A new three-dimensional regularization for finite fault source inversions

Navid Kheirdast[1], Anooshiravan Ansari[1], Susana Custódio[2]
[1]International Institute of Earthquake Engineering and Seismology(IIEES), Tehran, Iran
[2]Instituto Dom Luiz, Faculdade de Ciencias, Universidade de Lisboa, Lisbon, Portugal

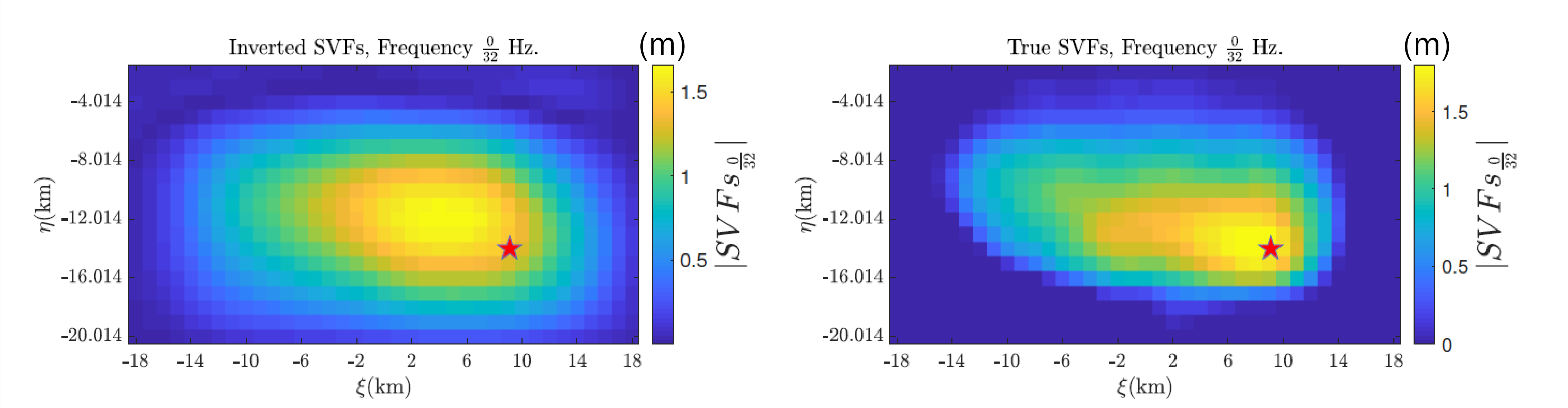
the source function at a given frequency can be found by inverting the seismic data at that frequency

for example: consider the 0Hz
(static displacement → static fault dislocation)

the spatial slip distributions

Inverted using the fuzzy approximation method [Kheirdast et al, under review]

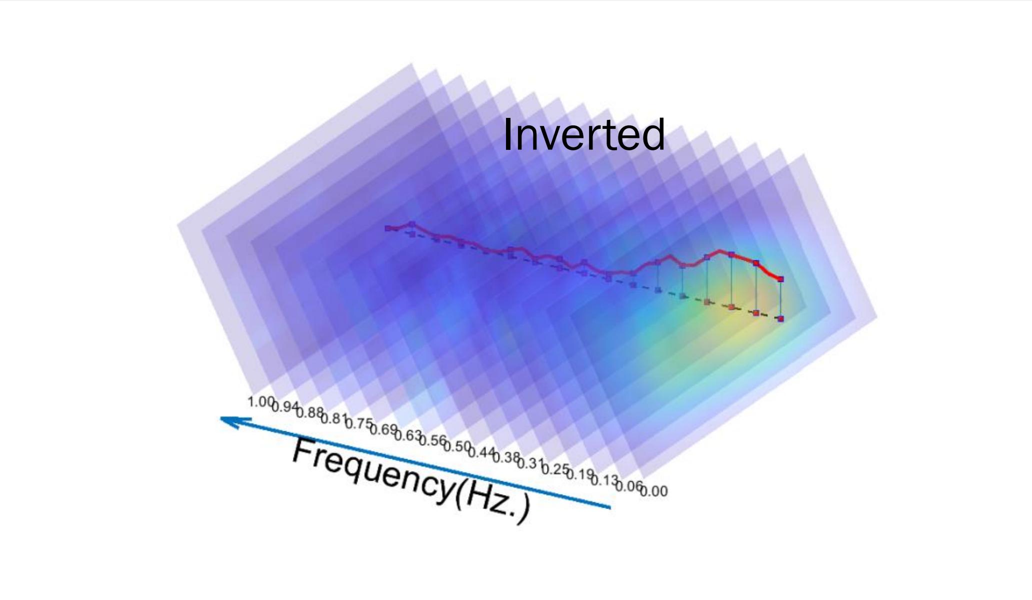
True

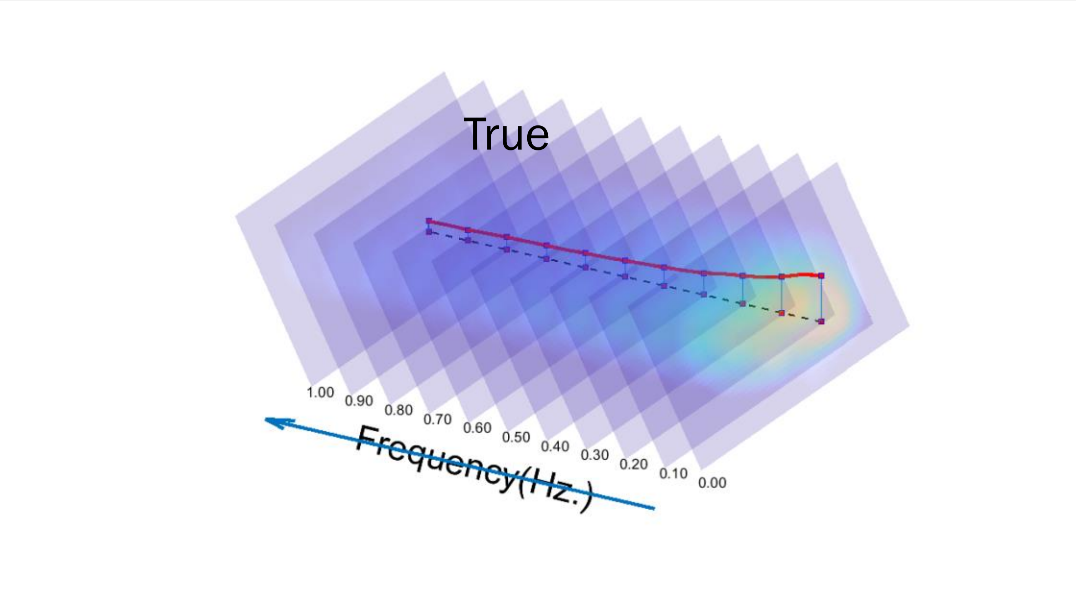


The SIV1-Benchmark

Mai et al, 2016

Something is not right!
The inverted spectrum is not smooth.



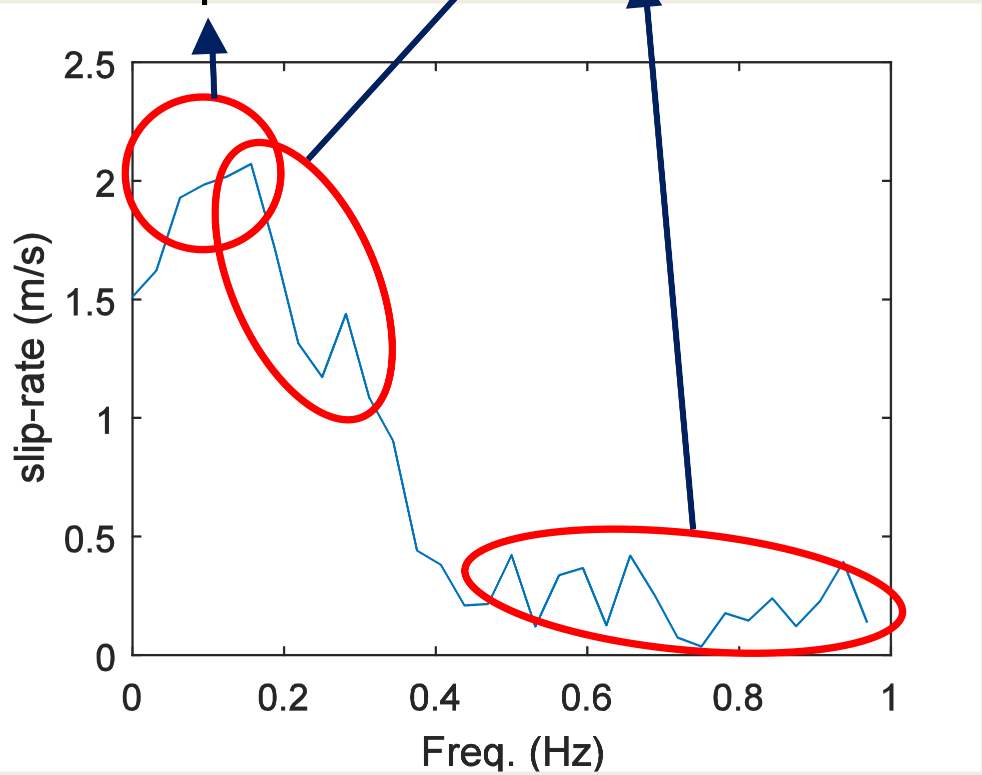


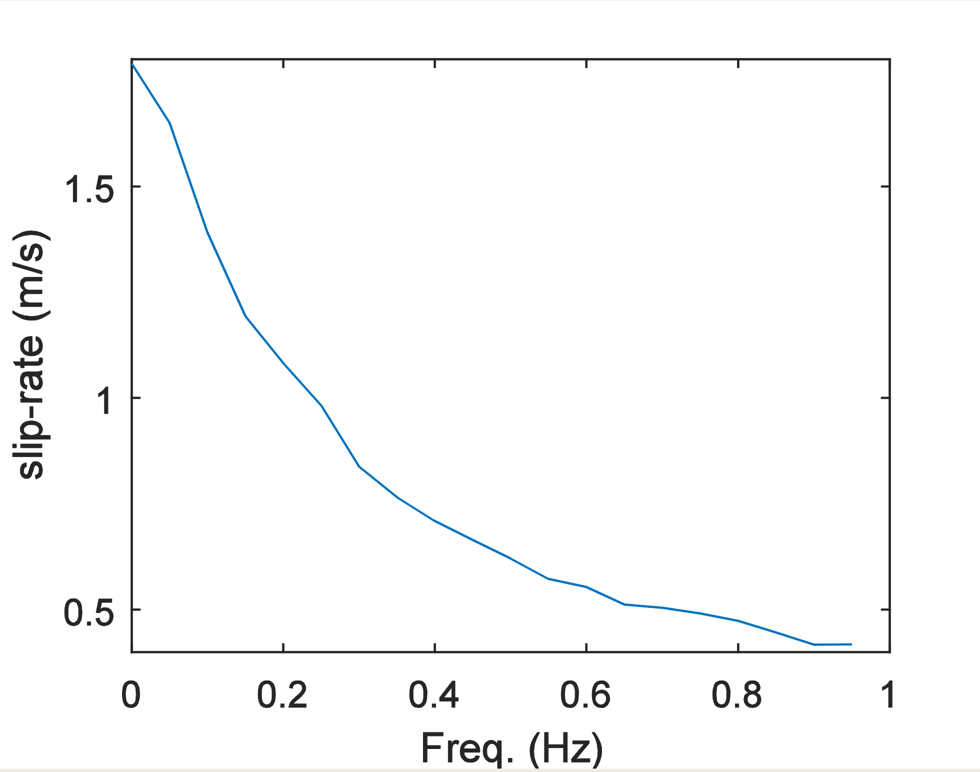
In the frequency-domain inversions: The slip is regularized in space, but not in frequency.

The frequency domain spectrum should be smooth

How can we avoid these **saw tooth**?
Why the spectrum is not as **smooth** as the True SVFs

What is this **peak**?





We need further regularization In the frequency domain

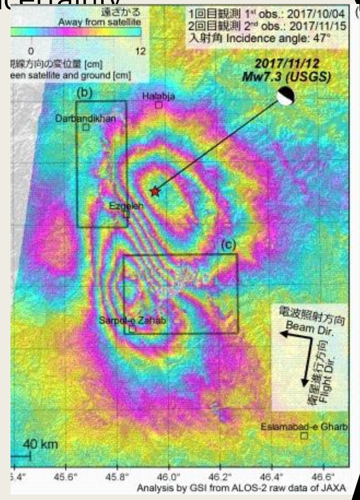
- For example minimizing the first order derivative of the spectrum with this well-known operator: L1 is the first-order derivative

$$\mathbf{L}_1 = \begin{bmatrix} -1 & 1 & & & & \\ & -1 & 1 & & & \\ & & \ddots & \ddots & & \\ & & & -1 & 1 & \\ & & & & -1 & 1 \end{bmatrix}.$$

EGU General Assembly 2020

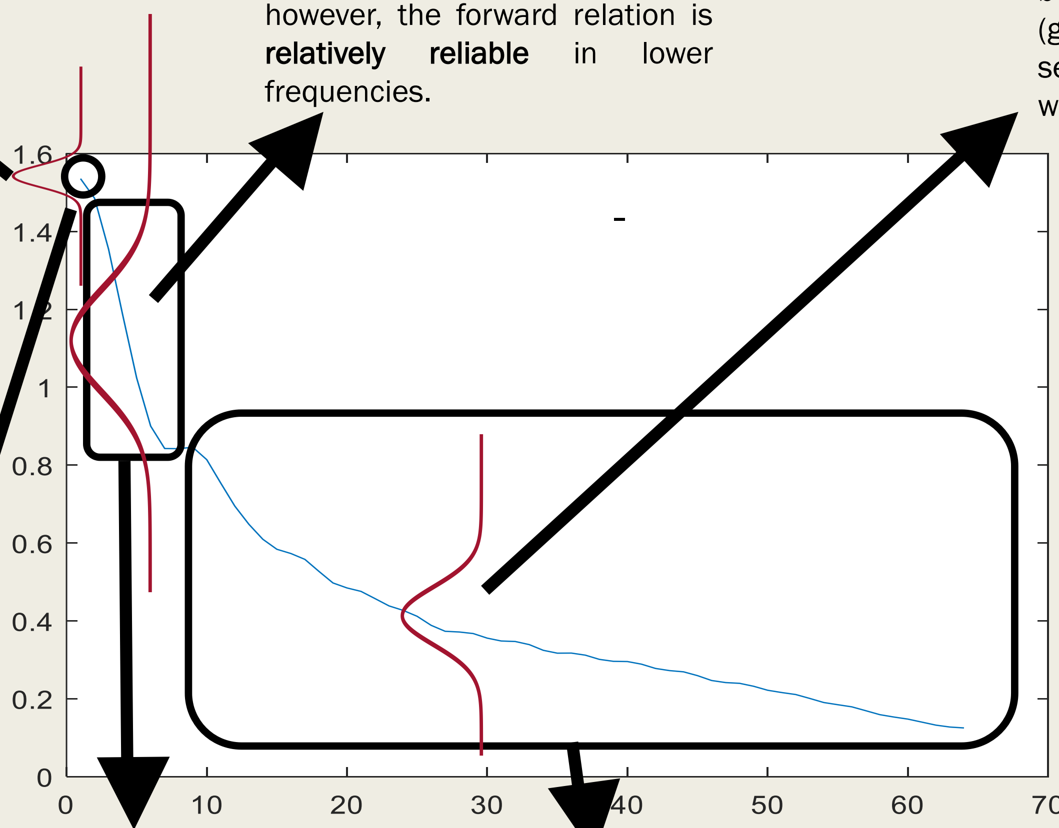
Benefits of further regularization:
Transferring knowledge from one frequency to another

dense acquisition in near fault region (e.g.. InSAR), data is **densely acquised, forward relation** is less ill-posed , the model parameters has [illegible] uncertainty



Sparse data acquisition networks do not cover densely, however, the forward relation is **relatively reliable** in lower frequencies.

Still **sparse data acquisition**, the forward relation becomes less reliable with increasing the frequency, because the fundamental solutions (green functions) become more sensitive to small perturbations of the wave-field material.



InSAR High-rate GPS Strong-motion

How can we apply the further regularization? By constraining the forward operator

Forward Block: that is more or less the **Rep. theorem.**

Data vector has its own **inherent limitations**

$$
\begin{bmatrix} d_{frq1} \\ d_{frq2} \\ \vdots \\ d_{nfrq} \\ \mathbf{0} \end{bmatrix}
=
\begin{bmatrix}
\boldsymbol{G}_{frq1} & & & \\
& \boldsymbol{G}_{frq2} & & \\
& & \ddots & \\
& & & \boldsymbol{G}_{nfrq} \\
\alpha_{frq1}\nabla^2 & & & \\
& & \ddots & \\
& & & \alpha_{nfrq}\nabla^2 \\
& \alpha_{spec}L1 & &
\end{bmatrix}
\begin{bmatrix} slip_{frq1} \\ slip_{frq2} \\ \vdots \\ slip_{nfrq} \end{bmatrix}
$$

Spatial Smoothing Block: already applied with **Laplacian operator**

We need to determine $\alpha_0, \ldots, \alpha_{nfrq}, \alpha_{spec}$

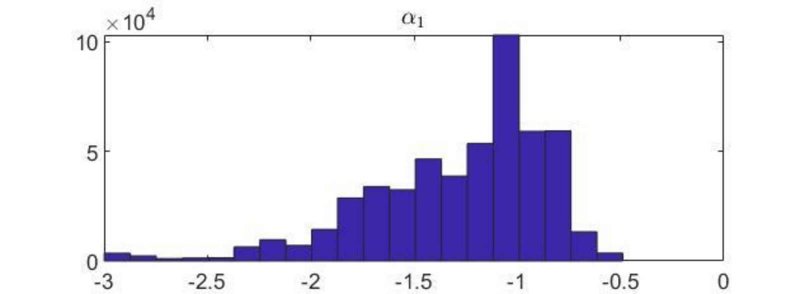
Spectral Smoothing Block: **the new development is placed here**

How to determine the regularizing parameters $\alpha_1, \ldots, \alpha_{nfrq}, \alpha_{spec}$?

This problem is a multi-parameter Tikhonov regularization

In this proposed method, we try to determine the probability distribution of $\alpha_0, \ldots, \alpha_{nfrq}$, α_{spec} using a **Bayesian** method by finding the PDF of the regularizing parameters:

$P(\alpha_1)$



From the PDF of $\alpha_0, \ldots, \alpha_{nfrq}, \alpha_{spec}$, we can then estimate their value using an estimator, for example:

- expected value estimator
- maximum likelihood estimator

Bayesian modelling

We can easily calculate this probability,
having the modelling error show before

$$P(\boldsymbol{\alpha}|data) = \frac{P(data|\boldsymbol{\alpha}) \times P(\boldsymbol{\alpha})}{P(data)}$$

Just scales the fraction, nothing important

Bayesian modeling

How to determine $P(data|\boldsymbol{\alpha})$?

We can easily calculate this probability, having the modeling error show before

$$P(\boldsymbol{\alpha}|data) = \frac{P(data|\boldsymbol{\alpha}) \times P(\boldsymbol{\alpha})}{P(data)}$$

We have no prior information, thus we consider it as uniformly distributed over a large set of values

Just scales the fraction, nothing important

How to determine $P(data|\boldsymbol{\alpha})$?

Morozov Discrepancy Principle:

- If we choose α in a way that:

$$||\mathbf{G}\mathrm{m}_{\alpha} - \mathrm{d}||^2 > \delta$$

All information in data would not used. We can explore more

- If we choose α in a way that:

$$||\mathbf{G}\mathrm{m}_{\alpha} - \mathrm{d}||^2 < \delta$$

We would over fitted the model to the noise.

- The best solution is: $||\mathbf{G}\mathrm{m}_{\alpha} - \mathrm{d}||^2 = \delta$

We need to characterize the error

d^o:true SIV data (blue)

d^S: our reproduced data from discretized model and Green functions from Axitra (orange)

Freq:3

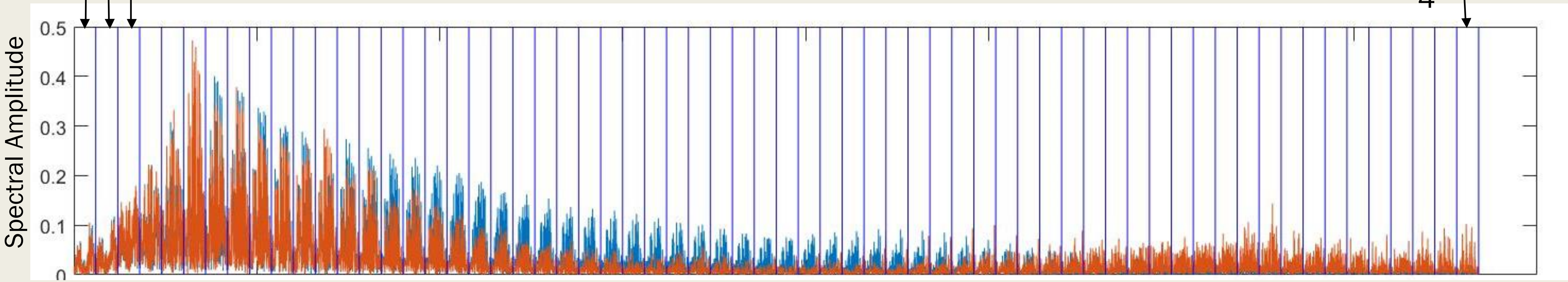
Freq:2

Freq:1

Error is frequency dependent

...

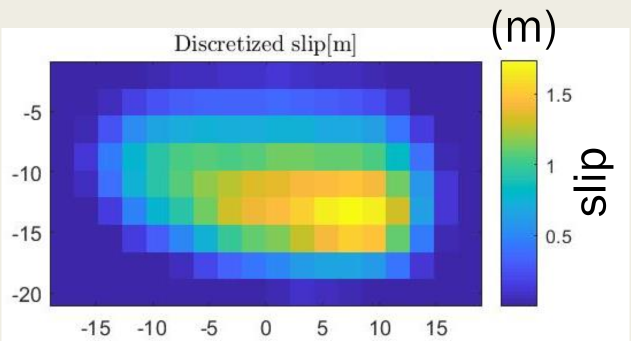
Freq:64



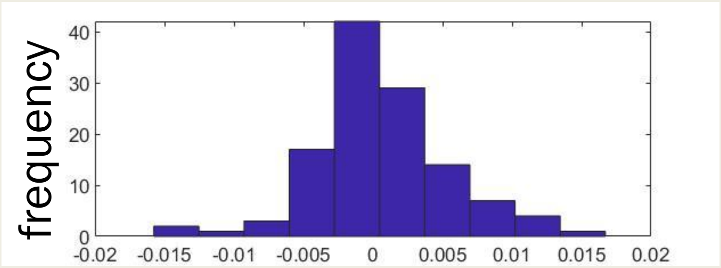
How can we apply discrepancy principle?

Assuming $P(data|\boldsymbol{\alpha})$ = *a priori* Noise/uncertainty model

True Solution :

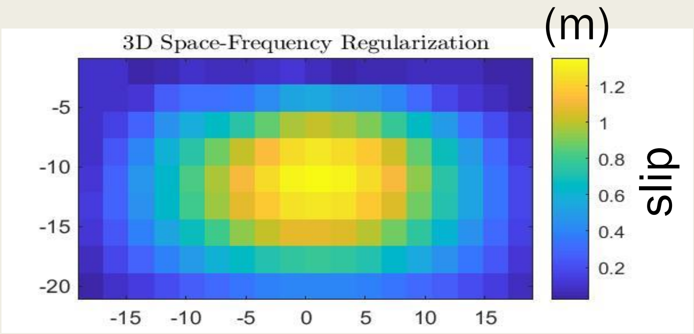


Has data error:



Error/misfit/noise

A Good solution

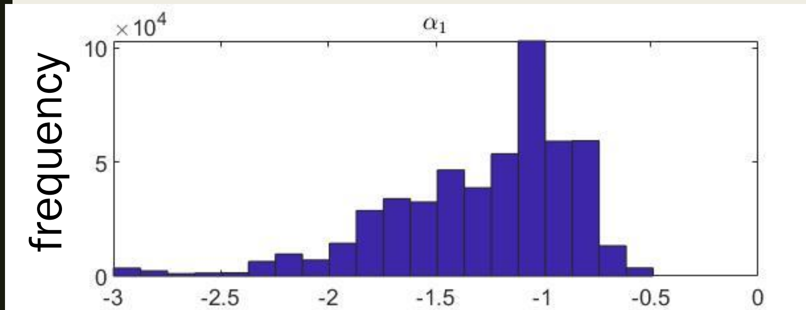


Follows the same data error:

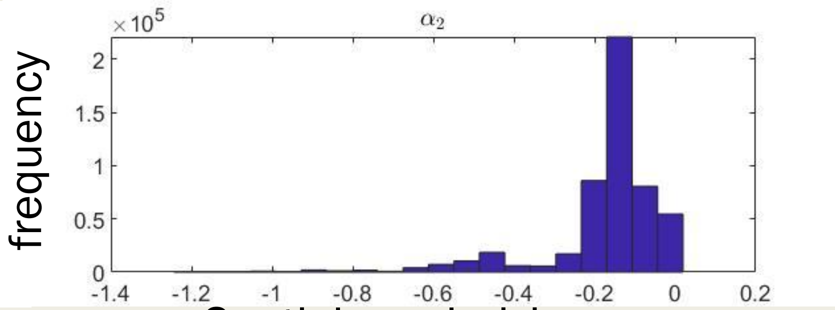
MCMC sampling

- To sample from $P(\boldsymbol{\alpha}|data)$ we adopt MCMC sampling,
- By means of this method, we explore a large parameter space (with a random walk strategy)
- We move toward the most probable part of the parameter space
- We have a larger number of samples from the most probable parameters

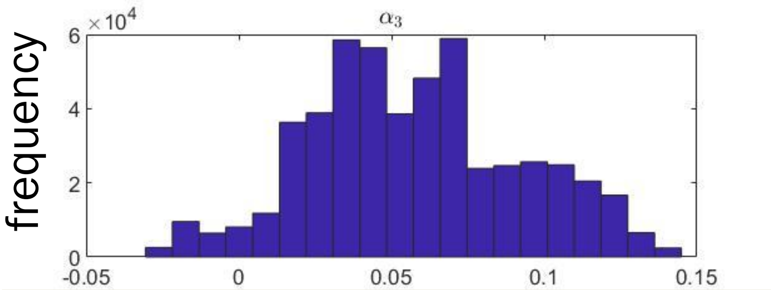
Synthetic Test on SIV1: 12 frequencies df:1/32Hz
Posterior distribution of regularizing parameters
After running MCMC with 500,000 sampling



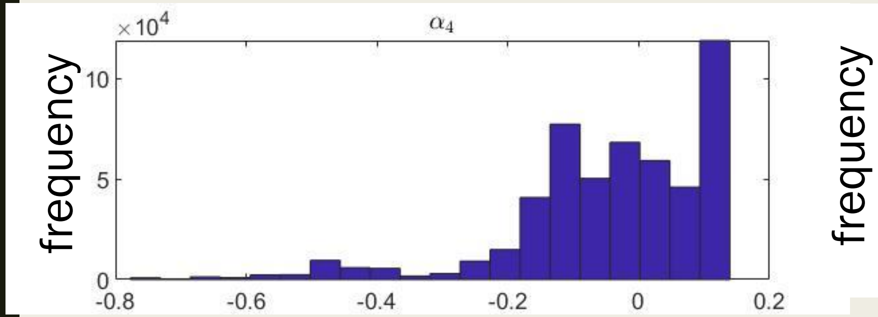
Spatial regularizing α_1



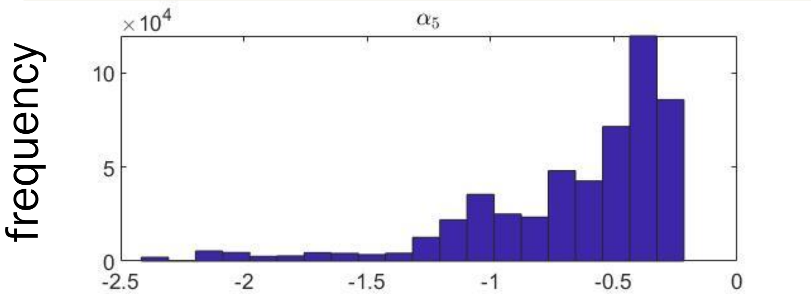
Spatial regularizing α_2



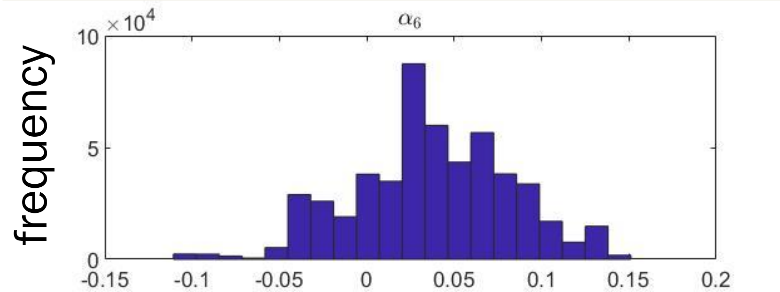
Spatial regularizing α_3



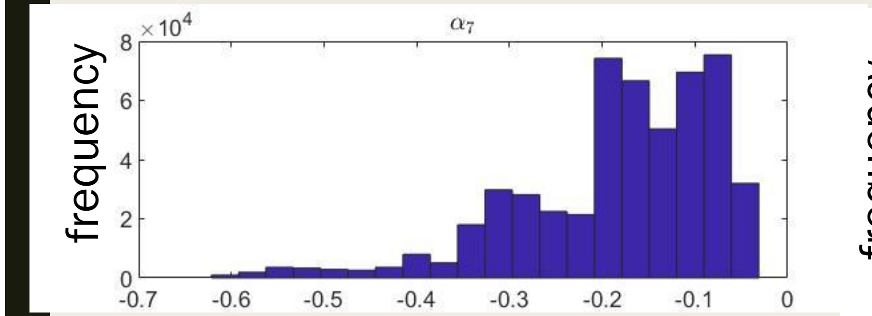
Spatial regularizing α_4



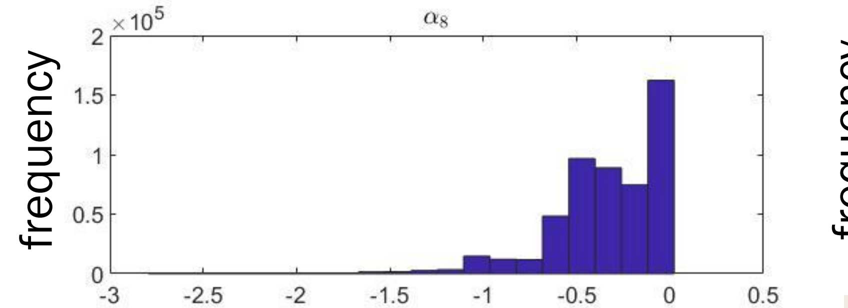
Spatial regularizing α_5



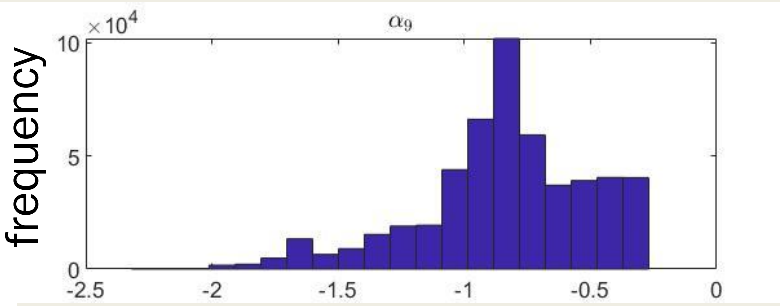
Spatial regularizing α_6



Spatial regularizing α_7

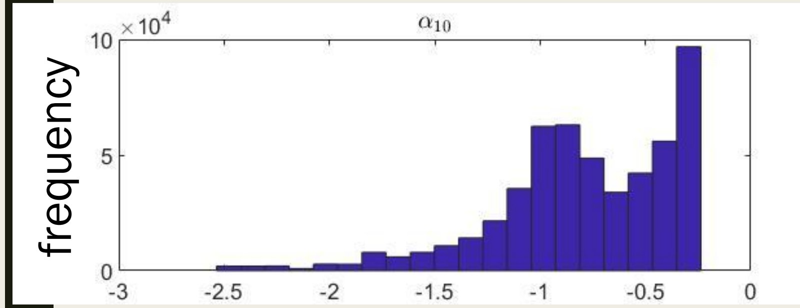


Spatial regularizing α_8

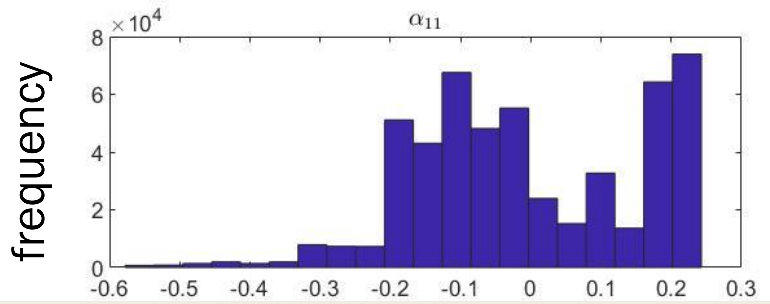


Spatial regularizing α_9

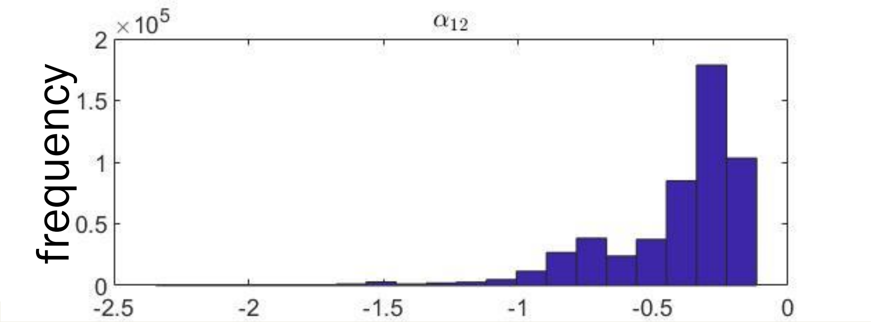
Synthetic Test on SIV1: 12 frequencies df:1/32Hz
Posterior distribution of regularizing parameters
After running MCMC with 500,000 sampling



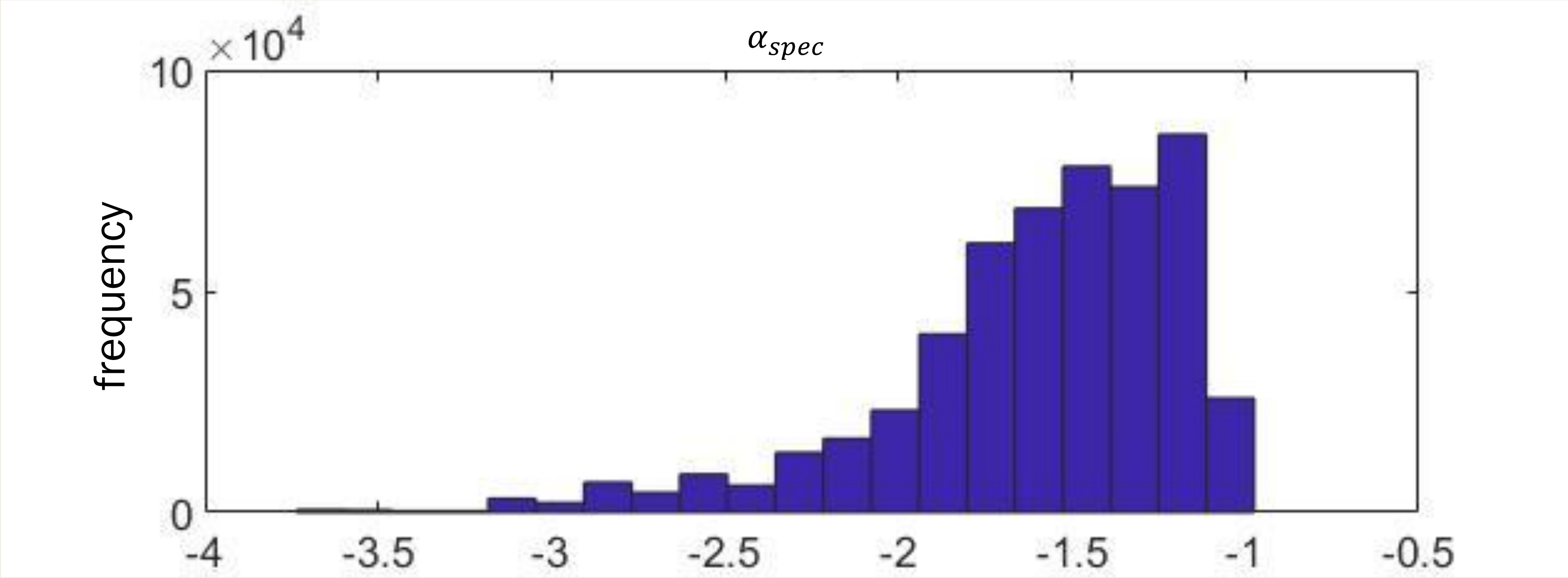
Spatial regularizing α_{10}



Spatial regularizing α_{11}



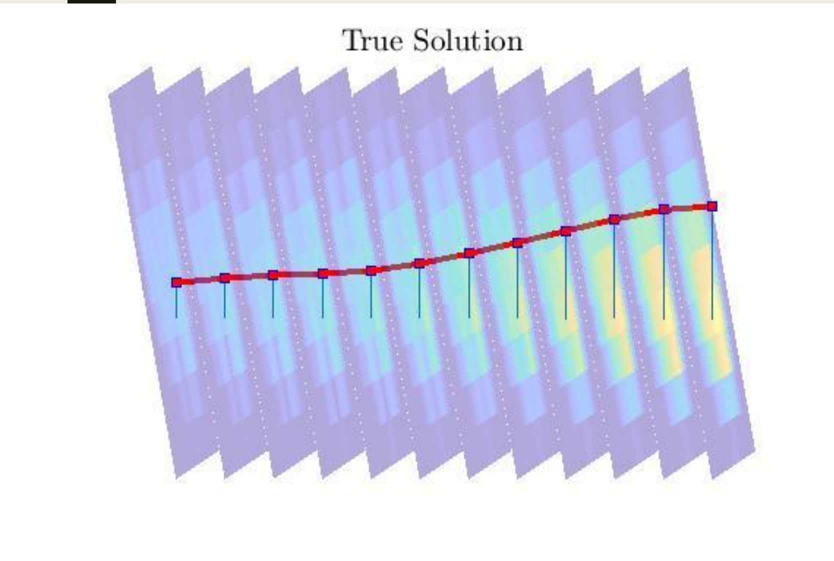
Spatial regularizing α_{12}



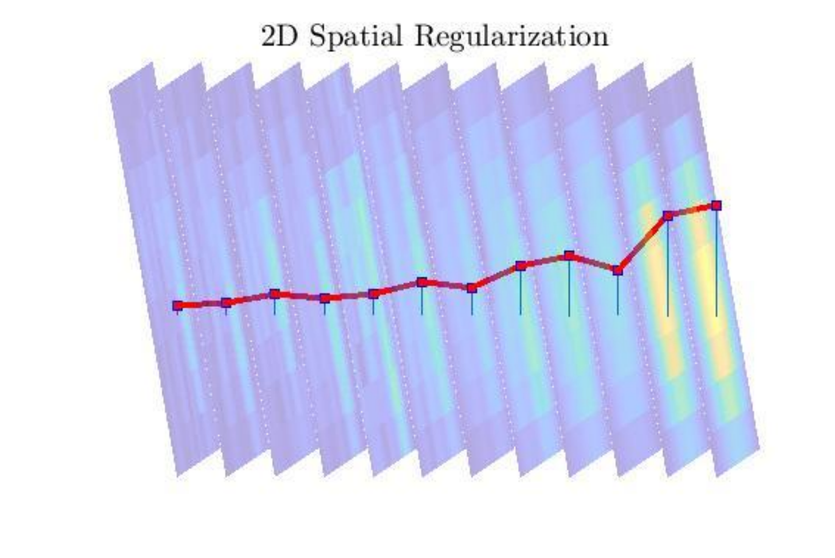
Spectral regularizing α_{spec}

Results (Tested on SIV-inv1)

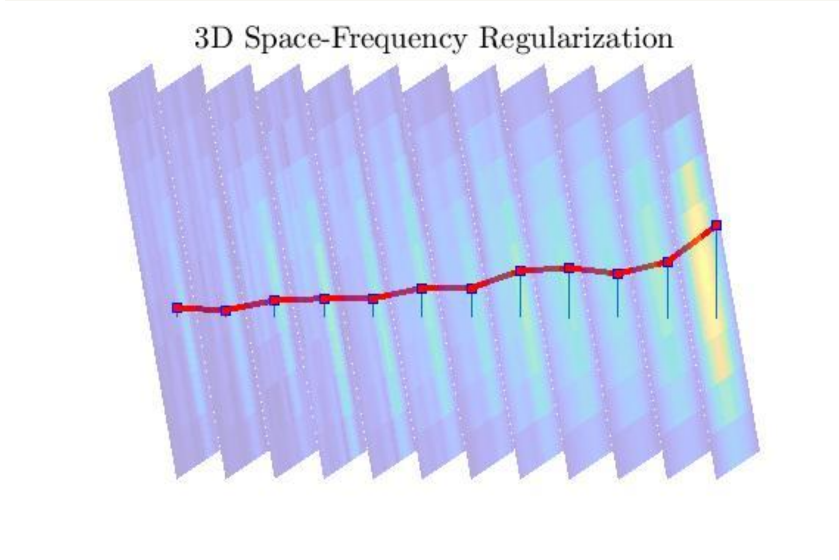
True model



Common FF Approach – 2D Regularization



New FF Approach – 3D Regularization



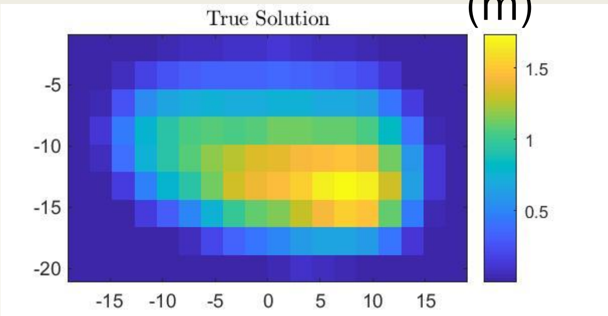
Results: |SVF| at different frequencies

True model

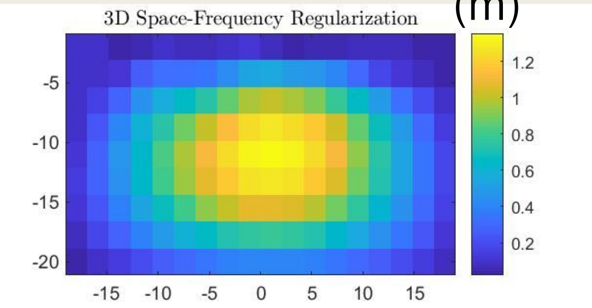
New FF Approach – 3D Regularization

Slip @ 0 Hz

(m)

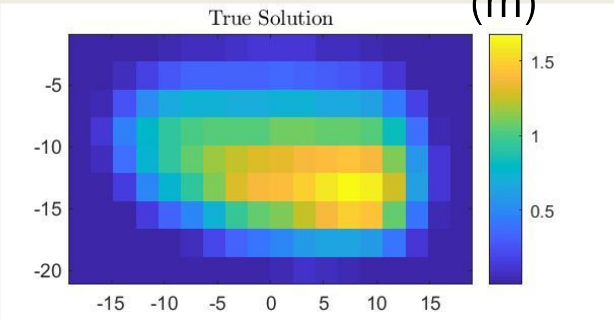


(m)

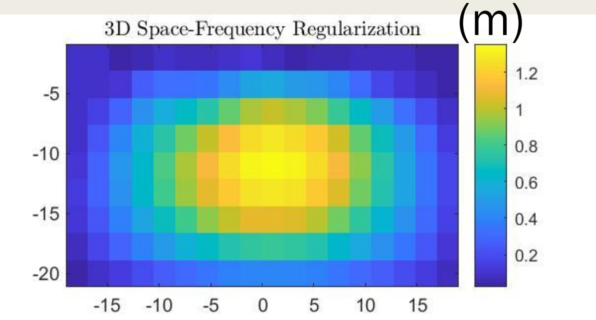


Slip @ 0.031 Hz

(m)



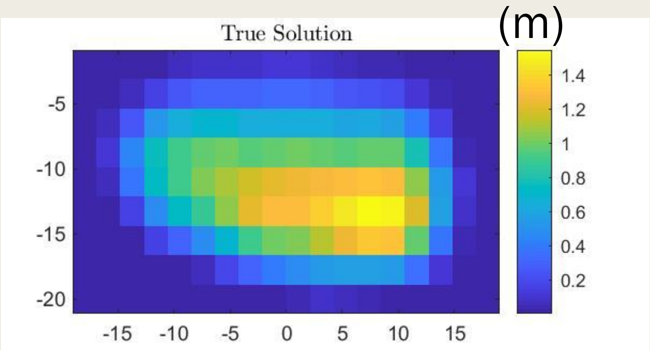
(m)

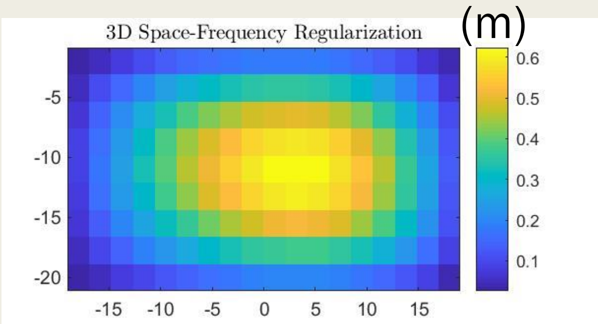


True model

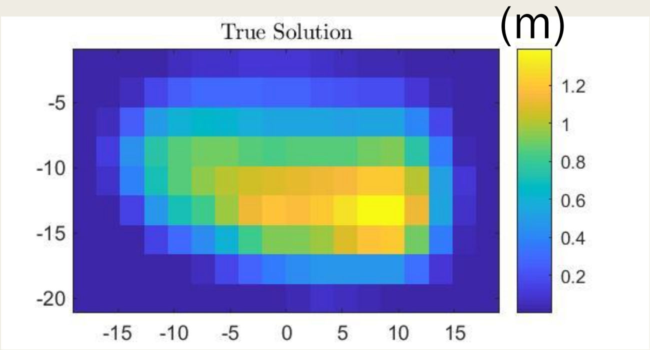
New FF Approach – 3D Regularization

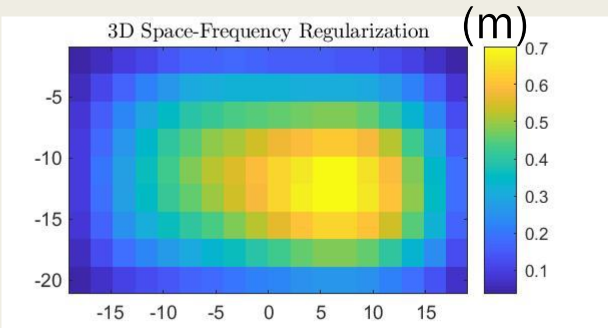
Slip @ 0.063 Hz



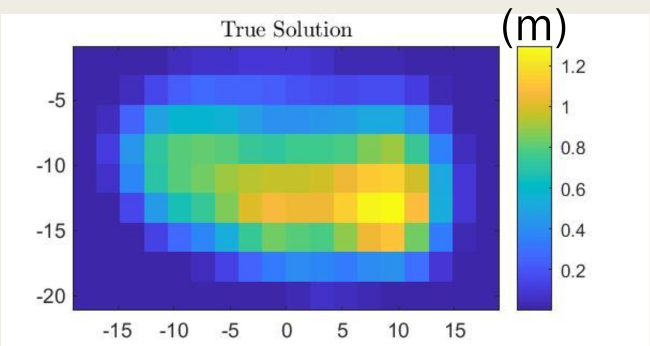


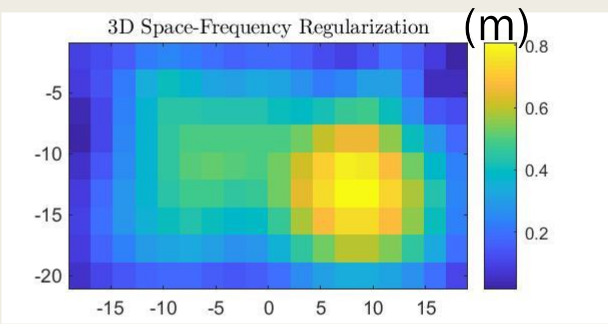
Slip @ 0.094 Hz





Slip @ 0.125 Hz

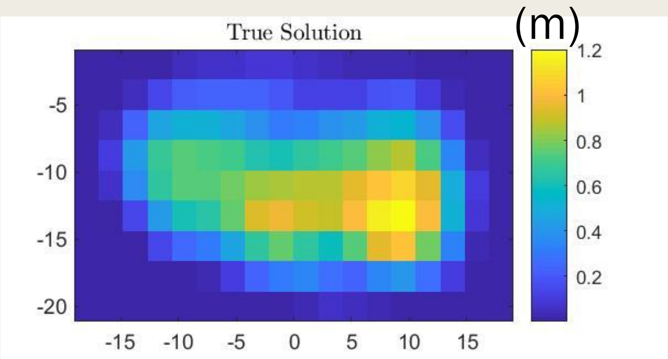


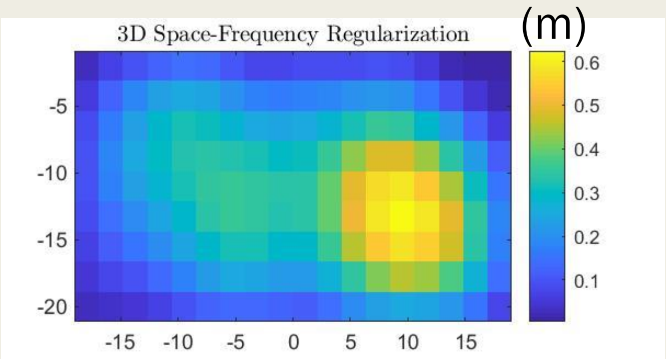


True Solution

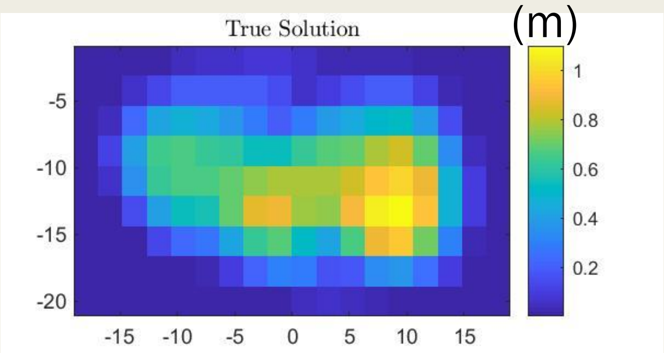
New FF Approach – 3D Regularization

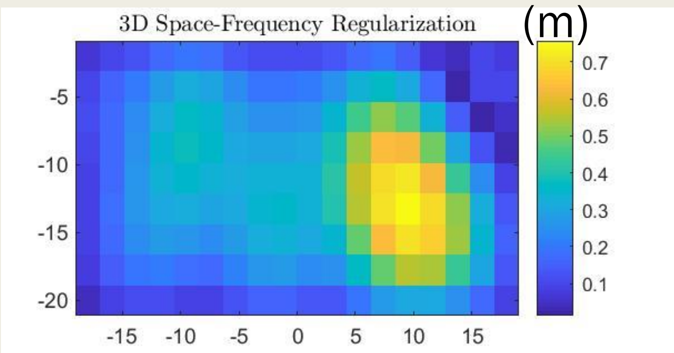
Slip @ 0.156 Hz



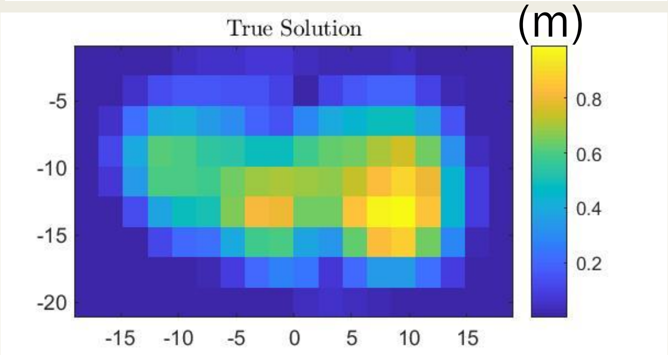


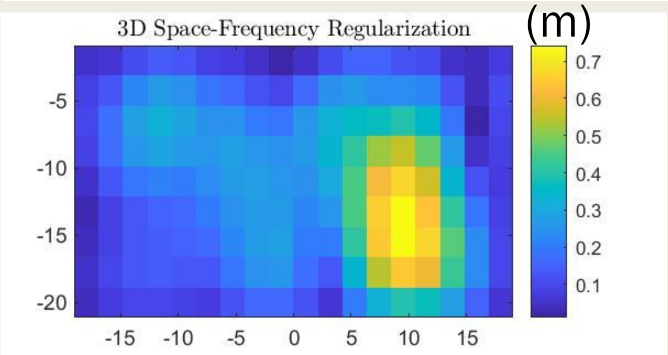
Slip @ 0.188 Hz





Slip @ 0.22 Hz





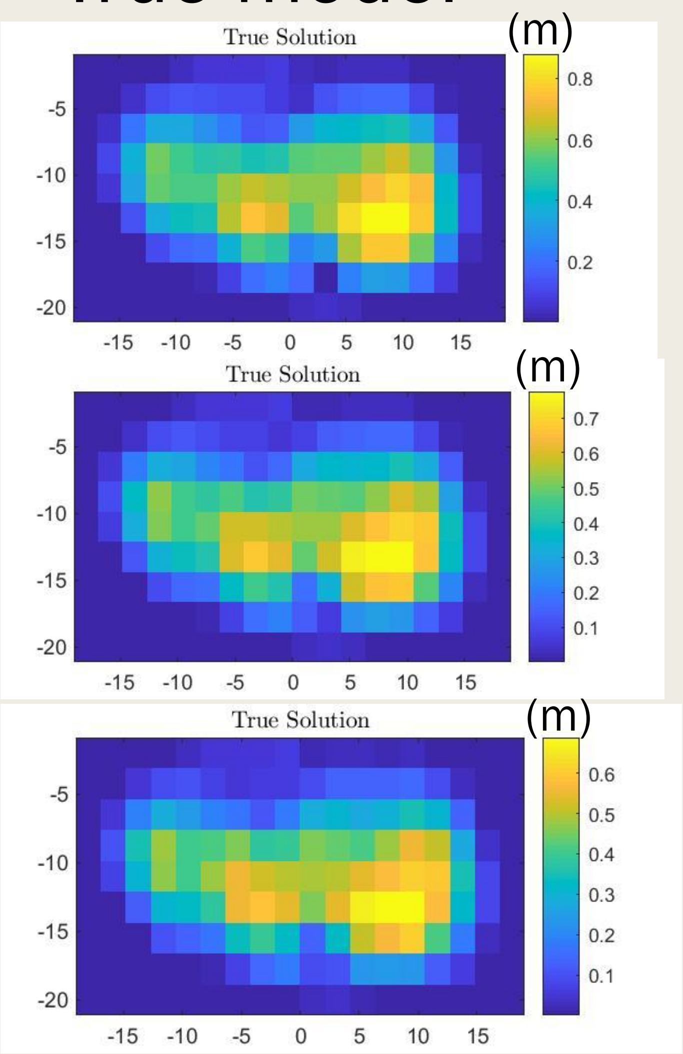
True model

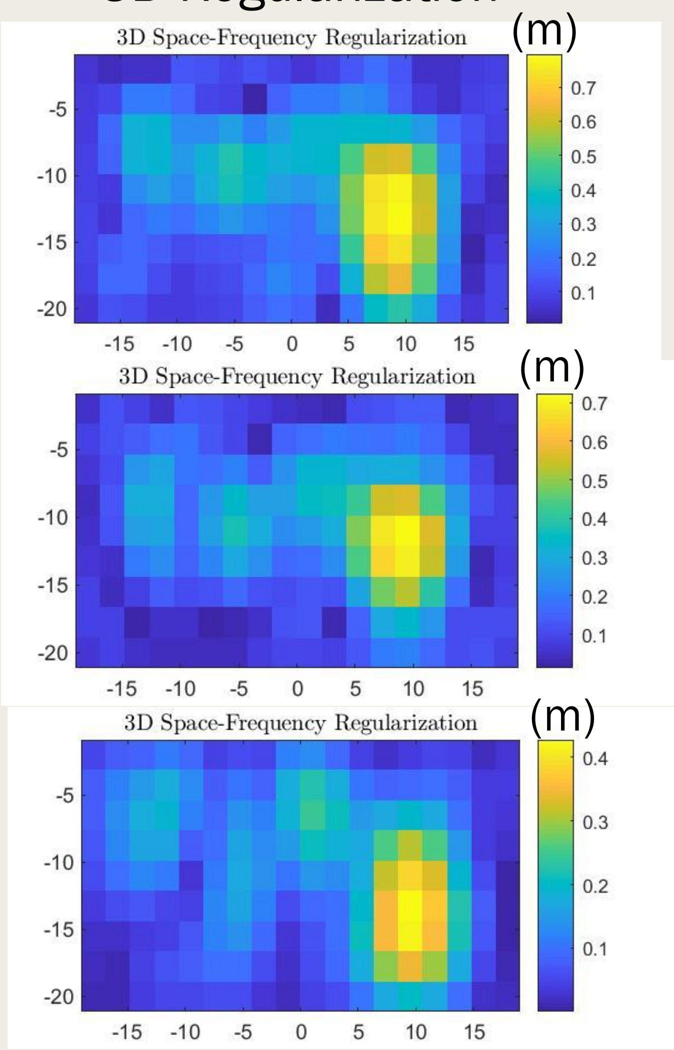
New FF Approach – 3D Regularization

Slip @ 0.25 Hz

Slip @ 0.281 Hz

Slip @ 0.313 Hz

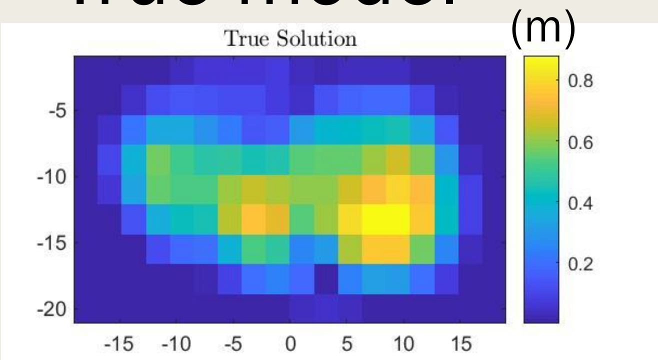


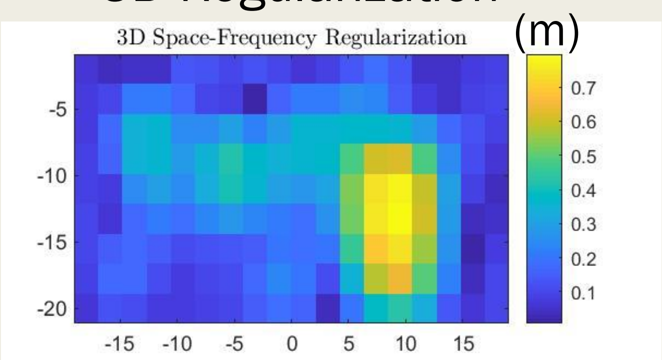


True model

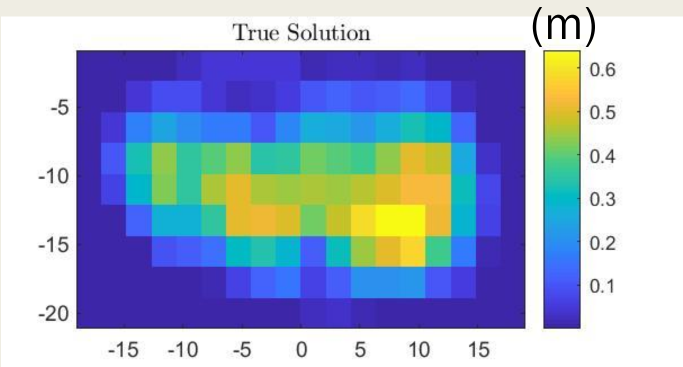
New FF Approach – 3D Regularization

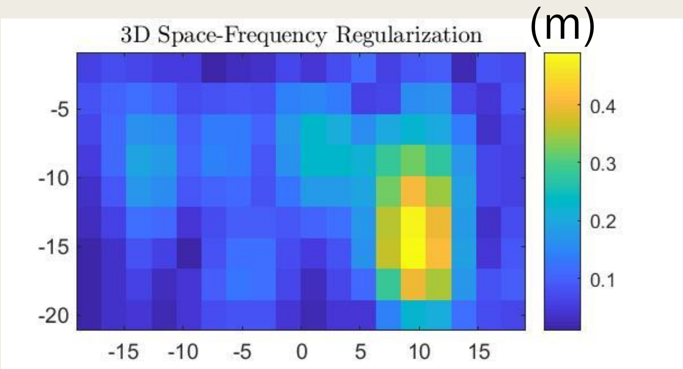
Slip @ 0.344 Hz





Slip @ 0.375 Hz





Conclusion

- We proposed a new regularization approach to take more realistic source functions, smooth in both space and frequency domains.
- The new operator helps us to transfer our inference from one frequency to another
- We applied a Bayesian method to determine regularizing parameter.