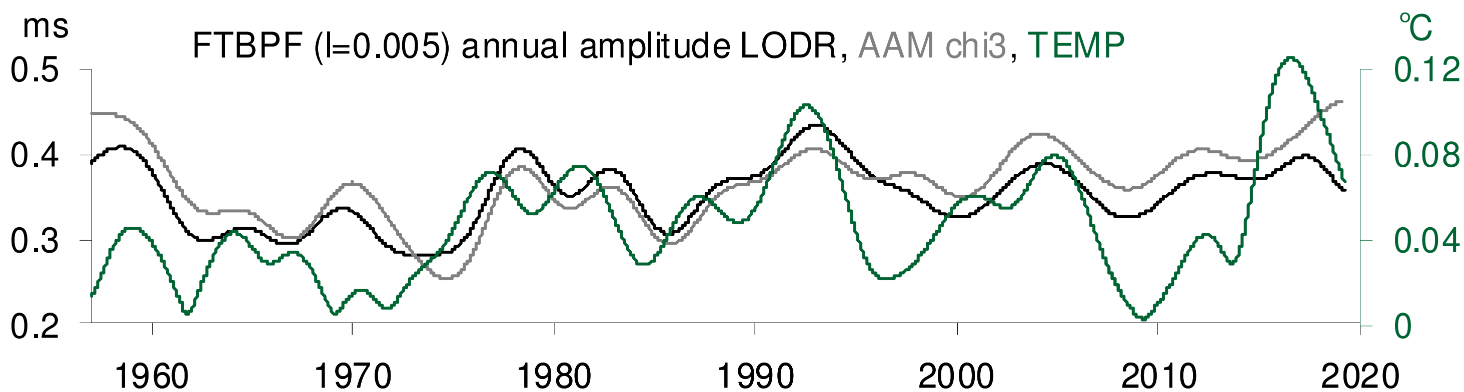
	LODR	AAMχ_3	SOI	Nino3.4	GMSL	TEMP
LODR	0.808	0.703 0.668	0.410 0.428	0.461 0.520	0.789 0.533	0.332 0.315
AAMχ_3		0.959	0.425 0.425	0.572 0.575	0.751 0.515	0.465 0.384
SOI			0.935	0.919 0.901	0.902 0.672	0.617 0.577
Nino3.4				0.947	0.904 0.649	0.632 0.614
GMSL					0.802	0.806 0.755
TEMP						0.852

FTPBF+HT (λ=0.004, λ=0.005) amplitudes of the annual oscillations (cutoff periods 0.83-1.25 years) in LODR residuals, AAMχ_3 and TEMP residuals

Correlation coefficients 1970-2019:

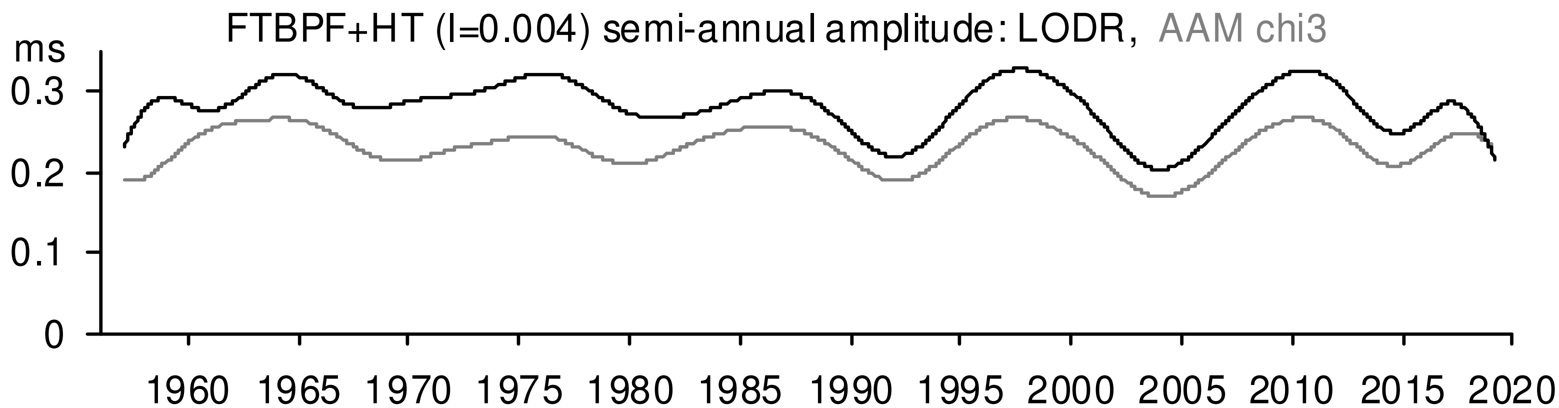


$\gamma_{LODR\text{-}temp}$=0.64
$\gamma_{\chi 3\text{-}temp}$=0.45
$\gamma_{LODR\text{-}\chi 3}$=0.75

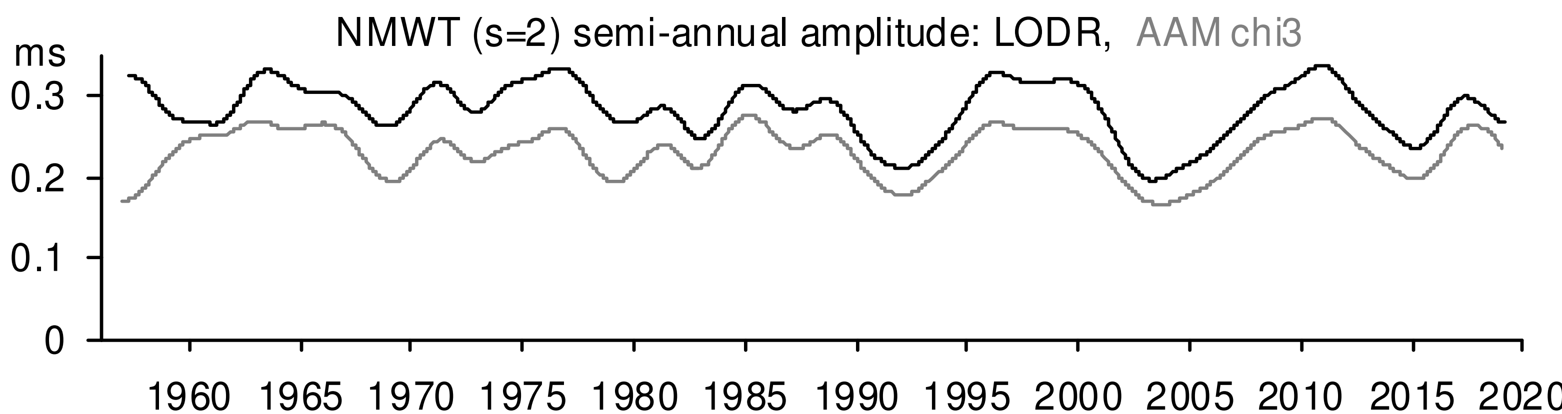


$\gamma_{LODR\text{-}temp}$=0.61
$\gamma_{\chi 3\text{-}temp}$=0.44
$\gamma_{LODR\text{-}\chi 3}$=0.79

FTPBF+HT (λ=0.004) and NMWT (σ=2) amplitude variations of the semi-annual oscillations in LODR residuals and AAMχ_3



γ=0.92



γ=0.92

$\gamma(\mathrm{LODR})_{\mathrm{FTBPF+HT\ \&\ NMWT}}=0.95$, $\gamma(\mathrm{AAM}\chi_3)_{\mathrm{FTBPF+HT\ \&\ NMWT}}=0.92$

CONCLUSIONS

- The NMWT and FTBPF+HT algorithms enable computation of instantaneous amplitudes of oscillations in real-valued time series as a function of their oscillation periods. The frequency resolution of amplitude spectra increases with the increase of the σ parameter in the NMWT and with the decrease of the λ parameter in the FTBPF+HT. The increase of the frequency resolution is associated with the decrease of time resolution of amplitude variations and their mean values.
- The mean amplitude spectra of LODR, AAMχ_3, SOI, Nino3.4, global mean sea level (GMSL), and global mean surface temperature (TEMP) data show annual and semi-annual signals in these data except the SOI data and wideband signal with periods from 2-7 years corresponding to ENSO phenomenon.
- The amplitude variations of signals with central periods of 3 and 4 years are very similar in GMSL, LODR, AAMχ_3, SOI, Nino3.4, and TEMP data. The maxima of amplitude variations correspond to El Niño events in 1972/73, 1982/83, 1997/98, 2009/10, and 2014-16 years. The correlation coefficients between these amplitude variations in 1970-2019 are of the order of 0.5-0.9 and are usually greater for oscillations with a central period of 3 years than for oscillations with a central period of 4 years.
- The amplitude variations of the annual oscillation computed by the FTBPF+HT in LODR, AAMχ_3, and TEMP data are similar. The correlation coefficients between these amplitude variations in 1970-2019 are of the order of 0.8, 0.6, and 0.4 for pairs LODR-AAMχ_3, LODR-TEMP, and AAMχ_3-TEMP, respectively.
- The amplitude variations of the semi-annual oscillations computed by the FTBPF+HT and NMWT in LODR and AAMχ_3 data are very similar.

Appendix: Combination of the FTBPF and Hilbert transform (FTBPF+HT)

$$z(t) = y(t) + i \cdot H\left[y(t)\right], \quad where \quad H[y(t)] = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{y(p)}{t-p}\,dp = \left\{\frac{1}{\pi p} * y(p)\right\}(t)$$

$$FT\left\{H\left[y(t)\right]\right\} = FT\left\{\frac{1}{\pi t} * y(t)\right\} = FT\left\{\frac{1}{\pi t}\right\} \cdot FT\left[y(t)\right]$$

$$FT\left[\frac{1}{\pi t}\right] = -i \cdot sign(\omega), \quad where \quad sign(\omega) = \begin{cases} 1 & for \quad \omega > 0 \\ 0 & for \quad \omega = 0 \\ -1 & for \quad \omega < 0 \end{cases}$$

$$FT\left[H\left[y(t)\right]\right] = -i \cdot sign(\omega) \cdot FT\left[y(t)\right]$$

$$FT\left[z(t)\right] = FT\left[y(t)\right] + i \cdot FT\left[H\left[y(t)\right]\right]$$

$$FT\left[z(t)\right] = FT\left[y(t)\right] + i\left[-i \cdot sign(\omega) FT\left[y(t)\right]\right]$$

$$FT\left[z(t)\right] = FT\left[y(t)\right] \cdot \left[1 + sign(\omega)\right]$$

$$FT^{-1}\left[FT\left[z(t)\right]\right] = FT^{-1}\left[FT\left[y(t)\right] \cdot \left[1 + sign(\omega)\right]\right]$$

$$\boxed{z(t) = FT^{-1}\left[FT\left[y(t)\right] \cdot \left[1 + sign(\omega)\right]\right]}$$

Appendix: Wavelet transform from time to frequency domain

$$\widehat{X}(b,a) = |a|^{-1/2} \int_{-\infty}^{\infty} x(t)\overline{\varphi}((t-b)/a)dt$$

$$\varphi(t) = \frac{1}{\sigma\sqrt{2\pi}} \exp(ipt)[\ \exp(-t^2/2\sigma^2) - \sqrt{2}\exp(-t^2/\sigma^2)\exp(-p^2\sigma^2/4)\ ]$$

– modified Morlet wavelet, p –frequency parameetr, b – translation parameter,
a - dilation parameter (scale), σ – decay parameter.

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\varphi(t-\tau)d\tau = x * \varphi \quad (t)$$

$$FT\left[y(t)\right] = FT\left[x(t)\right] \cdot FT\left[\varphi(t)\right]$$

$$FT\left[y(t)\right] = x(\omega) \cdot \varphi(\omega)$$

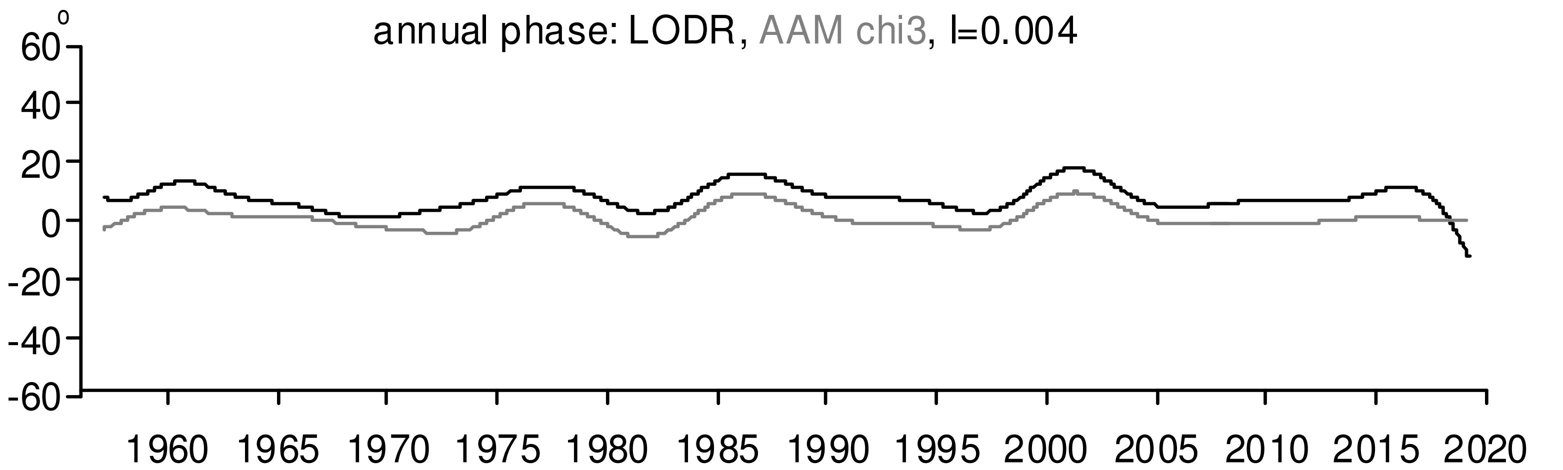
$$FT^{-1}\left\{FT\left[y(t)\right]\right\} = FT^{-1}\left\{x(\omega) \cdot \varphi(\omega)\right\}$$

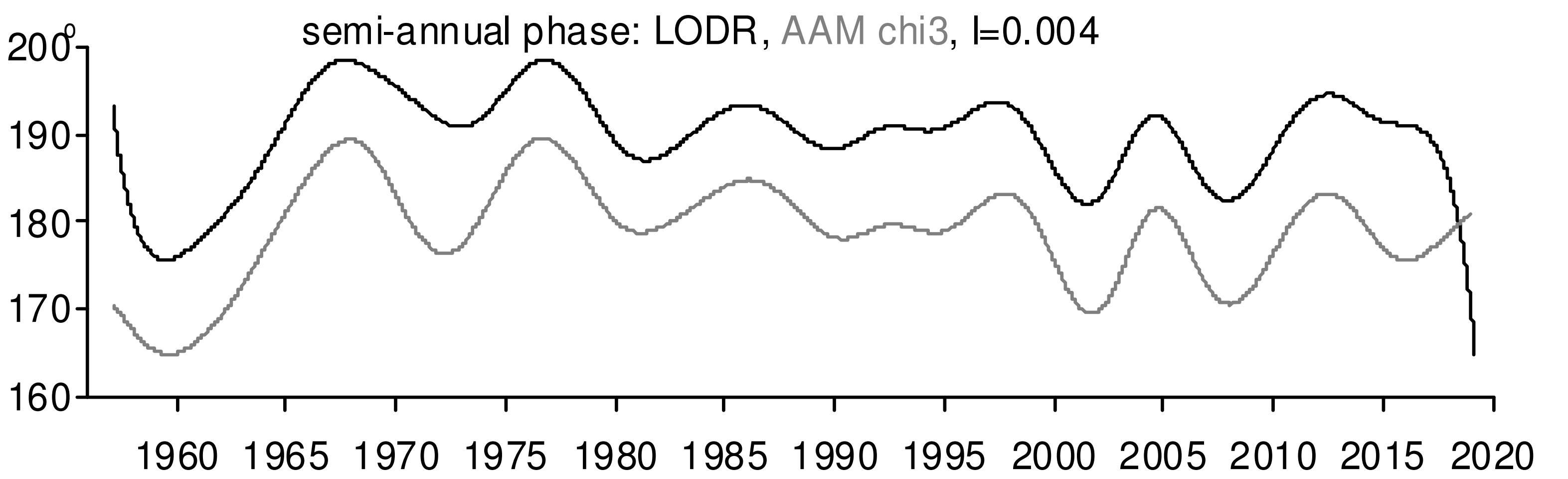
$$y(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(\omega) \cdot \varphi(\omega) e^{it\omega} d\omega$$

$$\widehat{X}(b,a) = \frac{1}{2\pi} |a|^{1/2} \int_{-\infty}^{\infty} \breve{x}(\omega)\overline{\breve{\varphi}}(a\omega)\exp(ib\omega)d\omega$$

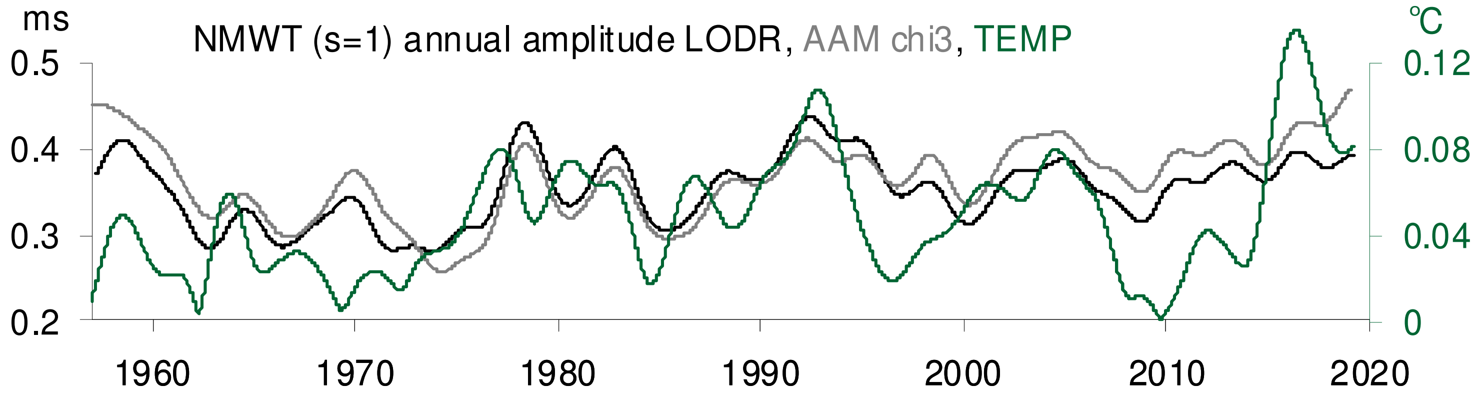
$$\breve{\varphi}(\omega) = CFT[\varphi(t)] = \exp(-(\omega - p)^2\sigma^2/2) - \exp(-(\omega - p)^2\sigma^2/4)\exp(-p^2\sigma^2/4)$$

FTPBF+HT (λ=0.004) phase variations of the annual and semi-annual oscillations in LODR residuals and AAMχ_3





NMWT (σ=1) and FTBPF+HT (λ=0.005) amplitudes of the annual oscillations in LODR residuals, AAM χ_3 and TEMP residuals

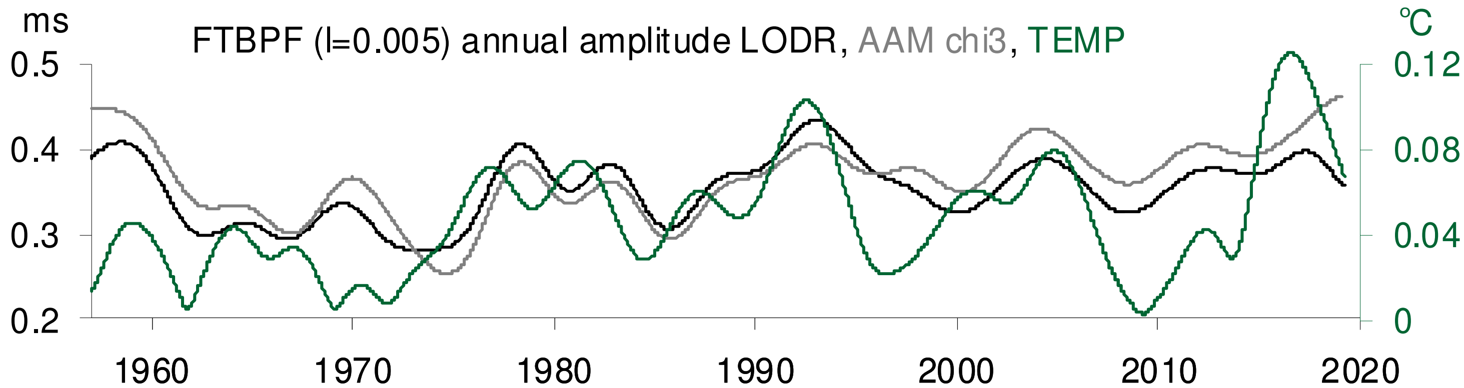


Correlation coefficients 1970-2019:

$\gamma_{\text{LODR-temp}}=0.54$

$\gamma_{\chi 3\text{-temp}}=0.39$

$\gamma_{\text{LODR-}\chi 3}=0.81$



$\gamma_{\text{LODR-temp}}=0.61$

$\gamma_{\chi 3\text{-temp}}=0.44$

$\gamma_{\text{LODR-}\chi 3}=0.79$

FTPBF+HT (λ=0.003) and NMWT (σ=0.5) amplitude spectra of LODR residuals, AAMχ_3, SOI, NINO3.4, GMSL residuals and TEMP residuals